WELCOME
TO
15-213
14-513
18-213
18-613
15-513

Bits, Bytes, and Integers – Part 2

15-213/18-213/14-513/15-513/18-613: Introduction to Computer Systems
3rd Lecture, September 8, 2020

Assignment Announcements

- **Lab 0 available via course web page and Autolab.**
  - Due Thurs. Sept. 10, 11:59:59pm ET
  - No grace days. No late submissions!
- **Lab 1 available after 5 pm via Autolab**
  - Due Thurs, Sept. 17, 11:59:59pm ET
  - Read instructions carefully: writeup, bits.c, tests.c
    - Quirky software infrastructure
  - Based on lectures 2, 3, and 4 (CS:APP Chapter 2)
  - After today you will know everything for the integer problems
  - Floating point covered Thursday Sept. 10
- **In-Person Recitations**
  - We will email students with their in-person recitation status based on the survey on Piazza (fill out before class 9/10 or be uncounted)
  - First recitations (in-person and remote) are 9/14

Bootcamps

- **Wednesday Sept 9 @ 7-9 pm ET**
  - GCC and Build Automation

- **Friday Sept 11 @ 7-9 pm ET**
  - Debugging and GDB

Summary From Last Lecture

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
- Representations in memory, pointers, strings
- Summary

Encoding Integers

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

Two's Complement Examples (w = 5)

		-16	8	4	2	1	
10	=	0	1	0	1	0	8+2 = 10

		-16	8	4	2	1	
-10	=	1	0	1	1	0	-16+4+2 = -10

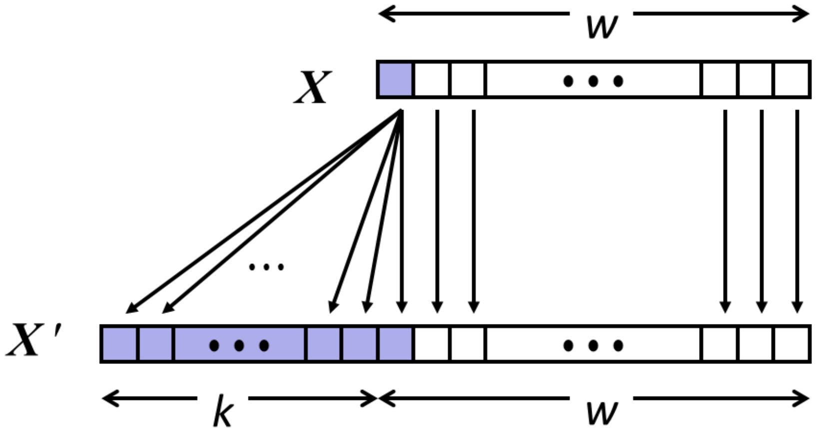
Unsigned & Signed Numeric Values

X	B2U(*X*)	B2T(*X*)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	–8
1001	9	–7
1010	10	–6
1011	11	–5
1100	12	–4
1101	13	–3
1110	14	–2
1111	15	–1

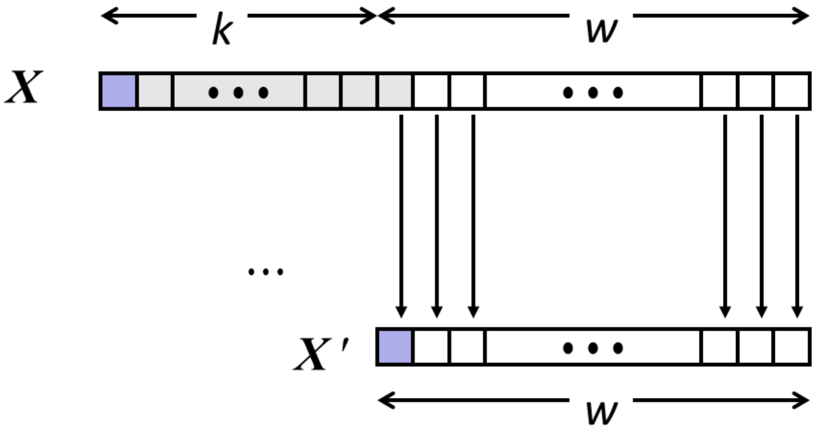
- **Equivalence**
  - Same encodings for nonnegative values
- **Uniqueness**
  - Every bit pattern represents unique integer value
  - Each representable integer has unique bit encoding
- **Expression containing signed and unsigned int:** `int` is cast to `unsigned`

