

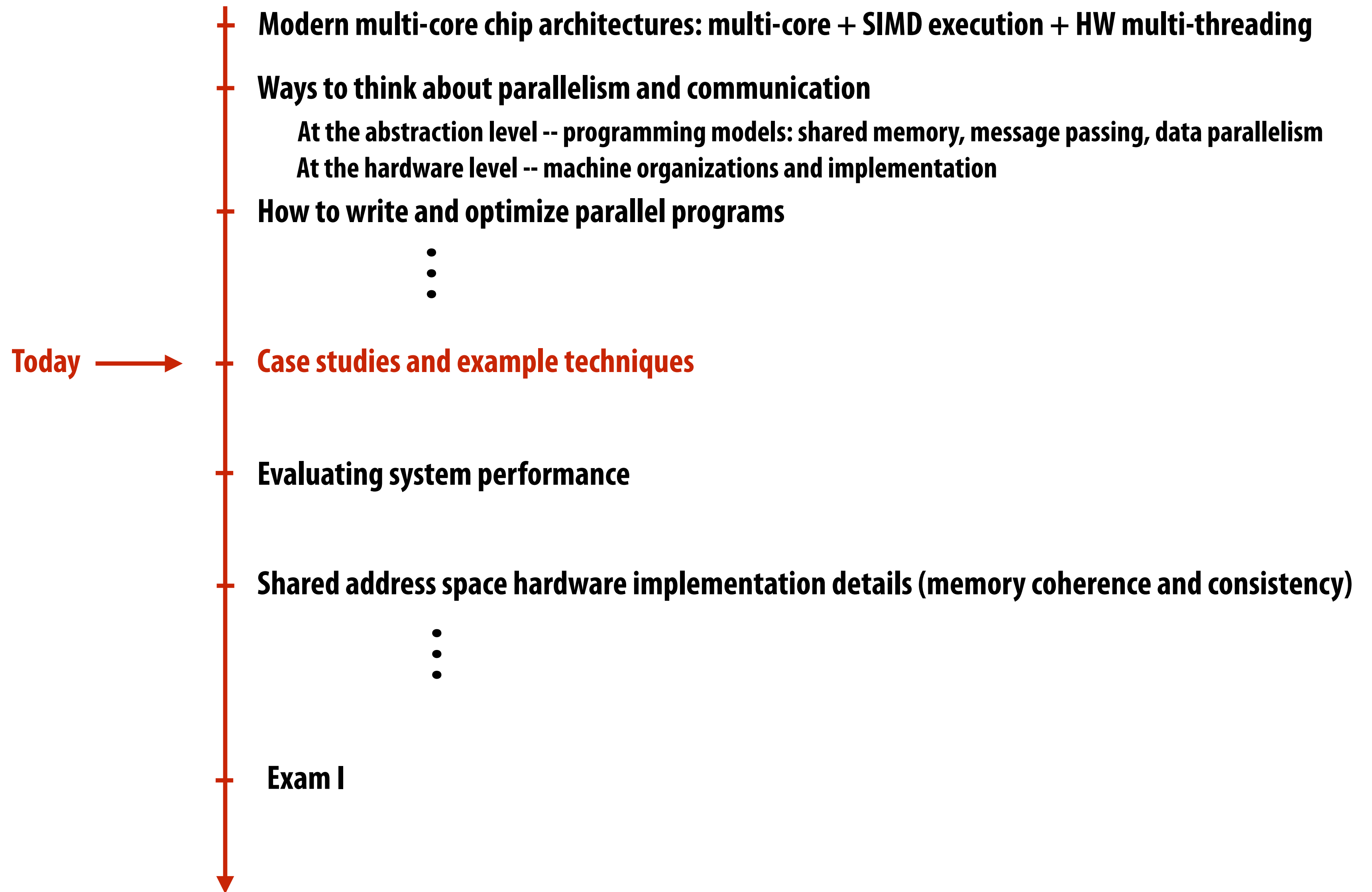
**Lecture 8:**

# **Parallel Programming Case Studies**

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**Parallel Computer Architecture and Programming  
CMU 15-418/15-618, Fall 2019**

# 15-418/618 course road map



# Today: case studies!

- **Several parallel application examples**
  - **Ocean simulation**
  - **Galaxy simulation** (Barnes-Hut algorithm)
  - **Parallel scan**
  - **Data-parallel segmented scan** (Bonus material!)
  - **Ray tracing** (Bonus material!)
- **Will be describing key aspects of the implementations**
  - **Focus on: optimization techniques, analysis of workload characteristics**

# Assumption: shared address space

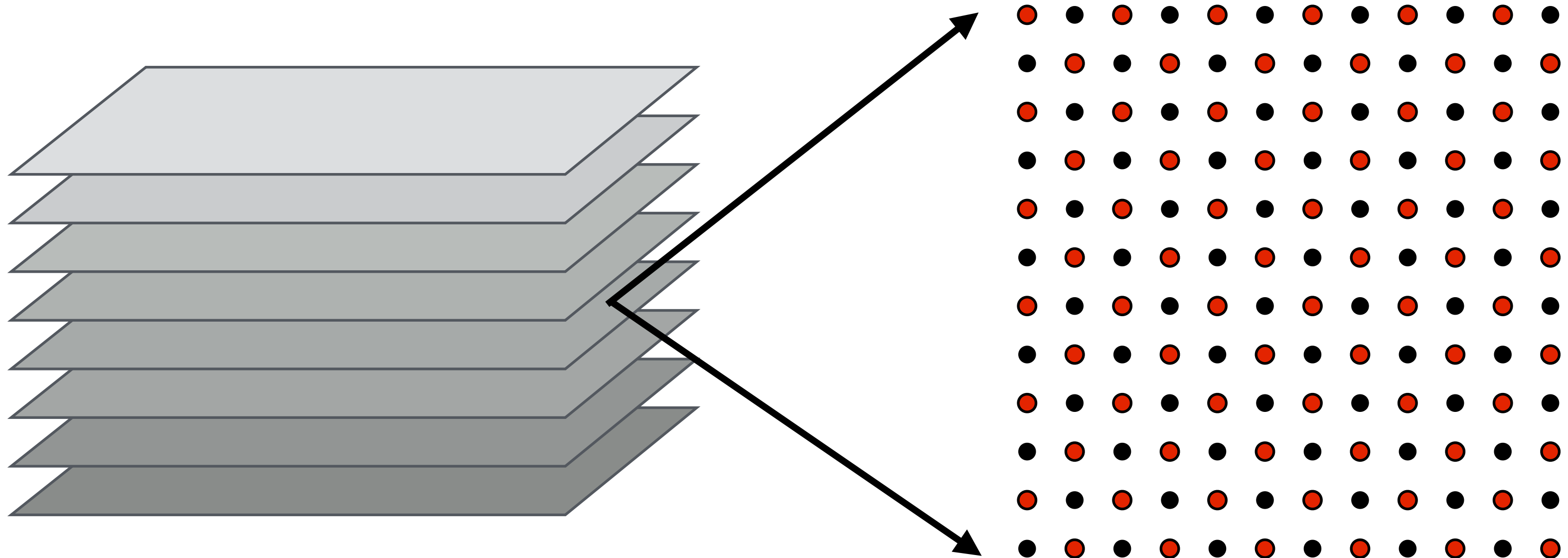
- For the purposes of today's lecture I encourage you to think about the example applications in the context of a large **NUMA shared address space** machine.

(single address space, but each processor can access a local region of the address space more quickly)

- But issues we discuss certainly also arise in a distributed address space setting.

# **Simulation of Ocean Currents (grid-based solver)**

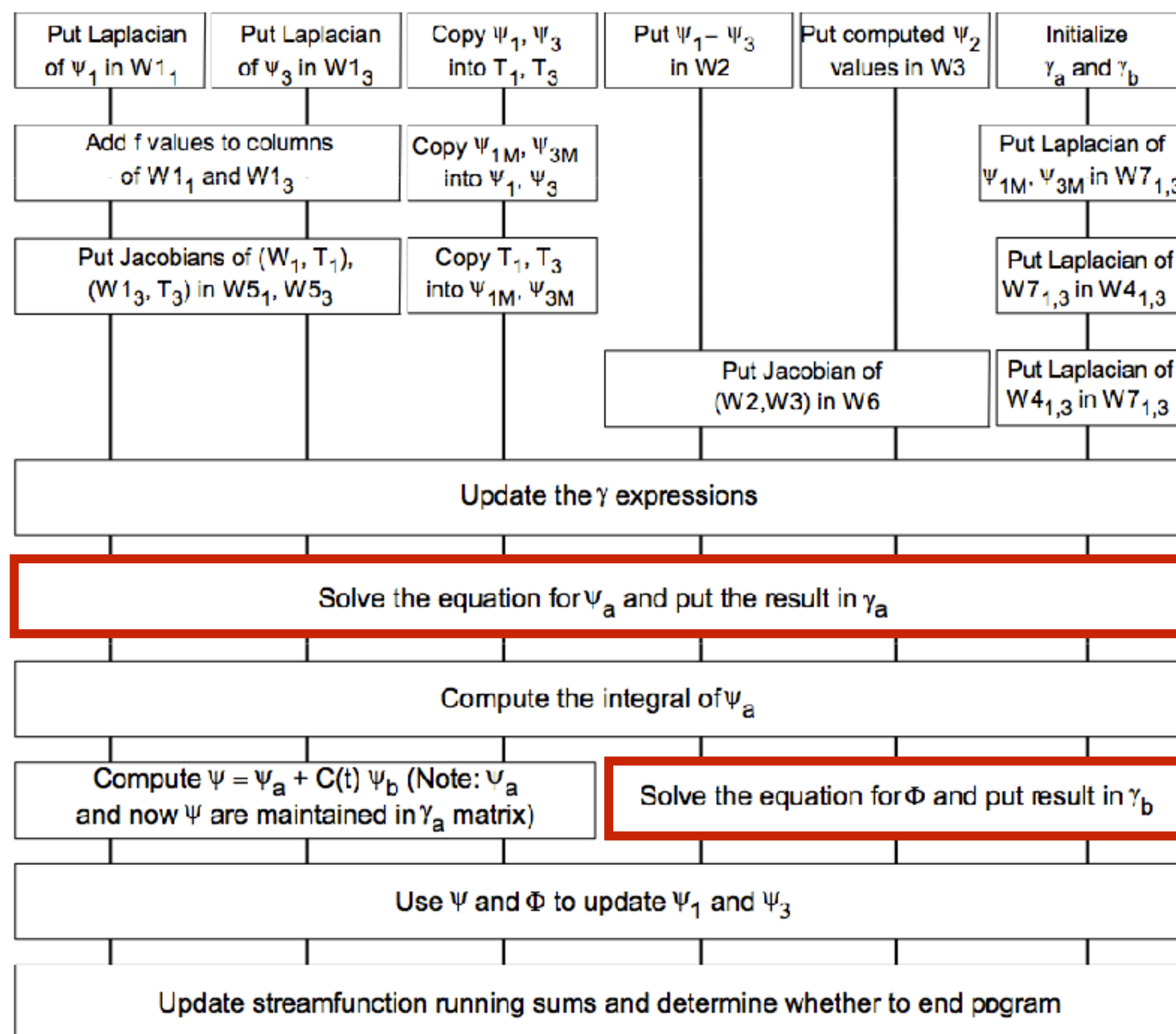
# Simulating of ocean currents



- Discretize 3D ocean volume into slices represented as 2D grids
- Discretize time evolution of ocean:  $\Delta t$
- High accuracy simulation requires small  $\Delta t$  and high resolution grids

# Where are the dependencies?

## Dependencies in one time step of ocean simulation



Boxes correspond to computations on grids

Lines express dependencies between computations on grids

The "grid solver" example corresponds to these parts of the application

**Parallelism within a grid (data-parallelism) and across operations on the different grids. The implementation only leverages data-parallelism (for simplicity)**

# Ocean implementation details

Recall shared-memory implementation discussed in previous classes:

## ■ Decomposition:

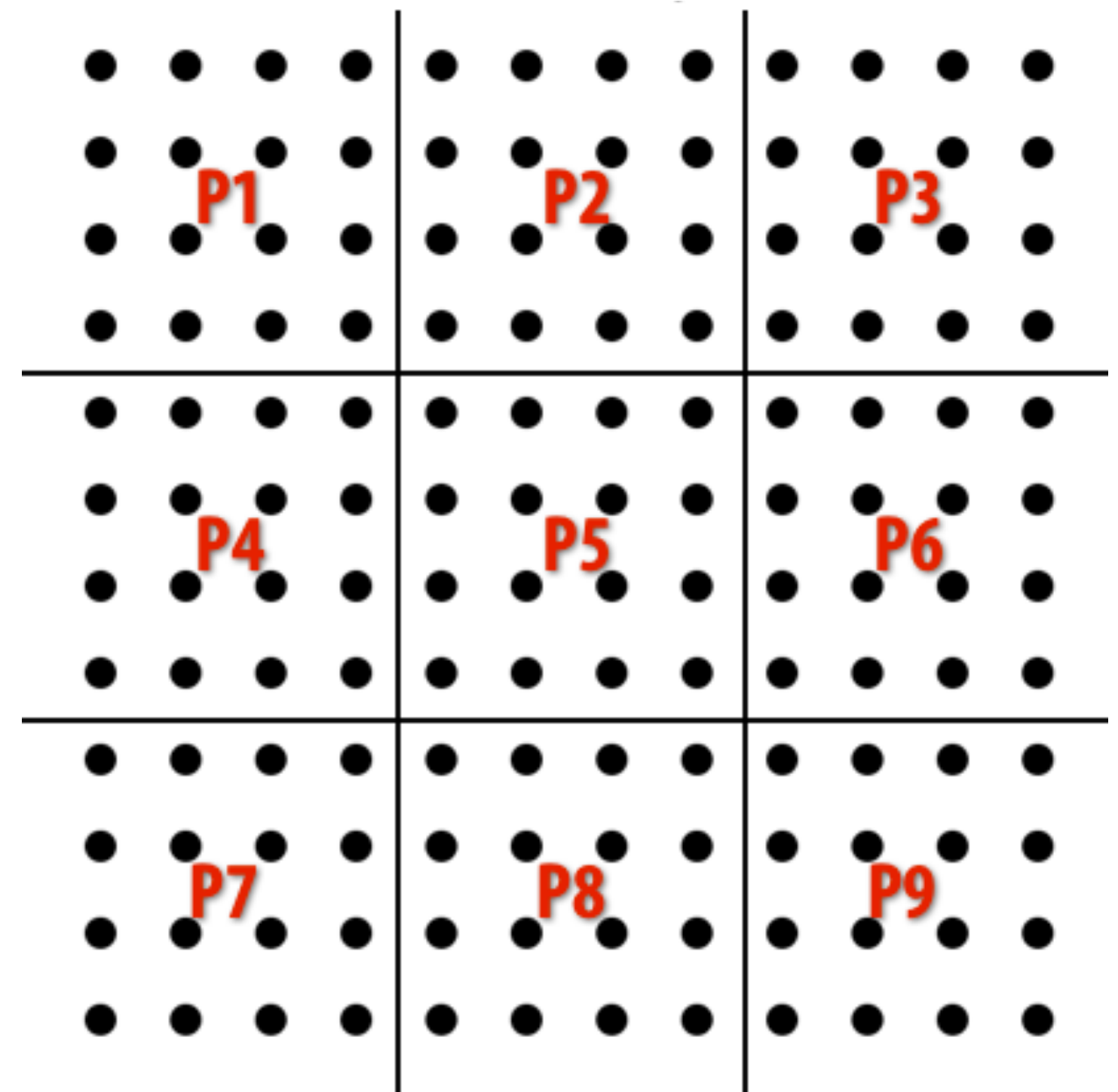
- Spatial partitioning of grid: each processor receives 2D tile of grid

## ■ Assignment

- Static assignment of tiles to processors

## ■ Synchronization

- Barriers (separate each pass over grid is a different phase of computation)
- Locks for mutual exclusion when updating shared variables (atomic update of 'diff')

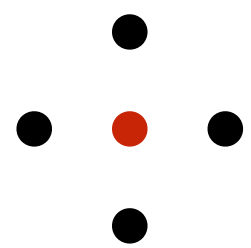




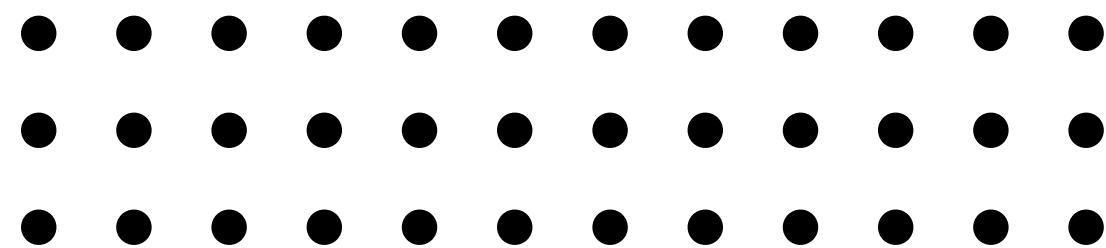
# Another question to ask: what are the **critical working sets**?

