

15-294 Rapid Prototyping Technologies:

The Pascaline

Dave Touretzky
Computer Science Dept.
Carnegie Mellon University



Blaise Pascal

- French mathematician, physicist, writer, philosopher.
- b. 1623 – d. 1662 (age 39)
- Father was a tax collector.
- Known for:
 - Theory of probability; Pascal's triangle
 - Study of fluids, pressure, and vacuum
 - Writings on philosophy and theology
 - First working mechanical calculator: the Pascaline



The Pascaline



Musée des arts et métiers-CNAM, Paris. Photo: J. C. Wetzel.

History of the Pascaline

- Pascal invented the device in 1642 (at age 19) to help his father with his tax computations.
- Widely viewed as the first mechanical calculator. Could add and (with a trick) subtract.
- Over 40 were built over several decades, in a variety of models.
- 9 survive today in museums or private collections.



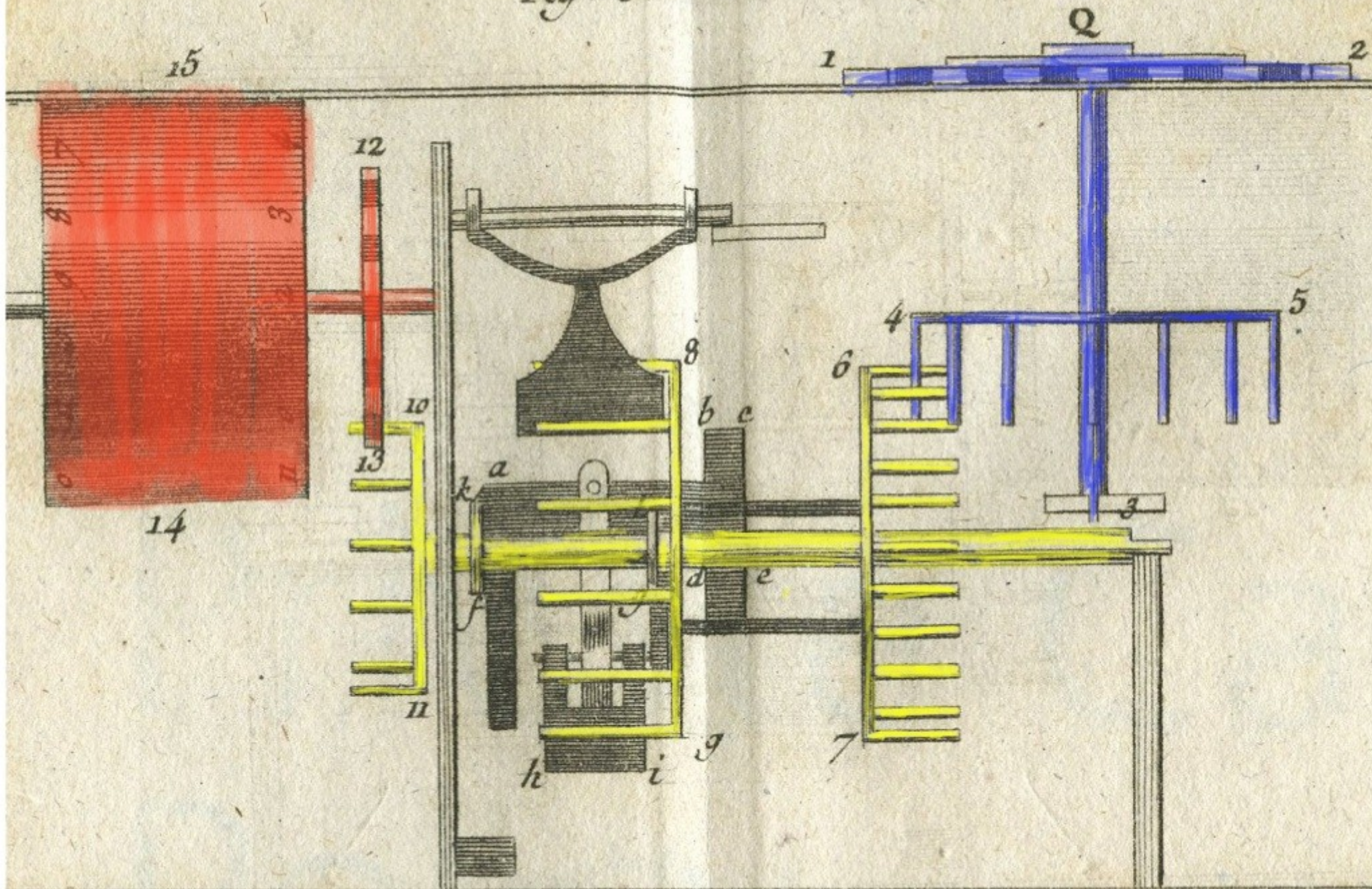
Input and Output



Input Close-Up



Fig. 3.



French Currency (17th century)



17th Cent. French Currency: Livres

- Deniers 12 deniers = 1 sol
- Sols 20 sols = 1 livre
- Nombres Simples “simple numbers”
- Dixaines tens
- Centaines hundreds
- Milles thousands
- Dixaines de Mille tens of thousands
- Centaines de Mille hundreds of thousands

Under the Hood

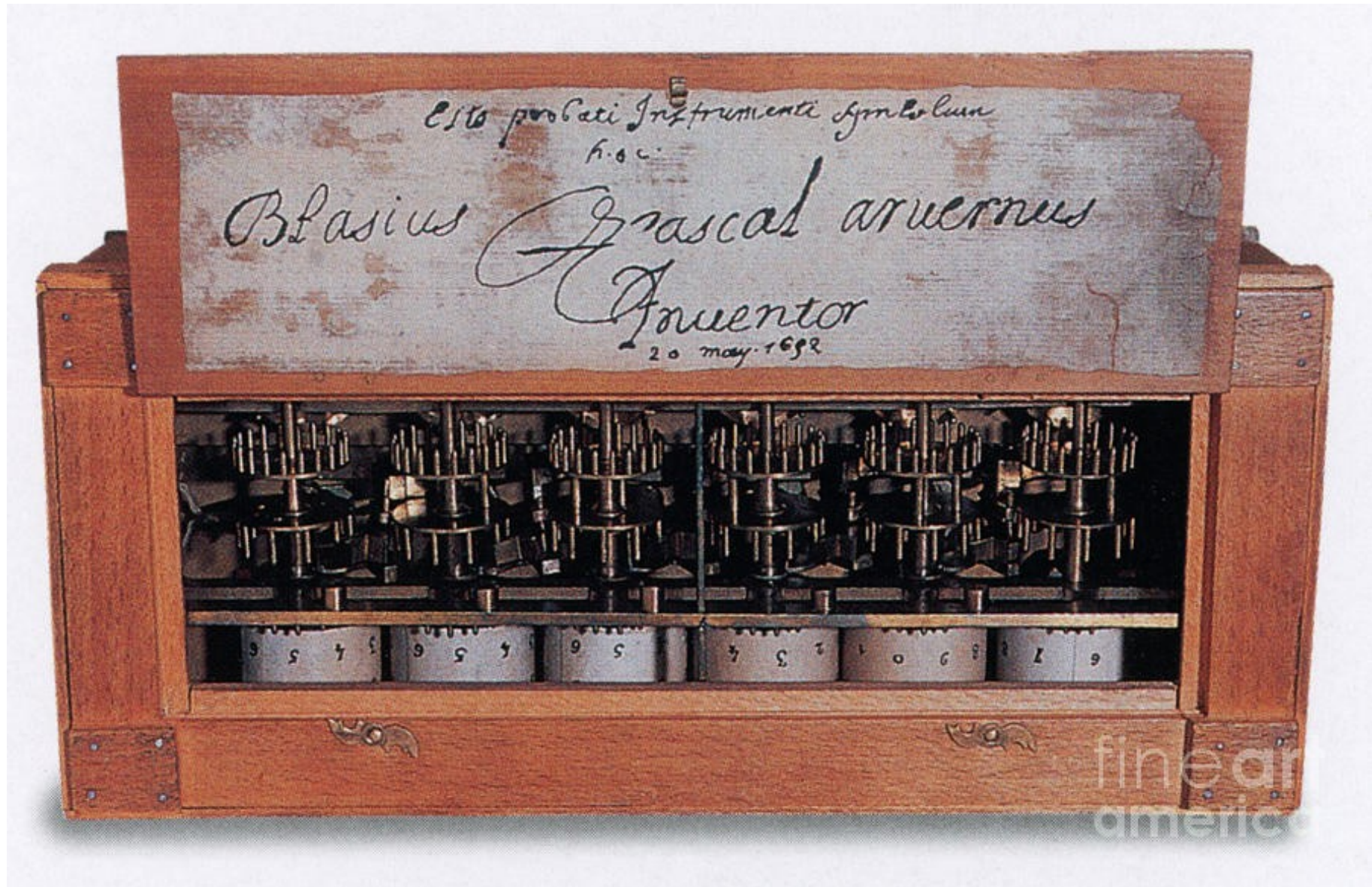


Esto probati Instrumenti Symbolum
not

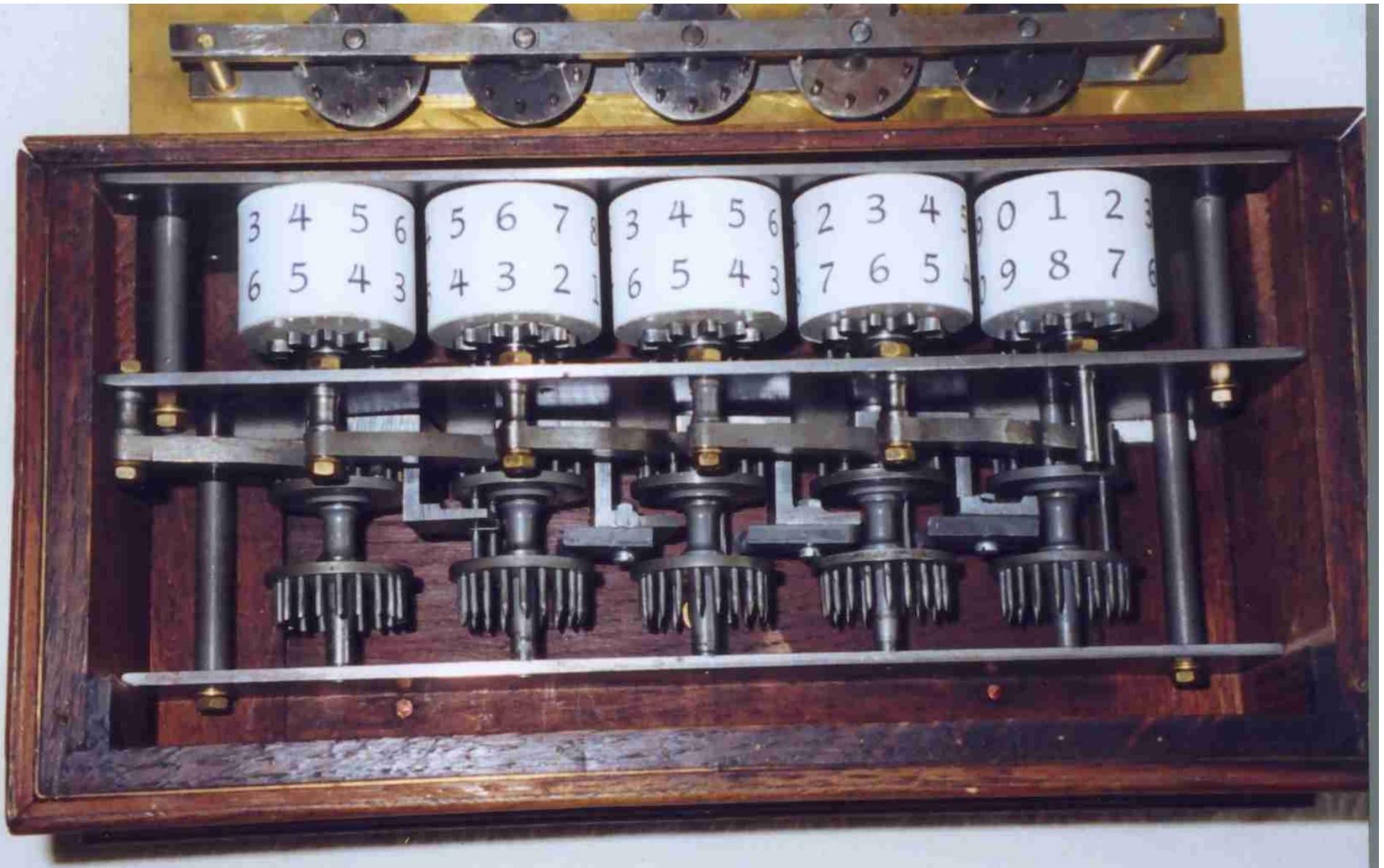
Blasius Gasca. arvernus
Inventor
20 may 1652



