# Distributed Model Predictive Control

Dong Jia and Bruce H. Krogh
Dept. of Electrical and Computer Engineering
Carnegie Mellon University
Pittsburgh, PA 15213, U.S.A.
Email: djia@andrew.cmu.edu, krogh@ece.cmu.edu

## Abstract

In this paper, we explored a *distributed model predictive control (DMPC)* scheme. The controllers apply *model predictive control* (MPC) policies to their local subsystems. They exchange their predictions by communication and incorporate the information from other controllers into their local MPC problem so as to coordinate with each other. For the full local state feedback and one-step delayed prediction exchange case, stability is ensured for controllable systems satisfying a *matching condition* by imposing *stability constraints* on the next state in the prediction. An example of multi-area load-frequency control is used as an example application for this DMPC scheme to show the performance of the scheme.

## 1 Introduction

*Model predictive control (MPC)*, also called *receding horizon control*, is a control scheme where the control input is obtained by solving an open-loop optimal control problem over a given horizon. Optimization produces an optimal control sequence. The first control in that sequence is applied to the physical system. At the next control instant, the controller obtains new measurements and solves an updated optimal control problem. MPC is very popular in the process industries because it is an optimal control strategy that incorporates operating constraints explicitly. Many successful MPC applications have been reported in the last two decades [2,12]. In the MPC literature, various methods have been proposed to assure stability for MPC strategies. In the dominant scheme, stability constraints are applied to the end state in the prediction [1, 10], but it is necessary for the prediction horizon to be long enough. Recently, Cheng and Krogh proposed a new scheme, *stability-constrained model predictive control (SC-MPC)*, in which a constraint propagated from previous steps is imposed on the first state in the prediction in the current optimization problem [4,5,6,7]. This *stability constraint* becomes a contractive constraint that is always feasible for linear-time invariant (LTI) controllable systems. We use this idea to guarantee stability in our distributed multi-controller MPC strategy.

MPC is normally implemented in a centralized fashion. One controller has the full knowledge about the system and computes all the control inputs for the system. In large-scale interconnected systems, such as power systems, water distribution systems, traffic systems, etc., such a centralized control scheme may not suitable or even possible for technical or commercial reasons. Decentralized or distributed control is required [14,16].

In this paper, we propose a scheme called *distributed model predictive control with stability constraint (DMPC-SC)* in which controllers coordinate with each other by exchanging their predictions. There is no centralized coordinator. In our scheme, the overall control problem is decomposed into several small ones, each of which is dealt with by a local MPC controller that uses local state measurements. The controllers exchange information about their measurements and predictions and incorporate this information in their local computations. The SC-MPC idea is used to guarantee the stability of the system. We show that if the system satisfies a structural property called a *matching condition*, the system controlled using the DMPC-SC policy with one-step delayed communication will be asymptotically stable.

The paper is organized as follows. Section II presents some preliminary lemmas for interconnected *linear time-invariant* (LTI) systems. Section III introduces the DMPC-SC algorithm and stability is proved for the local full state feedback and one-step delayed information exchange case. In section IV, the DMPC-SC scheme is applied to a two-area load-frequency control scenario and simulation results are given.

## 2 Interconnected LTI System

In this paper, we consider an LTI system composed of $M$ interconnected subsystems described by the following equation:

$$z(k+1) = Az(k) + Bu(k) + Ew(k) \qquad (1)$$

where $z(k) = \begin{bmatrix} z_1(k) \\ \vdots \\ z_M(k) \end{bmatrix} \in R^n$, $u(k) = \begin{bmatrix} u_1(k) \\ \vdots \\ u_M(k) \end{bmatrix} \in R^m$ $(n > m)$ and

$w(k) = \begin{bmatrix} w_1(k) \\ \vdots \\ w_M(k) \end{bmatrix} \in R^l$ are the state, the control and the

disturbance vectors of the system. The subvectors $z_i(k) \in R^{n_i}$, $u_i(k) \in R^{m_i}$ and $w_i(k) \in R^{l_i}$ are the state, control and disturbance vectors of the $i^{th}$ subsystem, where $\sum_{i=1}^{M} n_i = n$,

$$\sum_{i=1}^{M} m_i = m \quad \text{and} \quad \sum_{i=1}^{M} l_i = l \cdot \quad A = \begin{bmatrix} A_{11} & \cdots & A_{1M} \\ \vdots & \ddots & \vdots \\ A_{M1} & \cdots & A_{MM} \end{bmatrix} \in R^{n \times n}, \quad A_{ij} \in R^{n_i \times n_j},$$

$$B = \begin{bmatrix} B_1 & & \\ & \ddots & \\ & & B_M \end{bmatrix} \in R^{n \times m}, \quad B_i \in R^{n_i \times m_i} \quad (n_i > m_i), \quad E = \begin{bmatrix} E_1 & & \\ & \ddots & \\ & & E_M \end{bmatrix} \in R^{n \times n},$$

$E_i \in R^{n_i \times l_i}$. If this system satisfies the structural condition in the following lemma, the control input to one subsystem does not have direct effect on other subsystems, that is, the coupling among subsystems is only through state interactions.