1. Local neighborhood for cell
2. Three rows of a processor's local partition of grid
3. Processor's local partition of grid

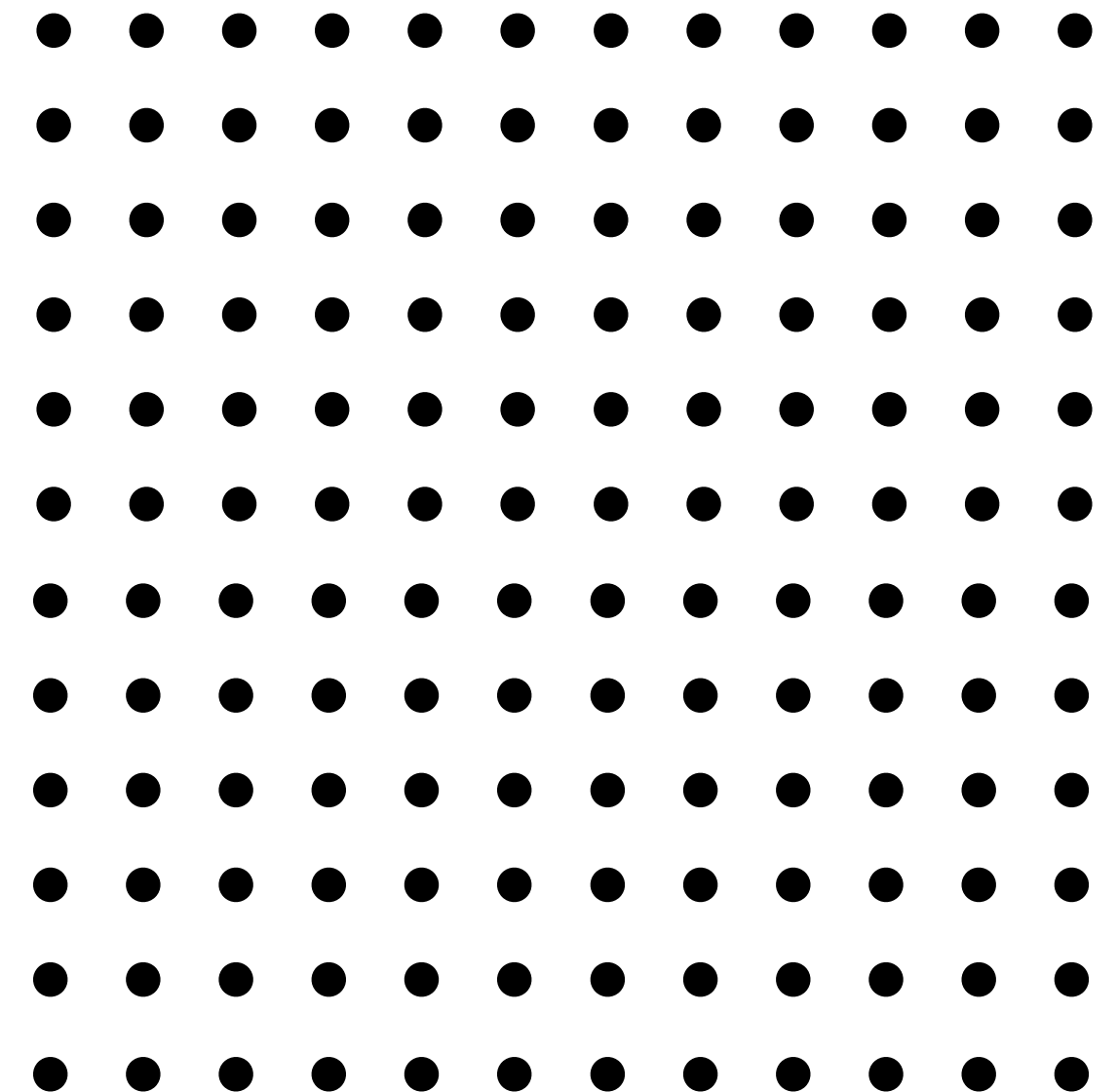
1.



2.

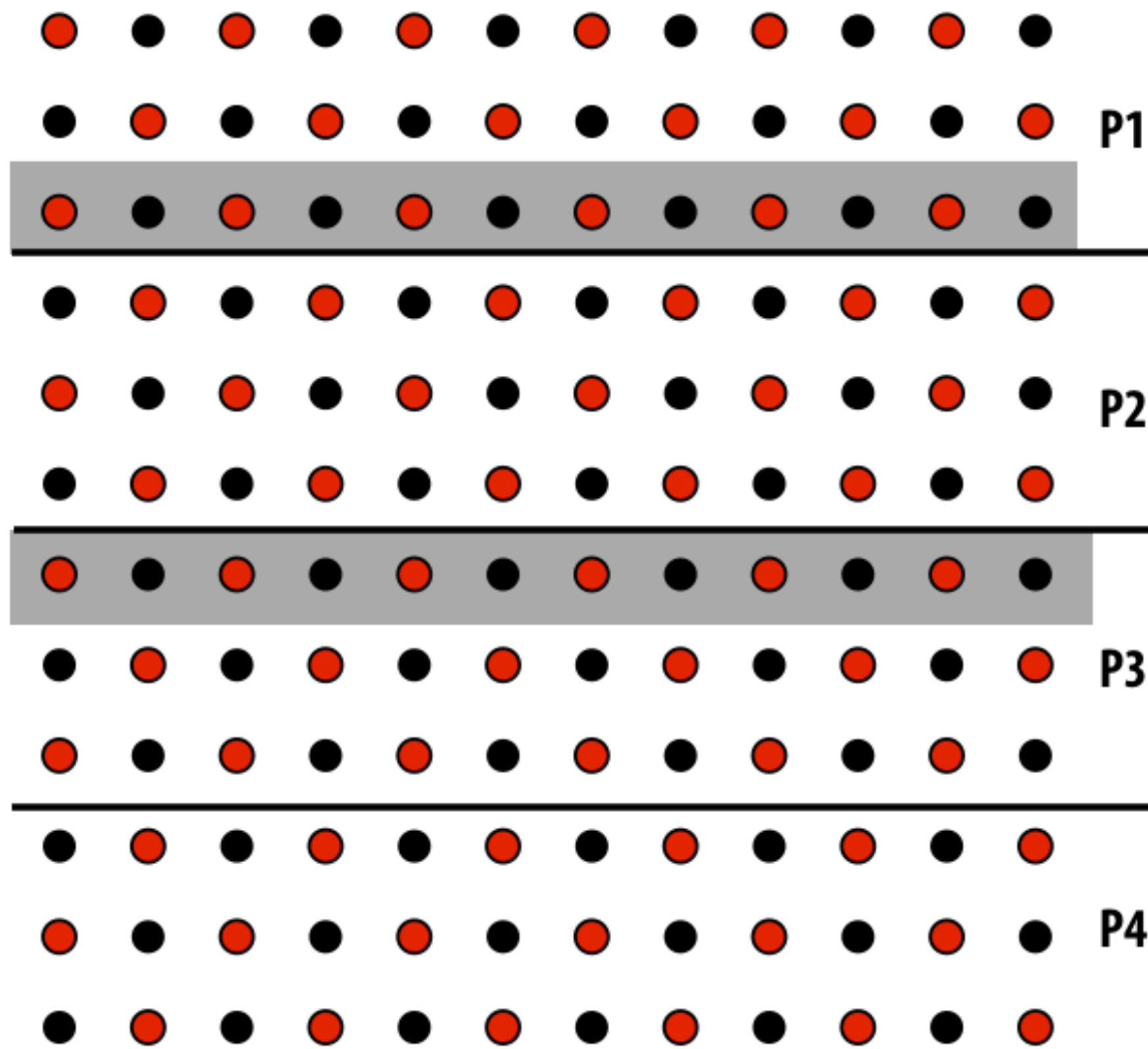


3.

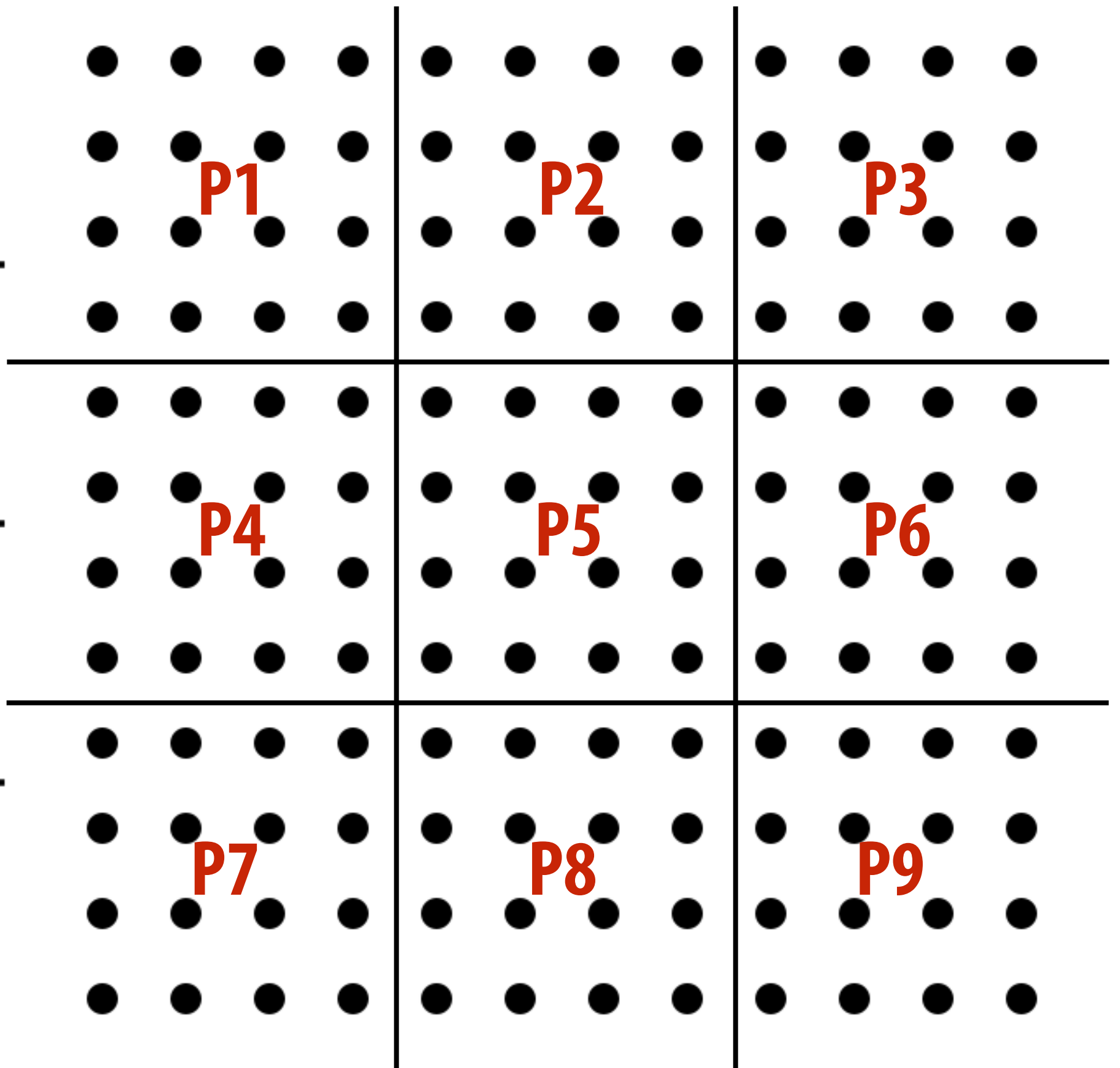


# Recall: Two Static Partitioning Schemes

## Strips



## Blocks

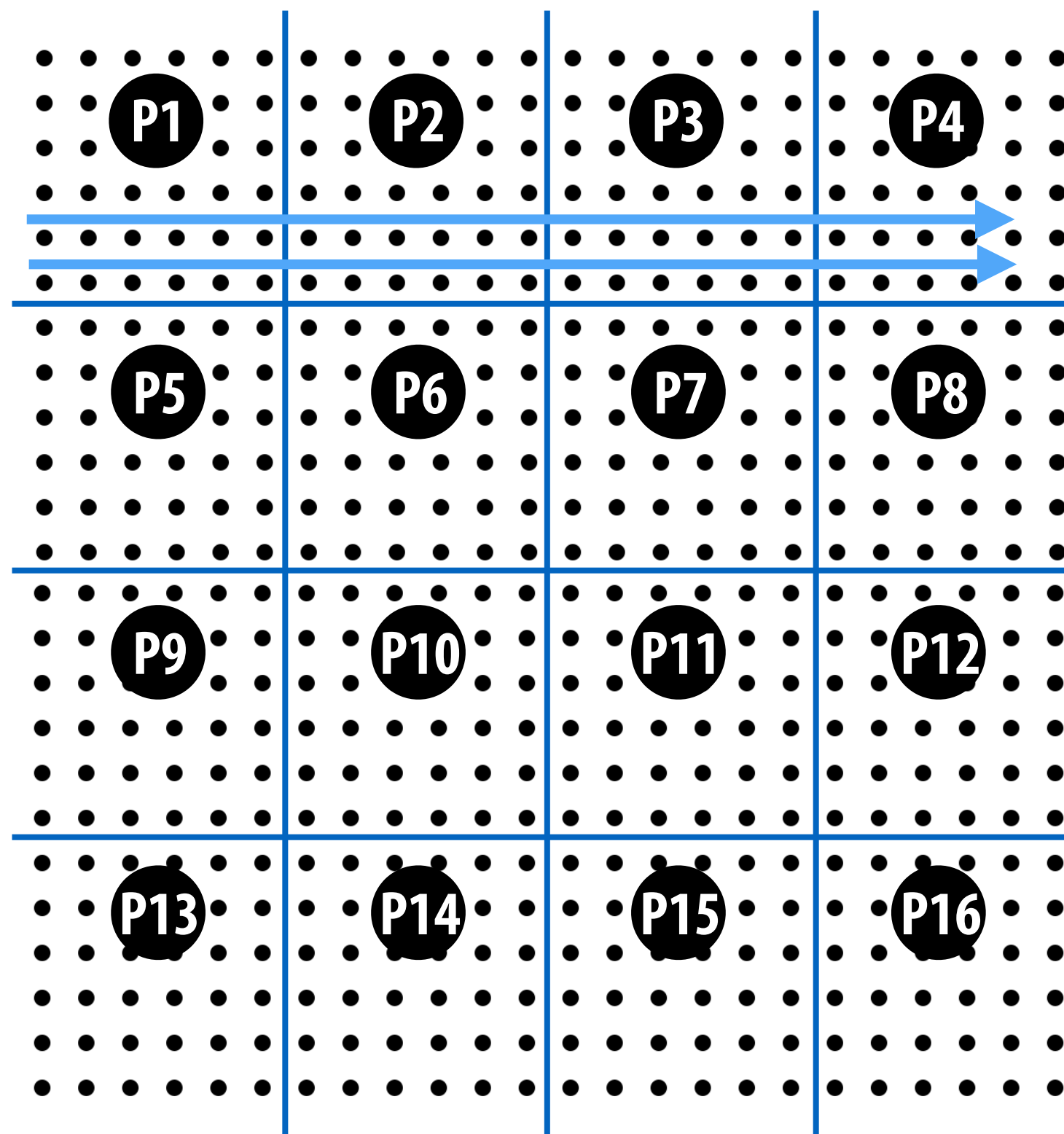


■ Which one is better?