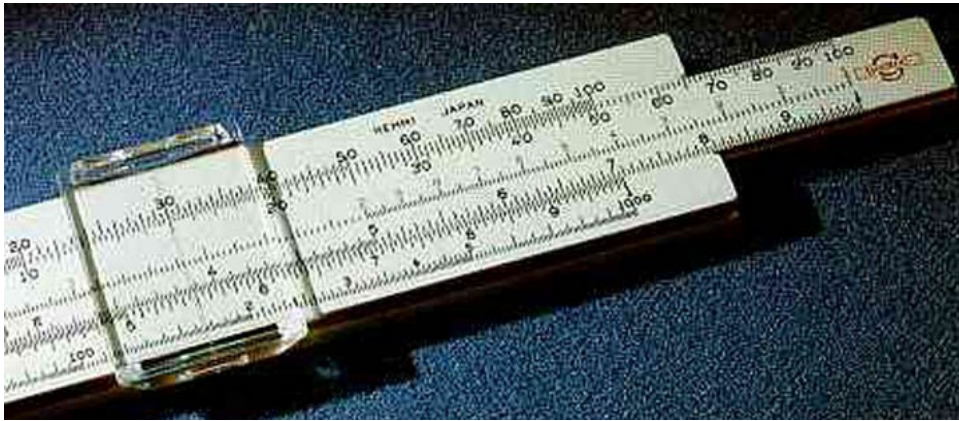
Early Version, From 1642



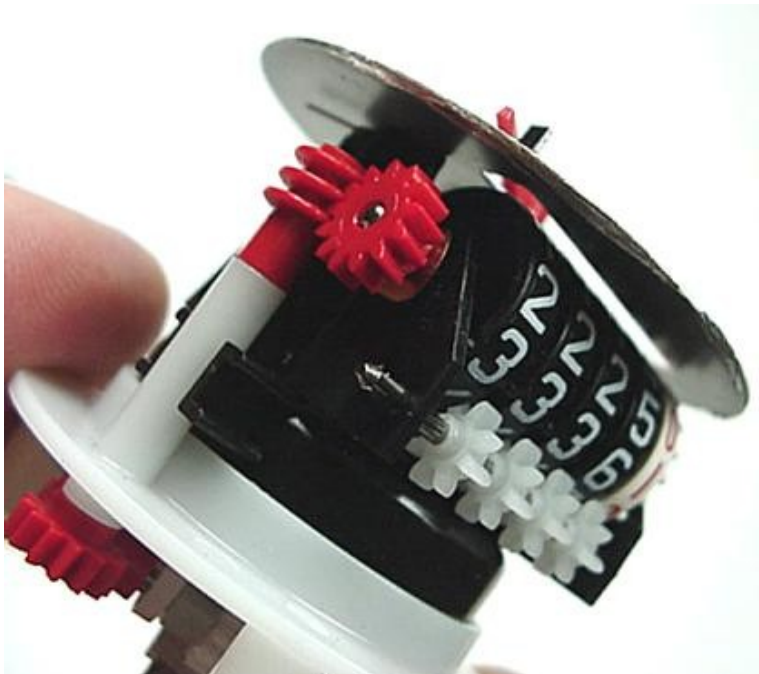
Why Do the Read-Out Wheels Have Two Sets of Numbers?



What Makes A Device “Digital”?



A slide rule is analog: it has a continuous state space, so an infinite number of states.

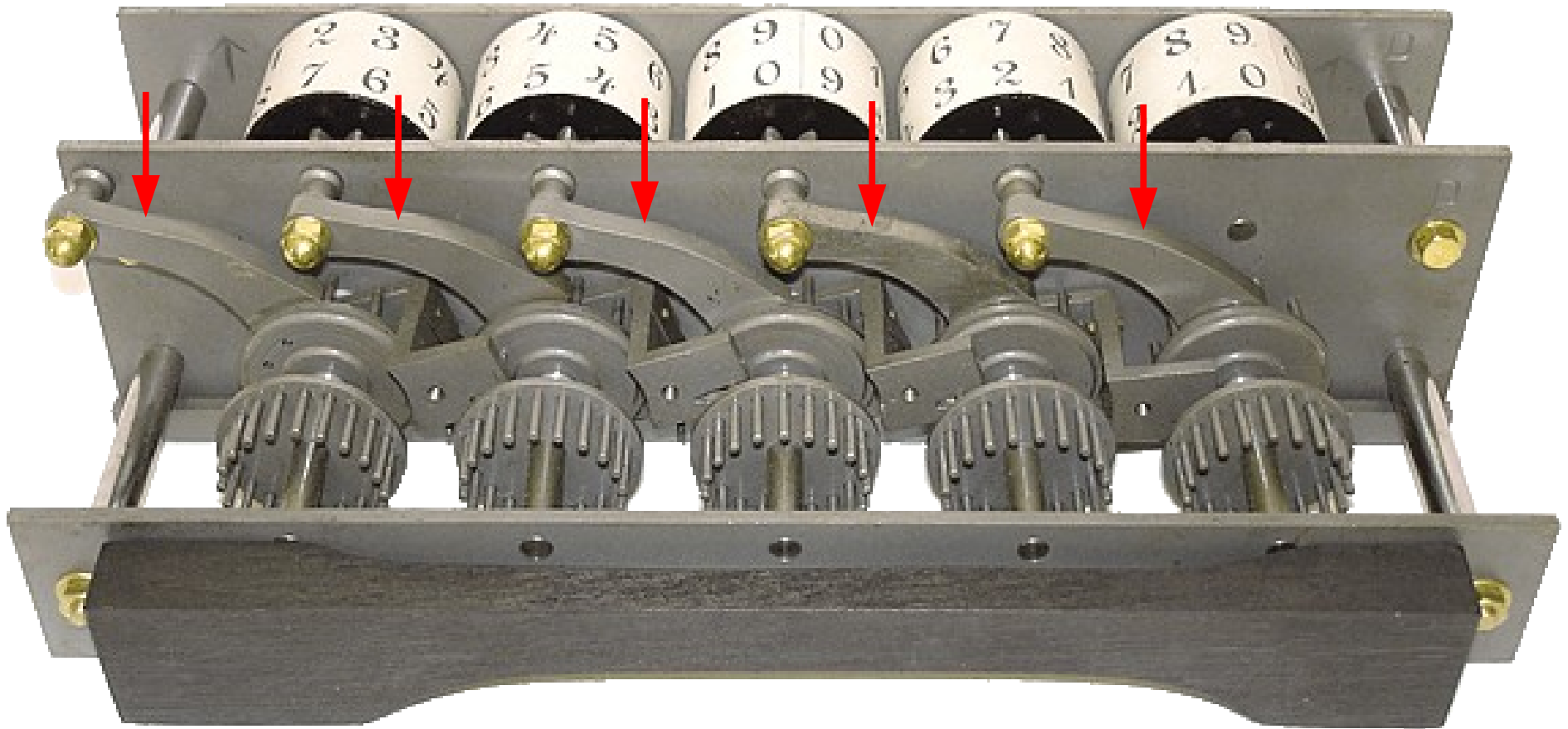


Digital devices have *discrete* state spaces, and a physical non-linearity to force clean transitions from one state to another.