**Lemma 1:** *Consider systems in the form of (1). Suppose* $(A_{ii}, B_i)$ $(i = 1, \cdots, M)$ *are controllable and* $B_i$ *is of full rank. If*
$$rank \begin{pmatrix} B_i & A_{i1} & \cdots & A_{i,i-1} & A_{i,i+1} & \cdots & A_{iM} & E_i \end{pmatrix} = m_i$$
$(i = 1, \cdots, M)$, *there exist a similarity transform matrix* $P = diag(P_1, P_2, \cdots, P_M)$, *where* $P_i \in R^{n_i \times n_i}$, *such that*

$$\begin{bmatrix} x_1(k+1) \\ \vdots \\ x_M(k+1) \end{bmatrix} = \begin{bmatrix} \bar{A}_{11} & \cdots & \bar{A}_{1M} \\ \vdots & \ddots & \vdots \\ \bar{A}_{M1} & \cdots & \bar{A}_{MM} \end{bmatrix} \begin{bmatrix} x_1(k) \\ \vdots \\ x_M(k) \end{bmatrix} + \begin{bmatrix} \bar{B}_1 & & \\ & \ddots & \\ & & \bar{B}_M \end{bmatrix} \begin{bmatrix} u_1(k) \\ \vdots \\ u_M(k) \end{bmatrix} + \begin{bmatrix} \bar{E}_1 & & \\ & \ddots & \\ & & \bar{E}_M \end{bmatrix} \begin{bmatrix} w_1(k) \\ \vdots \\ w_M(k) \end{bmatrix} \quad (2)$$

*where* $Pz = \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix}$, $x_i = \begin{bmatrix} x_i^1 \\ x_i^2 \end{bmatrix}$, $x_i^1 \in R^{m_i}$ *and* $x_i^2 \in R^{n_i - m_i}$.

$$PAP^{-1} = \bar{A} = \begin{bmatrix} \bar{A}_{11} & \cdots & \bar{A}_{1M} \\ \vdots & \ddots & \vdots \\ \bar{A}_{M1} & \cdots & \bar{A}_{MM} \end{bmatrix}, \quad \bar{A}_{ii} = \begin{bmatrix} 0 & I_{ii} \\ A_i^1 & A_i^2 \end{bmatrix}, \quad \bar{A}_{ij} = \begin{bmatrix} 0 & 0 \\ A_{ij}^1 & A_{ij}^2 \end{bmatrix}$$

$(j \neq i)$. $I_{ii} \in R^{(n_i - m_i) \times (n_i - m_i)}$, $A_{ij}^1 \in R^{m_i \times (n_j - m_j)}$ *and* $A_{ij}^2 \in R^{m_i \times n_j}$

$(i, j = 1, \cdots, M)$. $\quad PB = \bar{B} = \begin{bmatrix} \bar{B}_1 & & \\ & \ddots & \\ & & \bar{B}_M \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} 0 \\ B_i^2 \end{bmatrix}$,

$B_i^2 \in R^{m_i \times m_i}$. $\quad PE = \bar{E} = \begin{bmatrix} \bar{E}_1 & & \\ & \ddots & \\ & & \bar{E}_M \end{bmatrix}, \quad \bar{E}_i = \begin{bmatrix} 0 \\ E_i^2 \end{bmatrix}$,

$E_i^2 \in R^{m_i \times l_i}$. *The form (2) will be called the decentralized controllable companion form.*

**Proof.** This lemma follows directly from the controllable form in [6].

The system discussed in Lemma 1 satisfies $span(A_{ij}) \subset span(B_i)$ $(j \neq i)$, called the *matching condition* in decentralized control literature [15]. This condition on the structure of the system guarantees that the system can be stabilized by local state feedback. To design the feedback matrix for the distributed controller, only the knowledge of the local structure is needed.

**Remark.** We note that the similarity transformation in Lemma 1 can be computed and implemented in a distributed fashion. That is, if each controller is designated to control a subsystem, the distributed controllers can do the transformation individually so that the model of the overall system is transformed to the decentralized controllable companion form (2).