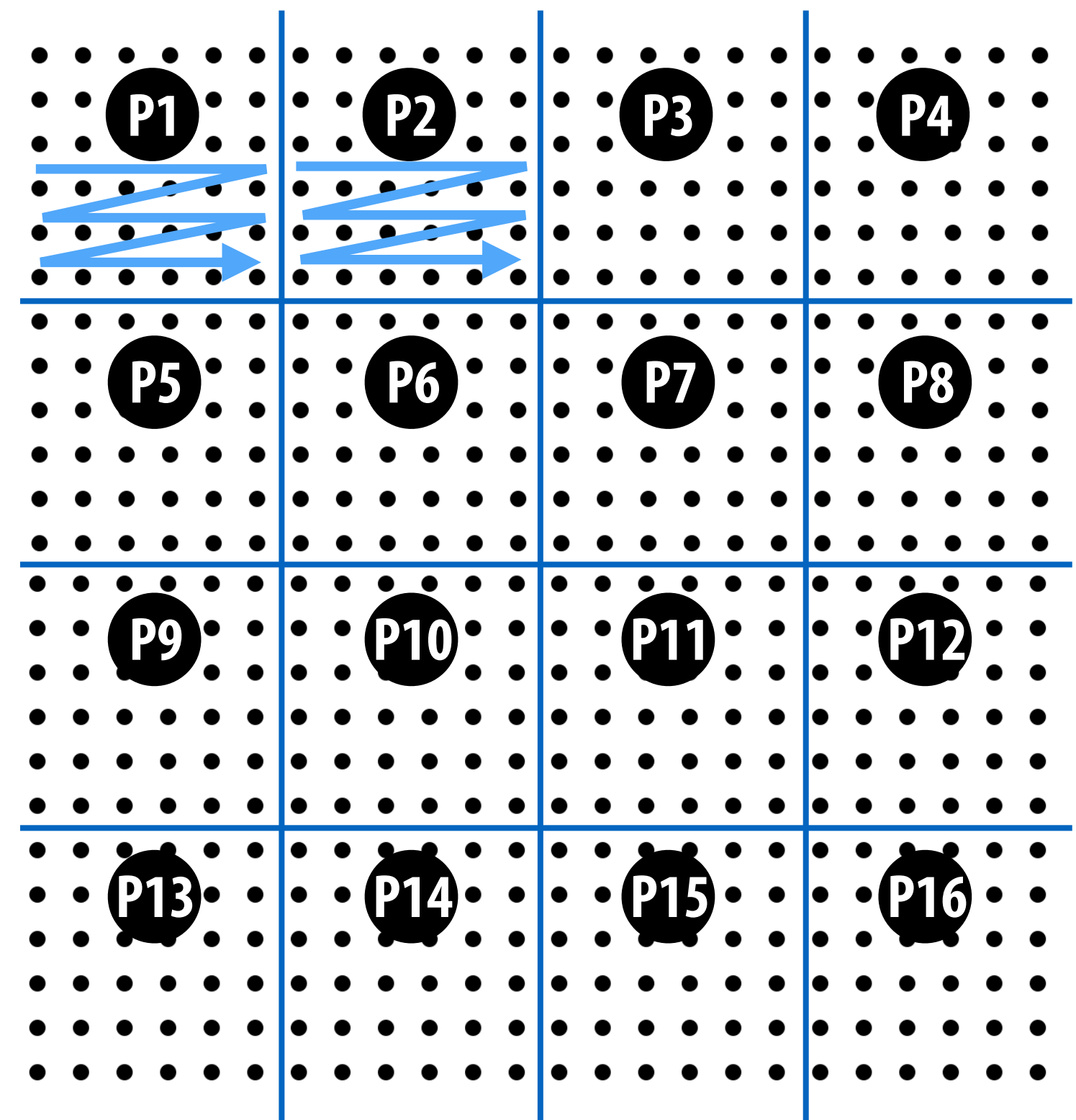
# Recall: two layouts of 2D grid in address space

(Blue lines indicate consecutive memory addresses)

## 2D, row-major array layout

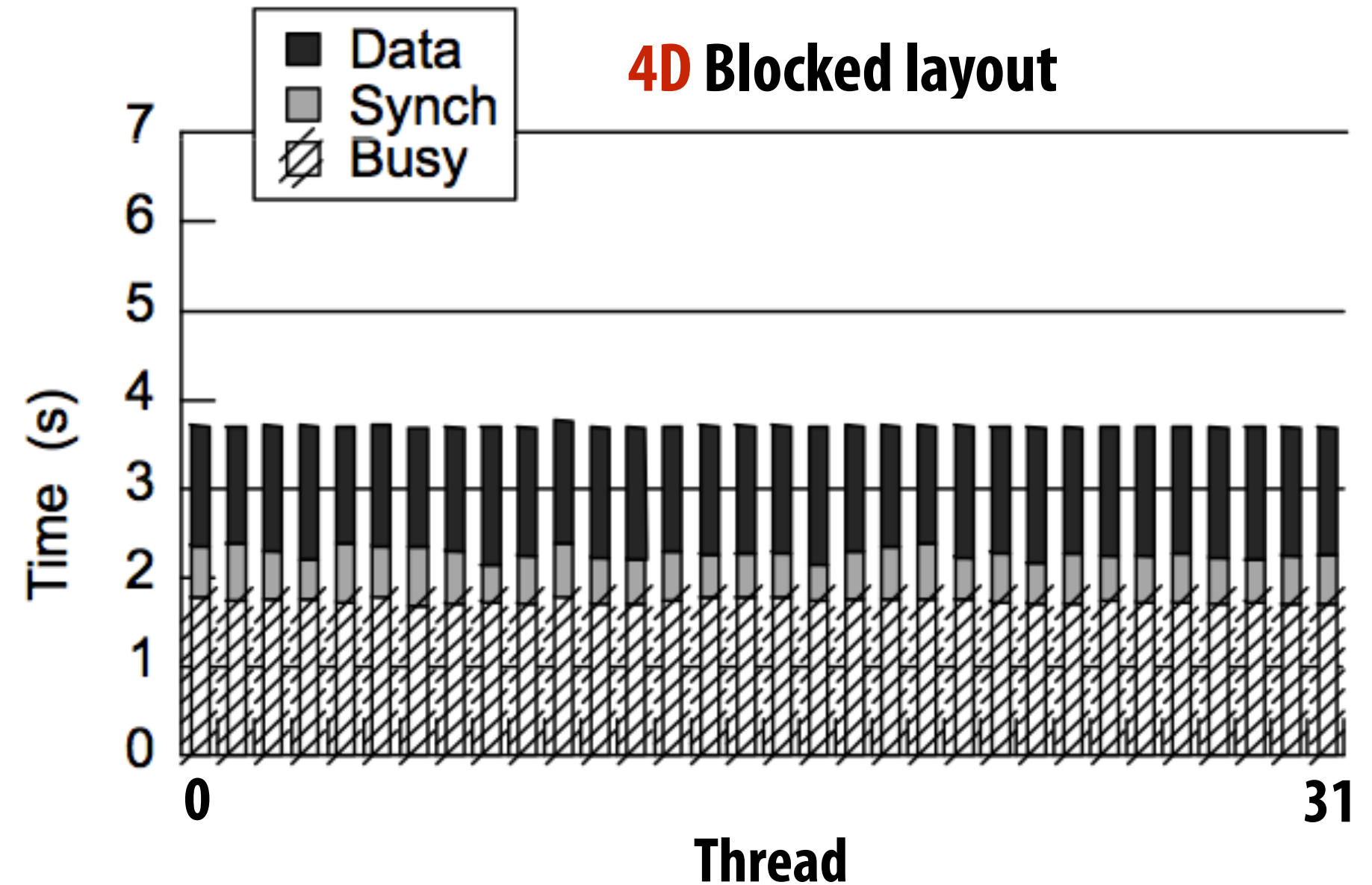
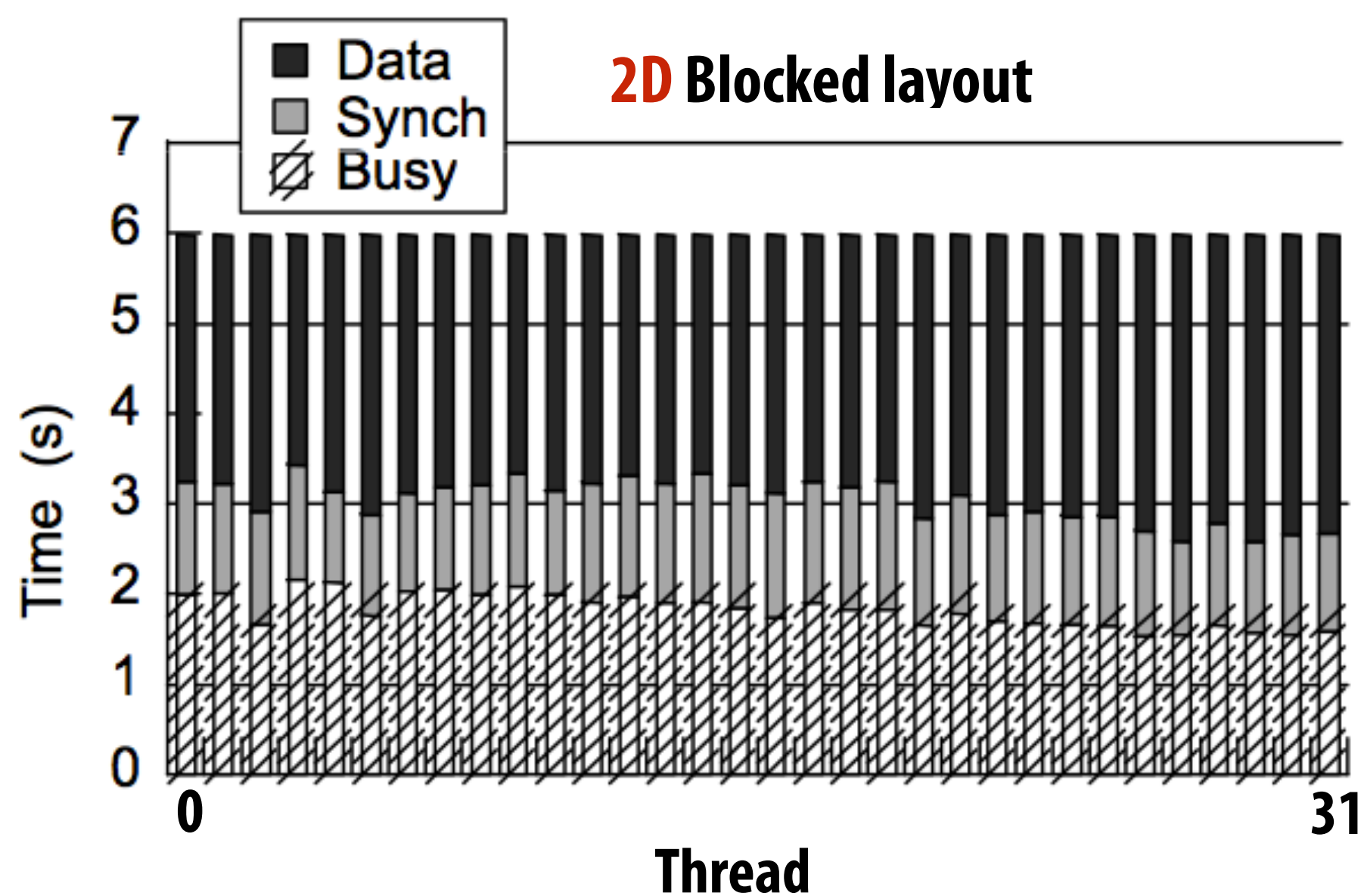


## 4D array layout (block-major)



# Ocean: execution time breakdown

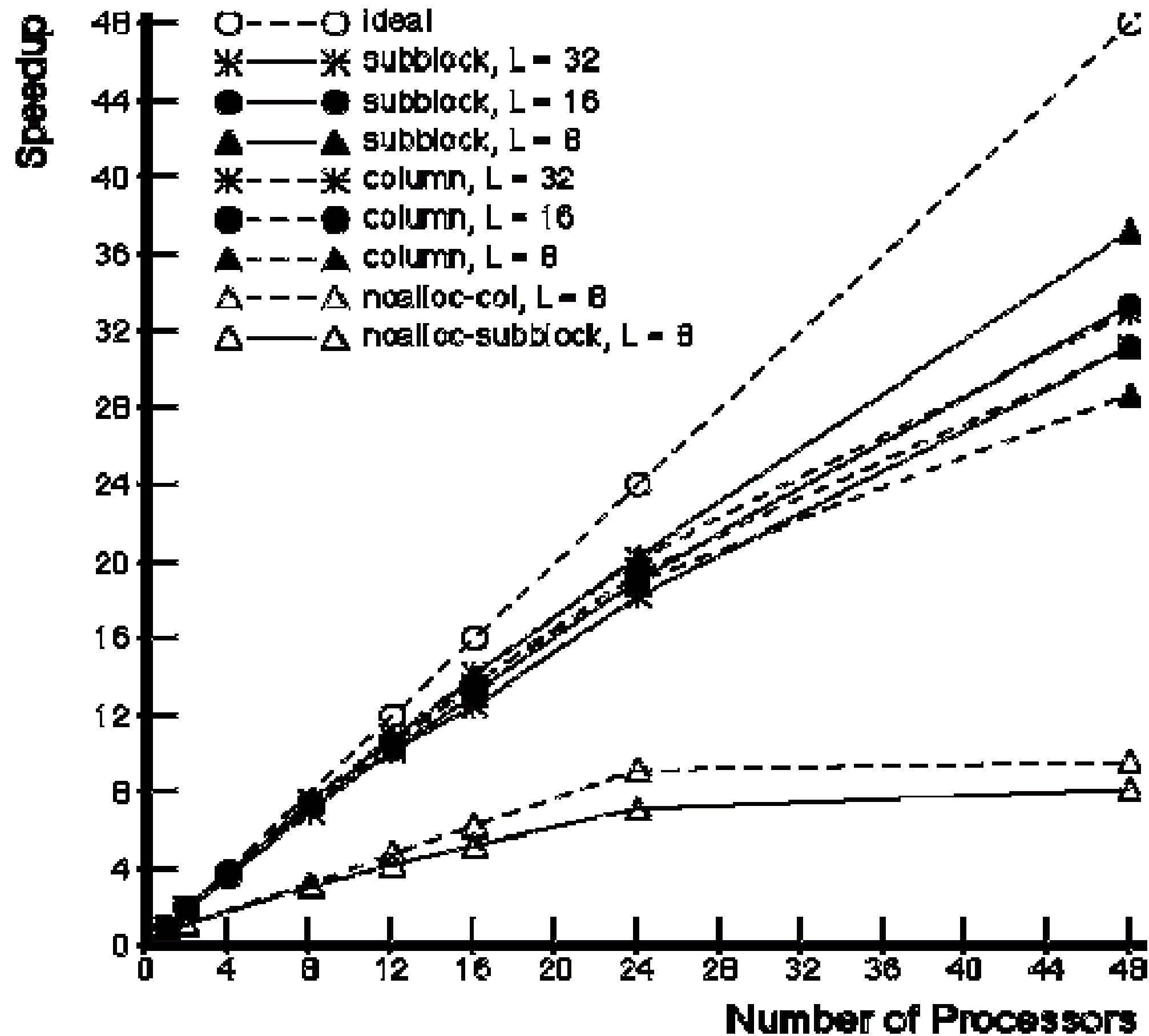
Execution on 32-processor SGI Origin 2000 (1026 x 1026 grids)



## Observations:

- **Static assignment is sufficient** (approximately equal busy time per thread)
- **4D blocking of grid reduces time spent on communication** (reflected on graph as data wait time)
- Synchronization cost is largely due to waiting at **barriers**

# Impact of Line Size and Data Distribution



(a) 16 KByte Cache, Grid\_98

- no-alloc = round-robin page allocation; otherwise, data assigned to local memory.
- L = cache line size.