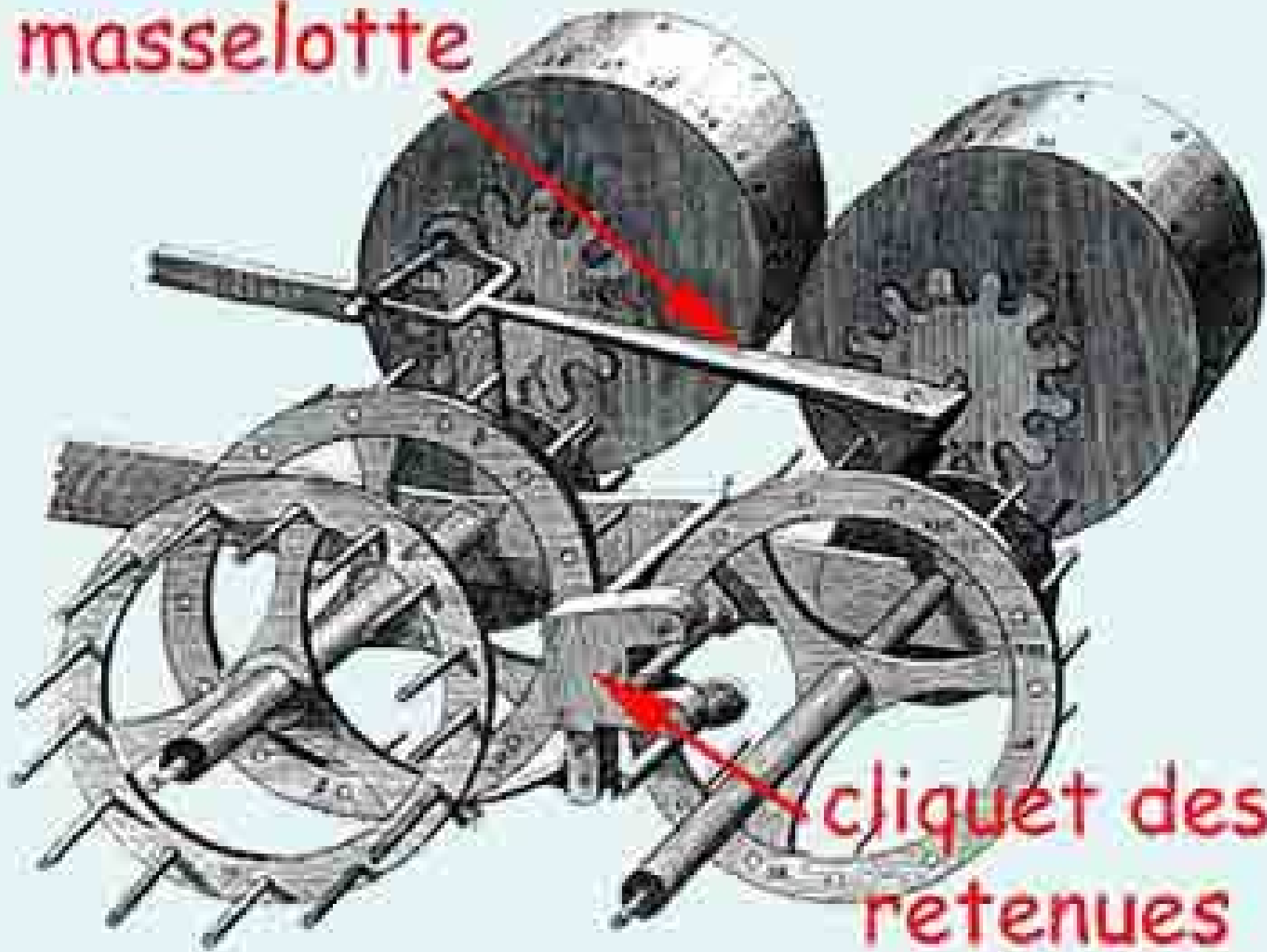
Digits Don't Guarantee Discreteness



The Backstop Pawl

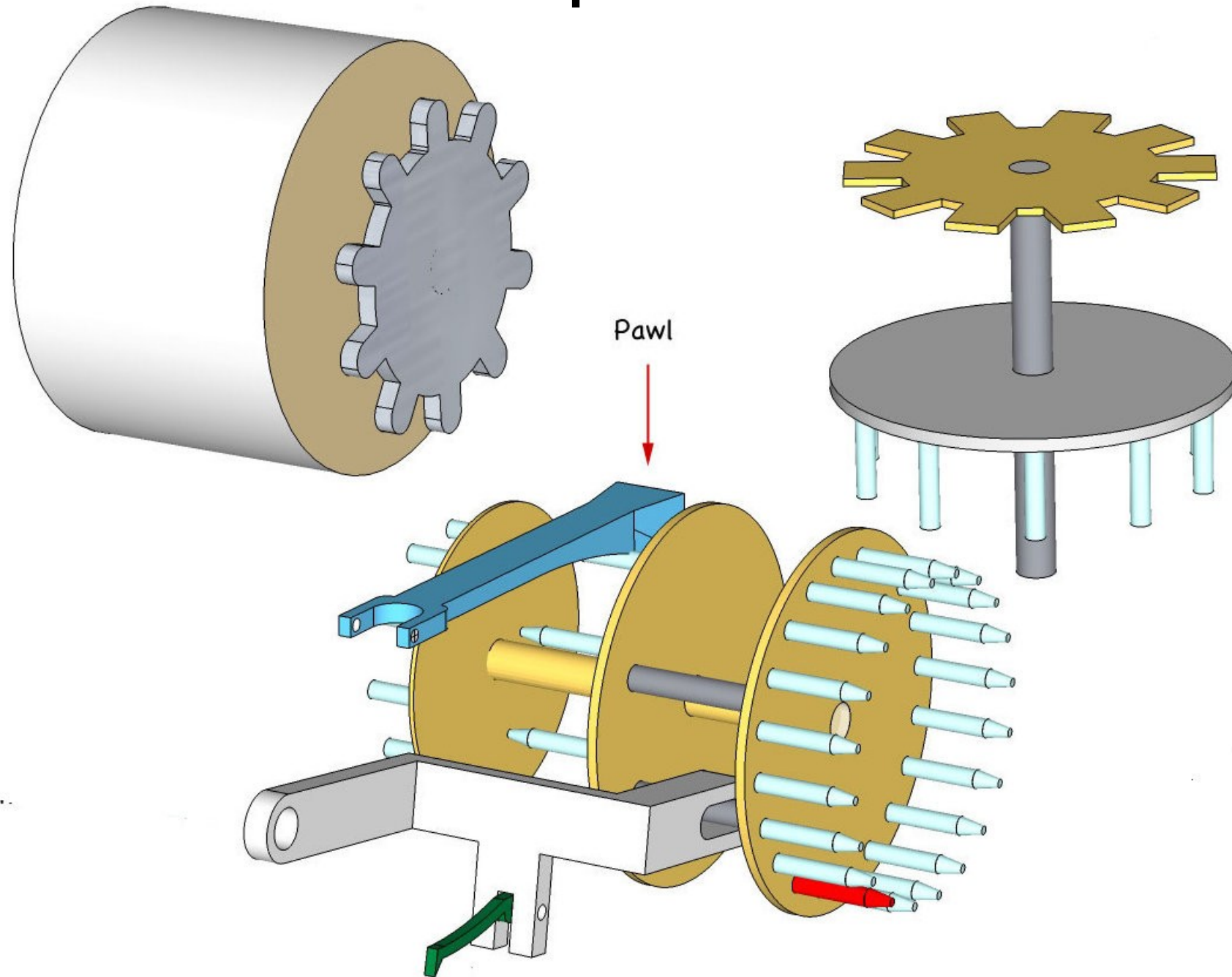


masselotte



cliquet des
retenues

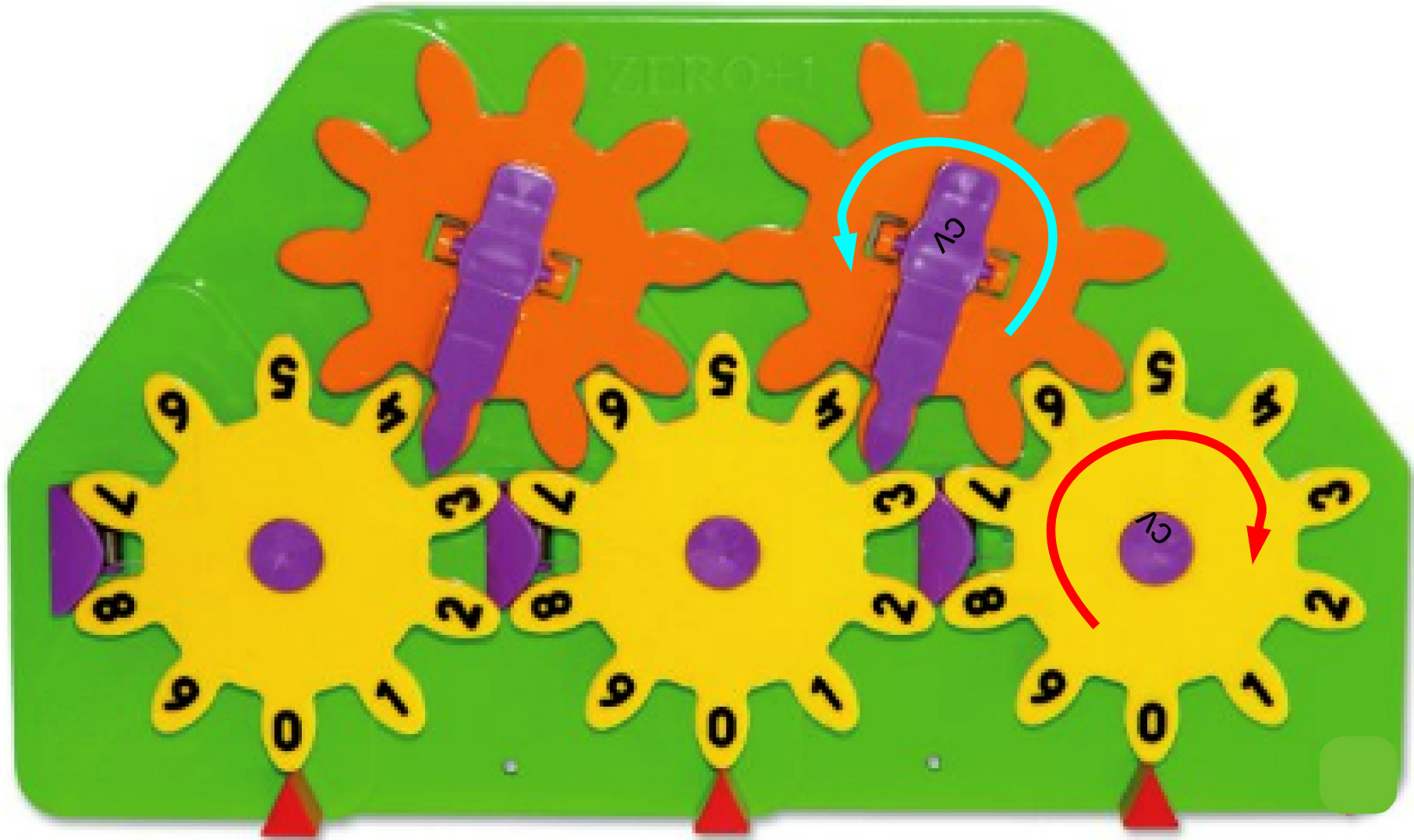
Backstop Pawl and Sautoir



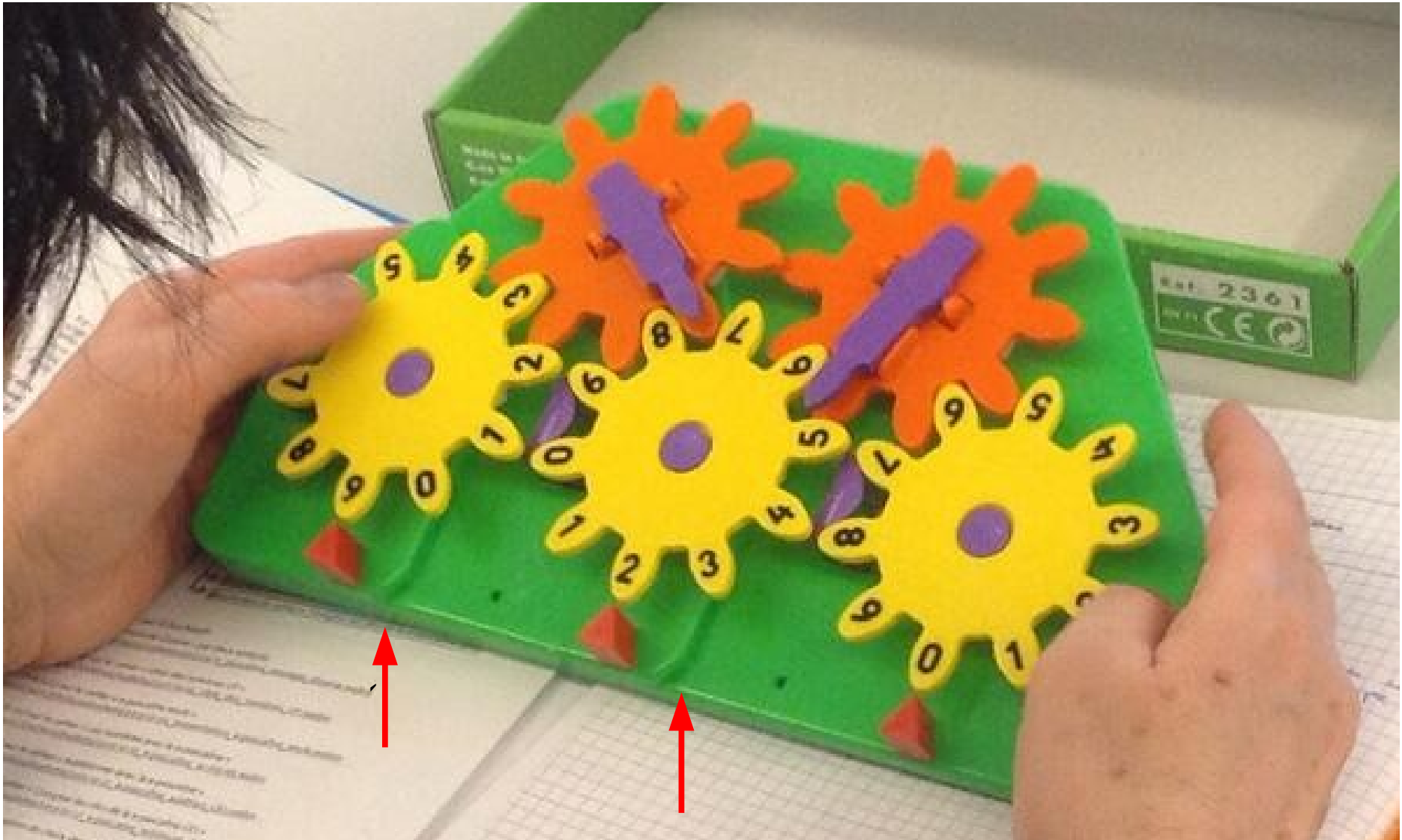
Carry Operations

- Carrying is what makes addition difficult.
- How do you get one digit to affect the digit next to it?
- Carry can propagate:
 $099999 \rightarrow 100000$
- Mechanically, this is a nightmare.
- Two solutions:
 - One-toothed gear (doesn't chain well; can jam)
 - The sautoir (“jumper”) – Pascal's invention