Sign Extension and Truncation

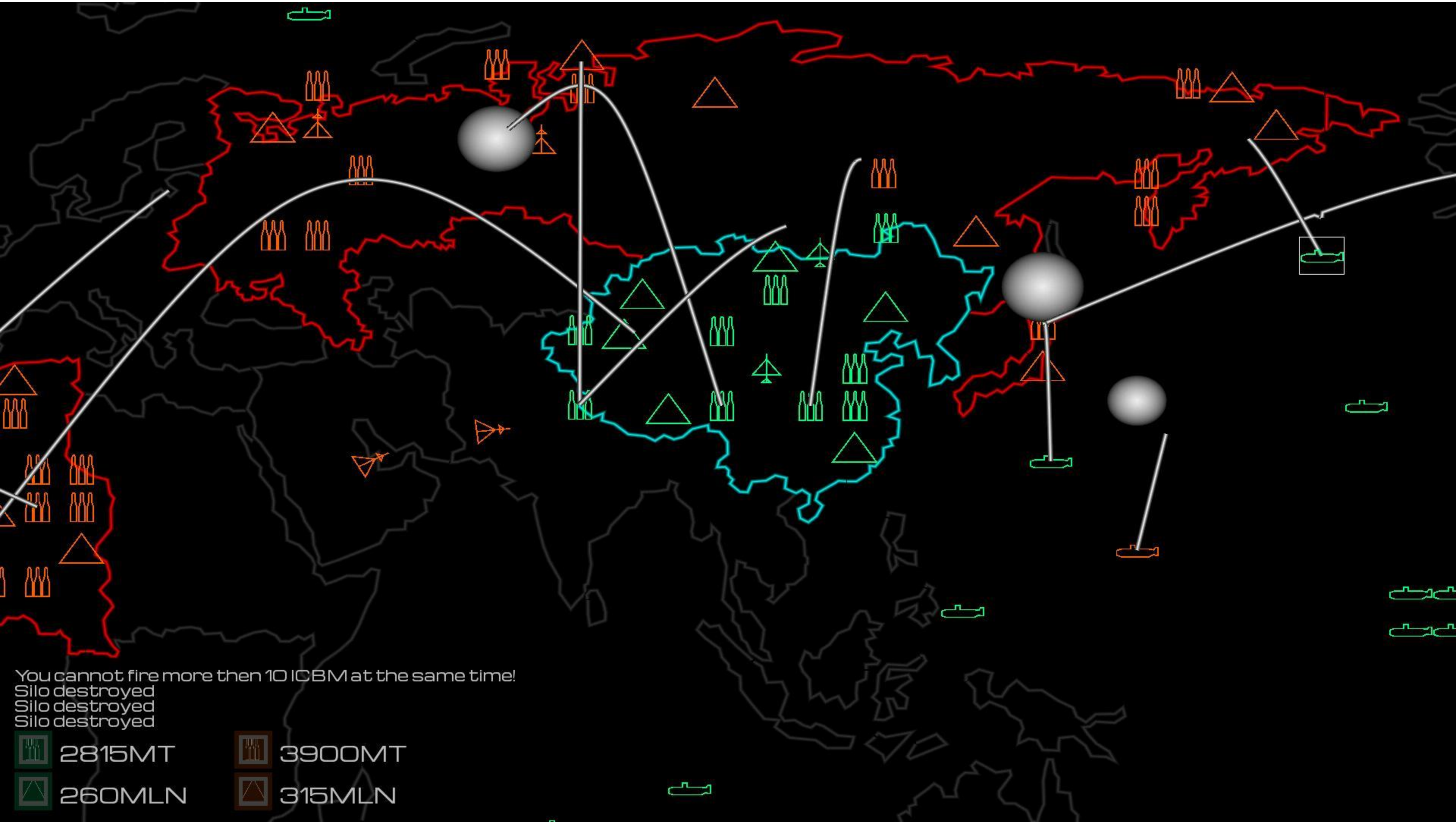
- **Sign Extension**



- **Truncation**

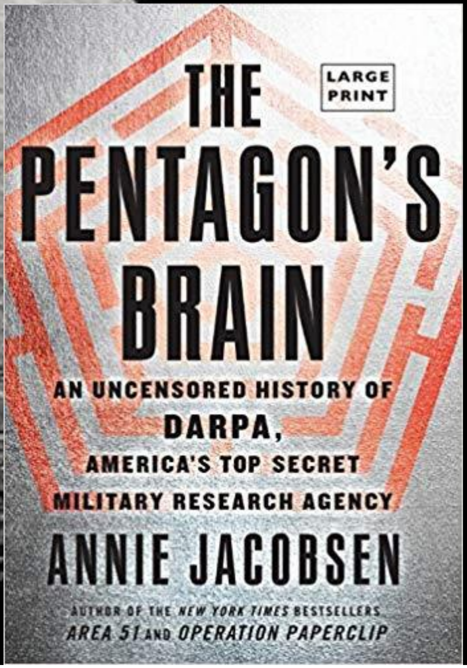


Global Thermonuclear War



You cannot fire more then 10 ICBM at the same time!
Silo destroyed
Silo destroyed
Silo destroyed

- **Misunderstanding integers can lead to the end of the world as we know it!**
- Thule (Qaanaaq), Greenland
- US DoD “Site J” Ballistic Missile Early Warning System (BMEWS)
- 10/5/60: world nearly ends
- Missile radar echo: 1/8s
- BMEWS reports: 75s echo(!)
- 1000s of objects reported
- NORAD alert level 5:
  - Immediate incoming nuclear attack!!!!



THE PENTAGON'S BRAIN

LARGE PRINT

AN UNCENSORED HISTORY OF DARPA, AMERICA'S TOP SECRET MILITARY RESEARCH AGENCY

ANNIE JACOBSEN

AUTHOR OF THE *NEW YORK TIMES* BESTSELLERS *AREA 51* AND *OPERATION PAPERCLIP*

- **Kruschev was in NYC 10/5/60 (weird time to attack)**
  - someone in Qaanaaq said “why not go check outside?”
- **“Missiles” were actually THE MOON RISING OVER NORWAY**
- **Expected max distance: 3000 mi; Moon distance: .25M miles!**
- **.25M miles % sizeof(distance) = 2200mi.**
- **Overflow of distance nearly caused nuclear apocalypse!!**

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - **Addition, negation, multiplication, shifting**
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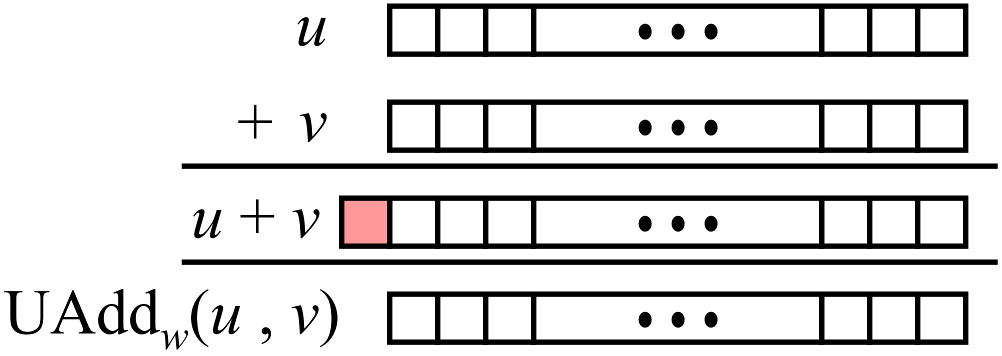
Unsigned Addition



Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits

- **Standard Addition Function**
  - Ignores carry output
- **Implements Modular Arithmetic**

$$s \quad = \quad \text{UAdd}_w(u , v) \quad = \quad u + v \mod 2^w$$

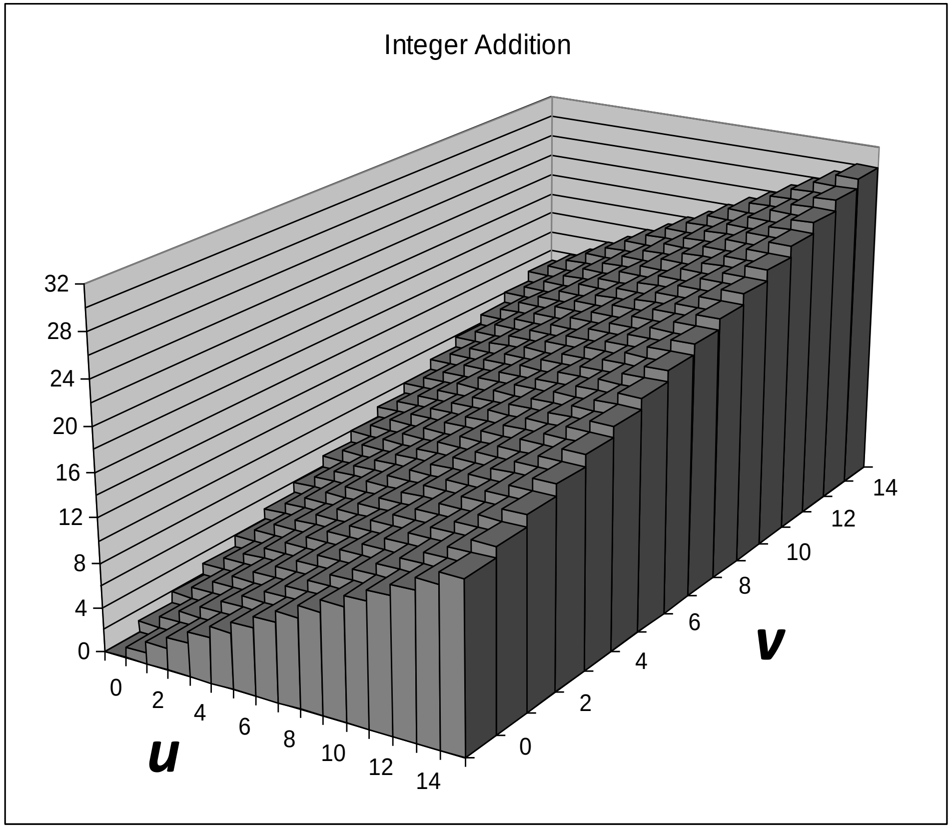
`unsigned char`	1110 1001	E9	223
	+ 1101 0101	+ D5	+ 213
	1 1011 1110	1BE	446
	1011 1110	BE	190

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Visualizing (Mathematical) Integer Addition

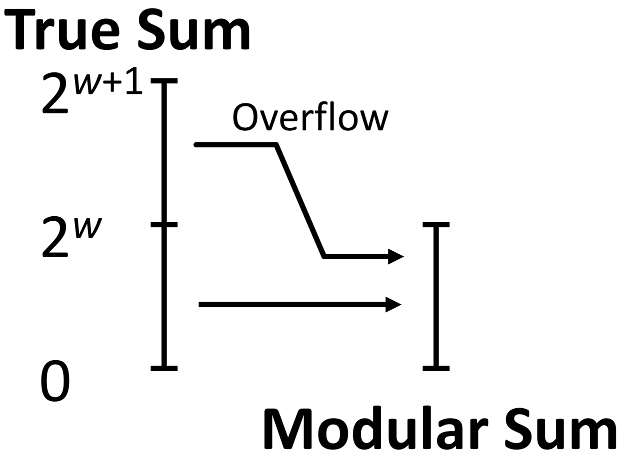
- **Integer Addition**
  - 4-bit integers *u*, *v*
  - Compute true sum $\text{Add}_4(u , v)$
  - Values increase linearly with *u* and *v*
  - Forms planar surface

