Min-Max Feedback Model Predictive Control For Distributed Control With Communication

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Abstract

This paper concerns a distributed model predictive control (DMPC) strategy in which each controller views the signals from other subsystems as disturbance inputs in its local model. The DMPC controllers exchange predictions on the bounds of their state trajectories and incorporate this information into their local DMPC problems. They also impose their own predicted state bounds as constraints in subsequent DMPC iterations to guarantee their subsystem satisfies the bounds broadcast to the other controllers. Each controller solves a local min-max problem on each iteration to optimize performance with respect to worst-case disturbances. Parameterized state feedback is introduced into the DMPC formulation to obtain less conservative solutions and predictions. The paper presents sufficient conditions for feasibility and stability. The approach is illustrated with an example.

1 Introduction

For large-scale systems, decentralized or distributed control is often implemented because a centralized solution is either too complex or impractical. For example, in today's deregulated power systems, there may be market barriers to implementing centralized control, since the local operating companies are not interested in revealing all of their local objectives and constraints. Nevertheless, there are global objectives of concern to all the controllers (such as system stability), so it is useful to introduce some mechanisms for coordinating the distributed control actions.

In this paper, we assume each controller has a model of its local dynamics with the influence of other subsystems represented as disturbance inputs to the local system. We consider MPC strategies for solving the local control problems. The motivation for using MPC for decentralized control is the same as for the centralized case: constraints and objectives are explicit in MPC, making it possible to incorporate important features of the problem directly into the on-line computations. To guarantee the local problems remain feasible, we assume the controllers have bounds on the disturbance inputs (and other uncertainties) and formulate a min-max optimization problem at each control step. This is a standard approach taken in the MPC literature to deal with uncertainties [8]. To avoid the overly conservative solutions generated by the standard open-loop formulation of the MPC optimization problem, we introduce parameterized feedback policies as proposed in [2, 10].

[1]This work was supported in part by the EPRI/DoD Complex Interactive Networks/Systems (CINS) Initiative under contract number EPRI-W08333-05

The steps performed by the local controllers at each control instant are illustrated in Figure 1. We consider a scheme where each controller broadcasts information about its predicted state trajectories so that other controllers can use this information to update the bounds on the disturbances in their local models. To ensure that such a scheme is viable, each controller must guarantee that the future behavior of its local system remains within the bounds it has broadcast to the other controllers. It is also necessary to show that the optimization problems for all the controllers remain feasible indefinitely. Finally, stability of the overall system under the distributed control actions needs to be demonstrated.

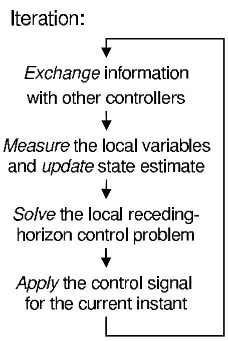

Figure 1: Local steps for distributed MPC with coordination.

This paper is organized as follows. Section 2 introduces *min-max feedback with constraint set* $\mathcal{C}$ ($\mathrm{M}^2\mathrm{F}(\mathcal{C})$) optimization for controlling systems with bounded uncertainties. Section 3 uses the $\mathrm{M}^2\mathrm{F}(\mathcal{C})$ optimization to formulate the *min-max feedback DMPC* (M^2F-DMPC) problem and discusses the related feasibility and stability issues. Section 4 illustrates our scheme for an example of distributed control of an electromechanical system. The paper concludes with a discussion of future research directions.

2 MPC With Bounded Uncertainties

Since each controller handles effects from other subsystems as bounded disturbances, rather than coordinating with other controllers to solve the global MPC problem distributedly, each local control problem is an MPC problem with bounded disturbances. The effect of the coordination signals is to introduce time-varying bounds on the disturbances. In this section we consider the formulation and solution of the resulting MPC problems with time-varying, bounded uncertainty. Following the approach in [2, 10], we introduce a *parameterized feedback control law* into the MPC optimization problem given by $u = h(x,\theta)$, where $\theta \in \Xi$ is a parameter vector subject to the constraint set $\Xi \subset \Re^{n_\theta}$. The purpose of θ is to obtain less conservative solutions for the min-max optimization, relative to the open-loop formulations in standard MPC.