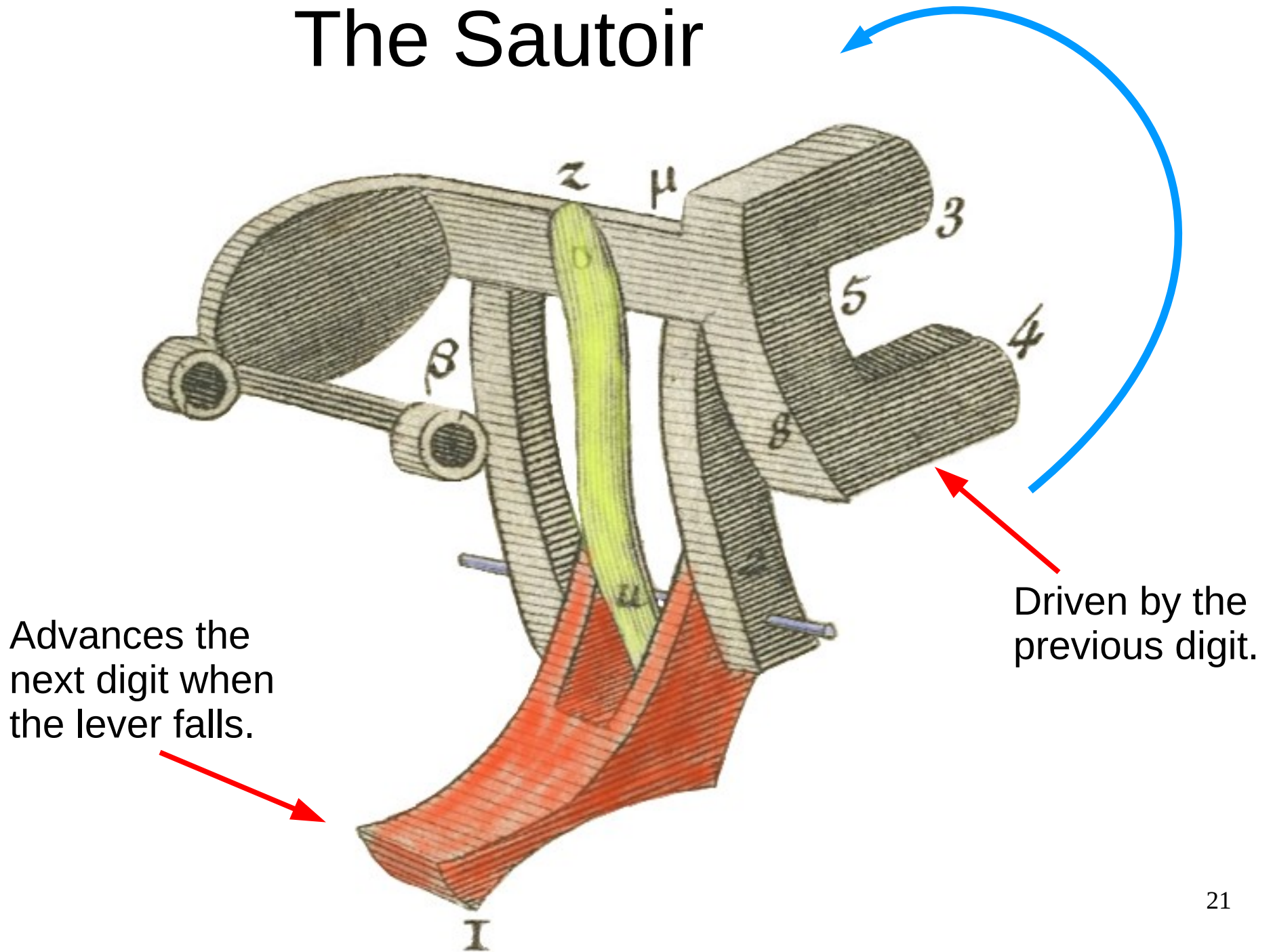
One-Tooth Gear (Purple) For Carry



Trick: Elevate Successive Digits



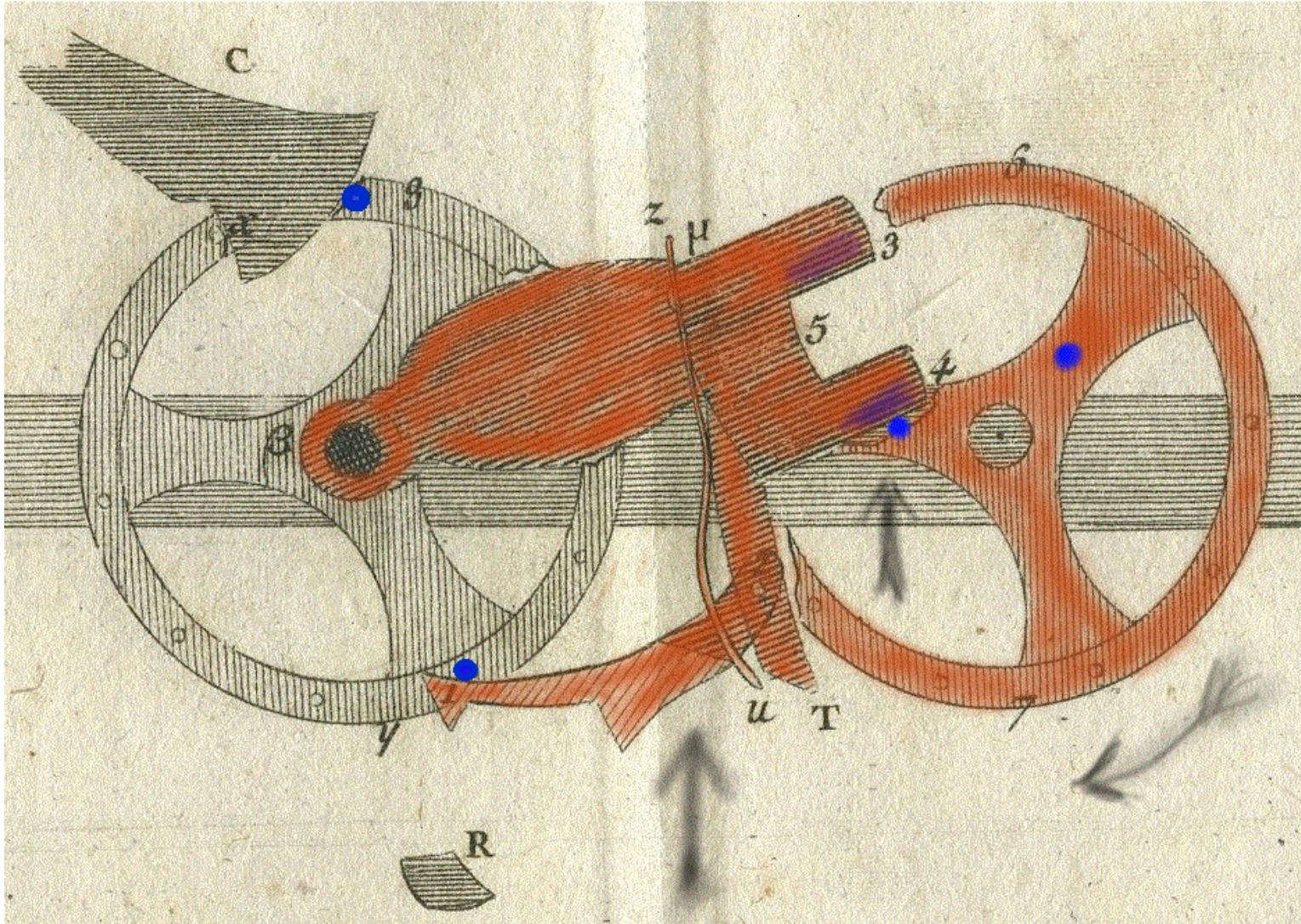
The Sautoir



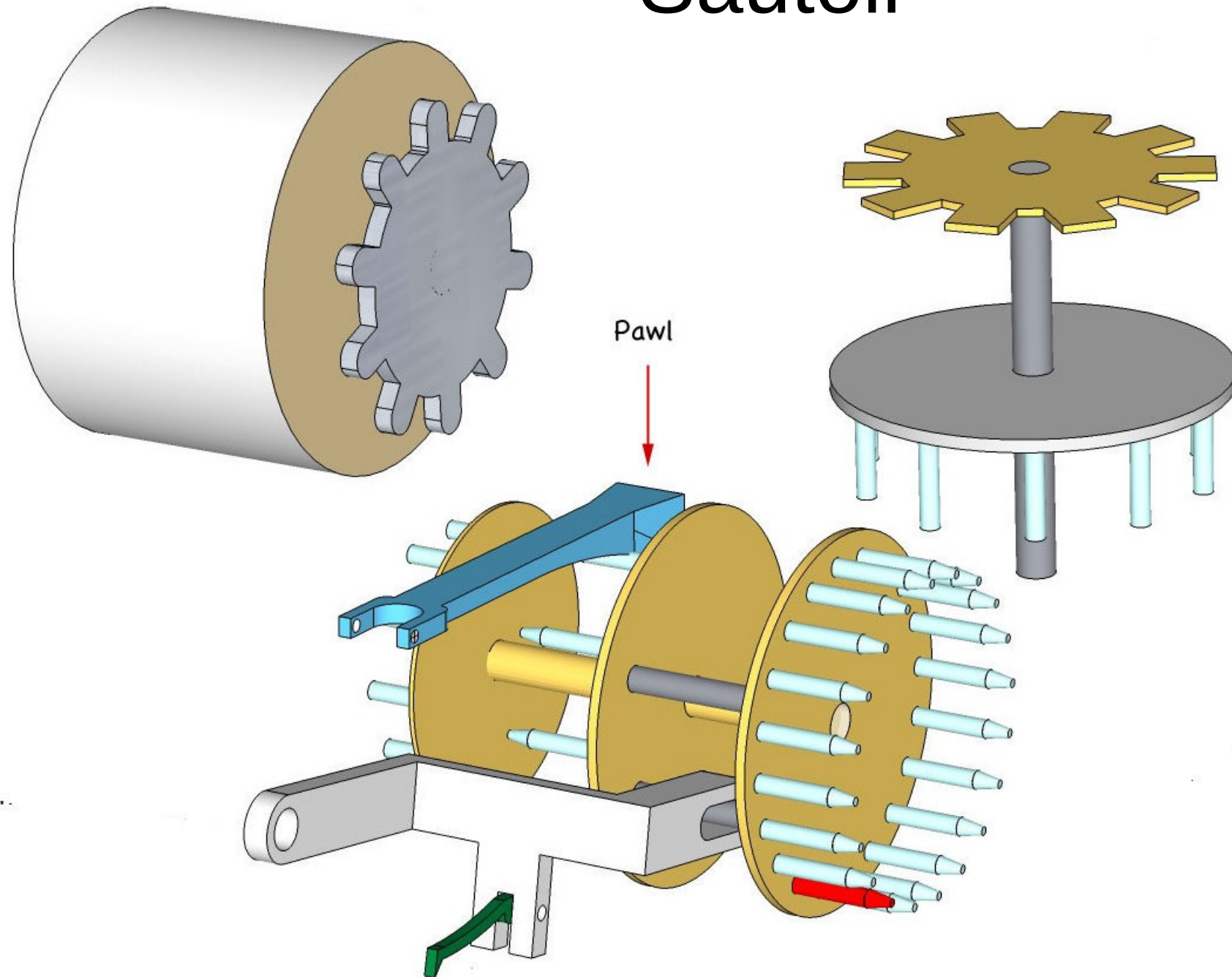
Advances the next digit when the lever falls.