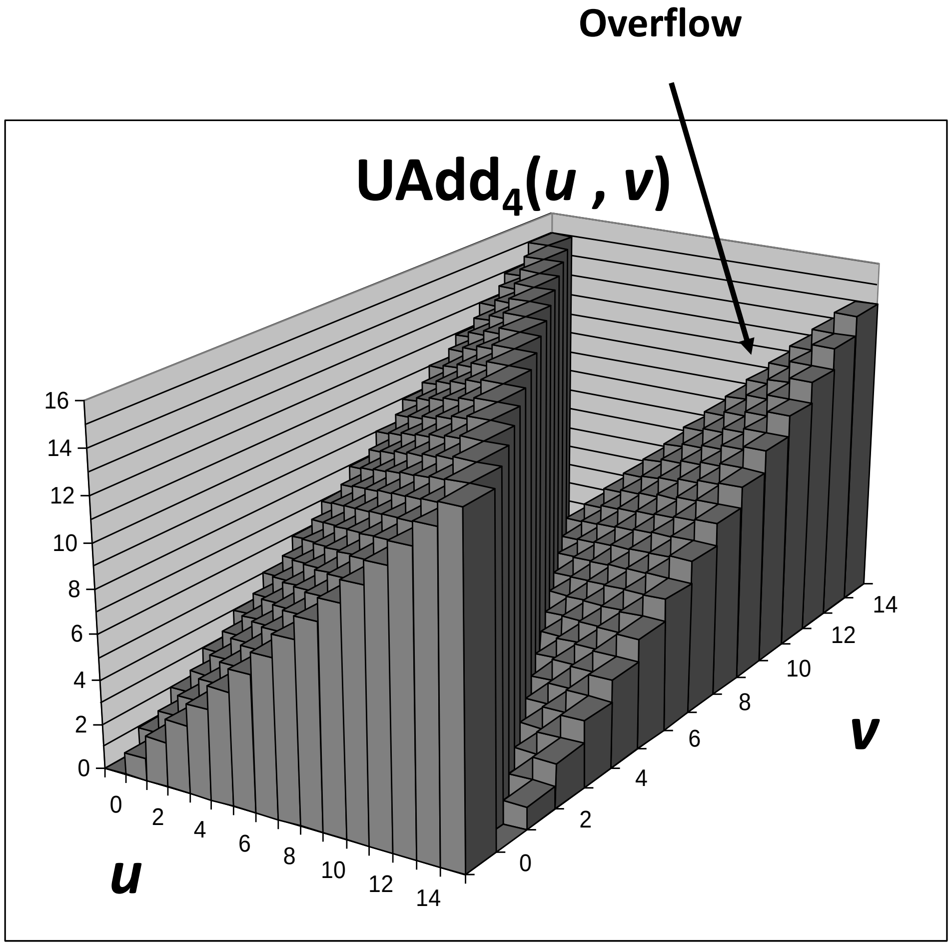
$\text{Add}_4(u , v)$



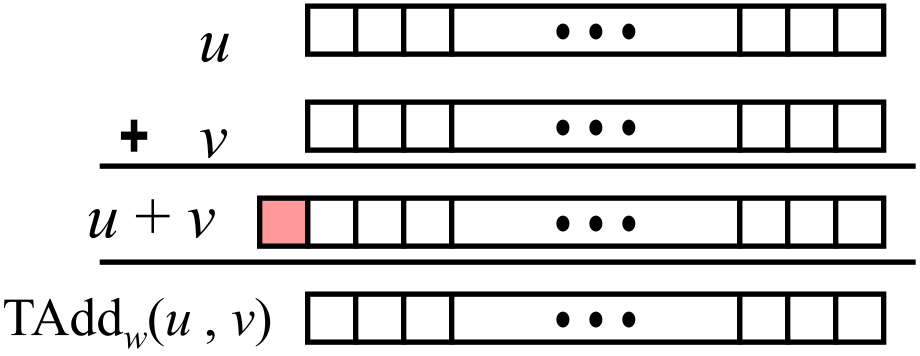
Visualizing Unsigned Addition

- **Wraps Around**
  - If true sum ≥ 2^w
  - At most once





Two’s Complement Addition



Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits

- **TAdd and UAdd have Identical Bit-Level Behavior**
  - Signed vs. unsigned addition in C:

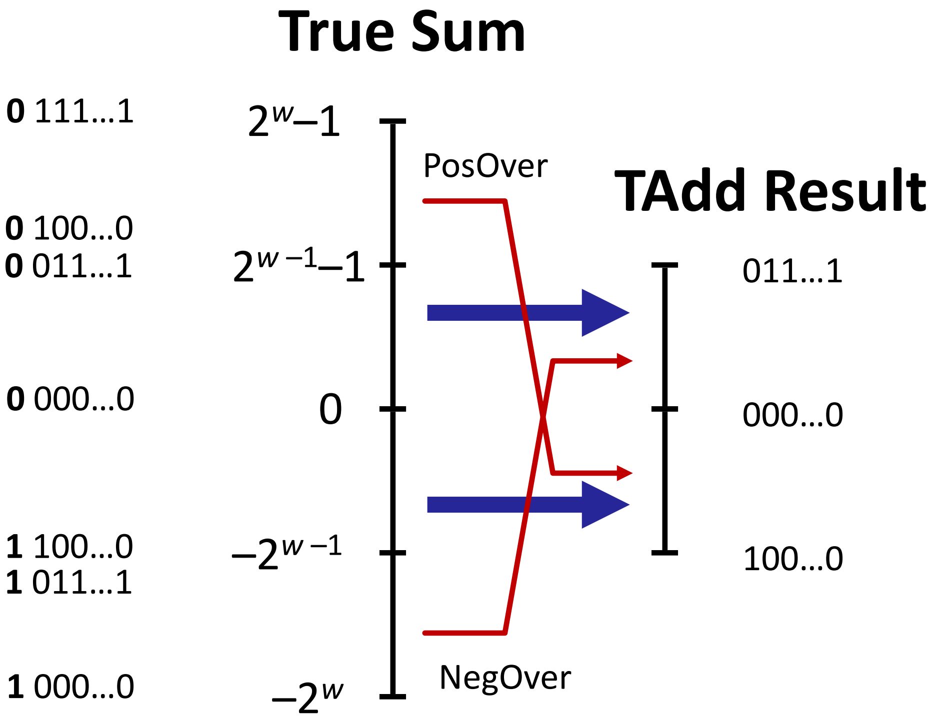
```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

  - Will give `s == t`

1110 1001	E9	-23
+ 1101 0101	+ D5	+ -43
1 1011 1110	1BE	-66
1011 1110	BE	-66

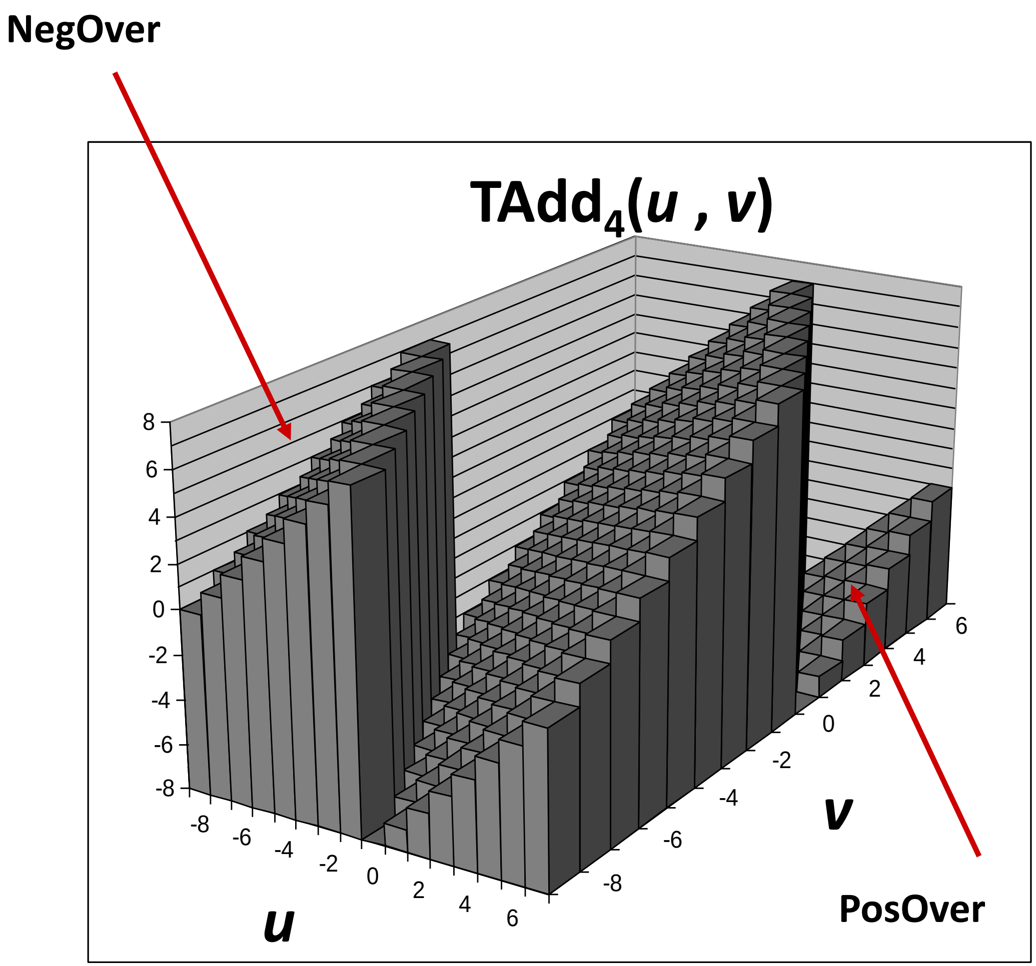
TAdd Overflow

- **Functionality**
  - True sum requires *w*+1 bits
  - Drop off MSB
  - Treat remaining bits as 2's comp. integer



Visualizing 2’s Complement Addition

- **Values**
  - 4-bit two’s comp.
  - Range from -8 to +7
- **Wraps Around**
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once

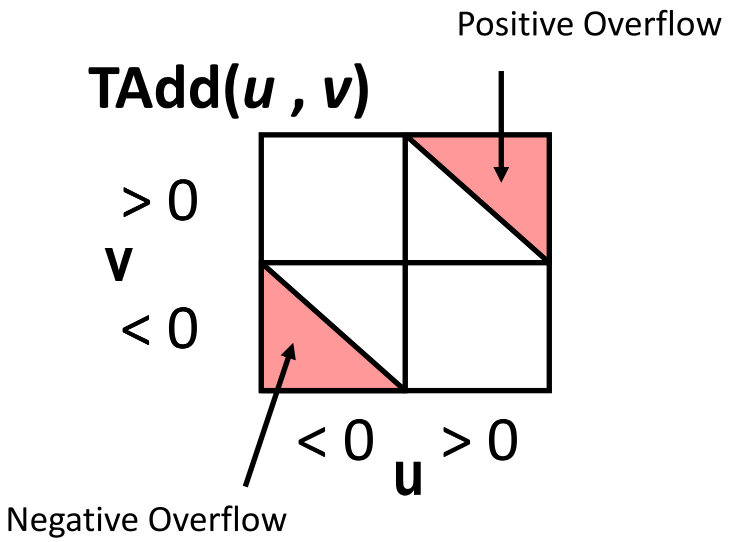


NegOver

PosOver

Characterizing TAdd

- **Functionality**
    - True sum requires *w*+1 bits
    - Drop off MSB
    - Treat remaining bits as 2's comp. integer

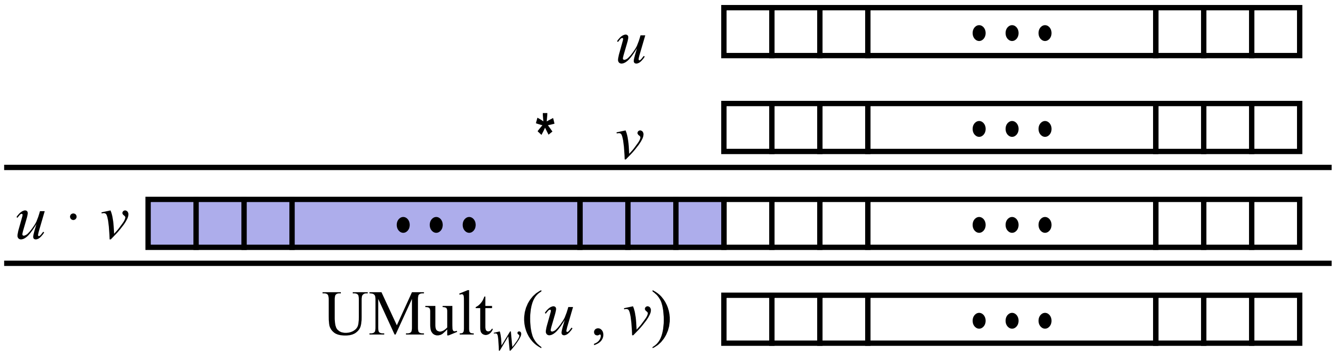


$$TAdd_w(u,v) = \begin{cases} u+v+2^w & u+v < TMin_w \quad \textbf{(NegOver)} \\ u+v & TMin_w \le u+v \le TMax_w \\ u+v-2^w & TMax_w < u+v \quad \textbf{(PosOver)} \end{cases}$$

Multiplication

- **Goal: Computing Product of *w*-bit numbers *x*, *y***
  - Either signed or unsigned
- **But, exact results can be bigger than *w* bits**
  - Unsigned: up to 2*w* bits
    - Result range: $0 \le x * y \le (2^{w} – 1)^{2} = 2^{2w} – 2^{w+1} + 1$
  - Two's complement min (negative): Up to 2*w*-1 bits
    - Result range: $x * y \ge (–2^{w–1})*(2^{w–1}–1) = –2^{2w–2} + 2^{w–1}$
  - Two's complement max (positive): Up to 2*w* bits, but only for $(TMin_{w})^{2}$
    - Result range: $x * y \le (–2^{w–1})^{2} = 2^{2w–2}$
- **So, maintaining exact results…**
  - would need to keep expanding word size with each product computed
  - is done in software, if needed
    - e.g., by "arbitrary precision" arithmetic packages

Unsigned Multiplication in C



Operands: *w* bits

True Product: 2**w* bits

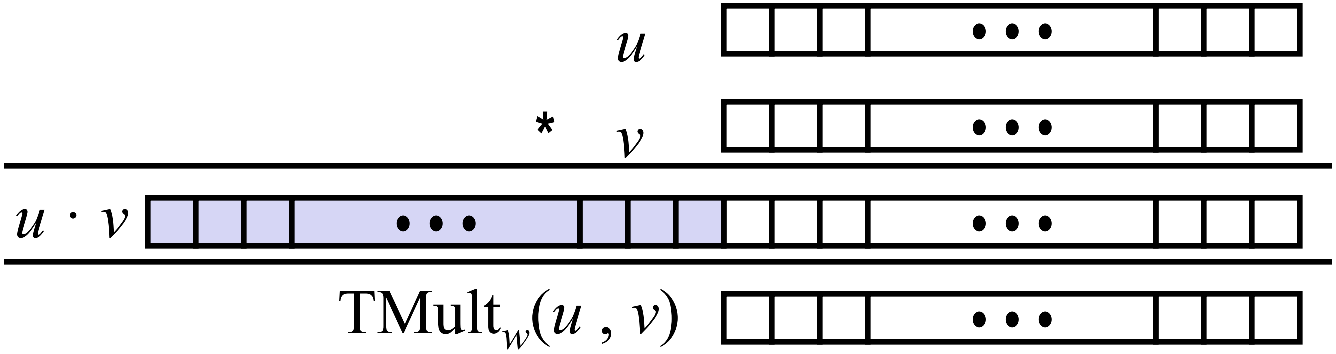
Discard *w* bits: *w* bits

- **Standard Multiplication Function**
  - Ignores high order *w* bits
- **Implements Modular Arithmetic**

$\mathrm{UMult}_w(u , v) = u \cdot v \bmod 2^w$