Consider the following constrained discrete-time nonlinear time-invariant system:

$$x(k+1) = f(x(k), u(k), v(k)) \tag{1}$$

subject to

$$\begin{aligned} x(k) &\in \mathcal{X} \subset \Re^n && (2)\\ v(k) &\in \mathcal{V} \subset \Re^l && (3)\\ G(x(k), u(k), v(k)) &\leq 0 && (4) \end{aligned}$$

where $x(k) \in \Re^n$, $u(k) \in \Re^m$ and $v(k) \in \Re^l$ are the state, the control and the uncertainty of the system, respectively. The input v represent all kinds of uncertainties in the system, including unknown disturbance inputs and uncertainties in the model. We assume that the system state is observable.

In the following, we use a variation of the notion of control invariant sets [6]. We say $\mathcal{T} \subseteq \mathcal{X}$ is a *θ-control invariant* set of the system described by the equations (1)–(4) if there exists a control parameter $\theta^{\mathcal{T}} \in \Xi$ such that

$$x \in \mathcal{T} \Longrightarrow f\left(x, h\left(x, \theta^{\mathcal{T}}\right), v\right) \in \mathcal{T}, \forall v \in \mathcal{V}.$$

Throughout the remainder of the paper, $\mathcal{T}$ will denote a θ-control invariant set.

In the formulation of the MPC problem we let N denote the prediction horizon and let $\mathcal{C}$ denote the set of constraints on the states and disturbances, that is,

$$\mathcal{C} = \{\mathcal{X}(1), \cdots, \mathcal{X}(N-1), \mathcal{V}(0), \cdots, \mathcal{V}(N-1)\},$$

where $\mathcal{X}(j) \subset \Re^n$, $j = 1, \cdots, N-1$ are constraints on the state variables, which are updated from stage to stage in our framework to ensure that the future behavior is consistent with the information the controller broadcasted to other controllers, and $\mathcal{V}(j) \subset \Re^l$, $j = 0, \ldots, N-1$, are bounds on uncertainties that can be updated by system identification techniques or by information sources (which in our case will be bounds from other controllers).

At each control step k, the controller solves the following *min-max feedback with constraint set* $\mathcal{C}$ ($\mathrm{M}^2\mathrm{F}(\mathcal{C})$) optimization problem:

$\mathbf{M}^2\mathbf{F}(\mathcal{C})$ Optimization Problem
Given the constraint set $\mathcal{C}$,
Performance Index:
$\min_{\Theta} \left[\max_{V} J(\Theta, V)\right]$
where
Parameters for Feedback Policy:
$\Theta = \{\theta_0, \theta_1, \cdots, \theta_{N-1}\}$
Uncertainties:
$V = \{v(0), v(1), \cdots, v(N-1)\}$
subject to
Initial Condition:
$\hat{x}(0) = x(k)$
Prediction:
$\hat{x}(j+1) = f(\hat{x}(j), u(j), v(j))$, $j = 0, 1, \cdots, N-1$
Feedback Policy:
$u(j) = h(\hat{x}(j), \theta_j)$, $j = 0, 1, \cdots, N-1$
Parameter Constraints:
$\Theta \in \Xi^N$
State Constraints:
$\hat{x}(j) \in \mathcal{X}(j)$, $j = 1, 2, \cdots, N-1$
$\hat{x}(N) \in \mathcal{T}$
Bounds on Uncertainties:
$v(j) \in \mathcal{V}(j)$, $j = 0, 1, \cdots, N-1$
Constraints for All Possible Uncertainties:
$\forall \tilde{v}(j) \in \mathcal{V}(j), j = 0, 1, \cdots, N-1$
$\tilde{x}(j+1) = f(\tilde{x}(j), h(\tilde{x}(j), \theta_j), \tilde{v}(j)) \in \mathcal{X}(j+1), j = 0, 1, \cdots, N-2$
$\tilde{x}(N) = f(\tilde{x}(N-1), h(\tilde{x}(N-1), \theta_{N-1}), \tilde{v}(N-1)) \in \mathcal{T}$
$G(\tilde{x}(j), h(\tilde{x}(j), \theta_j), \tilde{v}(j)) \leq 0$, $j = 0, 1, \cdots, N-1$

