15213 Recitation Section C

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Oct. 21, 2002

Outline

- Loop Unrolling
- Blocking

Lab 4 Reminders

- Due: Thursday, Oct 24, 11:59pm
- Submission is online *NOT* automatic
  - Class web page > Labs > L4
  - http://www.cs.cmu.edu/afs/cs/academic/class/15213-f02/www/L4.html

Loop Unrolling

```
void combine5(vec_ptr v, int *dest)
{
  int length = vec_length(v);
  int limit = length-2;
  int *data = get_vec_start(v);
  int sum = 0;
  int i;
  /* Combine 3 elements at a time */
  for (i = 0; i < limit; i+=3) {
    sum += data[i] + data[i+1]
           + data[i+2];
  }
  /* Finish any remaining elements */
  for (; i < length; i++) {
    sum += data[i];
  }
  *dest = sum;
}
```

- Combine multiple iterations into single loop body
- Amortizes loop overhead across multiple iterations
- Finish extras at end

Practice Problem

- Problem 5.12 and 5.13

Solution 5.12

```
void inner5(vec_ptr u, vec_ptr v, data_t *dest)
{ int i;
  int length = vec_length(u);
  int limit = length-3;
  data_t *udata = get_vec_start(u);
  data_t *vdata = get_vec_start(v);
  data_t sum = (data_t) 0;

  /* Do four elements at a time */
  for (i = 0; i < limit; i+=4) {
     sum += udata[i]*vdata[i] + udata[i+1]*vdata[i+1]
        + udata[i+2]*vdata[i+2] + udata[i+3]*vdata[i+3];
  }
  /* Finish off any remaining elements */
  for (; i < length; i++)
    sum += udata[i] * vdata[i];
  *dest = sum;
}
```

Solution 5.12

A. We must perform two loads per element to read values for udata and vdata. There is only one unit to perform these loads, and it requires one cycle.

B. The performance for floating point is still limited by the 3 cycle latency of the floating-point adder.

Solution 5.13

```
void inner6(vec_ptr u, vec_ptr v, data_t *dest)
{ int i;
  int length = vec_length(u);
  int limit = length-3;
  data_t *udata = get_vec_start(u);
  data_t *vdata = get_vec_start(v);
  data_t sum0 = (data_t) 0;
  data_t sum1 = (data_t) 0;
  /* Do four elements at a time */
  for (i = 0; i < limit; i+=4) {
        sum0 += udata[i] * vdata[i];
        sum1 += udata[i+1] * vdata[i+1];
        sum0 += udata[i+2] * vdata[i+2];
        sum1 += udata[i+3] * vdata[i+3];
  }
  /* Finish off any remaining elements */
  for (; i < length; i++)
     sum0 = sum0 + udata[i] * vdata[i];
   *dest = sum0 + sum1;
}
```

Solution 5.13

- For each element, we must perform two loads with a unit that can only load one value per clock cycle.
- We must also perform one floating-point multiplication with a unit that can only perform one multiplication every two clock cycles.
- Both of these factors limit the CPE to 2.

Summary of Matrix Multiplication

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

```
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```


Improving Temporal Locality by Blocking

- Example: Blocked matrix multiplication
  - "block" (in this context) does not mean "cache block".
  - Instead, it mean a sub-block within the matrix.
  - Example: N = 8; sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Key idea: Sub-blocks (i.e., $\mathbf{A_{xy}}$) can be treated just like scalars.

$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Blocked Matrix Multiply (bijk)

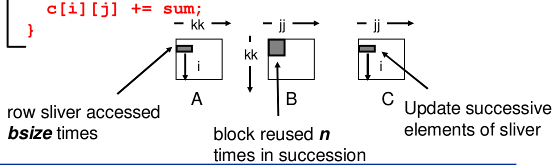
```
for (jj=0; jj<n; jj+=bsize) {
  for (i=0; i<n; i++)
    for (j=jj; j < min(jj+bsize,n); j++)
      c[i][j] = 0.0;
  for (kk=0; kk<n; kk+=bsize) {
    for (i=0; i<n; i++) {
      for (j=jj; j < min(jj+bsize,n); j++) {
        sum = 0.0
        for (k=kk; k < min(kk+bsize,n); k++) {
          sum += a[i][k] * b[k][j];
        }
        c[i][j] += sum;
      }
    }
  }
}
```

- Provides temporal locality as block is reused multiple times
- Constant cache performance

Blocked Matrix Multiply Analysis

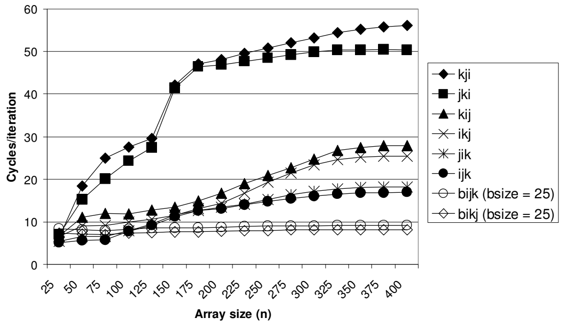
- Innermost loop pair multiplies a *1 X bsize* sliver of *A* by a *bsize X bsize* block of *B* and accumulates into *1 X bsize* sliver of *C*
- Loop over *i* steps through *n* row slivers of *A* & *C*, using same *B*

```
for (i=0; i<n; i++) {
  for (j=jj; j < min(jj+bsize,n); j++) {
    sum = 0.0
    for (k=kk; k < min(kk+bsize,n); k++) {
      sum += a[i][k] * b[k][j];
    }
    c[i][j] += sum;
  }
```



Pentium Blocked Matrix Multiply Performance

- Blocking (bijk and bikj) improves performance by a factor of two over unblocked versions (ijk and jik)
  - relatively insensitive to array size.



Summary

All systems favor "cache friendly code"

- Can get most of the advantage with generic optimizations:
  - Keep working set reasonably small (temporal locality)
  - Use small strides (spatial locality)
- Getting absolute optimum performance is very platform specific
  - Cache sizes, Line sizes, Associativities, etc.