**Lemma 2:** *Consider a system in decentralized controllable companion form (2). The state* $x(k)$ *can be represented in terms of components of the future states as*

$$x(k) = \begin{bmatrix} I_{n_1} & 0_{n_1 \times (m_1 \times l - n_1)} & & \\ & \ddots & & \\ & & I_{n_M} & 0_{n_M \times (m_M \times l - n_M)} \end{bmatrix} \begin{bmatrix} x_1^1(k) \\ \vdots \\ x_1^1(k+l-1) \\ \vdots \\ x_M^1(k) \\ \vdots \\ x_M^1(k+l-1) \end{bmatrix} \quad (3)$$

*where,*

$$l = \max_i \left( \begin{bmatrix} n_i \\ m_i \end{bmatrix} \right),$$

$$\begin{bmatrix} n_i \\ m_i \end{bmatrix} = \begin{cases} n_i / m_i & \text{if } n_i \text{ is a multiple of } m_i \\ int(n_i / m_i) & \text{otherwise} \end{cases}$$

**Proof.** Equation (3) follows from the definition of controllable for $(A, B)$.

**Lemma 3:** *Consider a system in decentralized controllable companion form (2). Suppose the local disturbance* $w_i(k)$ *is measurable by the* $i^{th}$ *controller. For each sub-system, for any* $x_i(k)$ *and* $0 < \beta_i < 1$, *there exists* $u_i(k)$ *such that*

$$\|x_i(k+1)\|^2 \leq \|x_i(k)\|^2 - \beta_i \|x_i^1(k)\|^2 \quad (4)$$

*Moreover, if* $u(k) = \begin{bmatrix} u_1(k) \\ \vdots \\ u_M(k) \end{bmatrix}$, *where* $u_i(k)$ *satisfies* (4), *and*

$\beta = \min(\beta_1, \cdots, \beta_M)$, *then*

$$\|x(k+1)\|^2 \leq \|x(k)\|^2 - \beta \|x^1(k)\|^2 \quad (5)$$

*where,* $x^1(k) = \begin{bmatrix} x_1^1(k) \\ \vdots \\ x_2^1(k) \end{bmatrix}$.

**Proof.** From the definition of the decentralized controllable companion form for $(A, B)$,

$$\|x_i(k+1)\|^2 = \left\| A_{ii} x_i(k) + B_i u_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^{M} A_{ij} x_j(k) + E_i w_i(k) \right\|^2$$

$$= \left\| x_i^2(k) \right\|^2 + \left\| \sum_{j=1}^{M} [A_{ij}^1 x_j^1(k) + A_{ij}^2 x_j^2(k)] + B_i^2 u_i(k) + E_i^2 w_i(k) \right\|^2$$

$$= \|x_i(k)\|^2 - \|x_i^1(k)\|^2 + \left\| \sum_{j=1}^{M} [A_{ij}^1 x_j^1(k) + A_{ij}^2 x_j^2(k)] + B_i^2 u_i(k) + E_i^2 w_i(k) \right\|^2$$

Substituting this equation into (4) yields the following equivalent inequality

$$\left\| \sum_{j=1}^{M} [A_{ij}^1 x_j^1(k) + A_{ij}^2 x_j^2(k)] + B_i^2 u_i(k) + E_i^2 w_i(k) \right\|^2 \leq (1 - \beta_i) \|x_i^1(k)\|^2$$

Obviously, $u_i(k) = (B_i^2)^{-1} \left\{ \sum_{j=1}^{M} [A_{ij}^1 x_j^1(k) + A_{ij}^2 x_j^2(k)] + E_i^2 w_i(k) \right\}$ is a feasible solution.

Consider the overall system. When (4) holds,

$$\|x(k+1)\|^2 = \sum_{i=1}^{M}\|x_i(k+1)\|^2$$

$$\leq \sum_{i=1}^{M}\left[\|x_i(k)\|^2 - \beta_i\|x_i^1(k)\|^2\right]$$

$$\leq \sum_{i=1}^{M}\left[\|x_i(k)\|^2 - \beta\|x_i^1(k)\|^2\right]$$

$$= \|x(k)\|^2 - \beta\|x^1(k)\|^2$$

Feasible solutions of subsystems consist a feasible solution to (5).

Lemma 3 shows that, when the constraint (5) is applied, the feasible control set is non-empty. It guarantees that, when distributed controllers are used to control the system with local constraint (4), the controls they find are feasible for the overall system.

## 3  Distributed Model Predictive Control Strategy

In MPC, control decisions are made at discrete time instants, which usually represent equally spaced time interevals. At each decision instant, the controller samples the current state of the system and then solves an optimization problem to find the control action. Currently, most applications of MPC are implemented in a centralized manner. The controller has the knowledge of the overall system and computes all the control inputs for the system. When there are multiple controllers, a distributed MPC scheme must be formulated. In the general problem formulation being considered, we propose that each controller follows the standard MPC strategy with some variation. Each controller must now include the behavior of other subsystems in its local optimization problem in order to coordinate with other controllers. So, for controller $i$, the basic optimal control problem must be modified as follows.

Performance Index:

$$\min_{X_i(k),U_i(k)} J_i(X_i(k),U_i(k))$$

where

$$X_i(k) = \{\hat{x}_i(k+1\,|\,k),\cdots,\hat{x}_i(k+N\,|\,k)\},$$
$$U_i(k) = \{\hat{u}_i(k\,|\,k),\cdots,\hat{u}_i(k+N-1\,|\,k)\}$$

s.t.