Θ is said to be a feasible solution to the $\mathrm{M}^2\mathrm{F}(\mathcal{C})$ problem if all constraints are satisfied, and the $\mathrm{M}^2\mathrm{F}(\mathcal{C})$ problem is said to be feasible if there exists a feasible solution Θ. We are interested in situations where it is possible to update the state constraints and the disturbance bounds at each step. The constraint set at stage k will be denoted by $\mathcal{C}^k$, and

$$\Theta^{k*} = \{\theta_0^{k*}, \theta_1^{k*}, \cdots, \theta_{N-1}^{k*}\}$$

will denote the solution to $\mathrm{M^2F}(\mathcal{C}^k)$. Given Θ^{k*}, the control applied to the system at step k is

$$u(k) = h(x(k), \theta_0^{k*}).$$

When there is no uncertainty in the system, the bound on disturbance becomes a series of known single value sets $\mathcal{V}(j) = \{v(j)\}$, $j = 0, 1, \cdots, N-1$. In other words, $\mathrm{M^2F}(\mathcal{C})$ becomes the conventional MPC optimization problem. Then, although a time-varying parameterized state feedback law is obtained, it is equivalent to an open-loop optimal control sequence because the future state trajectory is deterministic.

In following, we consider the feasibility of $\mathrm{M^2F}(\mathcal{C})$ problem with updated uncertainty bounds and state constraints that satisfies the following conditions.

Uncertainty Bounds Updates

$$\mathcal{V}^0(j) = \mathcal{V}, j = 0, 1, \cdots, N-1, \tag{5}$$

and for $k > 0$

$$\mathcal{V}^k(j) \subseteq \mathcal{V}^{k-1}(j+1), j=0,1,\cdots,N-2, \text{ and } \mathcal{V}^k(N-1) \subseteq \mathcal{V} \tag{6}$$

where $\mathcal{V}$ is a given initial bound on the disturbances.

State Constraint Updates

$$\mathcal{X}^0(j) = \mathcal{X}, j = 1, 2, \cdots, N-1, \text{ and } \mathcal{X}^0(N) = \mathcal{T} \tag{7}$$

and for $k > 0$

$$\hat{\mathcal{X}}(k+j|k-1) \subseteq \mathcal{X}^k(j) \subseteq \mathcal{X}^{k-1}(j+1), j=1,2,\cdots,N-1, \text{and } \mathcal{X}^k(N) \subseteq \mathcal{T} \tag{8}$$

where $\mathcal{X}$ is the given initial state constraints and $\hat{\mathcal{X}}(k+j|k-1)$ are the predicted bounds on the reachable states at each stage given by

$$\begin{aligned}\hat{\mathcal{X}}(k+j|k) = & \{x | x = f(x', h(x, \theta_j^{k*}), v), \\ & x' \in \hat{\mathcal{X}}(k+j-1|k), v \in \mathcal{V}^k(j-1)\},\end{aligned} \tag{9}$$

for all $j = 1, 2, \cdots, N$, where

$$\hat{\mathcal{X}}(k|k) \subseteq \hat{\mathcal{X}}(k|k-1). \tag{10}$$

$\mathcal{X}^k(j)$ is a conservative approximation of the future reachable set predicted recursively by the controller at the control step $k-1$ for the time $k+j$.

In the constraint set $\mathcal{C}^k$, $\mathcal{V}^k(0), \cdots, \mathcal{V}^k(N-1)$ are from identification methods or some information sources from outside world. Relation (6) imposes requirement on the identification algorithms and the information sources. The constraints $\mathcal{X}^k(1), \cdots, \mathcal{X}^k(N)$ are updated by the controller itself. The following lemma guarantees the feasibility of the condition (8) during control.

Lemma 2.1 *If $M^2F(\mathcal{C}^k)$ is feasible, then there exist $\mathcal{X}^{k+1}(1), \cdots, \mathcal{X}^{k+1}(N)$ such that*

$$\hat{\mathcal{X}}(k+j+1|k) \subseteq \mathcal{X}^{k+1}(j) \subseteq \mathcal{X}^k(j+1), j=1,2,\cdots,N-1. \tag{11}$$

Proof: That $\mathrm{M^2F}(\mathcal{C}^k)$ is feasible implies that the optimal solution

$$\Theta^{k*} = \{\theta_0^{k*}, \theta_1^{*k}, \cdots, \theta_{N-1}^{k*}\}$$

exists. From the state constraints in the $\mathrm{M^2F}(\mathcal{C})$ problem at the step k,

$$\hat{x}(j) \in \mathcal{X}^k(j), j = 1, 2, \cdots, N-1,$$

and the definition equation (10) for the predicted reachable sets,

$$\hat{\mathcal{X}}(k+j|k) \subseteq \mathcal{X}^k(j)$$

$\mathcal{X}^{k+1}(j) = \mathcal{X}^k(j+1)$, $j = 1, 2, \cdots, N-1$, is a trivial selection that satisfies (11). ∎