```
                 1110 1001        E9        233
*                1101 0101    *   D5    *   213
 1100 0001 1101 1101            C1DD      49629
                 1101 1101        DD        221
```

Signed Multiplication in C



Operands: *w* bits

True Product: 2**w* bits

Discard *w* bits: *w* bits

- **Standard Multiplication Function**
  - Ignores high order *w* bits
  - Some of which are different for signed vs. unsigned multiplication
  - Lower bits are the same

	1110 1001		E9		-23
*	1101 0101	*	D5	*	-43
	0000 0011 1101 1101		03DD		989
	1101 1101		DD		-35

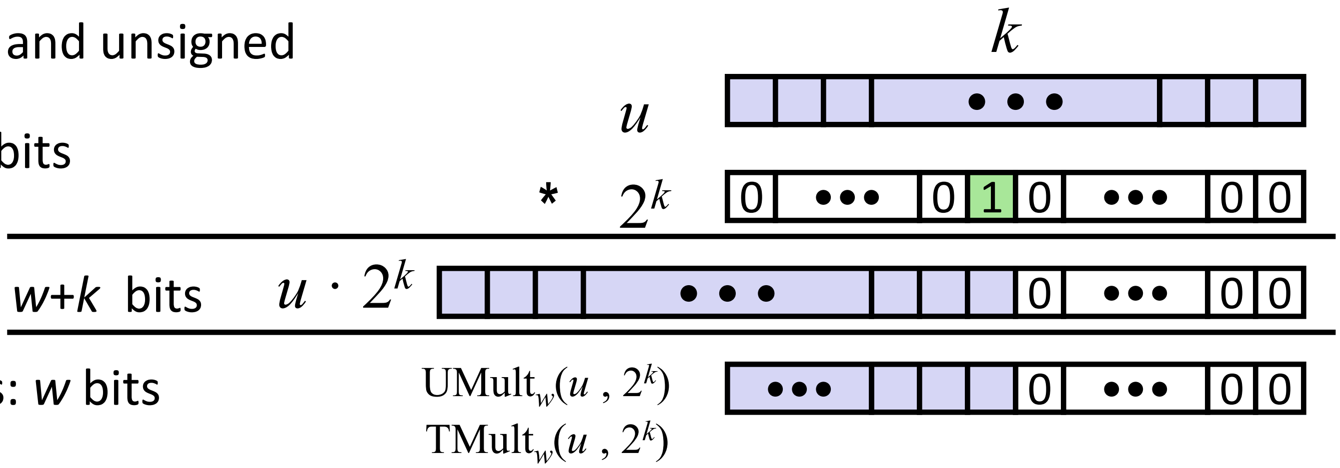
Power-of-2 Multiply with Shift

- **Operation**
  - `u << k` gives `u * 2`^{k}
  - Both signed and unsigned

Operands: *w* bits

True Product: *w*+*k* bits

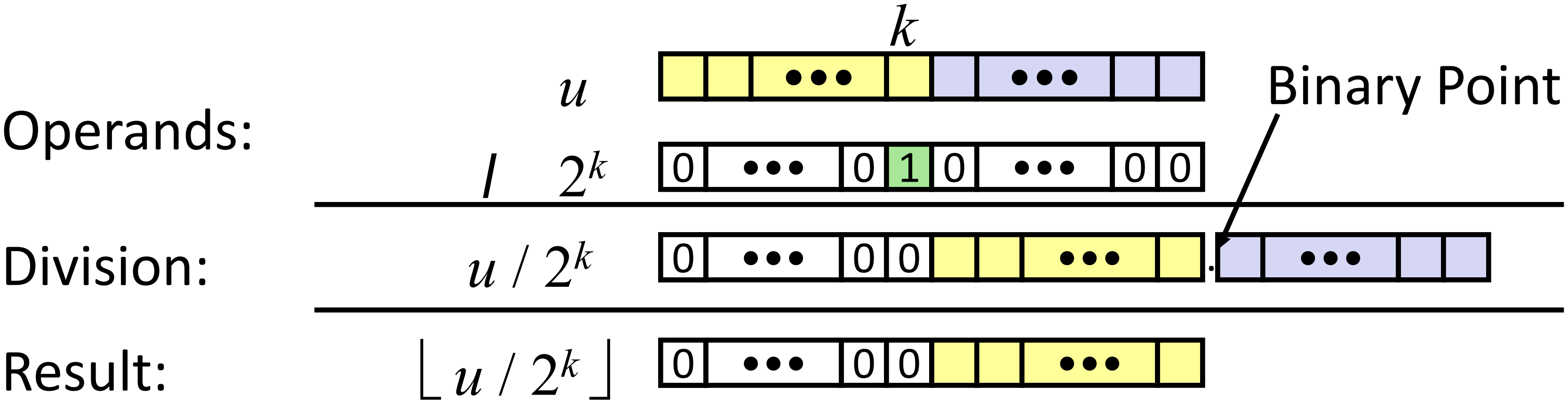
Discard *k* bits: *w* bits



- **Examples**
  - `u << 3 == u * 8`
  - `(u << 5) - (u << 3)== u * 24`
  - Most machines shift and add faster than multiply
    - Compiler generates this code automatically

Unsigned Power-of-2 Divide with Shift

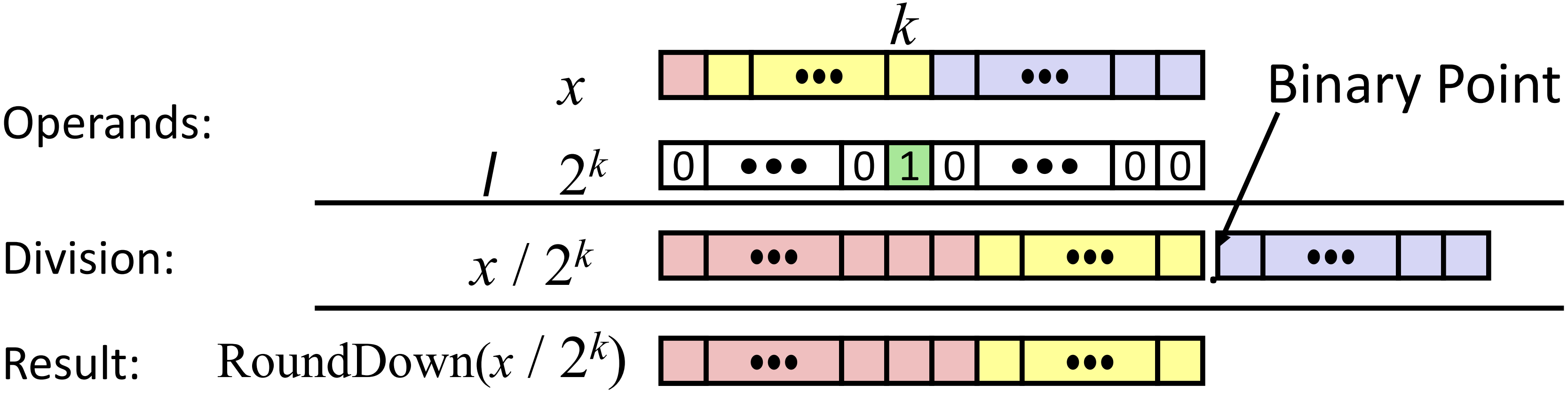
- **Quotient of Unsigned by Power of 2**
  - `u >> k` gives $\lfloor \texttt{u} / 2^k \rfloor$
  - Uses logical shift



	Division	Computed	Hex	Binary
`x`	15213	15213	`3B 6D`	`00111011 01101101`
`x >> 1`	7606.5	7606	`1D B6`	`00011101 10110110`
`x >> 4`	950.8125	950	`03 B6`	`00000011 10110110`
`x >> 8`	59.4257813	59	`00 3B`	`00000000 00111011`

Signed Power-of-2 Divide with Shift

- **Quotient of Signed by Power of 2**
  - `x >> k` gives $\lfloor \texttt{x} / 2^k \rfloor$
  - Uses arithmetic shift
  - Rounds wrong direction when `x < 0`

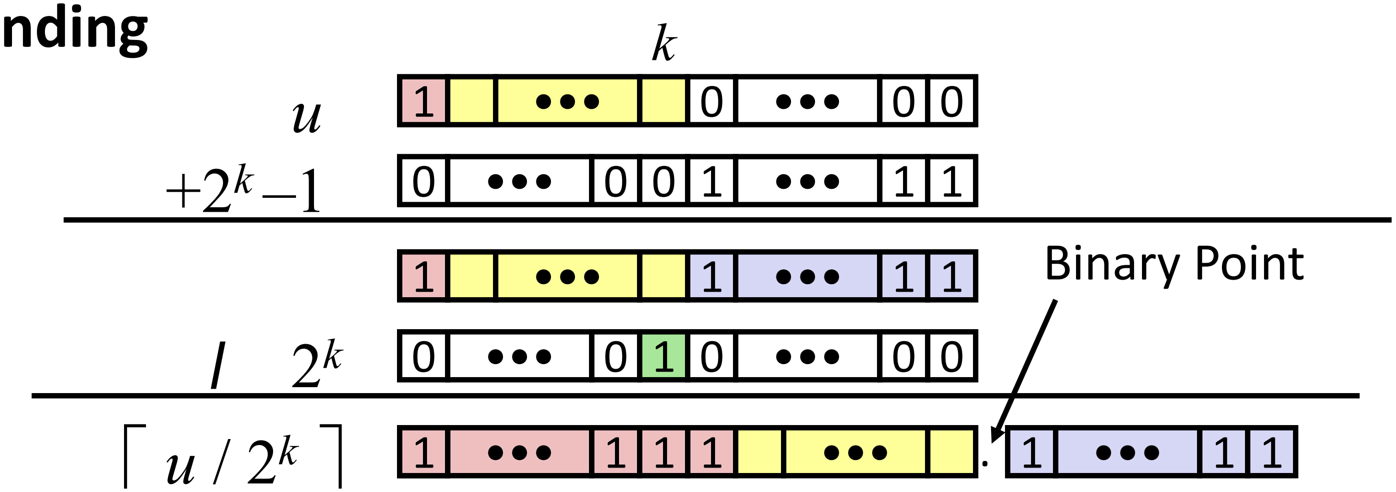


	Division	Computed	Hex	Binary
`x`	-15213	-15213	`C4 93`	`11000100 10010011`
`x >> 1`	-7606.5	-7607	`E2 49`	`11100010 01001001`
`x >> 4`	-950.8125	-951	`FC 49`	`11111100 01001001`
`x >> 8`	-59.4257813	-60	`FF C4`	`11111111 11000100`

Correct Power-of-2 Divide

- **Quotient of Negative Number by Power of 2**
  - Want $\lceil$ `x / 2`^{k} $\rceil$ (Round Toward 0)
  - Compute as $\lfloor$ `(x+2`^{k}`-1) / 2`^{k} $\rfloor$
    - In C: `(x + (1<<k)-1) >> k`
    - Biases dividend toward 0

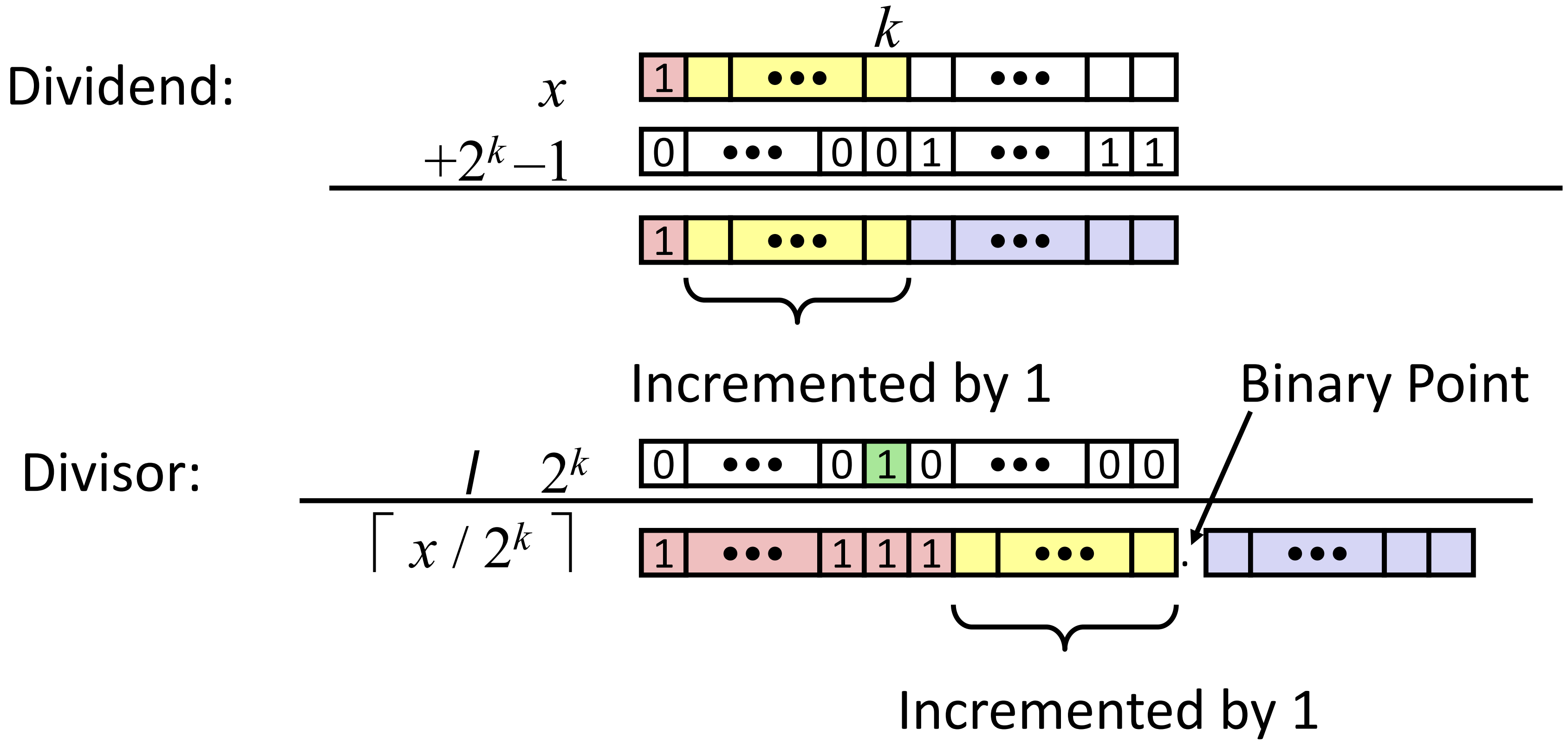
Case 1: No rounding



Biasing has no effect

Correct Power-of-2 Divide (Cont.)

Case 2: Rounding