Prediction:

$$\hat{x}_i(k+j+1\,|\,k) = F_i(\hat{x}_i(k+j\,|\,k),\hat{u}_i(k+j\,|\,k),\hat{v}_i(k+j\,|\,k),\hat{w}_i(k+j\,|\,k)),$$
$$(j = 0,\cdots,N-1)$$

Constraints:

$$G_i(\hat{x}_i(k+j\,|\,k),\hat{u}_i(k+j\,|\,k),\hat{v}_i(k+j\,|\,k)) \leq 0$$

Initial Condition:

$$\hat{x}_i(k\,|\,k) = x_i(k)$$

Here $\wedge$ means the predicted values of the corresponding variables. The predicted values of the disturbance input have no effect on the stability of the control. But these predictions can affect the performance of the system.

In this formulation, the control prediction horizon and the state prediction horizon are assumed to be the same. We assume that the state variables $x_i$ and the disturbance $w_i$ can be measured or estimated directly by the controller $i$. The control variables $u_i$ are controller $i$'s control actions, $u_i(k) = \hat{u}_i(k\,|\,k)$. The variables $v_i$ represent all variables outside the domain of the controller. In this paper, the system

with the decentralized controllable companion form (2) is considered so that

$$\hat{v}_j(k+i\,|\,k) = [\hat{x}_1^T(k+i\,|\,k-1) \ \cdots \ \hat{x}_{j-1}^T(k+i\,|\,k-1) \ \hat{x}_{j+1}^T(k+i\,|\,k-1) \ \cdots \ \hat{x}_M^T(k+i\,|\,k-1)]^T$$

This is a one-step delayed prediction exchange. In our DMPC scheme, the stability constraints (4) are incorporated to ensure the stability of the system. The input-output constraints will not be adressed in this paper, but can be incorporated directly into the local optimization problems.

### A.  Distributed Model Predictive Control with Stability Constraint (DMPC-SC)

For the $i^{th}$ controller, the algorithm is described as follows.

**Step 1.** *Communication:* Send out its previous predictions $X_j(k-1)$ to other controllers and also receive information $V_j(k) = \{\hat{v}_j(k\,|\,k),\cdots,\hat{v}_j(k+N-1\,|\,k)\}$ from other controllers.

**Step 2.** *Initialization:* Given measured $x_j(k)$ and $w_j(k)$ and $l_j(k)$ from the previous iteration (set $l_j(0)$ to be an arbitrary number), and $0 < \beta_j \leq 1$, define

$$\hat{l}_j(k) = \max\left\{l_j(k),\|x_j(k)\|^2\right\} - \beta_j\|x_j^1(k)\|^2.$$

Set $\hat{x}_j(k\,|\,k) = x_j(k)$ and $\hat{w}_j(k\,|\,k) = w_j(k)$. Predict the future disturbances $\hat{w}_j(k+1\,|\,k),\cdots,\hat{w}_j(k+N-1\,|\,k)$. [Note: These predictions have no influence on the stability results]

**Step 3.** *Optimization:* Solve the following optimal control problem.

$$\min J_j(X_j^P,U_j^P)$$

subject to

$$\hat{x}_j(k+i+1\,|\,k) = A_j\hat{x}_j(k+i\,|\,k) + B_j\hat{u}_j(k+i\,|\,k) + K_j\hat{v}_j(k+i\,|\,k) + E_j\hat{w}_j(k+i\,|\,k),$$
$$(i = 0,1,\cdots,N-1)$$
$$\|\hat{x}_j(k+1\,|\,k)\|^2 \leq \hat{l}_{j,k}$$

**Step 4.** *Assignment:* let

$$u_j(k) = \hat{u}_j(k\,|\,k), \quad l_j(k+1) = \|\hat{x}_j(k+1\,|\,k)\|^2$$

**Step 5.** *Implementation:* Application control $u(k)$. Set $k = k+1$ and return Step 1 at the next sample time.

**End**

**Remark.** When we implement this scheme, it is not necessary for each controller to communication with all the other controllers. Controller $i$ needs to send information to controller $j$ only if the corresponding sub-matrix in the decentralized controllable companion form is nonzero. If the system is loosely coupled, the amount of communication will not be large.

### B.  Stability of DMPC-SC

When we apply this DMPC-SC scheme, the stability of the system can be ensured if the coupling among the subsystems satisfies the condition in the following theorem.