# Galaxy Evolution using Barnes Hut

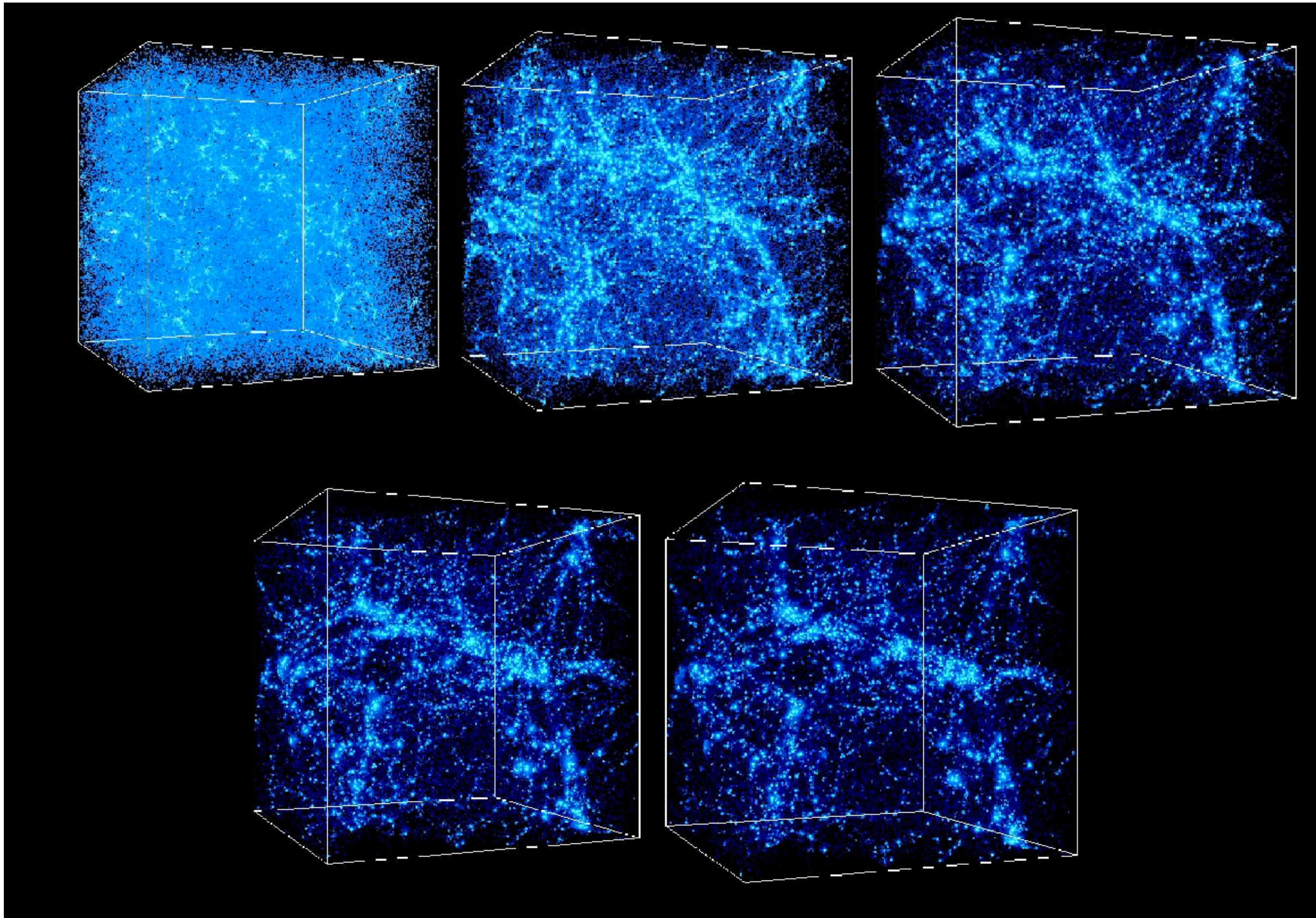
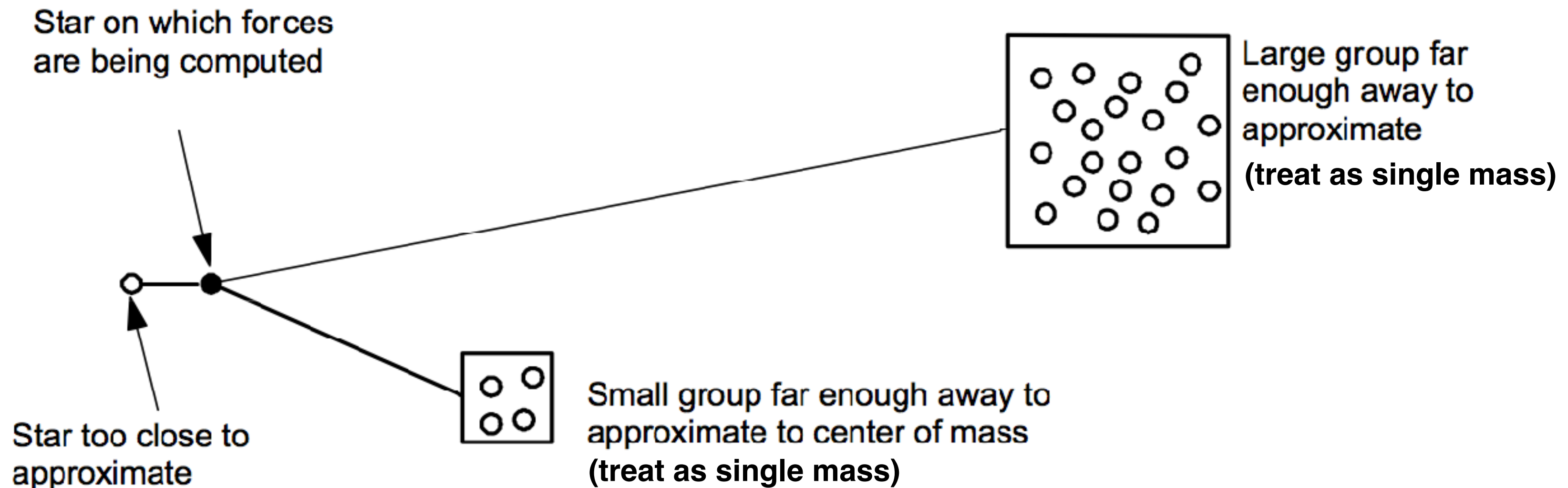


Image credit: [http://www.lsw.uni-heidelberg.de/users/mcamenzi/images/Universe\\_Box.gif](http://www.lsw.uni-heidelberg.de/users/mcamenzi/images/Universe_Box.gif)

Example taken from: Culler, Singh, and Gupta, Chapter 3

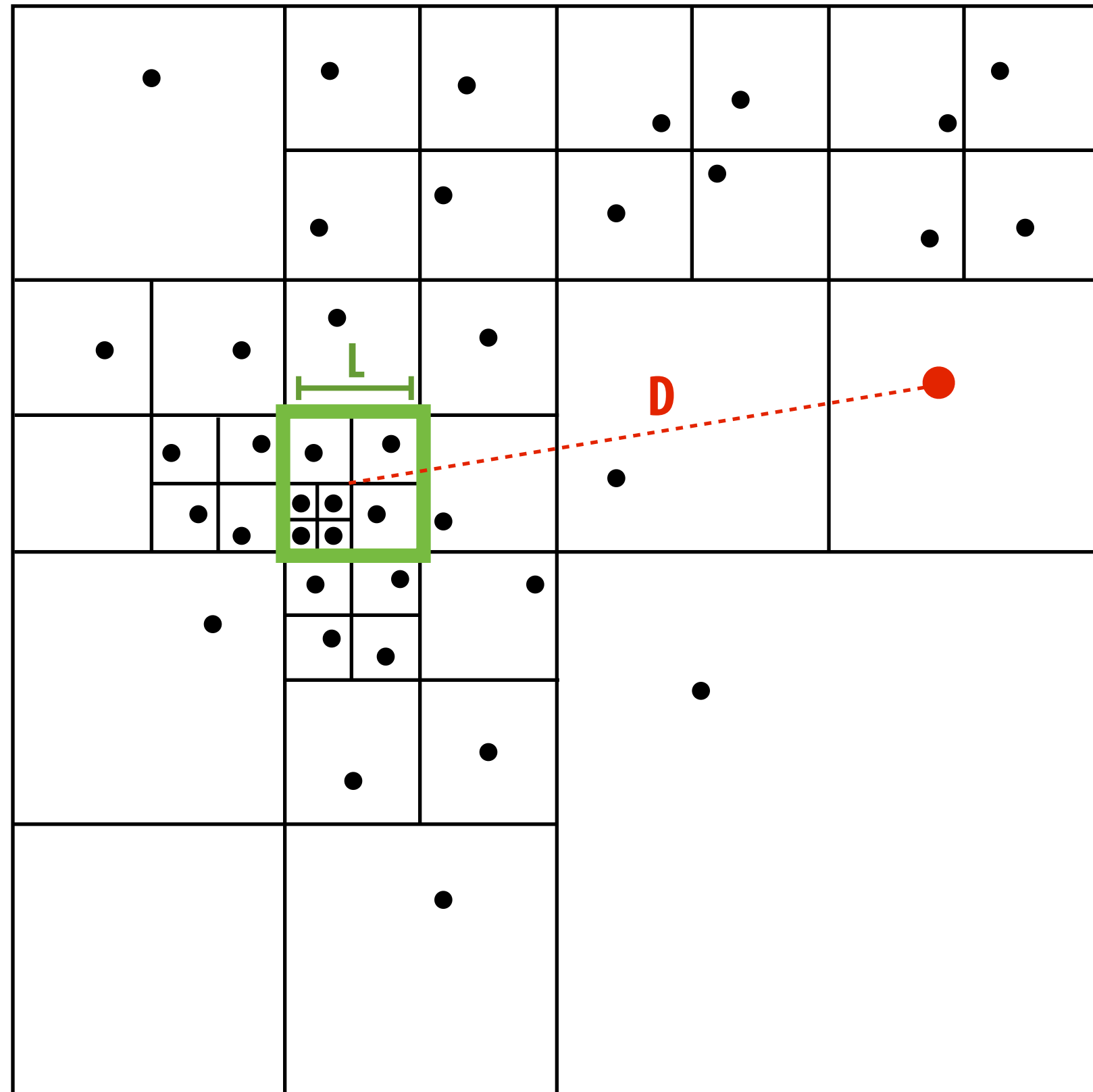
# Galaxy evolution

## Barnes-Hut algorithm

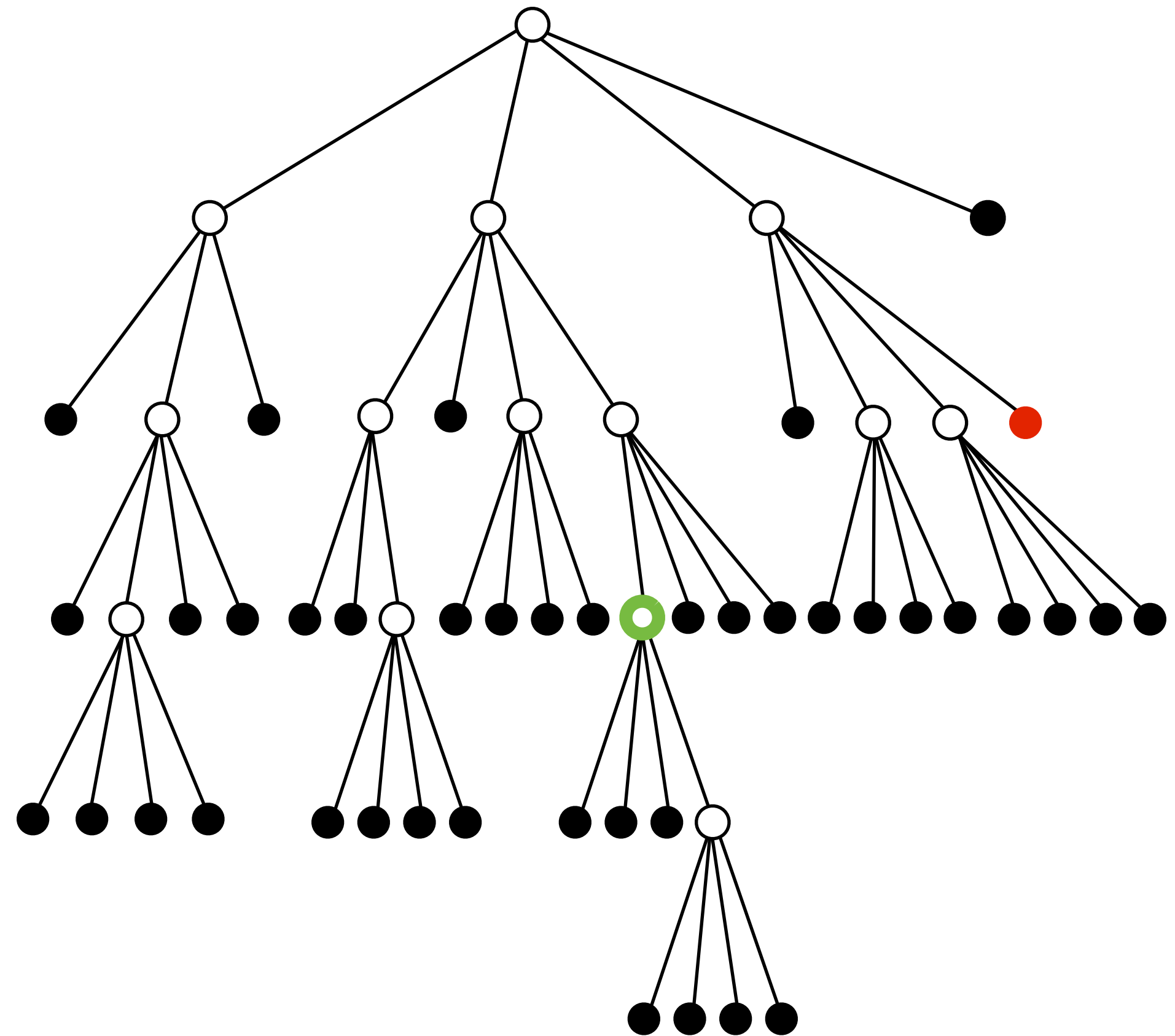


- Represent galaxy as a collection of **N particles** (think: particle = star)
- **Compute forces on each particle due to gravity**
  - Naive algorithm is  $O(N^2)$  — all particles interact with all others (gravity has infinite extent)
  - Magnitude of gravitational force falls off with distance (so algorithms approximate forces from far away stars to gain performance)
  - Result is an  $O(N \lg N)$  algorithm for computing gravitational forces between all stars

# Barnes-Hut tree



Spatial Domain



Quad-Tree Representation of Bodies

- Leaf nodes are star particles
- Interior nodes store center of mass + aggregate mass of all child bodies
- To compute forces on each body, traverse tree... accumulating forces from all other bodies
  - Compute forces using aggregate interior node if  $L/D < \Theta$ , else descend to children
- Expected number of nodes touched  $\sim \lg N / \Theta^2$



# Barnes-Hut application structure

for each time step in simulation:

build tree structure

compute (aggregate mass, center-of-mass) for interior nodes

for each particle:

traverse tree to accumulate gravitational forces

update particle position based on gravitational forces

## Challenges:

- Amount of **work** per body is **non-uniform**, **communication** pattern is **non-uniform** (depends on the local density of bodies)
  - The bodies move: so costs and communication patterns **change over time**
  - Irregular, fine-grained computation
- But, there is a lot of **locality** in the computation (bodies that are near in space require similar data to compute forces — it seems smart to co-locate these computations!)

# Work assignment

## ■ Challenge:

- **Equal number of bodies per processor  $\neq$  equal work per processor**
- **Want equal work per processor AND assignment should preserve locality**

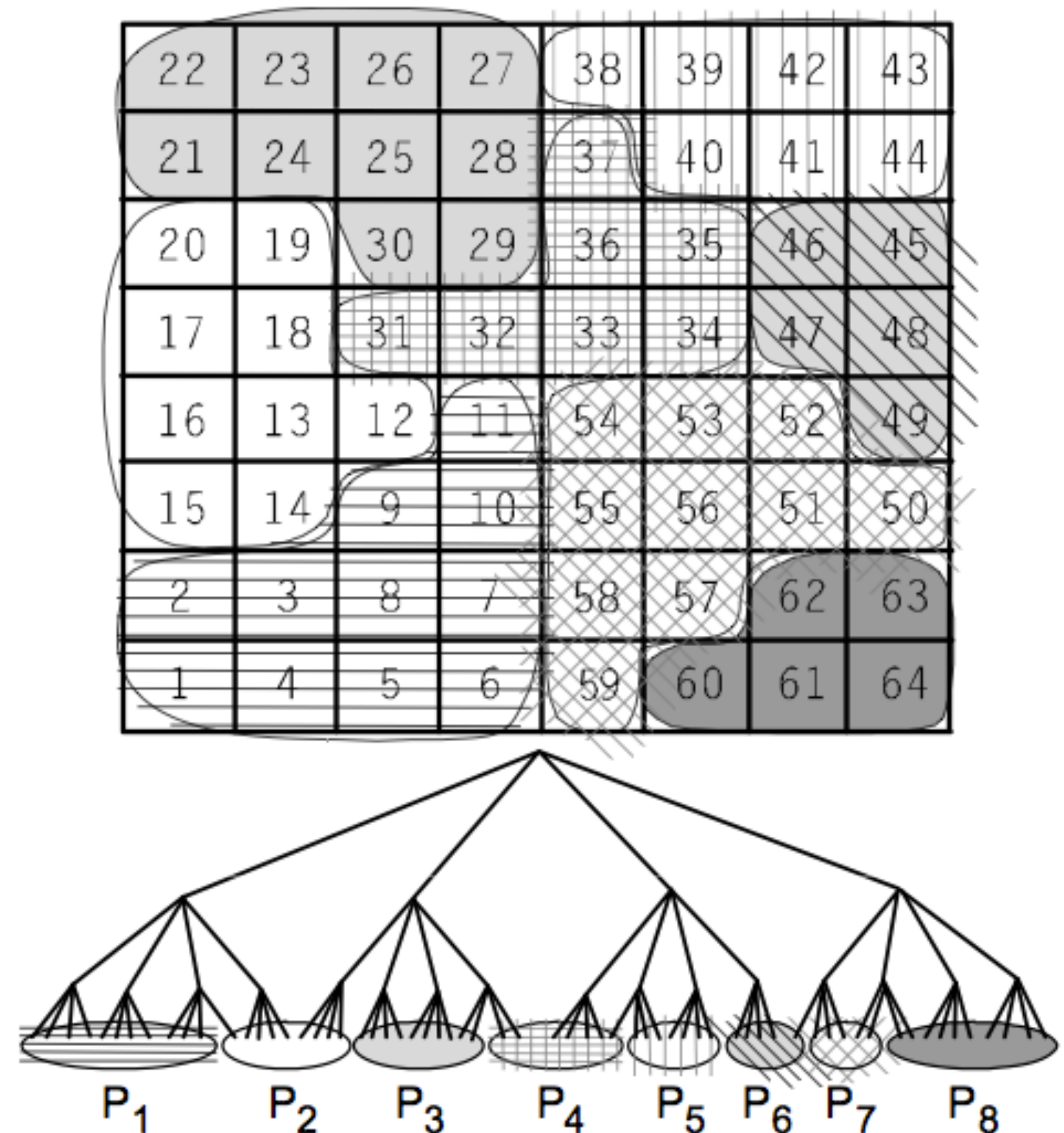
## ■ Observation: spatial distribution of bodies **evolves slowly**