Dividend: x

$+2^k-1$

Incremented by 1

Binary Point

Divisor: $/ \quad 2^k$

$\lceil x / 2^k \rceil$

Incremented by 1

Biasing adds 1 to final result

Negation: Complement & Increment

- **Negate through complement and increase**

  `~x + 1 == -x`

- **Example**
  - Observation: `~x + x == 1111…111 == -1`

	x	1	0	0	1	1	1	0	1
+	~x	0	1	1	0	0	0	1	0
	-1	1	1	1	1	1	1	1	1

x = 15213

	Decimal	Hex	Binary
`x`	15213	3B 6D	00111011 01101101
`~x`	-15214	C4 92	11000100 10010010
`~x+1`	-15213	C4 93	11000100 10010011
`y`	-15213	C4 93	11000100 10010011

Complement & Increment Examples

x = 0

	Decimal	Hex	Binary
0	0	00 00	00000000 00000000
~0	-1	FF FF	11111111 11111111
~0+1	0	00 00	00000000 00000000

x = TMin

	Decimal	Hex	Binary
`x`	-32768	80 00	10000000 00000000
`~x`	32767	7F FF	01111111 11111111
`~x+1`	-32768	80 00	10000000 00000000

Canonical counter example

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- **Integers**
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
  - **Summary**
- Representations in memory, pointers, strings

Arithmetic: Basic Rules

- **Addition:**
  - Unsigned/signed: Normal addition followed by truncate, same operation on bit level
  - Unsigned: addition mod 2^w
    - Mathematical addition + possible subtraction of 2^w
  - Signed: modified addition mod 2^w (result in proper range)
    - Mathematical addition + possible addition or subtraction of 2^w

- **Multiplication:**
  - Unsigned/signed: Normal multiplication followed by truncate, same operation on bit level
  - Unsigned: multiplication mod 2^w
  - Signed: modified multiplication mod 2^w (result in proper range)

Quiz Time!

Check out:

https://canvas.cmu.edu/courses/17808

Today: Bits, Bytes, and Integers

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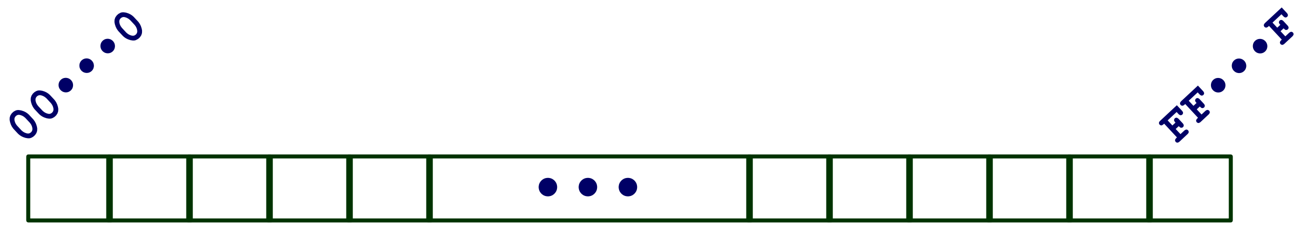
  - Summary
- **Representations in memory, pointers, strings**

Byte-Oriented Memory Organization



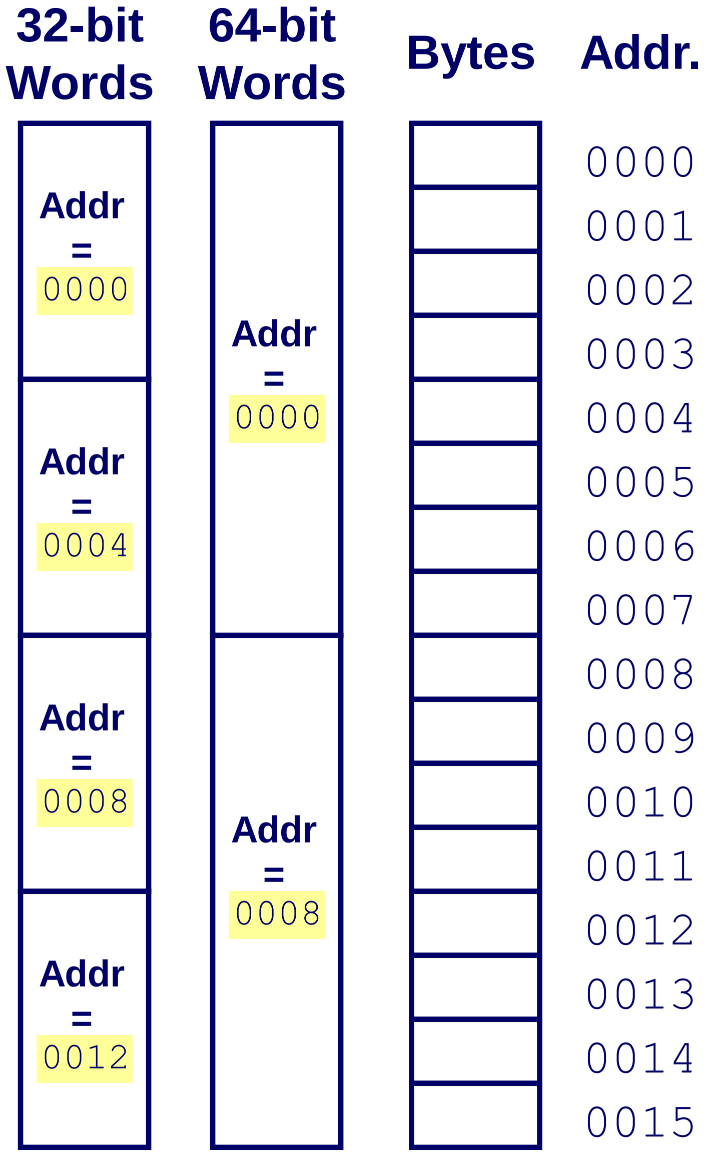
- **Programs refer to data by address**
  - Conceptually, envision it as a very large array of bytes
    - In reality, it's not, but can think of it that way
  - An address is like an index into that array
    - and, a pointer variable stores an address
- **Note: system provides private address spaces to each "process"**
  - Think of a process as a program being executed