Driven by the previous digit.

Sautoir



Sautoir



How To Add

1. Clear the machine:
 - (a) Set all digits to “9”.
 - (b) Add 1 to get all zeros: tests the ripple carry.
2. Dial in the digits of the first addend.
3. Dial in the digits of the second addend: this may cause carries to occur.
4. Read the result on the output wheels.

Representing Signed Numbers

- In a 3 digit machine $999 + 1 = 000$.
- So “999” is also -1.
- Which means “998” is also -2.
- We can choose between signed and unsigned representations:
 - Unsigned: 0 to 999 represented as “000” to “999”.
 - Signed: -500 to +499 represented as “500” to “999” followed by “000” to “499”.

Converting Positive to Negative

- Subtract the value from 999, then add 1.
- Example: how do we represent -14?

“999”

- 14

—

“985” + 1 = “986” is -14

Nines' Complement

- To form the nines' complement of a number, subtract every digit from 9.
- Note: there is never any borrowing or carrying, so this can be computed very quickly.
- Denote the machine representation of the nines' complement as $C(n)$.
- Verify for yourself:
$$C(C(n)) = n \quad \text{because } 9 - (9 - x) = (9 - 9) + x = 0 + x = x$$
- Denote the machine representation of negation as:
$$\text{Neg}(n) = C(n) + 1$$

Tens' Complement Subtraction

$$a - b = a + -b$$

$$= a + \text{Neg}(b)$$

$$= a + C(b) + 1 \quad \text{three operations}$$

But we can simplify this by taking the nines' complement of both sides.

Nines' Complement Subtraction

$$C(a-b) = \text{"9999"} - (a - b)$$

$$= (\text{"9999"} - a) + b \quad \textit{associative property}$$

$$= C(a) + b \quad \text{two operations}$$

Nines' Complement of a Sum

Proof that $C(a+b) = C(a) + C(b) + 1$:

$$\text{Neg}(a+b) = C(a+b) + 1$$

$$\begin{aligned}\text{Neg}(a+b) &= \text{Neg}(a) + \text{Neg}(b) \\ &= [C(a) + 1] + [C(b) + 1] \\ &= C(a) + C(b) + 2\end{aligned}$$

$$\text{So } C(a+b) + 1 = C(a) + C(b) + 2$$

$$\text{Therefore } C(a+b) = C(a) + C(b) + 1.$$

Nines' Complement Subtraction

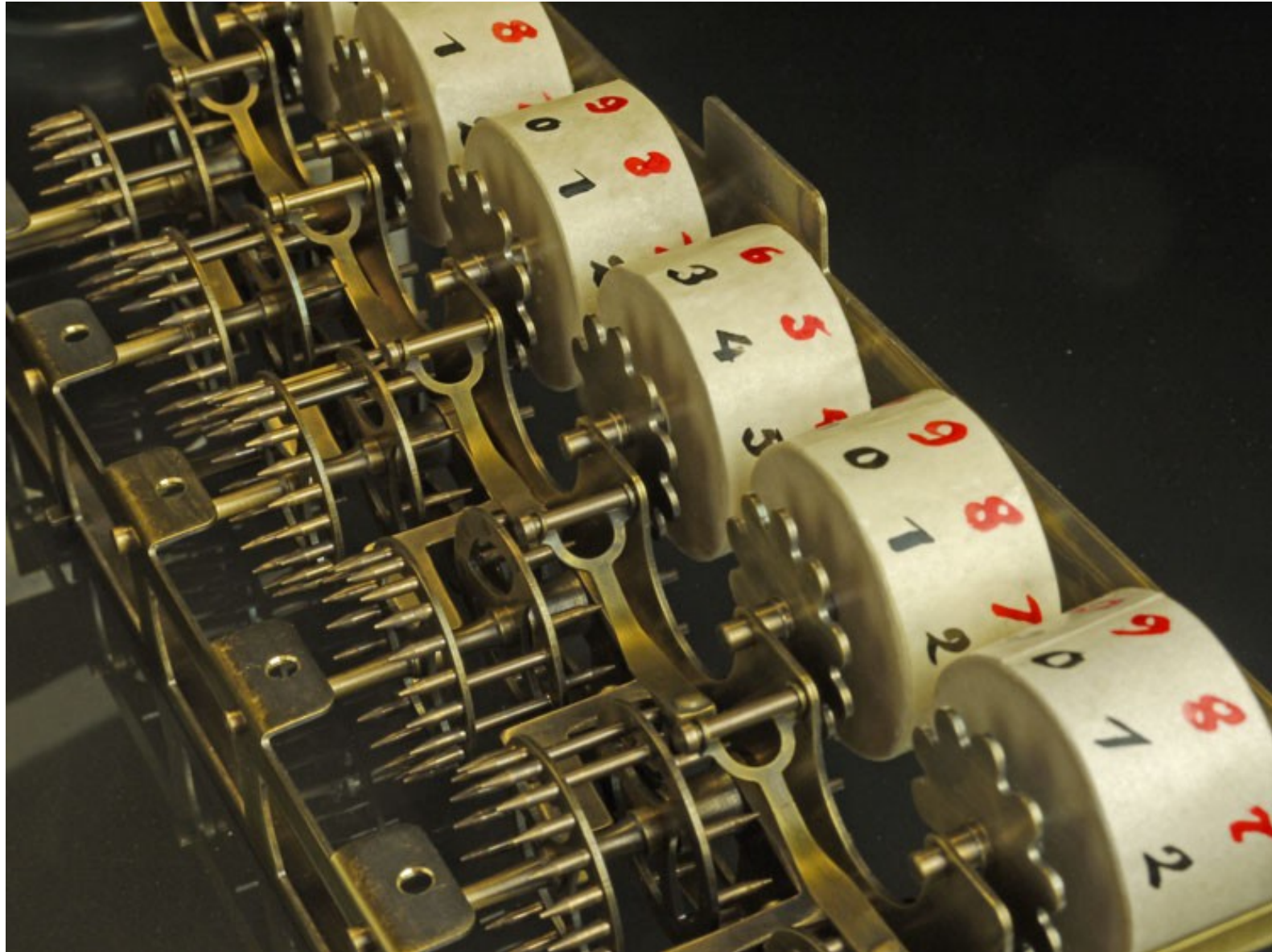
$$\begin{aligned}C(a-b) &= C(a + \text{Neg}(b)) \\ &= C(a + C(b) + 1) \\ &= C((a+1) + C(b)) \\ &= C(a+1) + C(C(b)) + 1 \\ &= C(a+1) + b + 1 \\ &= [C(a) + C(1) + 1] + b + 1 \\ &= C(a) + \text{"9998"} + 1 + b + 1 \\ &= C(a) + -2 + 1 + b + 1 \\ &= C(a) + b \quad \text{two operations}\end{aligned}$$

Subtraction in the Pascaline

Compute $a - b$ as $C(C(a) + b)$:

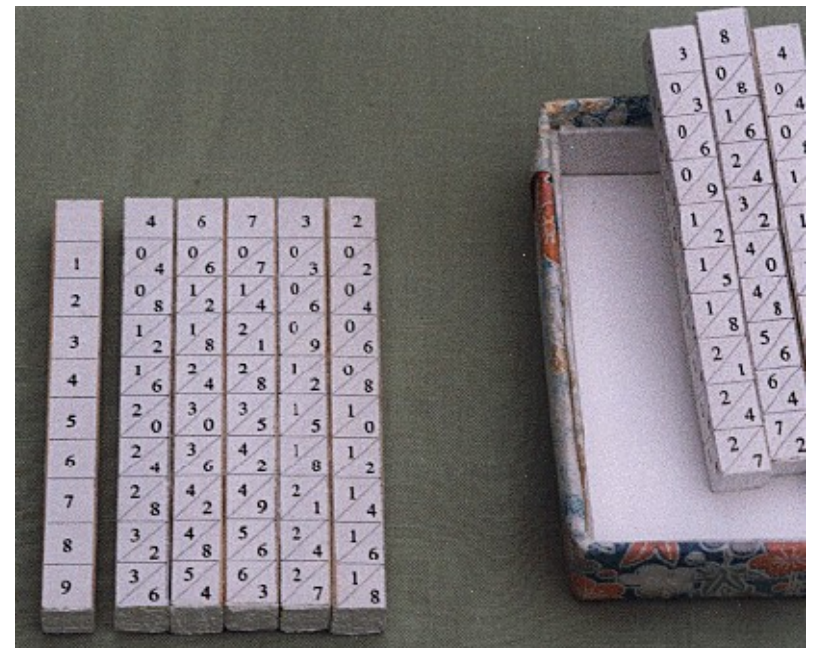
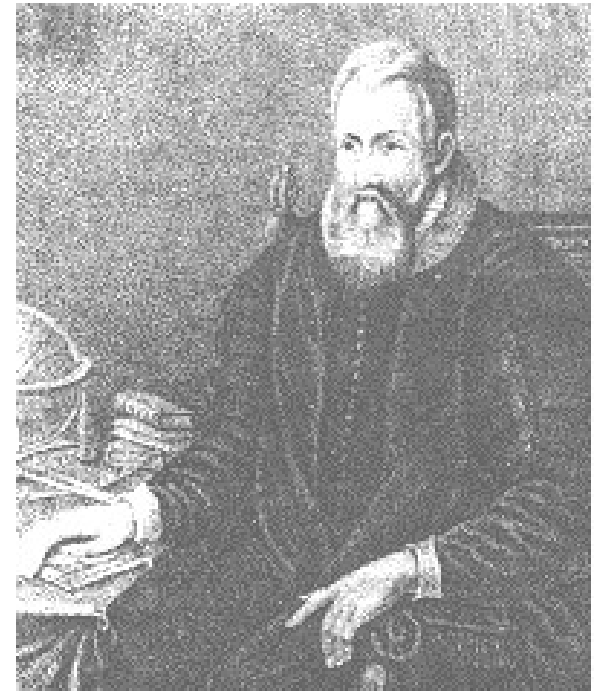
1. Clear the machine.
2. Enter $C(a)$ using complement digit marks.
3. Add in b using the regular digit marks.
 - This gives $C(a) + b$
4. Read the result on the complement number readout instead of the regular readout.
 - This gives $C(C(a) + b)$