## ■ Use **semi-static assignment**

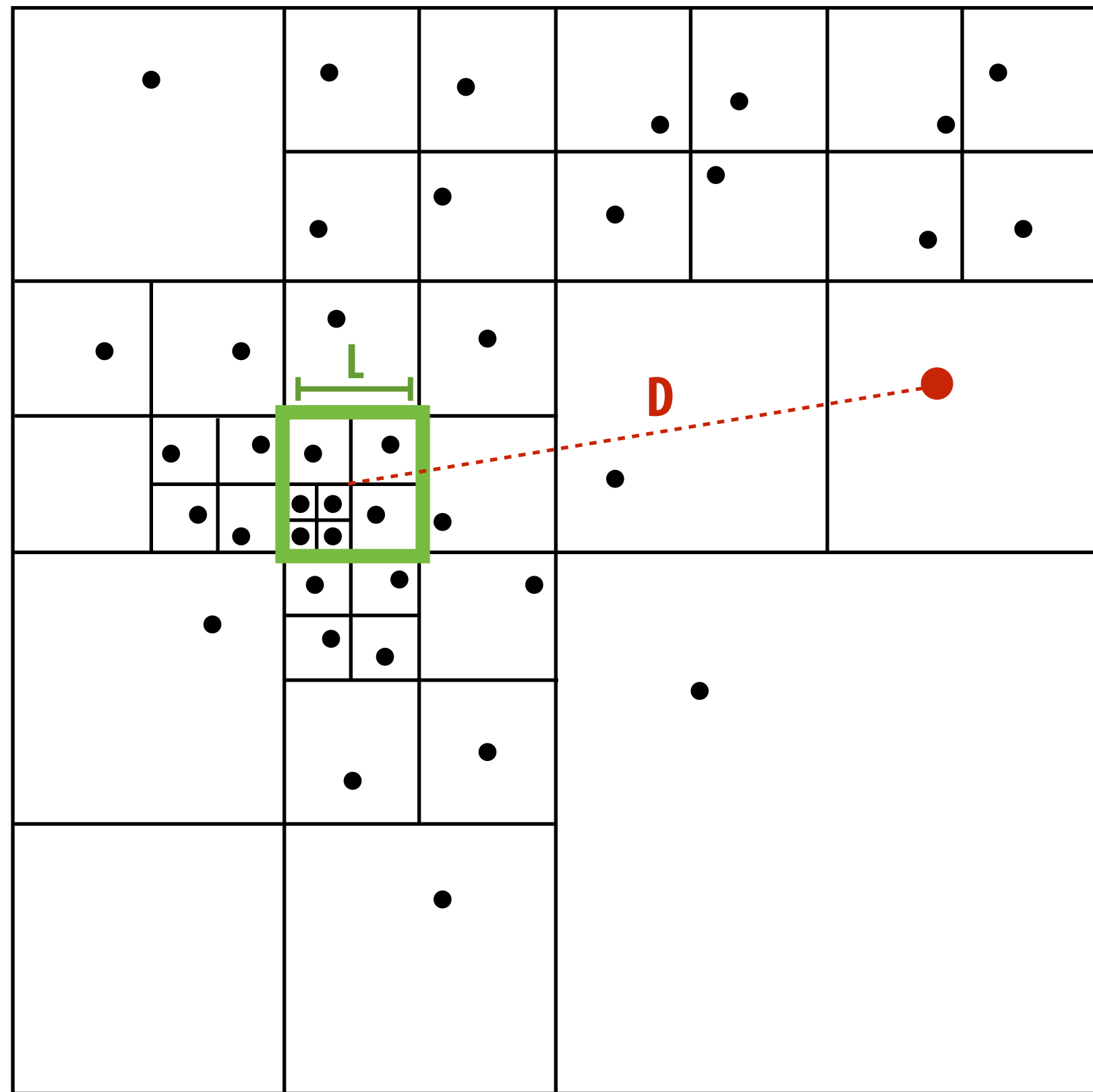
- **Each time step, for each body, record number of interactions with other bodies (the application profiles itself)**
  - **Cheap to compute. Just increment local per-body counters**
  - **Use values to **periodically recompute** assignment**

# Assignment using cost zones

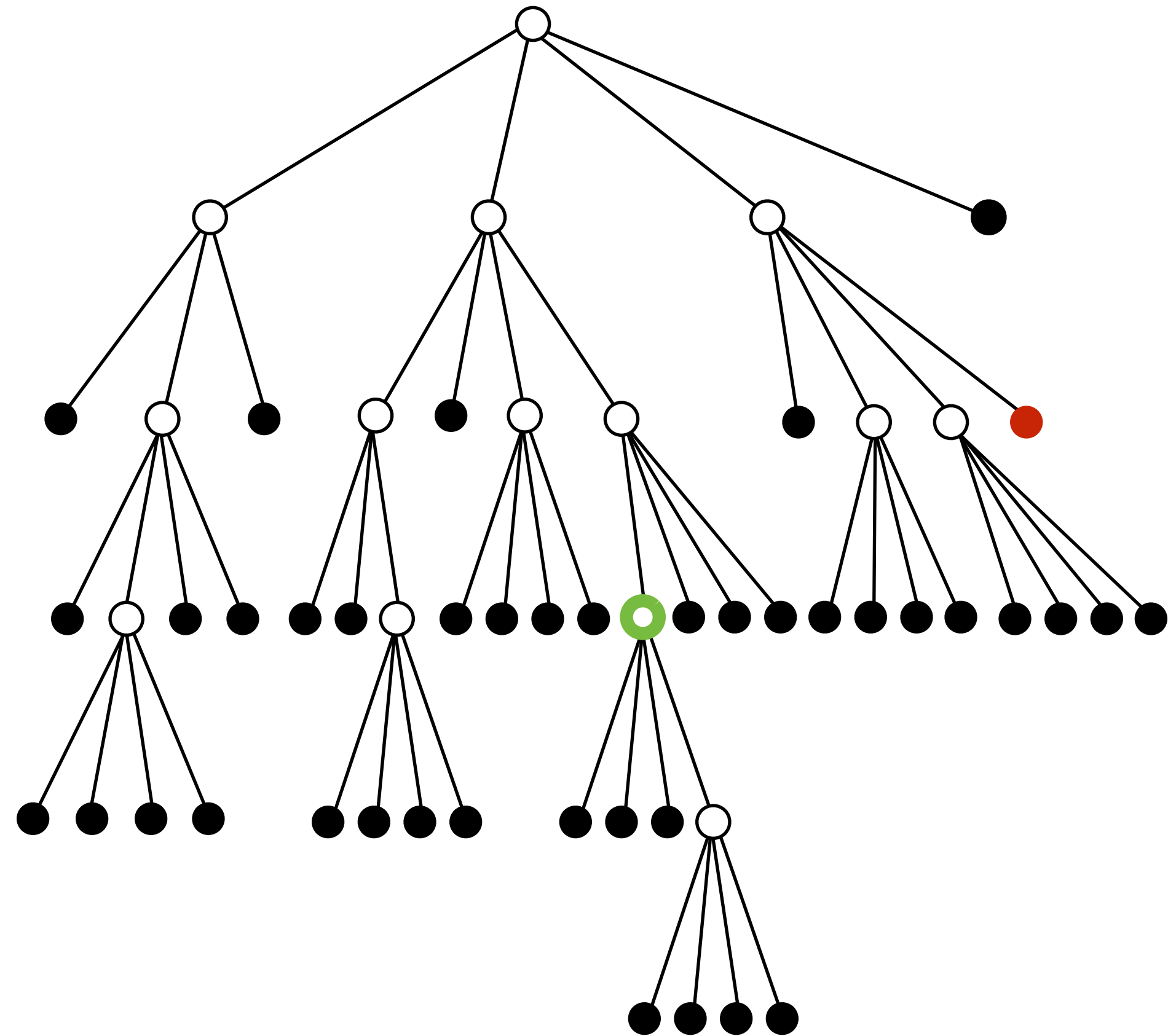
- Leverage locality inherent in tree
- Compute total work estimate  $W$  for all bodies (computed by summing per-body costs)
- Each processor is assigned  $W/P$  of the total work ( $P = \text{num processors}$ )
- Each processor performs depth-first (post-order) traversal of tree (accumulates work seen so far)
- Processor  $P_i$  responsible for processing bodies corresponding to work:  $iW/P$  to  $(i+1)W/P$
- Each processor can independently compute its assignment of bodies. (The only synchronization required is the sum reduction to compute total amount of work =  $W$ )



# Barnes-Hut: working sets



Spatial Domain

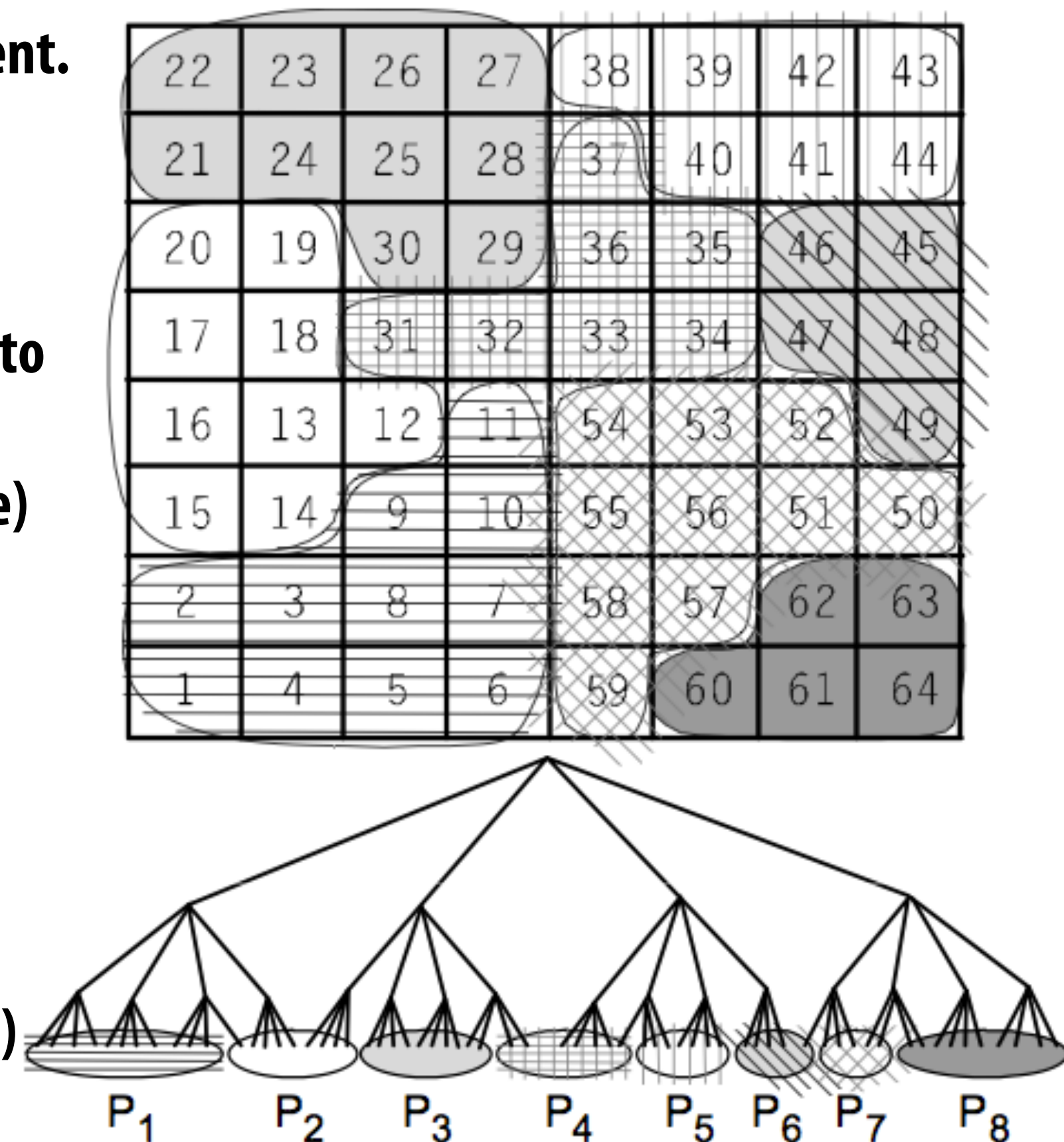


Quad-Tree Representation

- Working set 1: data needed to compute forces between body-body (or body-node) pairs
- Working set 2: data encountered in an entire tree traversal
  - Expected number of nodes touched for one body:  $\sim \lg N / \Theta^2$
  - Computation has high locality: consecutively processed bodies are nearby, so processing touches almost exactly the same nodes!

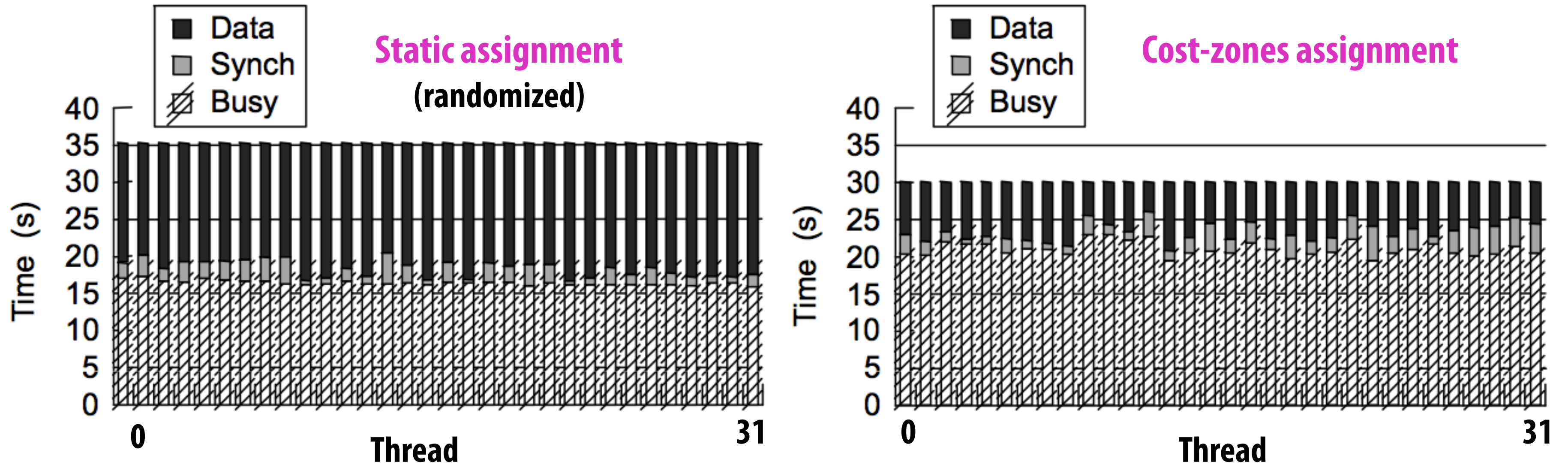
# Barnes-hut: data distribution

- **Cost zones technique computes a good work assignment. What about data distribution?**
- **Difficult to distribute data**
  - **Work assignment changes with time: would have to dynamically redistribute all simulation data**
  - **Data accessed at fine granularity (single tree node)**
- **Luckily: high temporal locality**
  - **Bodies assigned to same processor are nearby in space, so tree nodes accessed during force computations are very similar.**
  - **Data for traversal already in cache (Barnes-Hut benefits from large caches, smaller cache line size)**
- **Result: Unlike OCEAN, data distribution in Barnes-Hut does not significantly impact performance**
  - **Implementation uses static distribution (interleave particles throughout the machine)**