***Theorem 4:*** *Consider a system in decentralized controllable companion form (2). The control is computed at each control instant using DMPC-SC. The system asymptotically stable if the following matrix is stable (i.e., has all eigenvalues inside the unit circle)*

$$\tilde{A} = \begin{bmatrix} 0 & A_{12}^2 & \cdots & A_{1M}^2 \\ A_{21}^2 & 0 & \cdots & A_{2M}^2 \\ \vdots & \vdots & \ddots & \vdots \\ A_{M1}^2 & A_{M2}^2 & \cdots & 0 \end{bmatrix} \qquad (7)$$

***Proof.*** We first show that the error in the state predicted by each local controller (due to the delayed prediction information from the other controllers) goes to zero asymptotically. According to the DMPC-SC scheme, we have

$$\hat{x}_i(k+1\,|\,k) = \overline{A}_{ii}x_i(k) + \overline{B}_i u_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^M \overline{A}_{ij}\hat{x}_j(k\,|\,k-1) + \overline{E}_i w_i(k)$$

while the state of the system is

$$x_i(k+1) = \overline{A}_{ii}x_i(k) + \overline{B}_i u_i(k) + \sum_{\substack{j=1 \\ j \neq i}}^M \overline{A}_{ij}x_j(k) + \overline{E}_i w_i(k)$$

So the error of the prediction $x_{e_i}(k+1\,|\,k) = x_i(k+1) - \hat{x}_i(k+1\,|\,k)$ is

$$x_{e_i}(k\,|\,k-1) = \sum_{\substack{j=1 \\ j \neq i}}^p \overline{A}_{ij}x_{e_j}(k-1\,|\,k-2)$$

so that $x_e(k+1\,|\,k) = \hat{A}x_e(k\,|\,k-1)$, where $\hat{A} = \overline{A} - diag(\overline{A}_{11}, \cdots, \overline{A}_{MM})$. If $\hat{A}$ is stable, $x_e(k\,|\,k-1) \to 0$. Consider an eigenvalue $\lambda$ and corresponding eigenvector $v$, we have

$$\hat{A}v = \begin{bmatrix} 0 \\ \sum_{j \neq 1}(A_{1j}^1 v_j^1 + A_{1j}^2 v_j^2) \\ \vdots \\ 0 \\ \sum_{j \neq M}(A_{Mj}^1 v_j^1 + A_{Mj}^2 v_j^2) \end{bmatrix}, \quad v = \begin{bmatrix} v_1 \\ \vdots \\ v_M \end{bmatrix}, \quad v_i = \begin{bmatrix} v_i^1 \\ v_i^2 \end{bmatrix}, \quad v_i^1 \in R^{n_i - m_i} \text{ and } v_i^2 \in R^{m_i}$$

For non-zero eigenvalue, the corresponding eigenvector satisfies $v_i^1 = 0$. We have $\lambda$ is an eigenvalue of $\tilde{A}$ and $\begin{bmatrix} v_1^2 \\ \vdots \\ v_M^2 \end{bmatrix}$

is the corresponding eigenvector. So $\tilde{A}$ is stable, which implies that $\hat{A}$ is stable. Therefore, $x_e(k\,|\,k-1) \to 0$.

We now show the system is asymptotically stable. By definition,

$$\hat{l}_i(k) = \max\{l_i(k), \|x_i(k)\|^2\} - \beta_i\|x_i^1(k)\|^2$$

$$= \max\{\|\hat{x}_i(k\,|\,k-1)\|^2, \|\hat{x}_i(k\,|\,k-1) + x_{e_i}(k\,|\,k-1)\|^2\} - \beta_i\|x_i^1(k)\|^2$$

$$\leq \|\hat{x}_i(k\,|\,k-1)\|^2 + \|x_{e_i}(k\,|\,k-1)\|^2 - \beta_i\|x_i^1(k)\|^2$$

$$\leq \hat{l}_i(k-1) + \|x_{e_i}(k\,|\,k-1)\|^2 - \beta_i\|x_i^1(k)\|^2$$

Suppose, *ad absurdum*, there exists a subsystem $i$ for which the state $x_i^1(k)$ does not converge to zero. Then, there must exist positive number $\alpha$ and an infinite sequence of positive integers $0 < k_1 < k_2 < \cdots$ such that $\|x_i^1(k_j)\| > \alpha$ $(j = 1, 2, \cdots)$. Since $x_e(k\,|\,k-1) \to 0$, $x_{e_i}(k\,|\,k-1) \to 0$ must hold. There must exist $K > 0$ such that $\|x_{e_i}(k\,|\,k-1)\|^2 < \beta_i\alpha^2 - \gamma$ for all $j \geq K$. We have

$$\hat{l}_i(k_{K+s+1}) < \hat{l}_i(k_{K+s}) + \beta_i\alpha^2 - \gamma - \beta_i\alpha^2 < \hat{l}_i(k_{K+s}) - \gamma < \hat{l}_i(k_K) - s\gamma$$

When $s > \hat{l}_i(k_K)/\gamma$, $\hat{l}_i(k_{K+s}) < 0$. This contradicts the fact that $\hat{l}_{i,k} \geq 0$. So $x_i^1(k) \to 0$ for all $i = 1, \cdots, M$. By Lemma 2, we have $x(k) \to 0$.