  - So, a program can clobber its own data, but not that of others

Machine Words

- **Any given computer has a “Word Size”**
  - Nominal size of integer-valued data
    - and of addresses
  - Until recently, most machines used 32 bits (4 bytes) as word size
    - Limits addresses to 4GB (2^{32} bytes)
  - Increasingly, machines have 64-bit word size
    - Potentially, could have 18 EB (exabytes) of addressable memory
    - That’s 18.4 X 10^{18}
  - Machines still support multiple data formats
    - Fractions or multiples of word size
    - Always integral number of bytes

Word-Oriented Memory Organization

- **Addresses Specify Byte Locations**
  - Address of first byte in word
  - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Example Data Representations

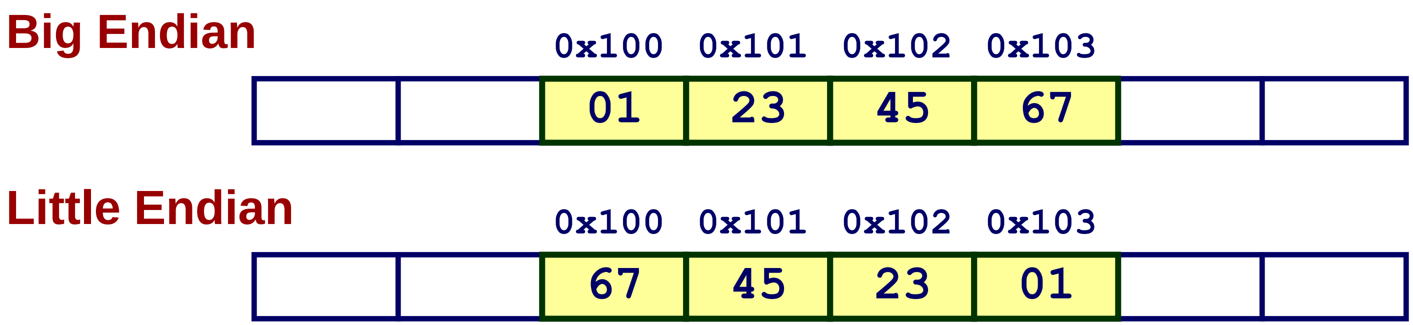
C Data Type	Typical 32-bit	Typical 64-bit	x86-64
`char`	1	1	1
`short`	2	2	2
`int`	4	4	4
`long`	4	8	8
`float`	4	4	4
`double`	8	8	8
pointer	4	8	8

Byte Ordering

- **So, how are the bytes within a multi-byte word ordered in memory?**
- **Conventions**
  - Big Endian: Sun (Oracle SPARC), PPC Mac, *Internet*
    - Least significant byte has highest address
  - Little Endian: *x86*, ARM processors running Android, iOS, and Linux
    - Least significant byte has lowest address

Byte Ordering Example

- **Example**
  - Variable x has 4-byte value of 0x01234567
  - Address given by &x is 0x100



Big Endian

		0x100	0x101	0x102	0x103		
		01	23	45	67		

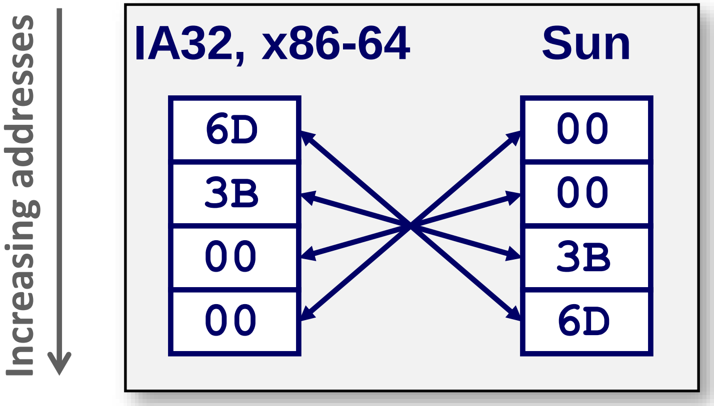
Little Endian

		0x100	0x101	0x102	0x103		
		67	45	23	01		

Representing Integers

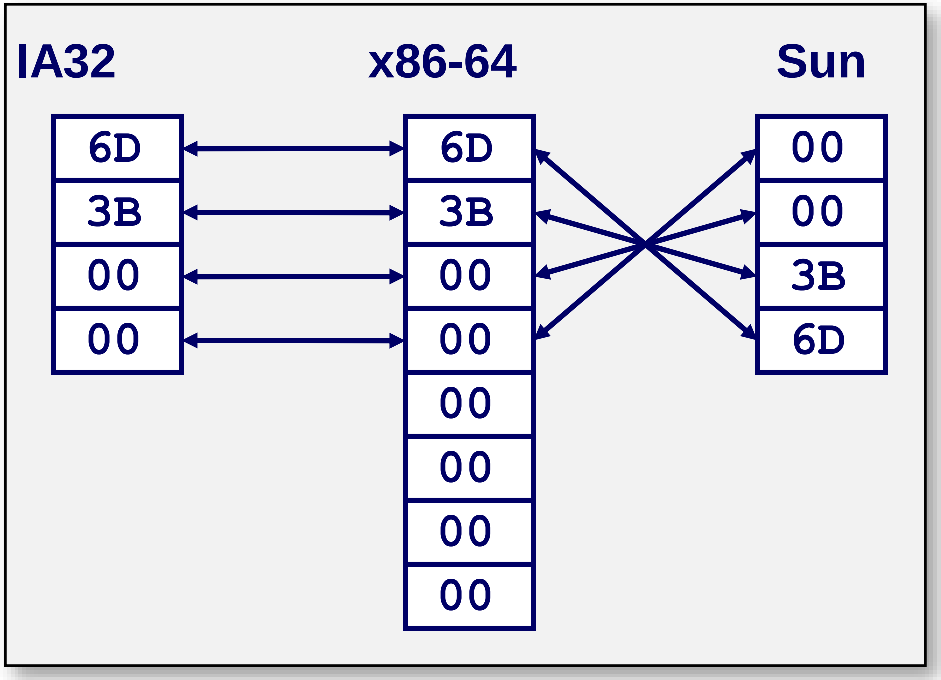
Decimal:	15213			
Binary:	0011	1011	0110	1101
Hex:	3	B	6	D

`int A = 15213;`



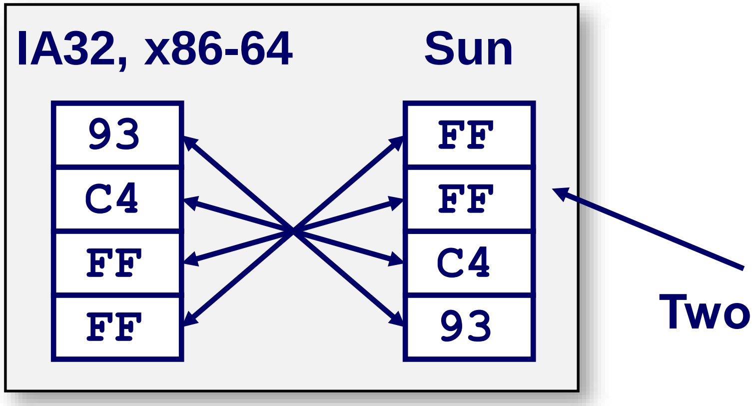
IA32, x86-64	Sun
6D	00
3B	00
00	3B
00	6D

`long int C = 15213;`



IA32	x86-64	Sun
6D	6D	00
3B	3B	00
00	00	3B
00	00	6D
	00	
	00	
	00	
	00	

`int B = -15213;`



IA32, x86-64	Sun
93	FF
C4	FF
FF	C4
FF	93

Two’s complement representation

Examining Data Representations

- **Code to Print Byte Representation of Data**
  - Casting pointer to unsigned char * allows treatment as a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, size_t len){
  size_t i;
  for (i = 0; i < len; i++)
    printf(”%p\t0x%.2x\n",start+i, start[i]);
  printf("\n");
}
```