Theorem 2.1 *If $M^2F(\mathcal{C}^0)$ is feasible, then $M^2F(\mathcal{C}^k)$ is feasible at all control steps $k \geq 0$. And furthermore,*

$$x(k) \in \mathcal{T}, k = N, N+1, \cdots.$$

Proof: Since $\mathrm{M^2F}(\mathcal{C}^0)$ is feasible, there exists the optimal solution

$$\Theta^{0*} = \{\theta_0^{0*}, \theta_1^{0*}, \cdots, \theta_{N-1}^{0*}\}$$

and the predicted bounds on the future reachable states satisfy,

$$\hat{\mathcal{X}}(j|0) \subseteq \mathcal{X}^0(j) = \mathcal{X}, j=1,2,\cdots,N-1, \text{ and } \hat{\mathcal{X}}(N|0) \subseteq \mathcal{T}.$$

Suppose $\mathrm{M^2F}(\mathcal{C}^k)$ is feasible.

$$\Theta^{k*} = \{\theta_0^{k*}, \theta_1^{k*}, \cdots, \theta_{N-1}^{k*}\}$$

and

$$\hat{\mathcal{X}}(k+j|k) \subseteq \mathcal{X}^k(j-1), j=1,2,\cdots,N-1, \text{ and } \hat{\mathcal{X}}(k+N|k) \subseteq \mathcal{T}.$$

At control stage $k+1$, consider the following selection of the constraint set

$$\hat{\mathcal{C}}^{k+1}=\left\{\mathcal{X}^{k+1}(0),\cdots,\mathcal{X}^{k+1}(N-1),\mathcal{V}^{k}(1),\cdots,\mathcal{V}^{k}(N-1),\mathcal{V}\right\}.$$

and Θ

$$\hat{\Theta}^{k+1}=\left\{\theta_1^{k*},\cdots,\theta_{N-1}^{k*},\theta^{\mathcal{T}}\right\}.$$

and the constraint set

$$\hat{\mathcal{C}}^{k+1}=\left\{\mathcal{X}^{k+1}(0),\cdots,\mathcal{X}^{k+1}(N-1),\mathcal{V}^{k}(1),\cdots,\mathcal{V}^{k}(N-1),\mathcal{V}\right\}.$$

For any $\hat{x}^{k+1}(0)\in\mathcal{X}^{k+1}(0)$ and $v^{k+1}(j)\in\mathcal{V}^{k}(j{+}1)$, $j=0,1,\cdots,N-2$, by the definition (10) for the predicted reachable set,

$$\begin{aligned}
&\hat{x}^{k+1}(0)\in\hat{\mathcal{X}}(k+1|s)\\
&\Rightarrow\left\{\begin{array}{l}\hat{x}^{k+1}(1)\in\hat{\mathcal{X}}(k+2|k)\subseteq\mathcal{X}^{k+1}(1)\\ G\left(\hat{x}^{k+1}(0),h\left(\hat{x}^{k+1}(0),\theta_1^{*}(k)\right),v(0)\right)\leq 0\end{array}\right.;\\
&\hat{x}^{k+1}(1)\in\hat{\mathcal{X}}(k+2|k)\\
&\Rightarrow\left\{\begin{array}{l}\hat{x}^{k+1}(2)\in\hat{\mathcal{X}}(k+3|k)\subseteq\mathcal{X}^{k+1}(2)\\ G\left(\hat{x}^{k+1}(1),h\left(\hat{x}^{k+1}(1),\theta_2^{*}(k)\right),v(1)\right)\leq 0\end{array}\right.;\\
&\cdots\\
&\hat{x}^{k+1}(N{-}2)\in\hat{\mathcal{X}}(k{+}N{-}1|k)\\
&\Rightarrow\left\{\begin{array}{l}\hat{x}^{k+1}(N{-}1)\in\hat{\mathcal{X}}(k{+}N|k)\subseteq\mathcal{X}^{k+1}(N{-}1)\subseteq\mathcal{T}\\ G\left(\hat{x}^{k+1}(N{-}2),h\left(\hat{x}^{k+1}(N{-}2),\theta_{N-1}^{*}\right),v(N{-}2)\right)\leq 0\end{array}\right.;\\
&\hat{x}^{k+1}(N-1)\in\mathcal{T}\\
&\Rightarrow\left\{\begin{array}{l}\hat{x}^{k+1}(N)\in\mathcal{T}\\ G\left(\hat{x}^{k+1}(N{-}1),h\left(\hat{x}^{k+1}(N{-}1),\theta^{\mathcal{T}}\right),v(N{-}1)\right)\leq 0\end{array}\right..
\end{aligned}$$