Pascaline Replicas (Many)



John Napier

- Scottish mathematician (1550-1617)
- Invented “Napier's bones”, used to perform multiplication using only addition.
- Napier's bones were very successful and widely used in Europe until the mid-1960s.
- Napier is also the inventor of logarithms.



$$7 \times 46785399 = ?$$

1	4	6	7	8	5	3	9	9	
2	0/8	1/2	1/4	1/6	1/0	0/6	1/8	1/8	
3	1/2	1/8	2/1	2/4	1/5	0/9	2/7	2/7	
4	1/6	2/4	2/8	3/2	2/0	1/2	3/6	3/6	
5	2/0	3/0	3/5	4/0	2/5	1/5	4/5	4/5	
6	2/4	3/6	4/2	4/8	3/0	1/8	5/4	5/4	
7	2/8	4/2	4/9	5/6	3/5	2/1	6/3	6/3	
8	3/2	4/8	5/6	6/4	4/0	2/4	7/2	7/2	
9	3/6	5/4	6/3	7/2	4/5	2/7	8/1	8/1	



2	4	4	5	3	2	6	6	3
8	2	9	6	5	1	3	3	
3	2	7	4	9	7	7	9	3

Multiplying Multi-Digit Numbers

- Do single digit multiplications, shift, and add:

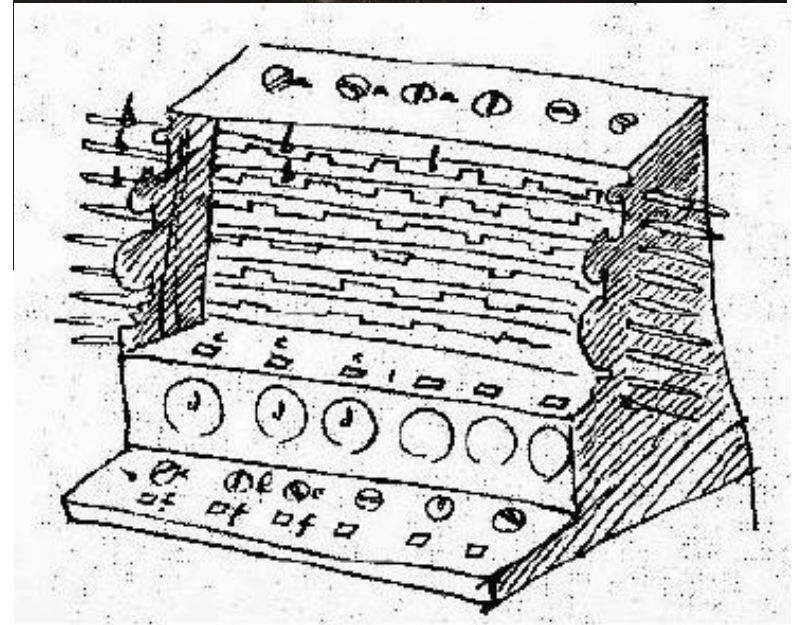
1	4	6	7	8	5	3	9	9
2	0/8	1/2	1/4	1/6	1/0	0/6	1/8	1/8
3	1/2	1/8	2/1	2/4	1/5	0/9	2/7	2/7
4	1/6	2/4	2/8	3/2	2/0	1/2	3/6	3/6
5	2/0	3/0	3/5	4/0	2/5	1/5	4/5	4/5
6	2/4	3/6	4/2	4/8	3/0	1/8	5/4	5/4
7	2/8	4/2	4/9	5/6	3/5	2/1	6/3	6/3
8	3/2	4/8	5/6	6/4	4/0	2/4	7/2	7/2
9	3/6	5/4	6/3	7/2	4/5	2/7	8/1	8/1

46785399	99
 081214161061818	× 96431
<hr/>	
46785399	→ 46785399
140356197	→ 140356197
187141596	→ 187141596
280712394	→ 280712394
+ 421068591	→ + 421068591
<hr/>	
4511562810969	4511562810969

As with the abacus, humans do most of the work.

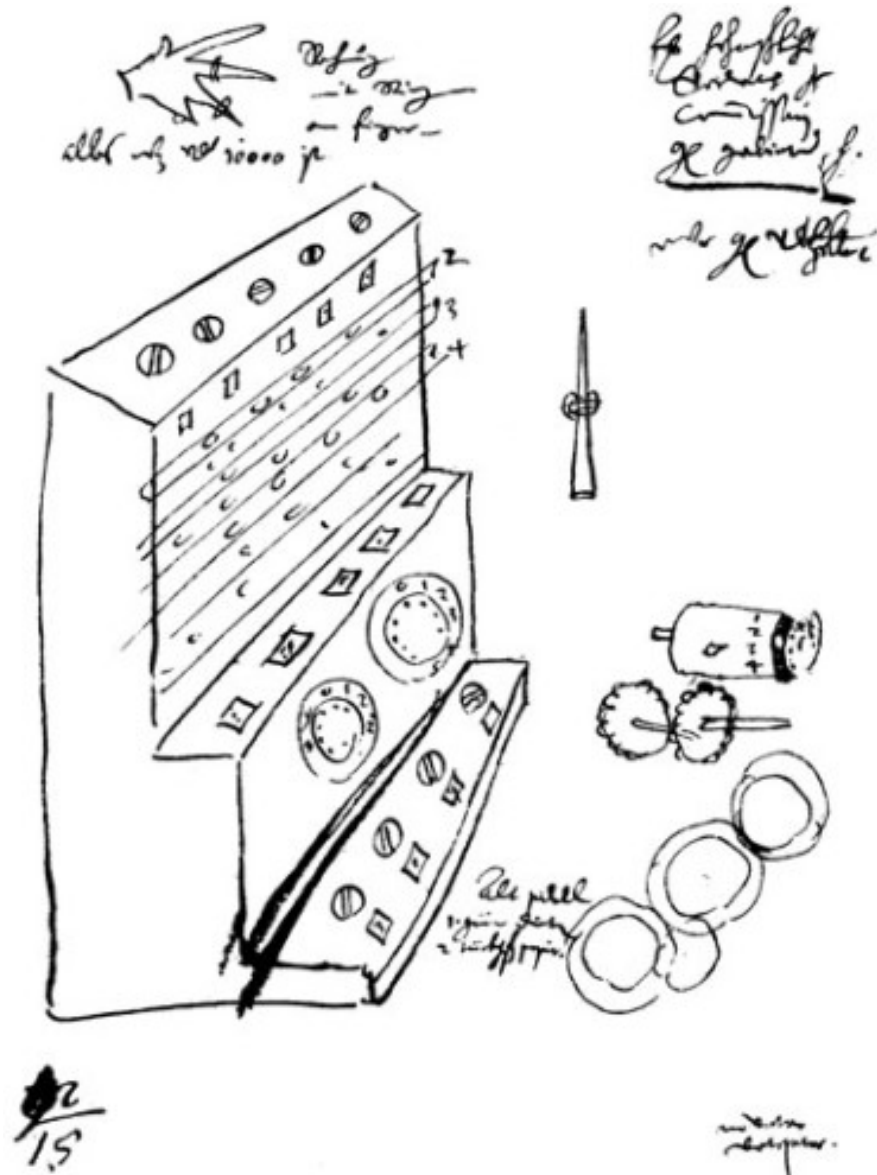
Wilhelm Schickard (1592 - 1635)

- Described a “calculating clock” in letters to his friend Johannes Kepler in 1623 (the year Pascal was born) and 1624, with sketches included.
- Claims the prototype worked, but it has not survived. Second, professionally-built version was destroyed in a fire before delivery.
- Addition by rotating wheels.
- Subtraction by moving wheels in opposite direction.
- Multiplication via Napier's bones (lookup table).



Schickard's Calculating Clock

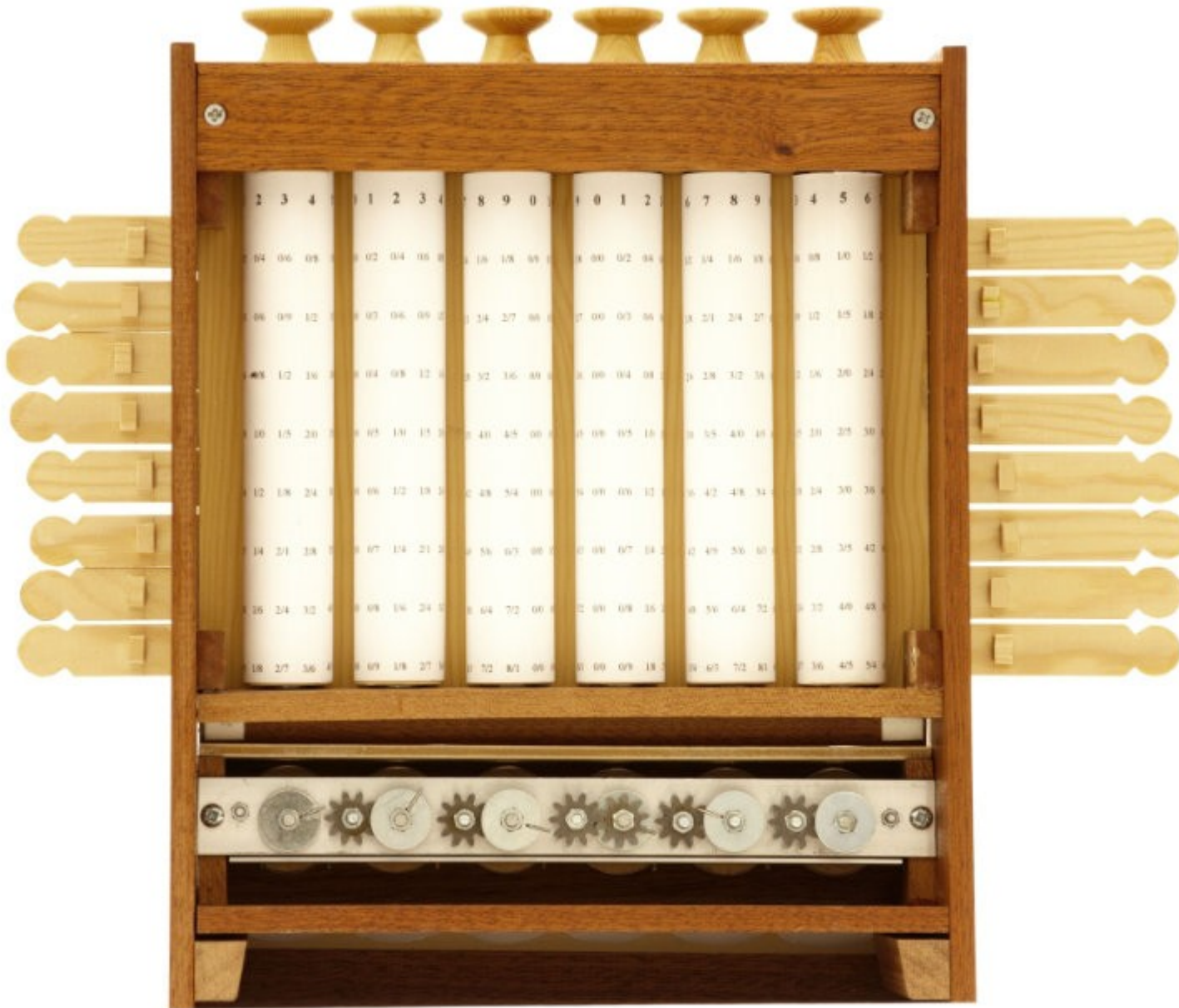
- The surviving notes don't describe a fully functional machine.
- Requires additional wheels and springs. Did he add them?
- Used single tooth carry gear, which doesn't work for many-digit carries.