Printf directives:

%p: Print pointer

%x: Print Hexadecimal

`show_bytes` Execution Example

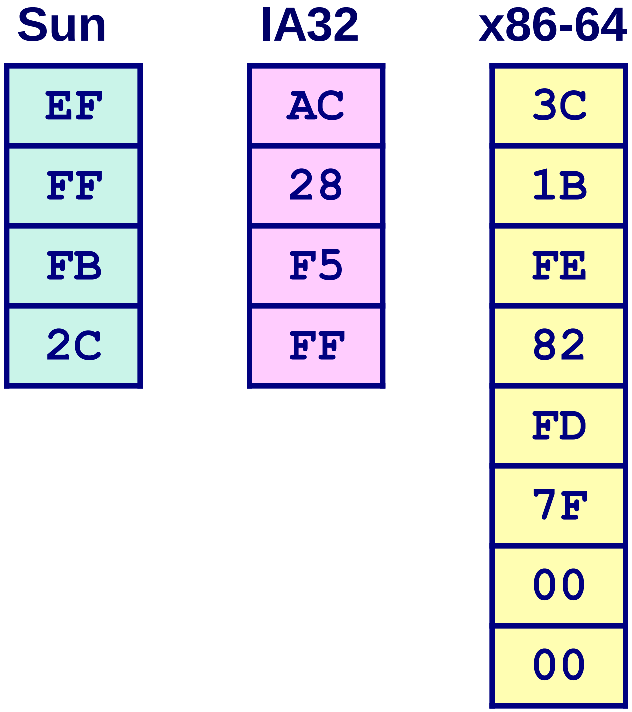
```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux x86-64):

```
int a = 15213;
0x7fffb7f71dbc     6d
0x7fffb7f71dbd     3b
0x7fffb7f71dbe     00
0x7fffb7f71dbf     00
```

Representing Pointers

```
int B = -15213;
int *P = &B;
```



Sun	IA32	x86-64
EF	AC	3C
FF	28	1B
FB	F5	FE
2C	FF	82
		FD
		7F
		00
		00

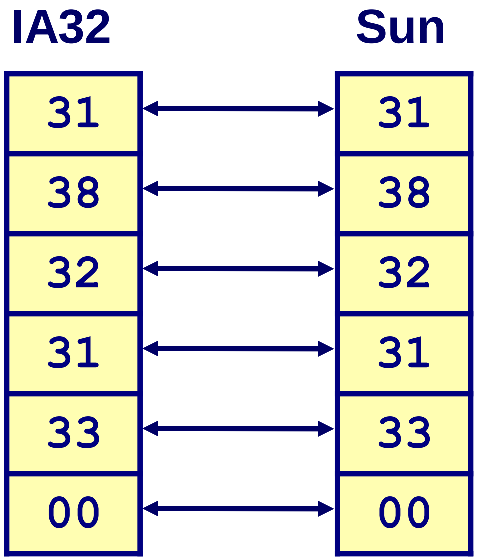
Different compilers & machines assign different locations to objects

Even get different results each time run program

Representing Strings

```
char S[6] = "18213";
```

- **Strings in C**
  - Represented by array of characters
  - Each character encoded in ASCII format
    - Standard 7-bit encoding of character set
    - Character “0” has code 0x30
      - Digit *i* has code 0x30+*i*
  - String should be null-terminated
    - Final character = 0
- **Compatibility**
  - Byte ordering not an issue



Reading Byte-Reversed Listings

- **Disassembly**
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- **Example Fragment**

```
Address       Instruction Code               Assembly Rendition
8048365:      5b                             pop     %ebx
8048366:      81 c3 ab 12 00 00              add     $0x12ab,%ebx
804836c:      83 bb 28 00 00 00 00           cmpl    $0x0,0x28(%ebx)
```

- **Deciphering Numbers**
  - Value: `0x12ab`
  - Pad to 32 bits: `0x000012ab`
  - Split into bytes: `00 00 12 ab`
  - Reverse: `ab 12 00 00`

Summary

- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
- Representations in memory, pointers, strings
- Summary