So, $\hat{\Theta}^{k+1}$ is a feasible solution to the $\mathrm{M}^2\mathrm{F}\left(\hat{\mathcal{C}}^{k+1}\right)$. Because the updated uncertainty bounds satisfy the condition (6), $\hat{\Theta}^{k+1}$ is also a feasible solution to $\mathrm{M}^2\mathrm{F}\left(\mathcal{C}^{k+1}\right)$. So, $\mathrm{M}^2\mathrm{F}\left(\mathcal{C}^{k}\right)$ is feasible for all control steps $k\geq 0$.

Following the state constraint updating condition (8), when $k\geq N$,

$$x(k)\in\hat{\mathcal{X}}(k|k)\subseteq\hat{\mathcal{X}}(k|k-1-1)\subseteq\cdots\subseteq\hat{\mathcal{X}}(k|k-N)\subseteq\mathcal{T}.$$

■

Theorem 2.1 shows that that the stability of the system (in the sense of remaining in $\mathcal{T}$) is ensured by updating the state constraints following the relation (8)–(10).

3 Distributed MPC with communication

We now consider interconnected systems where the couplings between M subsystems are through the state variable, i.e.

$$\begin{aligned}x_i(k{+}1)&=f_i(x_1(k),\cdots,x_M(k),u_i(k)),\ i{=}1,\cdots,M\\ &=f_i(x_i(k),u_i(k),v_i(k)),\end{aligned}$$

where

$$v_i(k)=\{x_1(k),\cdots,x_{i-1}(k),x_{i+1}(k),\cdots,x_M(k)\}.$$

The bounded uncertainties in the local models are bounded effects from other subsystems. These bounds are updated when new information is obtained from other controllers. Also each controller needs to impose the bounds it sends to other controllers as constraints on its own state to assure that its broadcasted bounds are maintained.

We propose that each controller directly adopts the $\mathrm{M}^2\mathrm{F}(\mathcal{C})$ optimization problem with updated state constraints and bound on uncertainties to handle uncertainties in the information from controllers, which is called *min-max feedback DMPC* (M^2F-DMPC). In the following discussion, we use the subscript $i, i=1,2,\cdots,M$, where M is the number of controllers in the system, to denote the problem and the variables corresponding to the i^{th} controller. Since disturbances to each subsystem are the effects from other subsystems and the predicted future reachable set is used as the bound, the uncertainty bound is updated in the following way,

$$\mathcal{V}_i^k(j)=\prod_{\substack{l=1\\ l\neq i}}^{M}\mathcal{X}_l^k(j),\ j=1,2,\cdots,N-1$$

and

$$\mathcal{V}_i^k(0)=\prod_{\substack{l=1\\ l\neq i}}^{M}\mathcal{X}_l^k(0),$$

where $\hat{\mathcal{X}}_l(k|k-1)\subseteq\mathcal{X}_l^k(0)\subseteq\mathcal{X}_l^{k-1}(1)$, $l=1,\cdots,i-1,i+1,\cdots,M$, is computed by the same method for $\mathcal{X}_l^k(j)$. We assume a one-step delay in the communication of the constraints.. In min-max optimization, each controller tries to find a feedback control policy to optimize the worst case performance. Thus, each controller should optimizes against the worst possible case from neighbor controllers' information about future reachable sets. For this purpose, each agent updates state constraints following the equations (8)–(10) and thus the uncertainty bound updating condition (6) can naturally be satisfied.

Theorem 3.1 (Feasibility) *If M^2F-DMPC is feasible at the first step $k=0$, it is feasible at all control steps $k\geq 0$. Furthermore, the state of the whole system goes into the set $\mathcal{T}_1\times\mathcal{T}_2\times\cdots\times\mathcal{T}_M$.*

Proof: This theorem follows from the feasibility result for the $\mathrm{M}^2\mathrm{F}(\mathcal{C})$ optimization problem with updated state constraints and bounds on the uncertainties. ■

Following the proof for the Theorem 2.1, the M^2F-MPC can be feasible even without communication, i.e. bound updating. When communication is available, the neighbor information is less conservative and the controllers can make less conservative decisions.