# Barnes-hut: execution time

Execution on 32-processor SGI Origin 2000 (512K bodies)

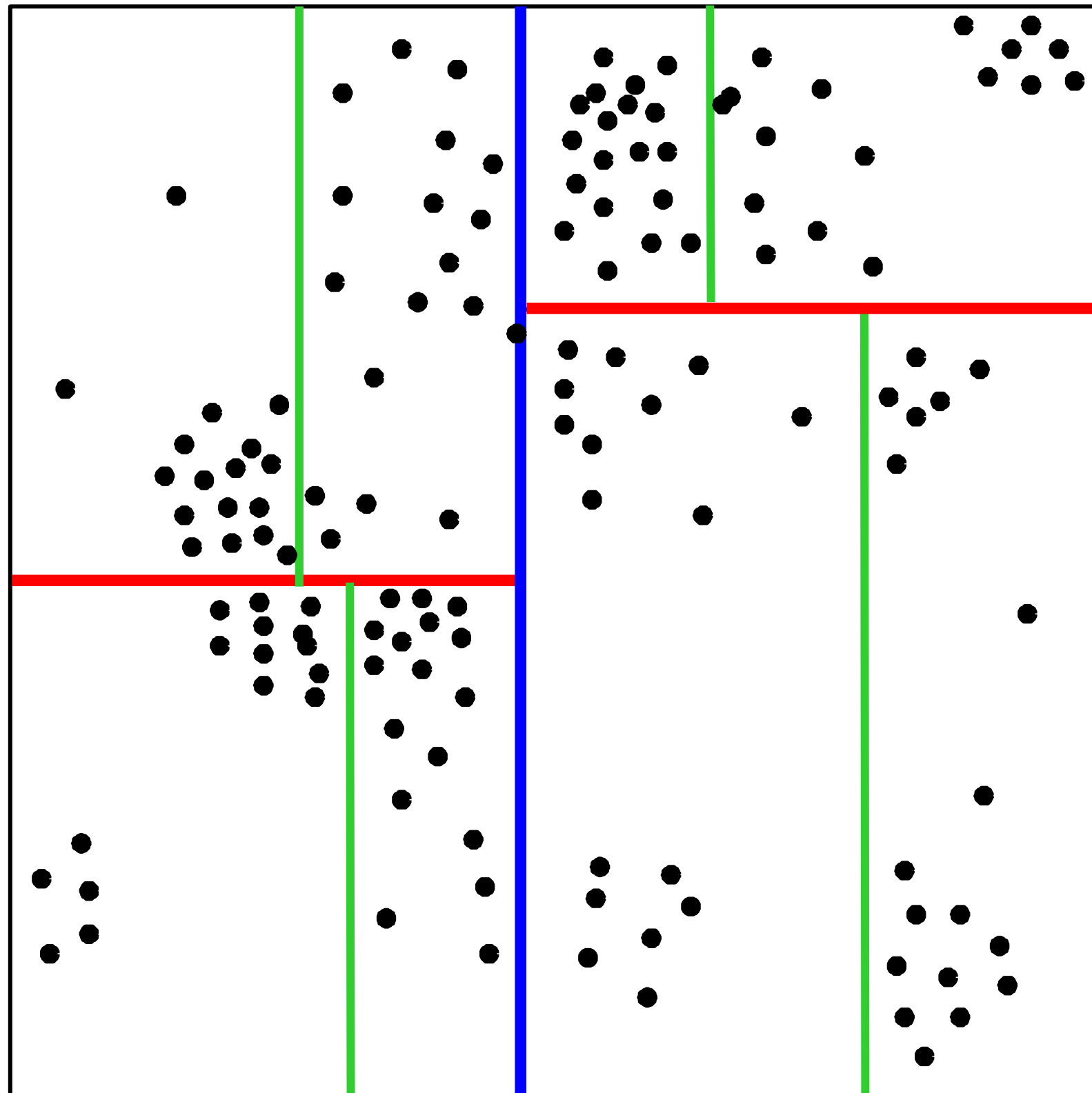


- Load balance is good even with static assignment because of random assignment
  - On average, each processor does approximately the same amount of work
- But **random assignment yields poor locality**
  - Significant amount of inherent communication
  - Significant amount of artifactual communication (fine-grained accesses to tree nodes)
- Common tension: **work balance vs. locality** (cost-zones get us both!)  
(similar to work balance vs. synchronization trade-offs in “work distribution” lecture)

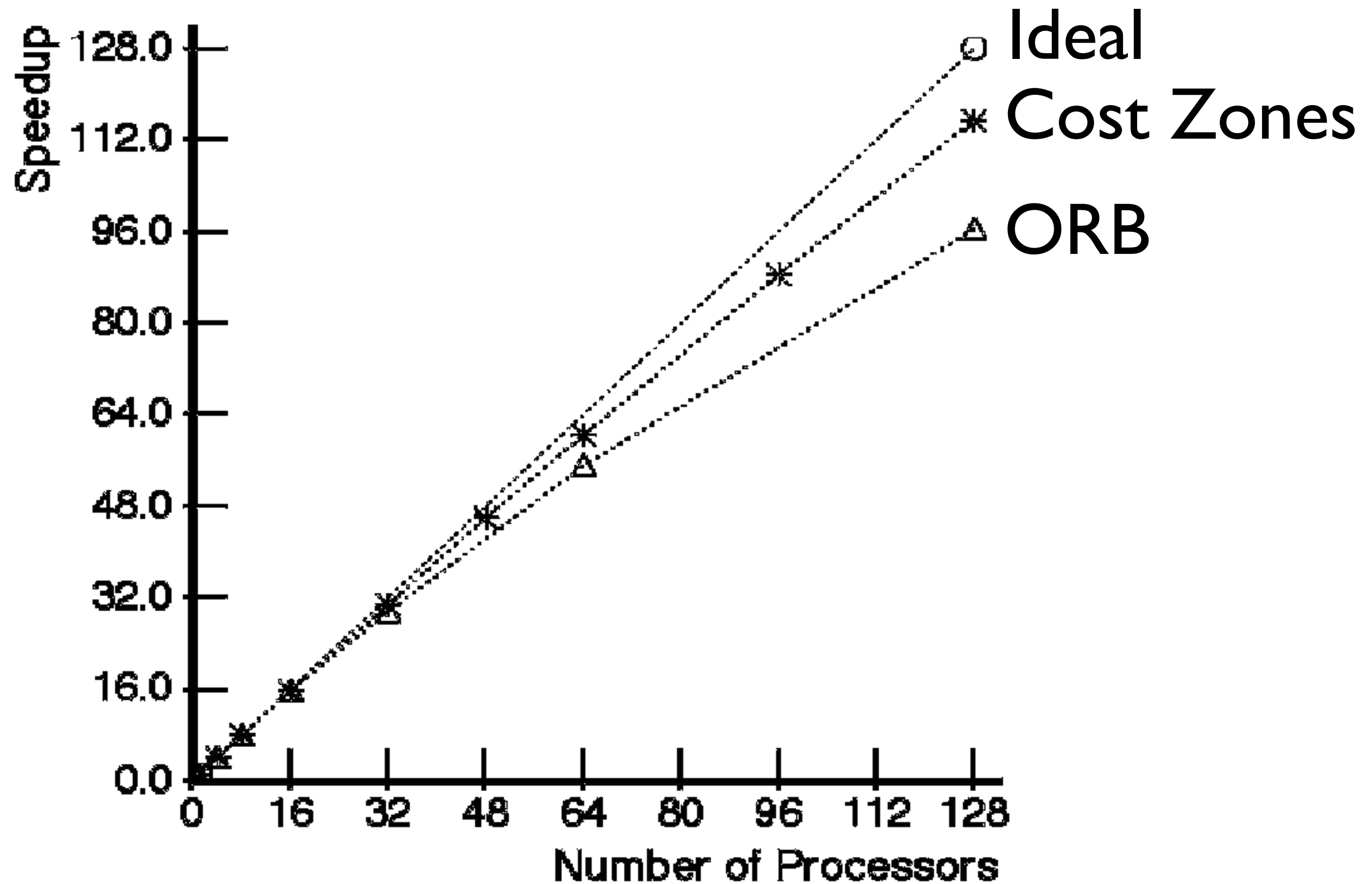
# Another Partitioning Approach: ORB

## ■ Orthogonal Recursive Bisection:

- **Recursively bisect** space into subspaces with **equal work**
- **Work is associated with bodies, as before**
- **Continue until one partition per processor**



# ORB vs. Cost Zones



- Extra work in ORB is the difference



# Summary

- **Today so far: two examples of parallel program optimization**
- **Key issues when discussing the applications**
  - **How to balance the work?**
  - **How to exploit locality inherent in the problem?**
  - **What synchronization is necessary?**

# Parallel Scan

# Data-parallel scan

**let**  $A = [a_0, a_1, a_2, a_3, \dots, a_{n-1}]$

**let**  $\oplus$  **be an associative binary operator with identity element**  $I$

**scan\_inclusive** $(\oplus, A) = [a_0, a_0 \oplus a_1, a_0 \oplus a_1 \oplus a_2, \dots$

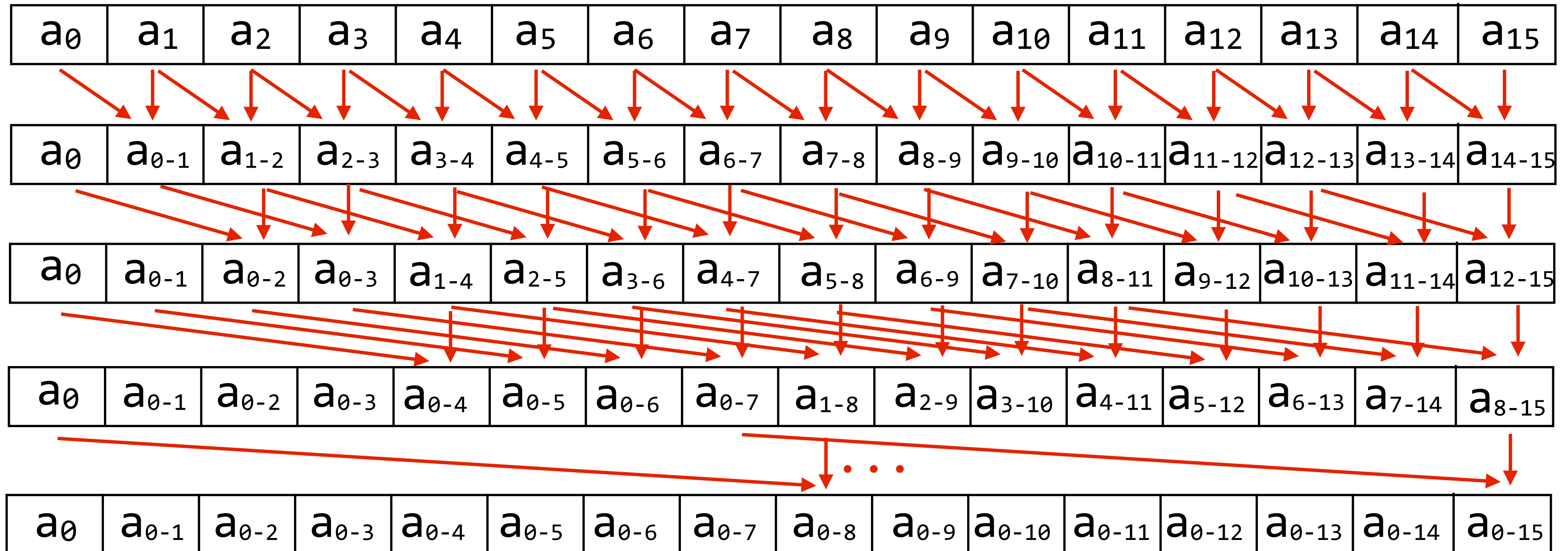
**scan\_exclusive** $(\oplus, A) = [I, a_0, a_0 \oplus a_1, \dots$

**If operator is**  $+$ , **then** **scan\_inclusive** $(+, A)$  **is a prefix sum**

**prefix\_sum** $(A) = [a_0, a_0 + a_1, a_0 + a_1 + a_2, \dots$

# Data-parallel inclusive scan

(Subtract original vector to get exclusive scan result: not shown)



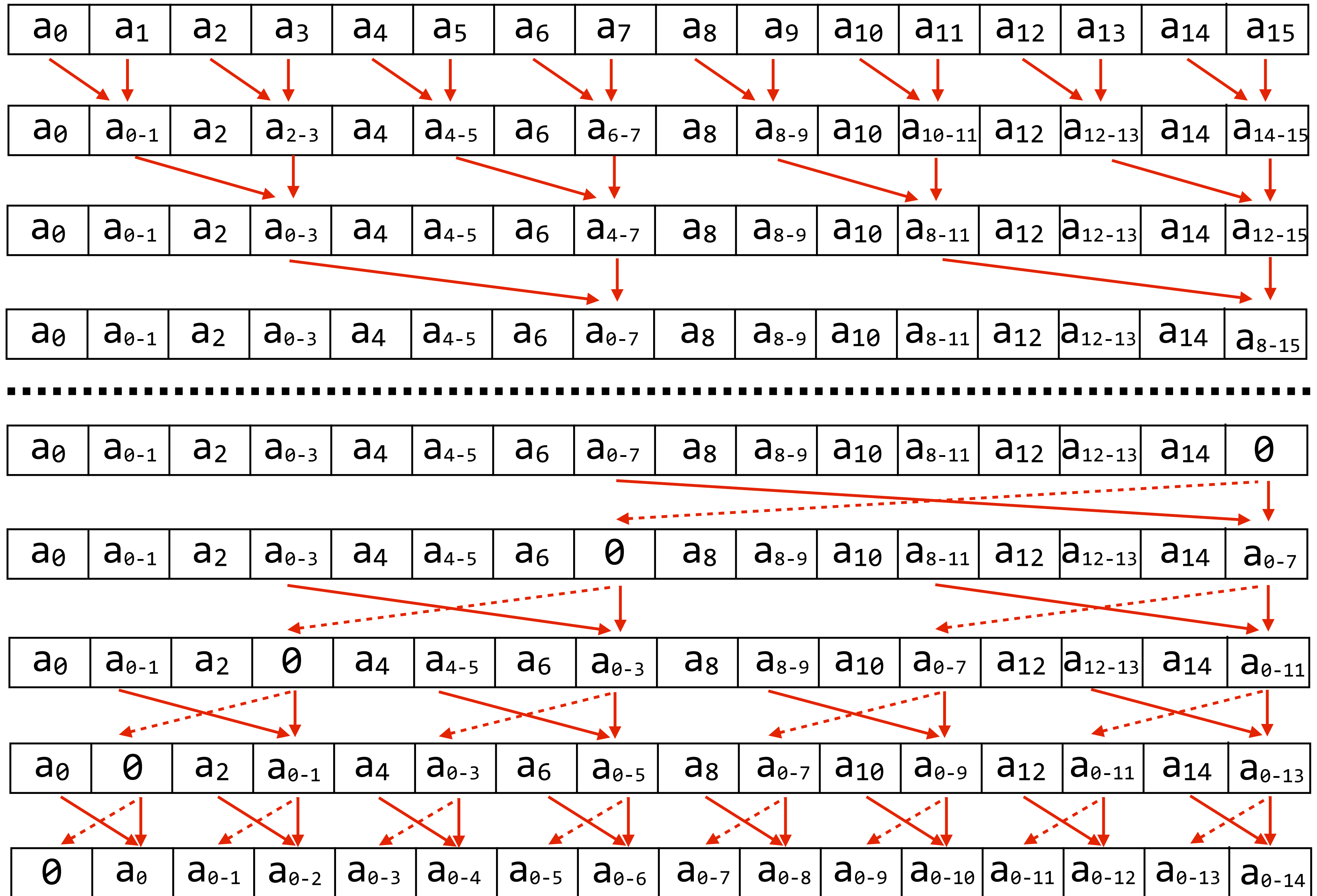
\* not showing all dependencies in last step

**Work:**  $O(N \lg N)$

**Span:**  $O(\lg N)$

**Inefficient compared to sequential algorithm!**

# Work-efficient parallel exclusive scan ( $O(N)$ work)



# Work efficient exclusive scan algorithm

(with  $\oplus = "+"$ )

## Up-sweep:

```
for d=0 to (log2n - 1) do
  forall k=0 to n-1 by 2d+1 do
    a[k + 2d+1 - 1] = a[k + 2d - 1] + a[k + 2d+1 - 1]
```