## 4    DMPC-SC Simulation Study

In a power system with two or more independently controlled areas, the generation within each area has to be controlled so as to keep the system frequency and maintain the scheduled power exchange between areas. This function is commonly referred to as *load-frequency control (LFC)*, in which centralized control is not practical. Many decentralized control schemes have been proposed for the LFC problem [8,9,11,13]. Here we handle the load-frequency control problem by our DMPC-SC scheme. We assign MPC controllers to control the generator power output directly. Additional turbine controllers can be used to control the turbines. Consequently, each area can be described as one equivalent generator in series with impedance. These model predictive controllers coordinate the generator outputs and set them to be the set points for the turbine controllers. In this paper, the turbine control is not included. The dynamic equation model of each area can be expressed as

$$\Delta\dot{\delta}_i(t) = 2\pi\Delta f_i(t) \qquad (8)$$

$$\Delta\dot{f}_i(t) = \frac{-\Delta f_i(t)}{T_{Pi}} + \frac{K_{Pi}\Delta P_{gi}(t)}{T_{Pi}} - \frac{K_{Pi}}{2\pi T_{Pi}}\left(\sum_{j\in N}K_{Sij}[\Delta\delta_i(t) - \Delta\delta_j(t)]\right) - \frac{K_{Pi}\Delta P_{di}(t)}{T_{Pi}} \qquad (9)$$

where

$\Delta\delta(i) =$ incremental phase angle deviation of the area bus in *radians*;

$\Delta f_i(t) =$ incremental frequency deviation in *Hz*;

$\Delta P_{g_i}(t) =$ incremental change in the generator output in *p.u.*;

$\Delta P_{d_i}(t) =$ load disturbance for the $i^{th}$ area in *p.u.*;

$T_{P_i} =$ system model time constant in *s*;

$K_{P_i} =$ system gain;

$K_{S_{ij}} =$ synchronizing coefficient of the tie-line between $i^{th}$ and $j^{th}$ area.

The object of load-frequency control is to drive the frequency deviation and the deviation of the power flow through tie lines to zero following a disturbance (e.g., a step-change in the system load). The deviation of power flow between areas is proportional to the difference of phase angle deviation between areas. So, the local performance index can be selected as

$$J_i(t) = \int_t^{t+T}\left\{\sum_{\substack{j=1 \\ j \neq i}}^M p_{ij}(\Delta\delta_i(\tau) - \Delta\delta_j(\tau))^2 + q_i\Delta f_i^2(\tau) + r_i\Delta P_{g_i}^2(\tau)\right\}d\tau \qquad (10)$$

In the simulation study, a two-area load-frequency scenario is constructed. Each controller is assumed to know the load disturbance in its local part. The parameters used in the simulation are: area #1: $T_{P_1} = 25s$, $K_{P_1} = 112.5$, $K_{S_{12}} = 0.5$; area #2: $T_{P_2} = 20s$, $K_{P_2} = 120$, $K_{S_{21}} = 0.5$. Other parameters for optimization are $p_{12} = p_{21} = 10$, $q_1 = q_2 = 100$ and $r_1 = r_2 = 10$. The constant for the stability constraints are $\beta_1 = \beta_2 = 0.8$. The load disturbance is a load increment of 0.01

p.u. in each area at time $t$=0. The control interval is set to 2 second, a typical sampling period in power systems. The prediction horizon is selected to be 1 to minimize the amount of computation and communication. Using an Euler approximation to the continuous-time state equations, a discrete-time state space model of the system is obtained that satisfies the matching condition in Lemma 1 and condition (7) in Theorem 4.
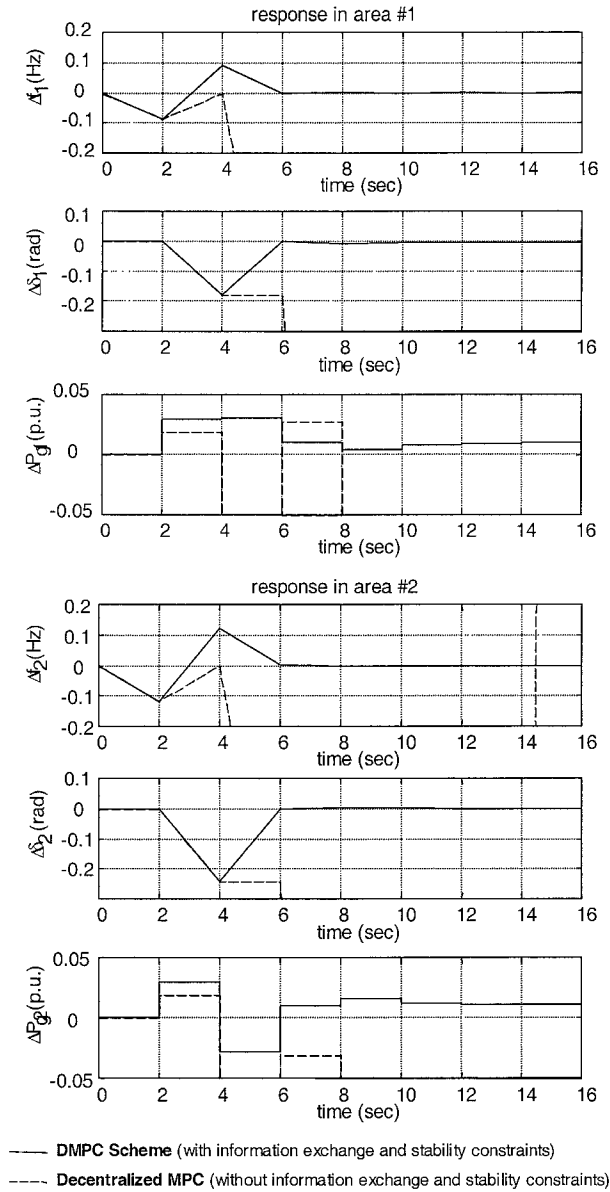