Schickard Replica

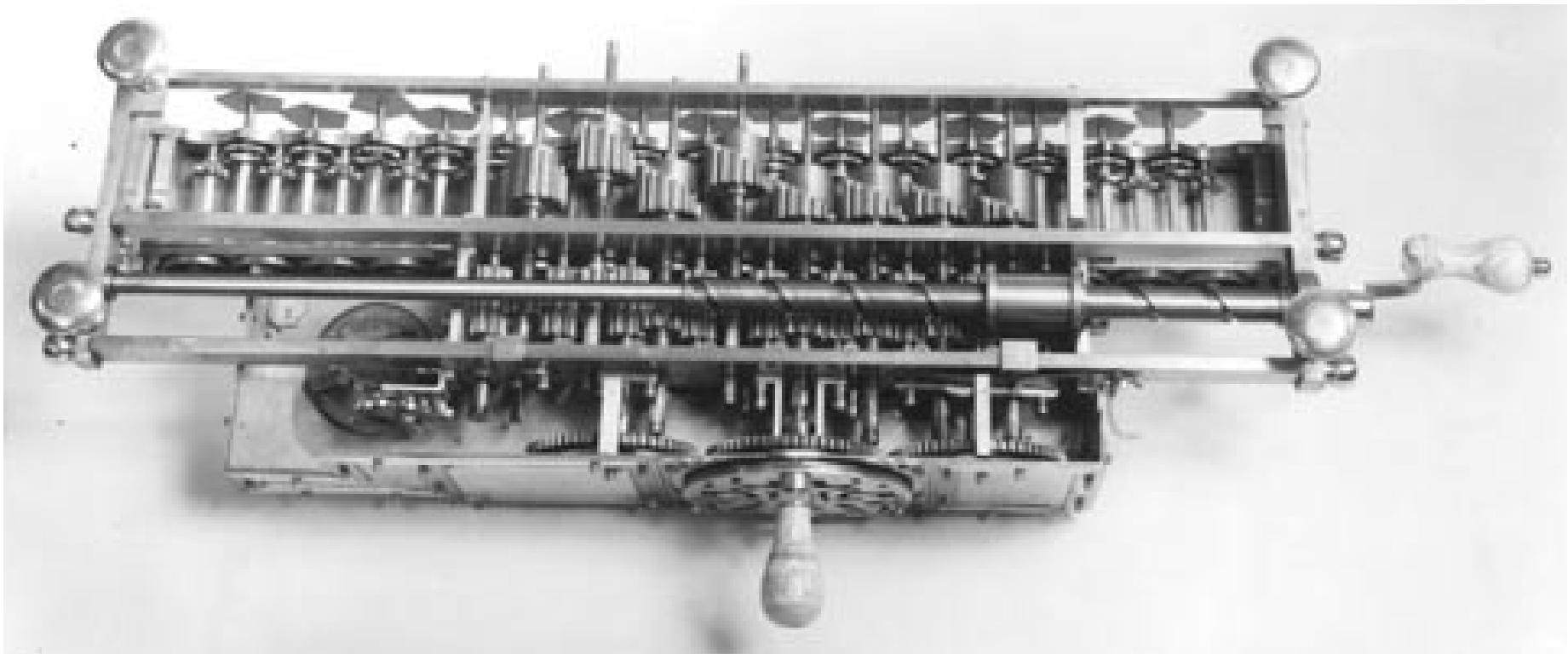


Back Side of the Replica



Leibniz Step Reckoner

- Successor to the Pascaline. Designed in 1673, completed in 1694.
- Could add and subtract automatically.
- Multiply and divide by shifting the carriage.



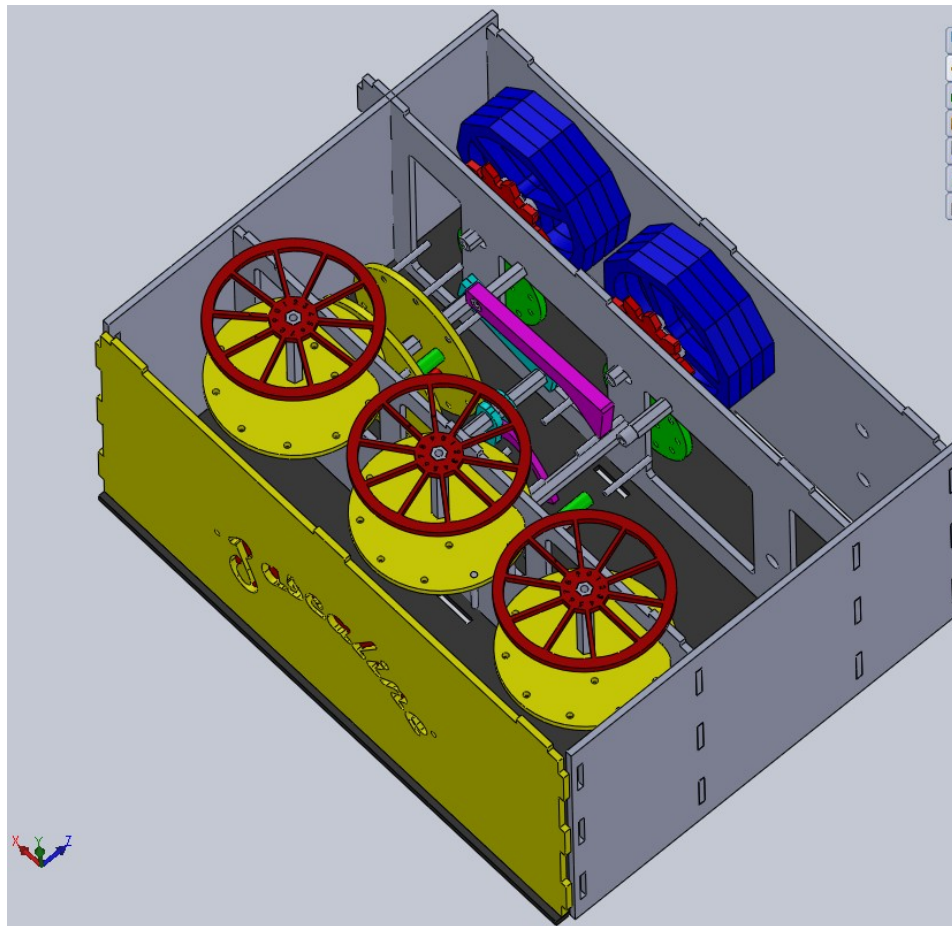
History of Computing

To learn more about the history of computing:

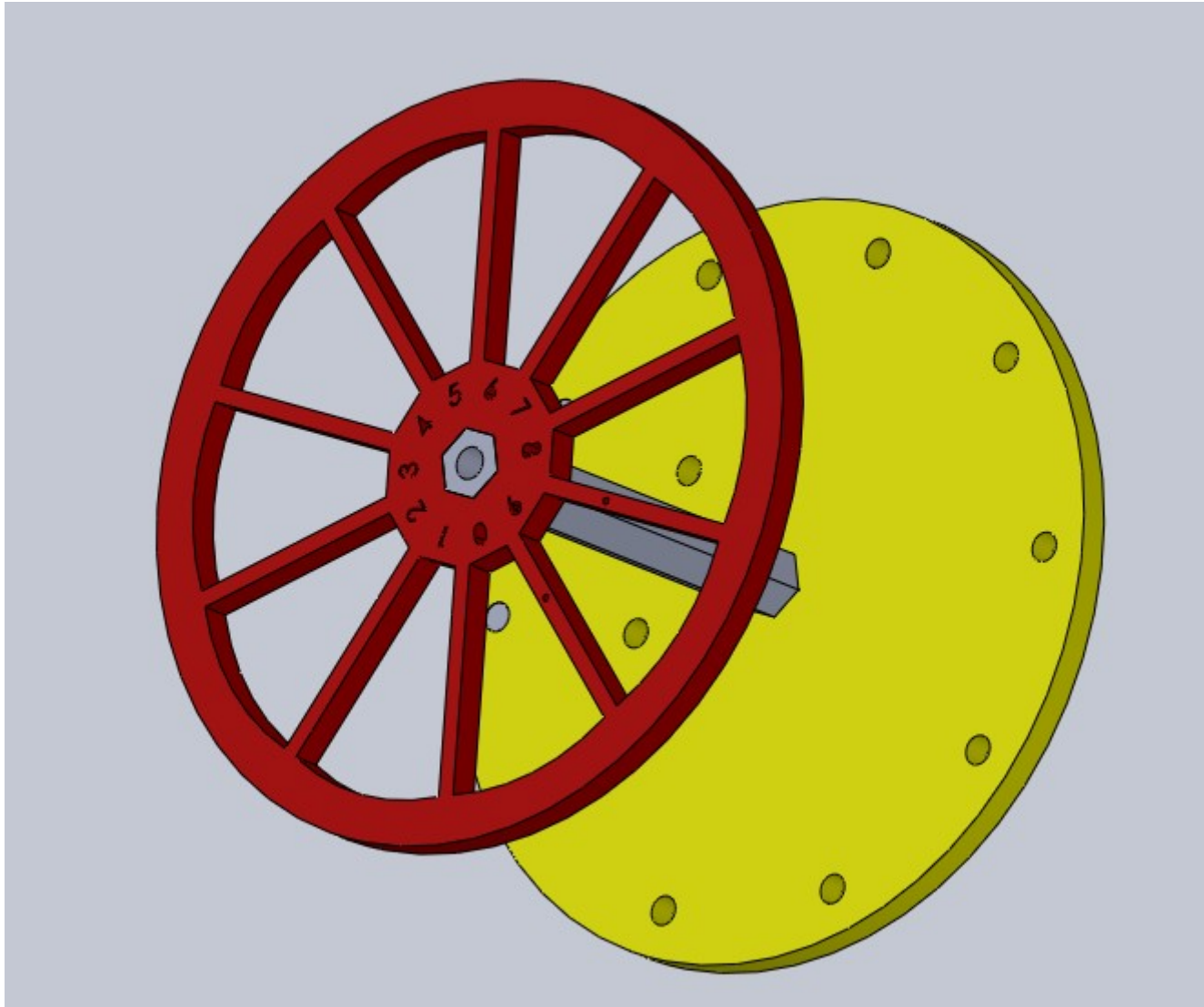
- Take Tom Cortina's mini-course:
15-292 History of Computing
- Visit the Computer History Museum at www.computerhistory.org or in person in Mountain View, California.