## Down-sweep:

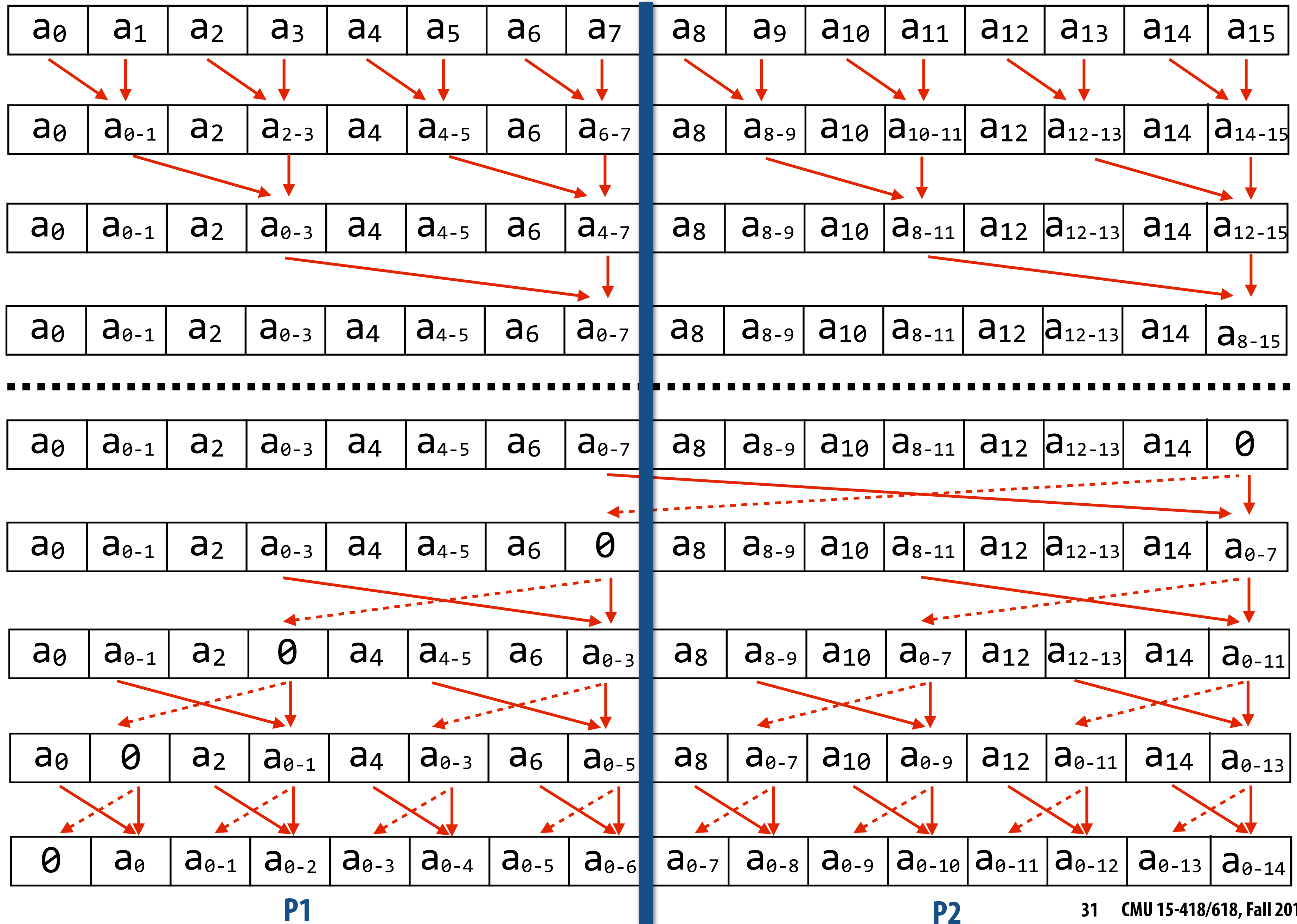
```
x[n-1] = 0
for d=(log2n - 1) down to 0 do
  forall k=0 to n-1 by 2d+1 do
    tmp = a[k + 2d - 1]
    a[k + 2d - 1] = a[k + 2d+1 - 1]
    a[k + 2d+1 - 1] = tmp + a[k + 2d+1 - 1]
```

**Work:**  $O(N)$  (but what is the constant?)

**Span:**  $O(\lg N)$  (but what is the constant?)

**Locality:** ??

# Now consider scan implementation on just **two cores**



# Exclusive scan: two processor implementation

$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$	$a_7$	$a_8$	$a_9$	$a_{10}$	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	$a_{15}$
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P1

Sequential scan on elements [0-7]

Let base =  $a_{0-7}$

Add base to elements  $a_8$  thru  $a_{8-11}$

P2

Sequential scan on elements [8-15]

Add base to elements  $a_{8-12}$  thru  $a_{8-15}$

**Work:  $O(N)$**  (but constant is now only 1.5)

**Data-access:**

- Very **high spatial locality** (contiguous memory access)
- P1's access to  $a_8$  through  $a_{8-11}$  may be more costly on large NUMA system, but on small-scale system access likely same cost as from P2



# Exclusive scan: SIMD implementation (in CUDA)

Example: perform exclusive scan on 32-element array: SPMD program, assume 32-wide SIMD execution

When `scan_warp` is run by a group of 32 CUDA threads, each thread returns the exclusive scan result for element `idx`

(also: upon completion `ptr [ ]` stores inclusive scan result)

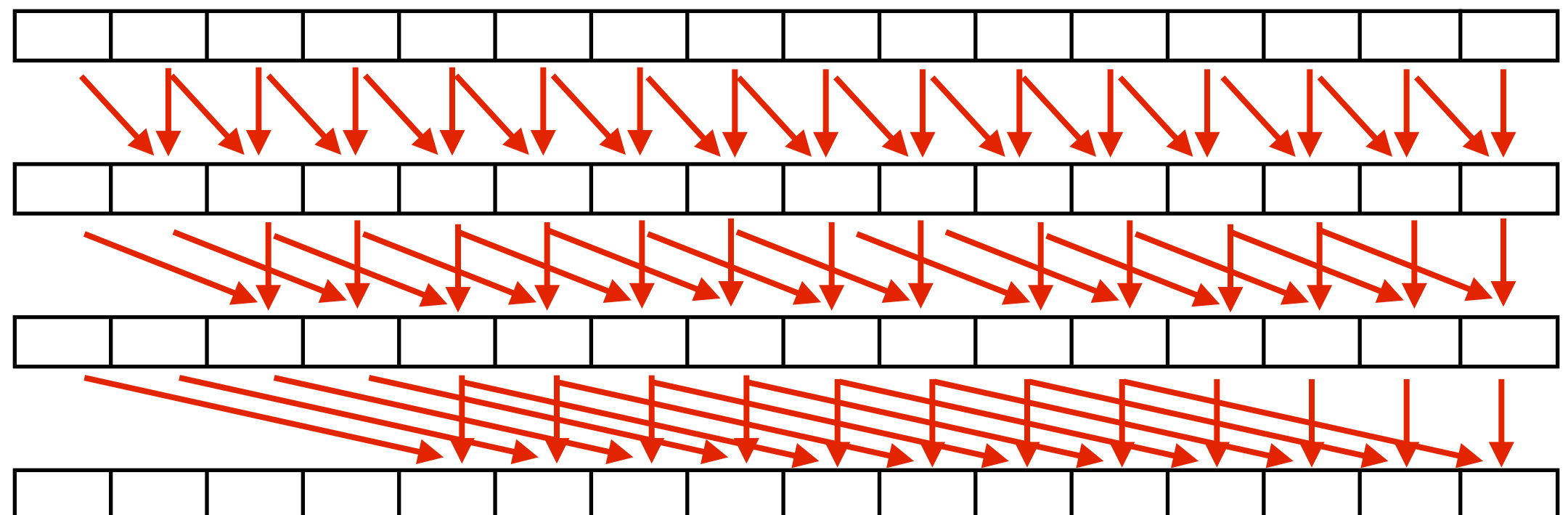
CUDA thread index of caller

```
__device__ int scan_warp(volatile int *ptr, const unsigned int idx)
{
    const unsigned int lane = idx & 31; // index of thread in warp (0..31)

    if (lane >= 1) ptr[idx] = ptr[idx - 1] + ptr[idx];
    if (lane >= 2) ptr[idx] = ptr[idx - 2] + ptr[idx];
    if (lane >= 4) ptr[idx] = ptr[idx - 4] + ptr[idx];
    if (lane >= 8) ptr[idx] = ptr[idx - 8] + ptr[idx];
    if (lane >= 16) ptr[idx] = ptr[idx - 16] + ptr[idx];

    return (lane > 0) ? ptr[idx-1] : 0;
}
```

Work: ??



# Exclusive scan: SIMD implementation (in CUDA)

CUDA thread  
index of caller



```
__device__ int scan_warp(volatile int *ptr, const unsigned int idx)
{
    const unsigned int lane = idx & 31; // index of thread in warp (0..31)

    if (lane >= 1) ptr[idx] = ptr[idx - 1] + ptr[idx];
    if (lane >= 2) ptr[idx] = ptr[idx - 2] + ptr[idx];
    if (lane >= 4) ptr[idx] = ptr[idx - 4] + ptr[idx];
    if (lane >= 8) ptr[idx] = ptr[idx - 8] + ptr[idx];
    if (lane >= 16) ptr[idx] = ptr[idx - 16] + ptr[idx];

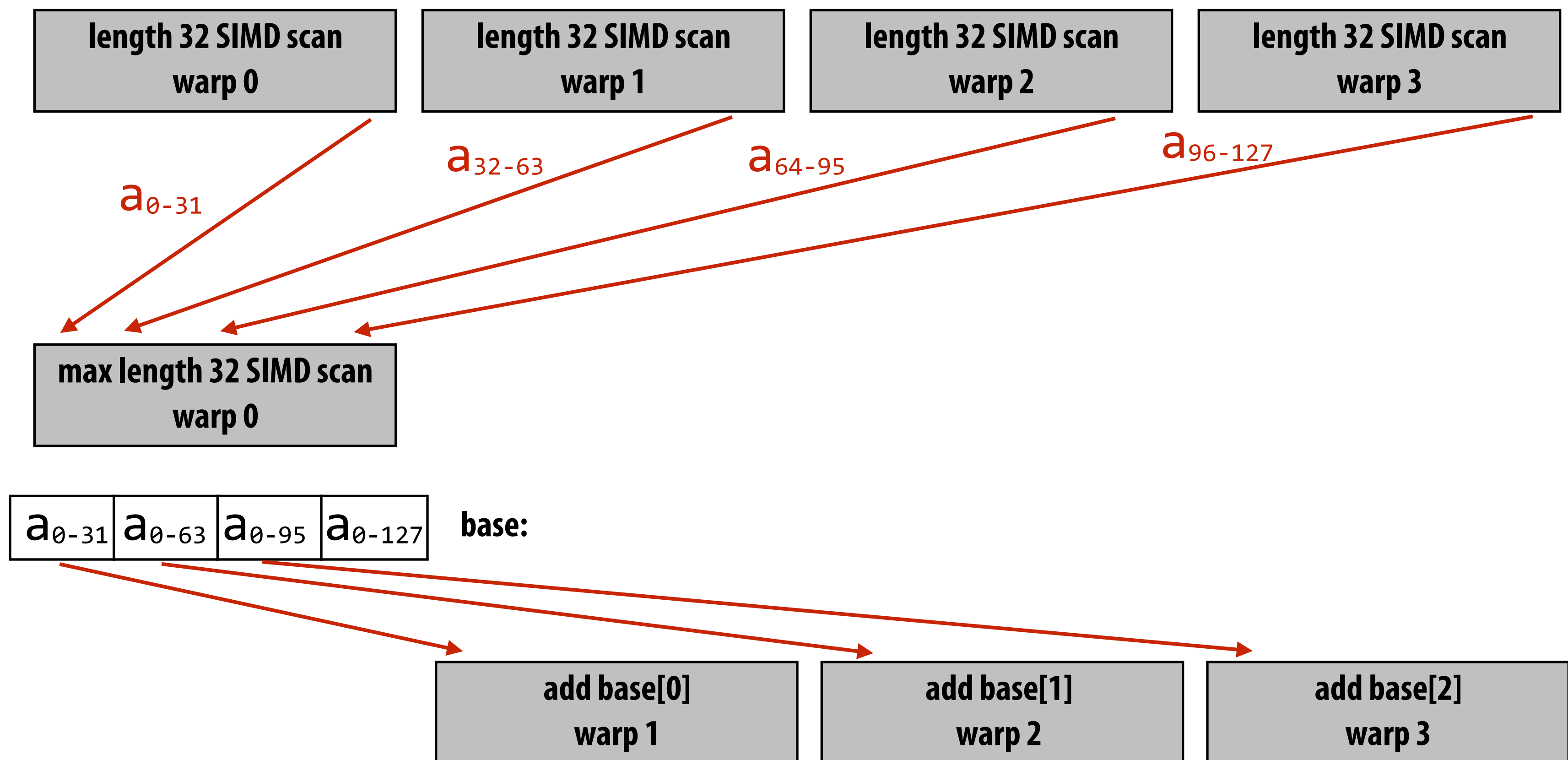
    return (lane > 0) ? ptr[idx-1] : 0;
}
```

**Work:  $N \lg(N)$**

**Work-efficient formulation of scan is not beneficial in this context because it results in low SIMD utilization. It would require more than 2x the number of instructions as the implementation above!**

# Building scan on larger array

Example: 128-element scan using four-warp thread block



# Multi-threaded, SIMD implementation

## Example: cooperating threads in a CUDA thread block perform scan

We provided similar code in assignment 2.

Code assumes length of array given by `ptr` is same as number of threads per block.

CUDA thread  
index of caller



```
__device__ void scan_block(volatile int *ptr, const unsigned int idx)
{
    const unsigned int lane = idx & 31;    // index of thread in warp (0..31)
    const unsigned int warp_id = idx >> 5; // warp index in block

    int val = scan_warp(ptr, idx);          // Step 1. per-warp partial scan
                                           // (Performed by all threads in block,
                                           // with threads in same warp communicating
                                           // through shared memory buffer 'ptr')

    if (lane == 31) ptr[warp_id] = ptr[idx]; // Step 2. thread 31 in each warp copies
    __syncthreads();                        // partial-scan bases in per-block
                                           // shared mem

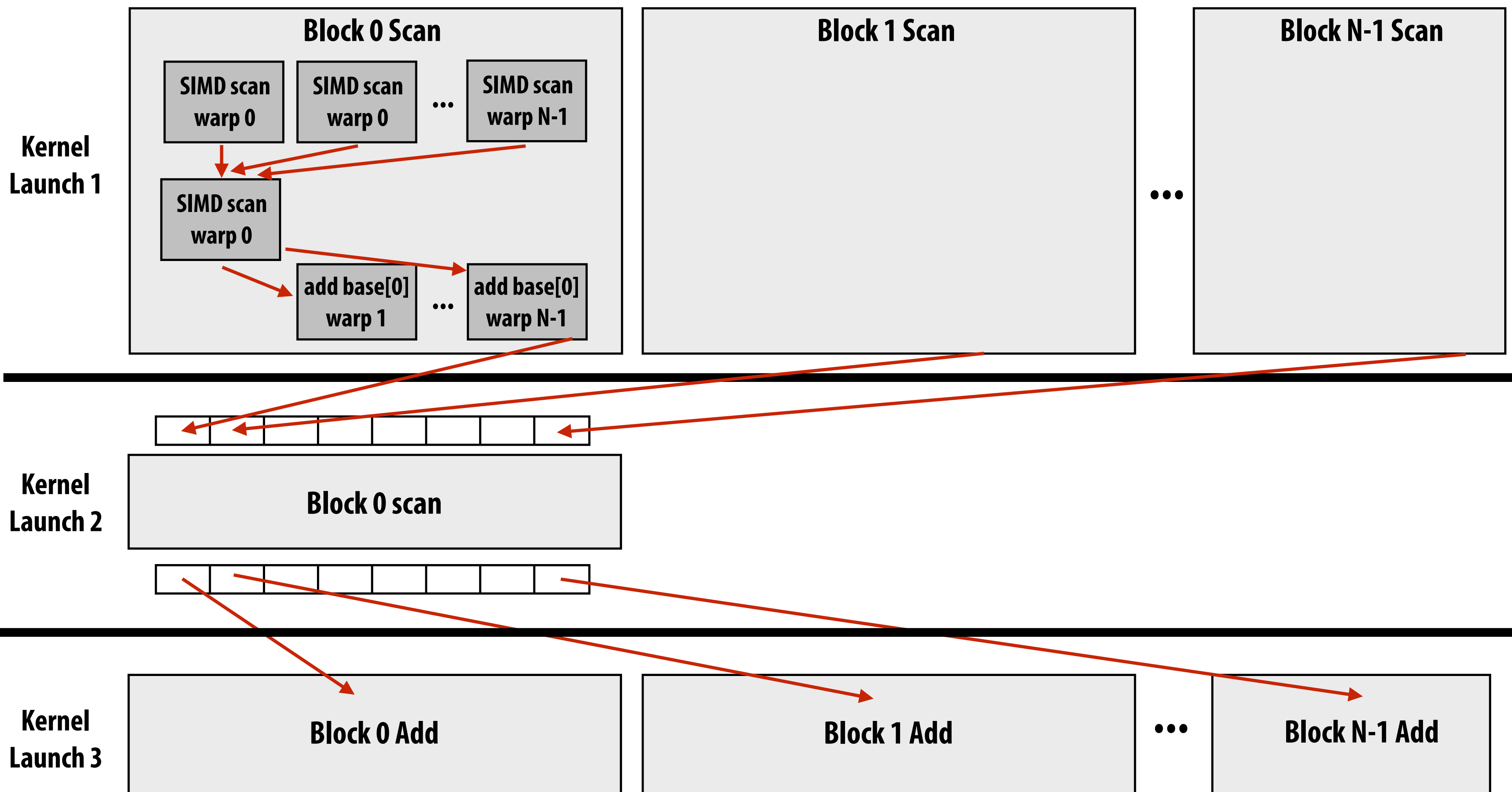
    if (warp_id == 0) scan_warp(ptr, idx);  // Step 3. scan to accumulate bases
    __syncthreads();                        // (only performed by warp 0)

    if (warp_id > 0)
        val = val + ptr[warp_id-1];        // Step 4. apply bases to all elements
    __syncthreads();                        // (performed by all threads in block)

    ptr[idx] = val;
}
```