Figure 1. Response of the LFC system for DMPC-SC and decentralized MPC without stability constraints or information exchange (simulation of the discrete-time model).

Figure 1 shows the results of simulating the discrete-time model for two decentralized control schemes. The solid curves are system behavior by the DMPC-SC scheme. The frequency

variations go back to zero and each sub-system provides enough power for their local load increment. The dash curves are system behavior by MPC in a completely decentralized fashion without information exchange or stability constraints. The system is unstable in this case. Centralized MPC with stability constraints was also tried (not shown in Fig. 1). The system performance is not much better than that by DMPC-SC. They are nearly the same. From the simulation we find that the DMPC-SC scheme can work as well as centralized MPC for systems with canonical structure. Although the model by Euler method is only a very rough approximation of the system dynamics, this simulation example illustrates the potential for applying DMPC-SC.
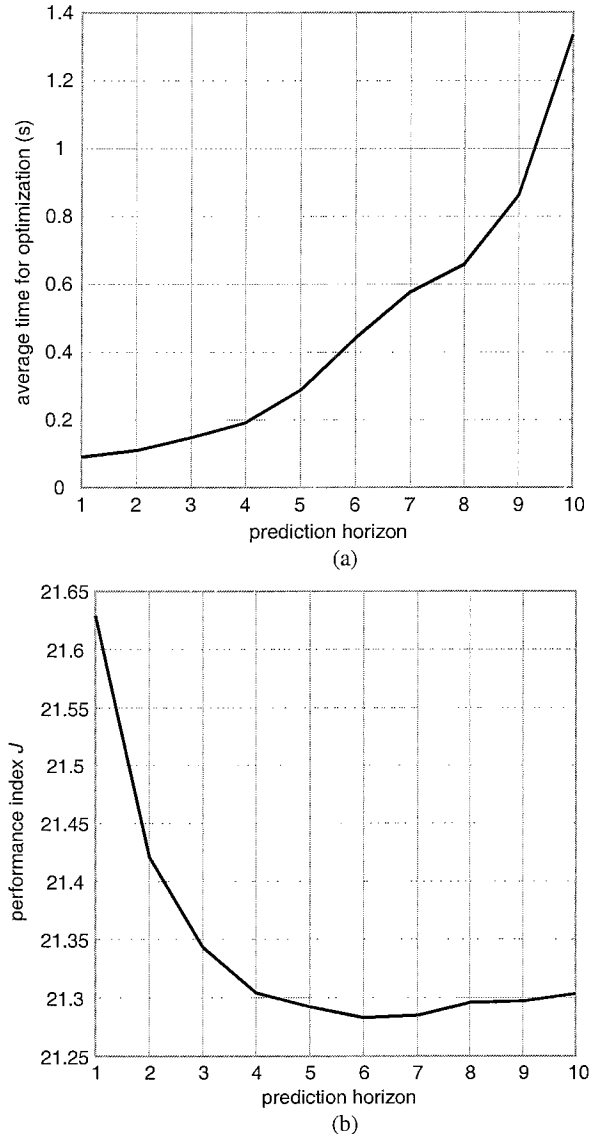


Figure 2. The computation time costed and the performance achieved by different prediction horizon. The performance is evaluated over the whole system. The computation time is the average time used in optimization in one agent.