4 An Example

Consider the spring-mass system, shown as Figure 2, consisting of four identical springs with spring constant k. The masses are three identical solenoids with the mass m, each of which is controlled by an MPC controller.

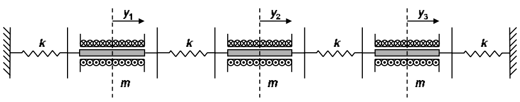

Figure 2: A solenoid example.

The dynamic equations for the system are:

$$\begin{aligned}
\begin{bmatrix} \dot{x}_{1,1} \\ \dot{x}_{1,2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -2\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_{1,1} \\ x_{1,2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u_1 \\
&+ \begin{bmatrix} 0 & 0 \\ \frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_{2,1} \\ x_{2,2} \end{bmatrix} \\
y_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{1,1} \\ x_{1,2} \end{bmatrix} \\
\begin{bmatrix} \dot{x}_{2,1} \\ \dot{x}_{2,2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -2\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_{2,1} \\ x_{2,2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u_2 \\
&+ \begin{bmatrix} 0 & 0 \\ \frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_{1,1} \\ x_{2,1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_{3,1} \\ x_{3,2} \end{bmatrix} \\
y_2 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{2,1} \\ x_{2,2} \end{bmatrix} \\
\begin{bmatrix} \dot{x}_{3,1} \\ \dot{x}_{3,2} \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -2\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_{3,1} \\ x_{3,2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u_3 \\
&+ \begin{bmatrix} 0 & 0 \\ \frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} x_{2,1} \\ x_{2,2} \end{bmatrix} \\
y_3 &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_{3,1} \\ x_{3,2} \end{bmatrix}
\end{aligned}$$

where the state variables $x_{i,1}$ and $x_{i,2}$ are the position and velocity of the i^{th} solenoid, for $i = 1, 2, 3$. A zero-order hold, with sampling time $T_s = 0.2$s, is used to obtain the discrete-time model with $m = 1$kg and $k = 10$N/m. In the following, the prediction horizon is $N = 2$ and the initial conditions are

$$\begin{bmatrix} x_{1,1}(0) \\ x_{1,2}(0) \end{bmatrix} = \begin{bmatrix} x_{2,1}(0) \\ x_{2,2}(0) \end{bmatrix} = \begin{bmatrix} x_{3,1}(0) \\ x_{3,2}(0) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}.$$

We first consider the case where $u_i^k(j) = \theta_i^k(j)$, i.e. using the open-loop MPC formulation,

Performance Index:

$$\min_{X_i^k, U_i^k} \left[\max_{V_i^k} J_i\left(X_i^k, U_i^k, V_i^k\right) \right]$$

where
State Prediction:
$X_i^k = \left\{x_i^k(0), x_i^k(1), \cdots, x_i^k(N)\right\}$
Control Prediction:
$U_i^k = \left\{u_i^k(0), u_i^k(1), \cdots, u_i^k(N-1)\right\}$

We use the ellipsoidal method of Kurzhanski and Vályi [7] to compute approximations of state constraint sets. For each reachable set, we compute a single ellipsoid $\tilde{\mathcal{X}}_i(k+j|k)$ as an external approximation, which has the minimal sum of squares of semi-axises. These ellipsoids are also used as $\mathcal{X}^k(j)$. Since the open-loop control sequence can only manipulate the centers of the reachable sets, we move the centers of all these ellipsoids to the origin so that it is easy to tell whether or not the MPC problem is feasible. In Figure 3, the left three plots are $\tilde{\mathcal{X}}_i(1|0)$ and the right three plots are $\tilde{\mathcal{X}}_i(2|0)$, $i = 1, 2, 3$. The solid ellipsoids give the approximate reachable sets and the dashed ellipsoids form the state constraints. It is obvious that no matter what controller 2 selects as the open-loop control sequence, the approximate reachable set $\tilde{\mathcal{X}}_2(2|0)$ cannot satisfy the state constraint.