Building Our Own Pascaline

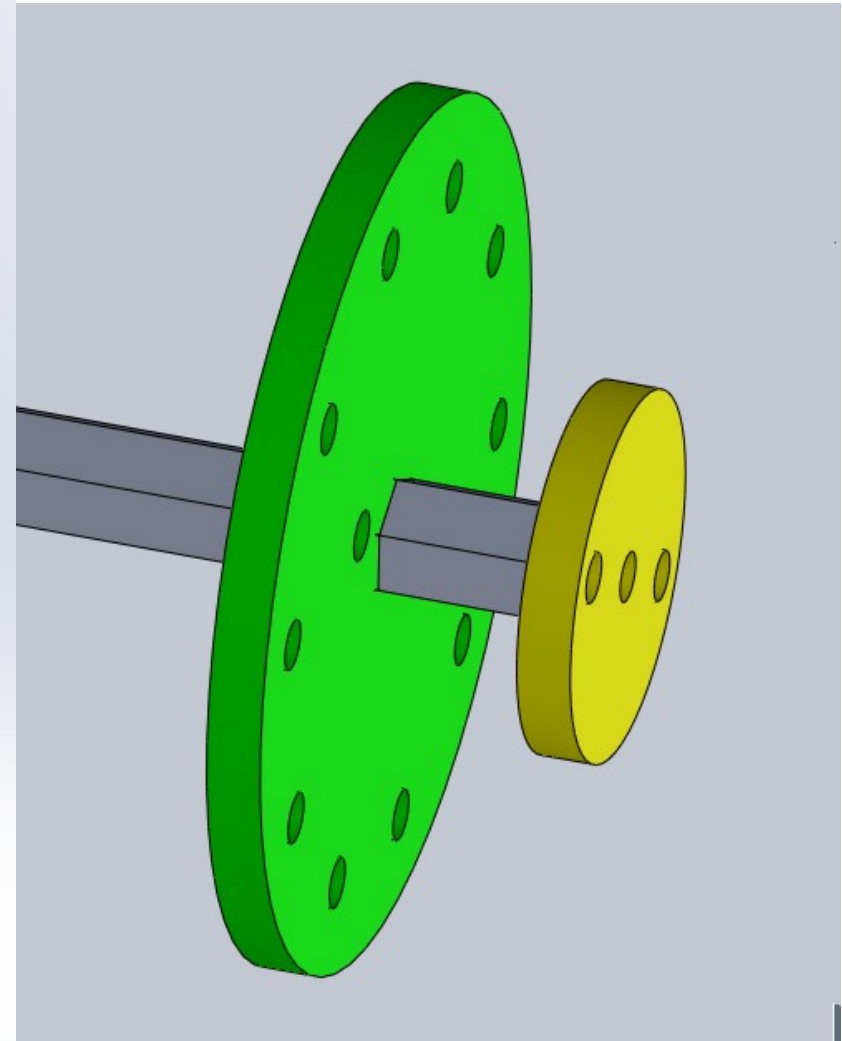
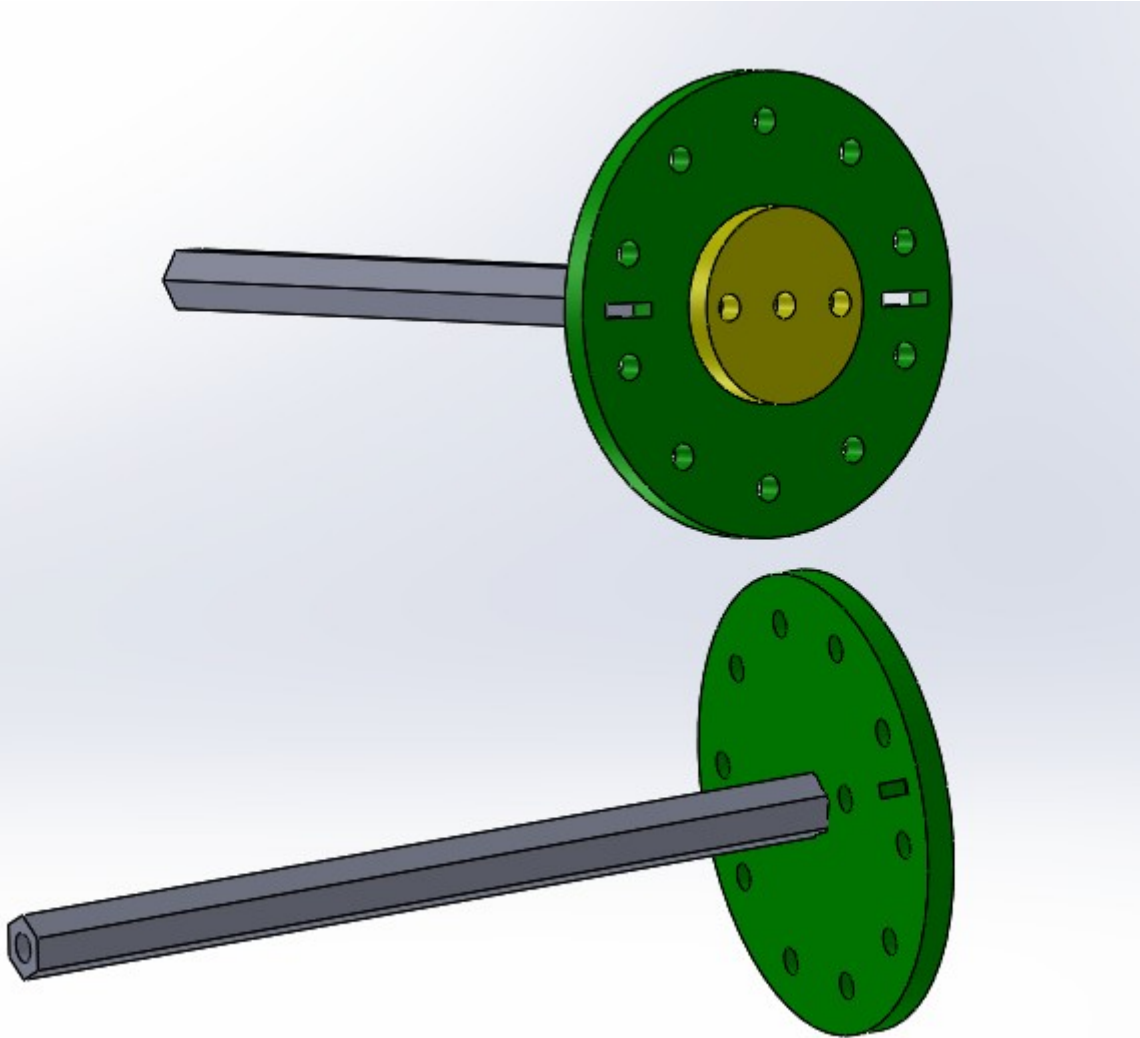
- Just laser-cut parts plus metal fasteners.
- No sawing. No drilling. No glue. Assemble with a screwdriver and pliers.



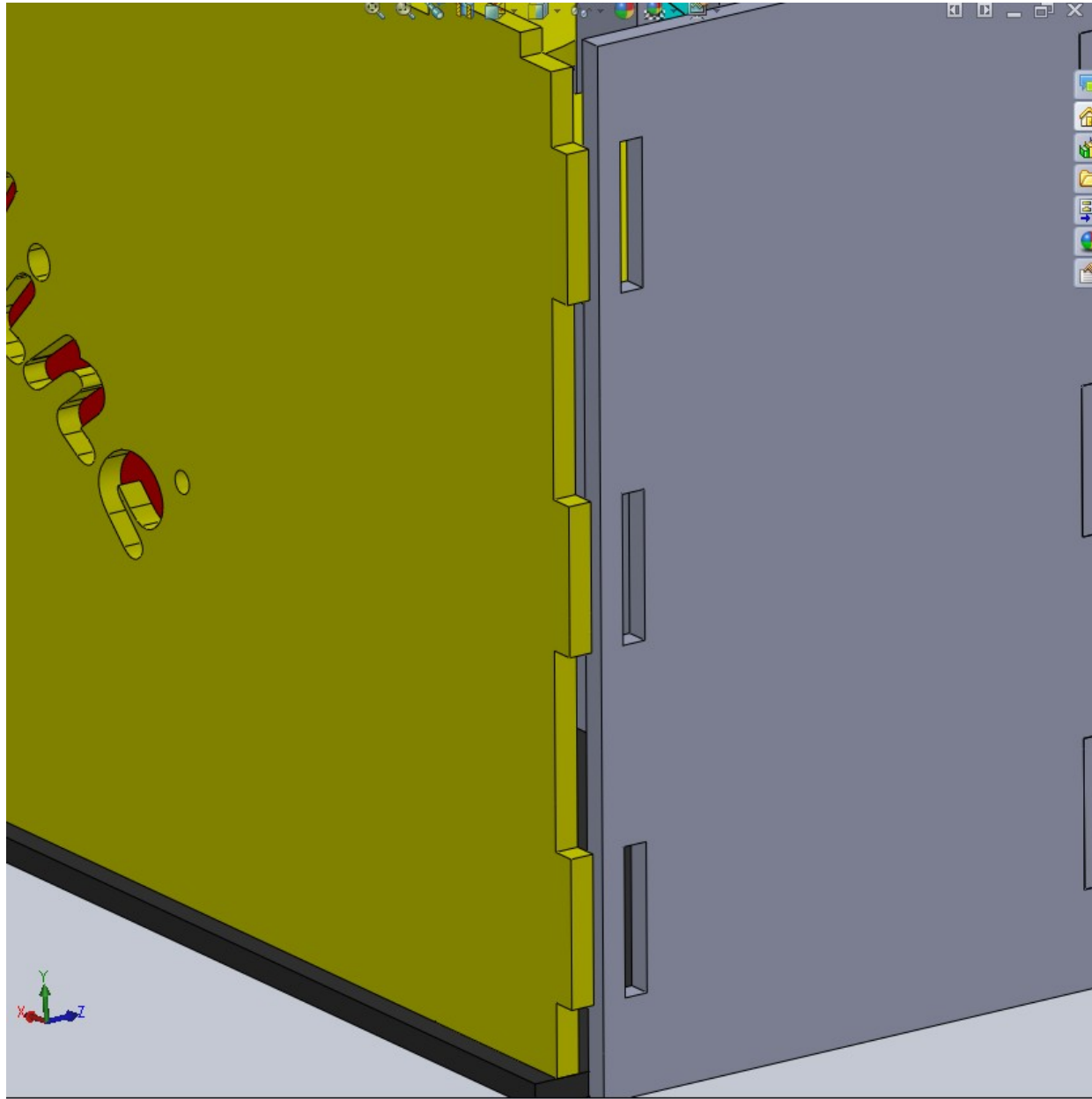
Hex Standoffs Prevent Rotation



End-Cap Holds Gear on Shaft



Slot and Tab Box Construction



Goal

- Make a Pascaline kit.
- Open source.
- Distribute via:
 - TechShop?
 - Instructables?
 - [your suggestions here]



Assignment 3: The Pascaline

- Will be done in groups of 2.
- Files due in a week.
- Pascaline “checkpoint” during recitation.
- Assembled machines will be tested for grading.
- What you need to do today:
 - Find a partner.
 - Email your pairings to Dave by Wednesday.