**2771**

As we mentioned in the introduction, our method applies no constraint on the selection of prediction horizon. This provides more flexibility in selecting the prediction horizon. Obviously, a short prediction horizon would require a smaller amount of computation time. But a properly chosen longer prediction horizon could improve the performance of the system. As shown in figure 2(a), the average time for optimization increase exponentially as the prediction horizon increases. But, in figure 2(b), the performance is improved as the prediction horizon increases till it reaches 6. We see that too long prediction can deteriorate the performance because the errors in the prediction are very large for too long prediction horizon. These two are conflicting factors. Thus, applying this distributed model predictive control scheme, one should find a compromise between the potential improvement in performance and the prediction errors.

## 5    Conclusions

In this paper, we present a new distributed MPC scheme in which the controllers coordinate their actions by exchanging their predictions about the future behavior of the local subsystem. For the full local state measurement and one-step delayed information exchange case, asymptotic stability is proved for LTI systems satisfying two structural conditions. The first condition (Lemma 3) is a common matching condition in the literature on decentralized control. The second condition (Theorem 4) requires the interconnection matrix to exhibit a particular stability property. A simulation example shows that the system is stable under delayed information exchange for a case when strictly decentralized MPC cannot stabilize the system.

Currently, we are investigating possibilities for relaxing the sufficient conditions for stability developed in this paper. Simulation studies have shown that stability can be achieved when the matching condition is not satisfied exactly (i.e., when the interaction terms required to be zero in the matching condition are small). For applications it is important to relax the requirement that the disturbances be measurable. Robustness when the model is not known exactly is also an important issue. Finally, we are interested in extending this approach to situations where the controllers act asynchronously, perhaps at different sampling rates. We believe the stability constraint approach shows promise as a general technique for dealing with all of these issues since the constraint is generated online based on the previous real-time computations, rather than being computed off-line as proposed in other contraction-based methods for MPC.

### Acknowledgment

### References

[1] Bemporad, A. and Morari, M. (1999). Robust Model Predictive Control: A Survey, *Robustness in Identification and Control, Lecture Notes in Control and Information Sciences*, vol. 245, Springer-Verlag.

[2] Camacho, E.F. and Bordons, C. (1995). *Model Predictive Control in the Process Industry*. Springer.

[3] Chawdhry, P.K. and Ahson, S.I. (1982). Application of Interaction-Prediction Approach to Load-Frequency Control (LFC) Problem, *IEEE Transactions on Systems, Man and Cybernetics*, Vol.SMC12, No.1, 66-71.

[4] Cheng, X. (1996). *Model Predictive Control and Its Application to Plasma Enhanced Chemical Vapor Deposition*. Ph.D. Thesis, Carnegie Mellon University.

[5] Cheng, X. and Krogh, B.H. (1997). Stability Constrained Model Predictive Control with State Estimation, *Proceedings of the American Control Conference*.

[6] Cheng, X. and Krogh, B.H. (1998). Stability Constrained Model Predictive Control for Nonlinear Systems, *Proceedings of the American Control Conference*.

[7] Cheng, X. and Krogh, B.H. (1998). Nonlinear Stability-Constrained Model Predictive Control with Input and State Constraints, *Proceedings of the 36$^{th}$ Conference on Decision and Control*.

[8] Jiang, H., Cai, H., Dorsey, J.F. and Qu, Z. (1997). Toward a Globally Robust Decentralized Control for Large-Scale Power Systems, *IEEE Transactions on Control Systems Technology*, Vol.5, No.3, 309-319.

[9] Lim, K.Y., Wang, Y., Guo, G. and Zhou, R. (1998). A New Decentralized Robust Controller Design for Multi-Area Load-Frequency Control via In Complete State Feedback, *Optimal Control Applications and Methods*, Vol.19, 345-361.

[10] Mayne, D. Q., Rawlings, J. B., Rao, C. V. and Scokaert, P. O. M., (2000). Constrained Model Predictive Control: Stability and Optimality, *Automatica*, Vol. 36, 789-814.

[11] Miniesy, S.M. and Bohn, E.V. (1990). Two-Level Control of Interconnected Power Plants, *IEEE Transactions on Power Apparatus and Systems*, Vol.PAS90, No.6, 2742-2748.

[12] Morari, M. and Lee, J.H. (1999). Model predictive control: past, present and future, *Computers and Chemical Engineering*, Vol.23, 667-682.

[13] Rubaai, A. and Momoh, J.A. (1991). Three-Level Control Scheme for Load-Frequency Control of Interconnected Power Systems, *TENCON'91 IEEE Region 10 International Conference on EC3-Energy, Computer, Communication and Control Systems*.

[14] Sandell, N.R., Varaiya, P., Athans, M. and Safonov, M.G. (1978). Survey of Decentralized Control Methods for Large Scale Systems, *IEEE Transactions on Automatic Control*, Vol.AC23, No.2, 108-128.

[15] Siljak, D. D. (1990). *Decentralized Control of Complex Systems*, Academic Press.

[16] Siljak, D. D. (1996). Decentralized Control and Computations: Status and Prospects, *Annual Review of Control*, Vol. 20, 131-141.