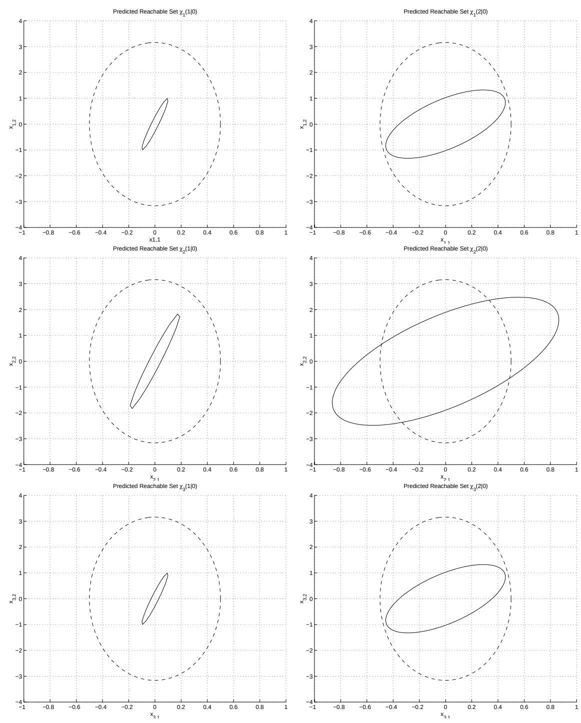

Figure 3: Feasibility Problem by Open-Loop Policy.

Now, consider a parameterized affine state feedback control policy, i.e. $u_i^k(j) = K_i^k(j)\, x_i^k(j) + \bar{u}_i^k(j)$, where $K_i^k(j)$ and $\bar{u}_i^k(j)$ are parameters subject to optimization, i.e. $\theta_i^k(j)$. The Figure 4 shows the simulation results for 20 steps. The left three plots give the responses of the three sub-systems. The curves with crosses are positions and the curves with circles are velocities of those solenoids. The right three plots are the predictions of the corresponding controllers which are used as the dynamic information by the neighbor controller(s). The largest ellipsoid in each plot is the state constraint. The solid ellipsoids are the approximate reachable set $\tilde{\mathcal{X}}_i(k+1|k)$ and the dashed ellipsoids are $\tilde{\mathcal{X}}_i(k+2|k)$. In this case, $\mathrm{M^2F}(\mathcal{C}^k)$ is feasible for all k, which is to be expected since $\mathrm{M^2F}(\mathcal{C}^0)$ is feasible. Each sub-system goes back to the equilibrium point and the uncertainties in the predictions of the controllers decrease throughout the simulation.

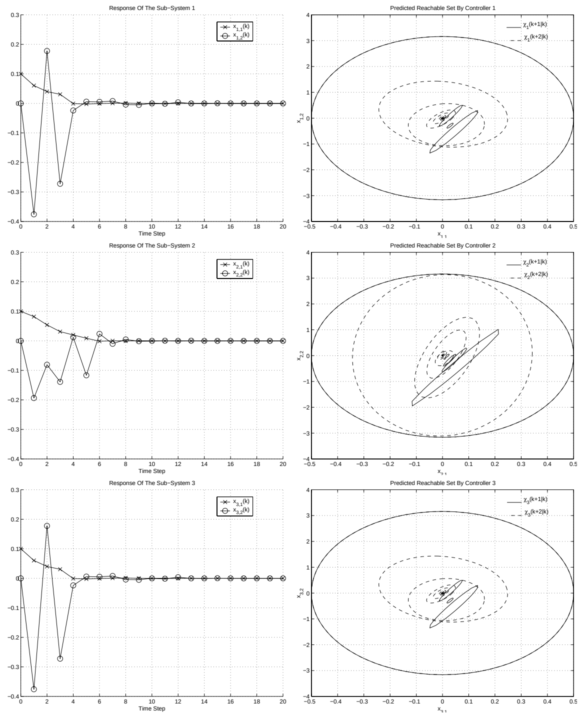

Figure 4: Simulation results.

5 Discussion

This paper presents a new distributed scheme to achieve coordination among decentralized MPC controllers with one-step delayed information exchange. Min-max optimization is used to handle the uncertainties in the disturbances from neighbor controllers and the feedback policy is introduced to obtain less conservative solutions. When the delay in communication cannot be ignored, the method in [1] can be directly applied to our scheme. We are currently investigating conditions for asymptotic stability, extending the results obtained in [5] for linear systems without uncertainty. We are also investigating set-membership state estimation [3, 4, 9] to handle uncertainties in the states and methods for set representation and approximation.

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