# Building a larger scan

Example: one million element scan (1024 elements per block)



Exceeding 1 million elements requires partitioning phase two into multiple blocks

# Scan implementation

## ■ Parallelism

- Scan algorithm features  $O(N)$  parallel work
- But efficient implementations only leverage as much parallelism as required to make good utilization of the machine
  - Goal is to reduce work and reduce communication/synchronization

## ■ Locality

- Multi-level implementation to match memory hierarchy  
(CUDA example: per-block implementation carried out in local memory)

## ■ Heterogeneity: different strategy at different machine levels

- CUDA example: Different algorithm for intra-warp scan than inter-thread scan
- Low core count CPU example: based largely on sequential scan

# Parallel Segmented Scan

# Segmented scan

- **Common problem: operating on sequence of sequences**
- **Examples:**
  - **For each vertex in a graph:**
    - **For each edge incoming to vertex:**
  - **For each particle in simulation**
    - **For each particle within cutoff radius**
- **Also there's two levels of parallelism in the problem that a programmer might want to exploit**
- **But its irregular: the size of edge lists, particle neighbor lists, etc, may be very different from vertex to vertex (or particle to particle)**



# Segmented scan

- Generalization of scan
- Simultaneously perform scans on arbitrary contiguous partitions of input collection

```
let A = [[1,2],[6],[1,2,3,4]]
```

```
let ⊕ = +
```

```
segmented_scan_exclusive(⊕,A) = [[0,1], [0], [0,1,3,6]]
```

**We'll assume a simple "head-flag" representation:**

```
A = [[1,2,3],[4,5,6,7,8]]
```

```
flag: 0 0 0 1 0 0 0 0
```

```
data: 1 2 3 4 5 6 7 8
```

# Work-efficient segmented scan

(with  $\oplus = "+"$ )

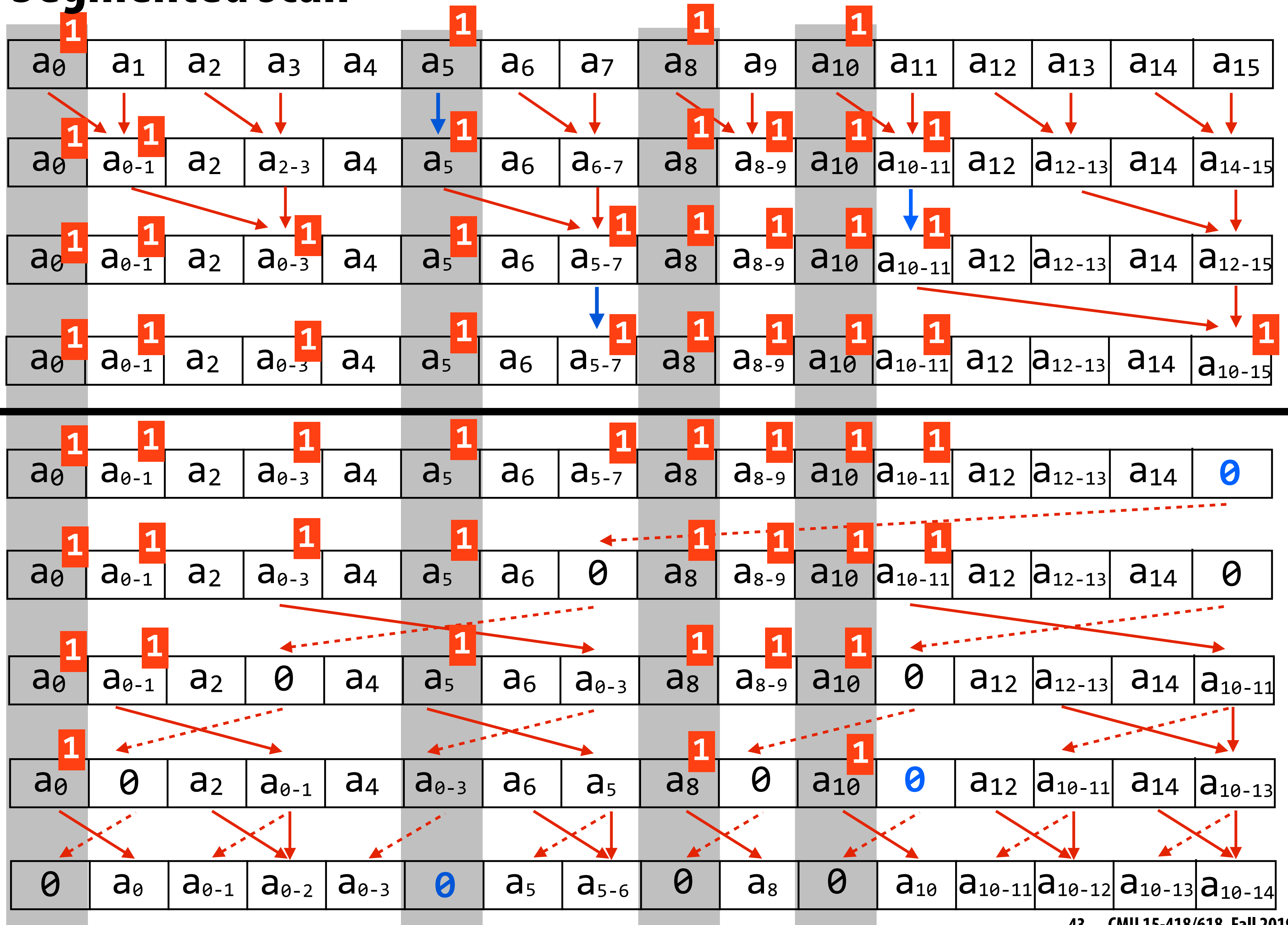
## Up-sweep:

```
for d=0 to (log2n - 1) do:
  forall k=0 to n-1 by 2d+1 do:
    if flag[k + 2d+1 - 1] == 0:
      data[k + 2d+1 - 1] = data[k + 2d - 1] + data[k + 2d+1 - 1]
    flag[k + 2d+1 - 1] = flag[k + 2d - 1] || flag[k + 2d+1 - 1]
```

## Down-sweep:

```
data[n-1] = 0
for d=(log2n - 1) down to 0 do:
  forall k=0 to n-1 by 2d+1 do:
    tmp = data[k + 2d - 1]
    data[k + 2d - 1] = data[k + 2d+1 - 1]
    if flag_original[k + 2d] == 1:           # must maintain copy of original flags
      data[k + 2d+1 - 1] = 0                 # start of segment
    else if flag[k + 2d - 1] == 1:
      data[k + 2d+1 - 1] = tmp
    else:
      data[k + 2d+1 - 1] = tmp + data[k + 2d+1 - 1]
    flag[k + 2d - 1] = 0
```

# Segmented scan



# Scan/segmented scan summary

## ■ Scan

- **Parallel implementation of (intuitively sequential application)**
- **Theory: parallelism linear in number of elements**
- **Practice: exploit locality, use only as much parallelism as necessary to fill the machine**
  - **Great example of applying different strategies at different levels of the machine**

## ■ Segmented scan

- **Express computation and operate on irregular data structures (e.g., list of lists) in a regular, data parallel way**

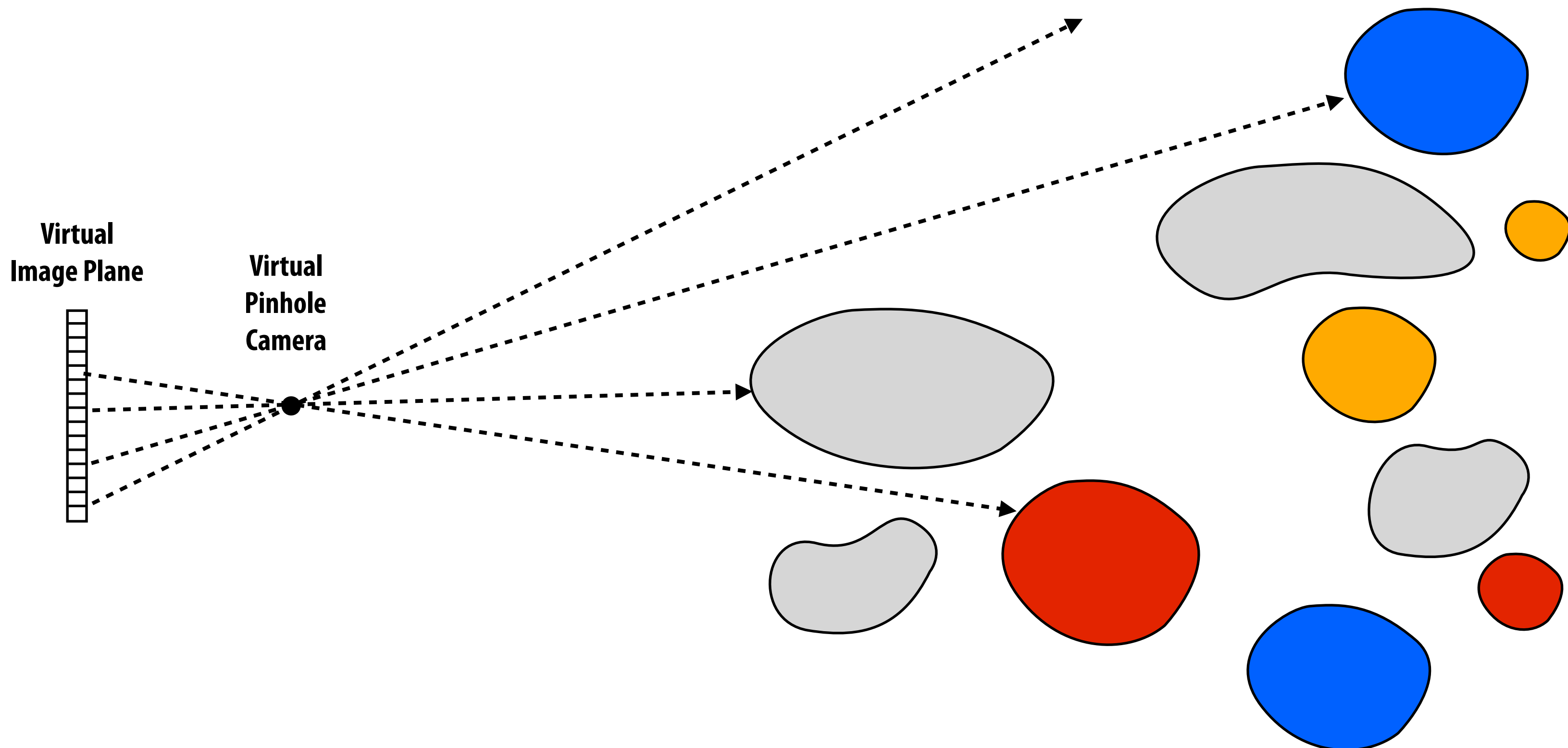
# **Parallel Ray Tracing on SIMD Architectures**

**(since many students always ask about parallel ray tracing)**

# Ray tracing

**Problem statement:**

**Given a “ray”, find closest intersection with scene geometry**



**Simplest ray tracer:**

**For each image pixel, shoot ray from camera through pixel into scene.**

**Color pixel according to first surface hit.**

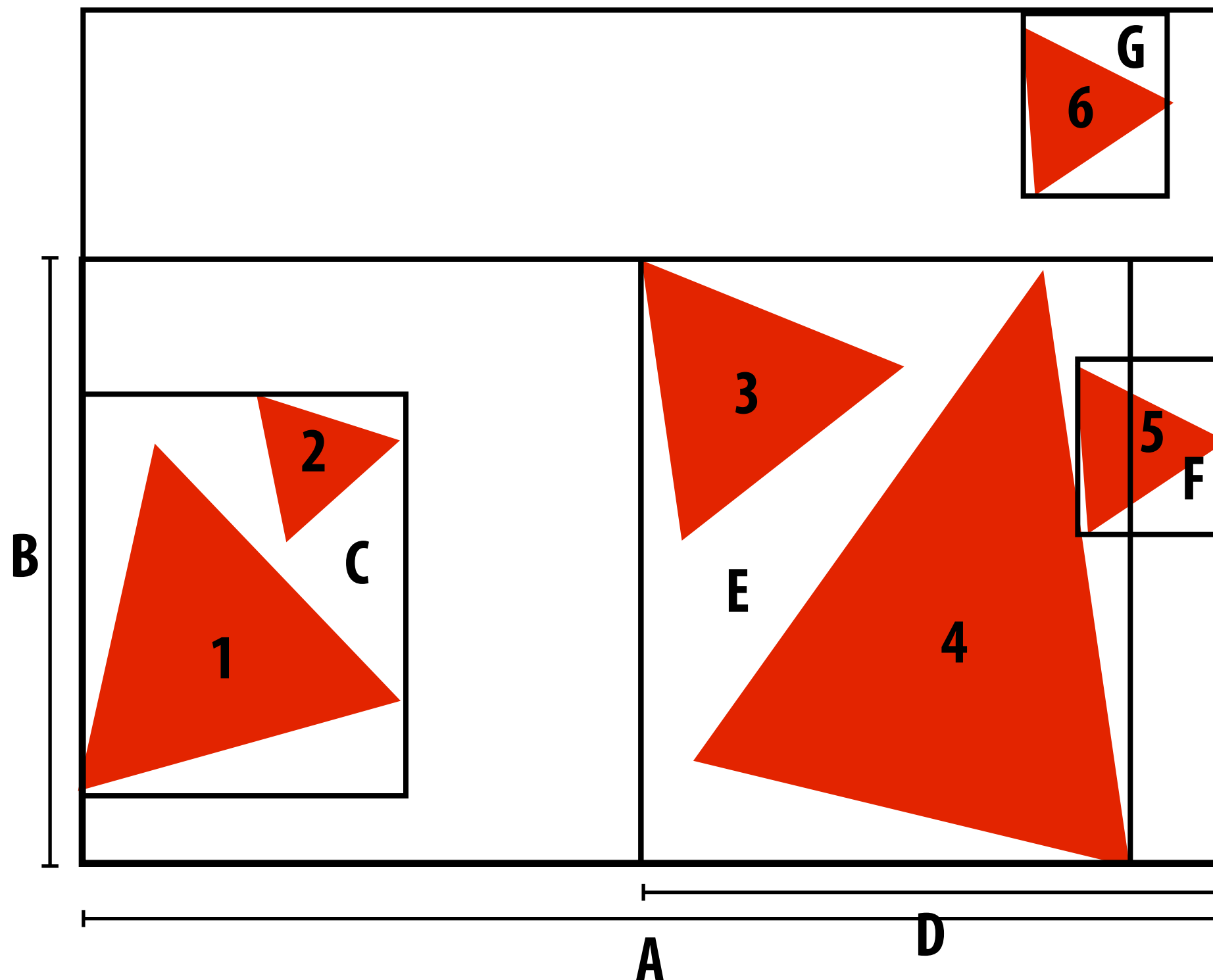
# Accelerating ray-scene intersection

Preprocess scene to build data structure that accelerates finding “closest” geometry along ray

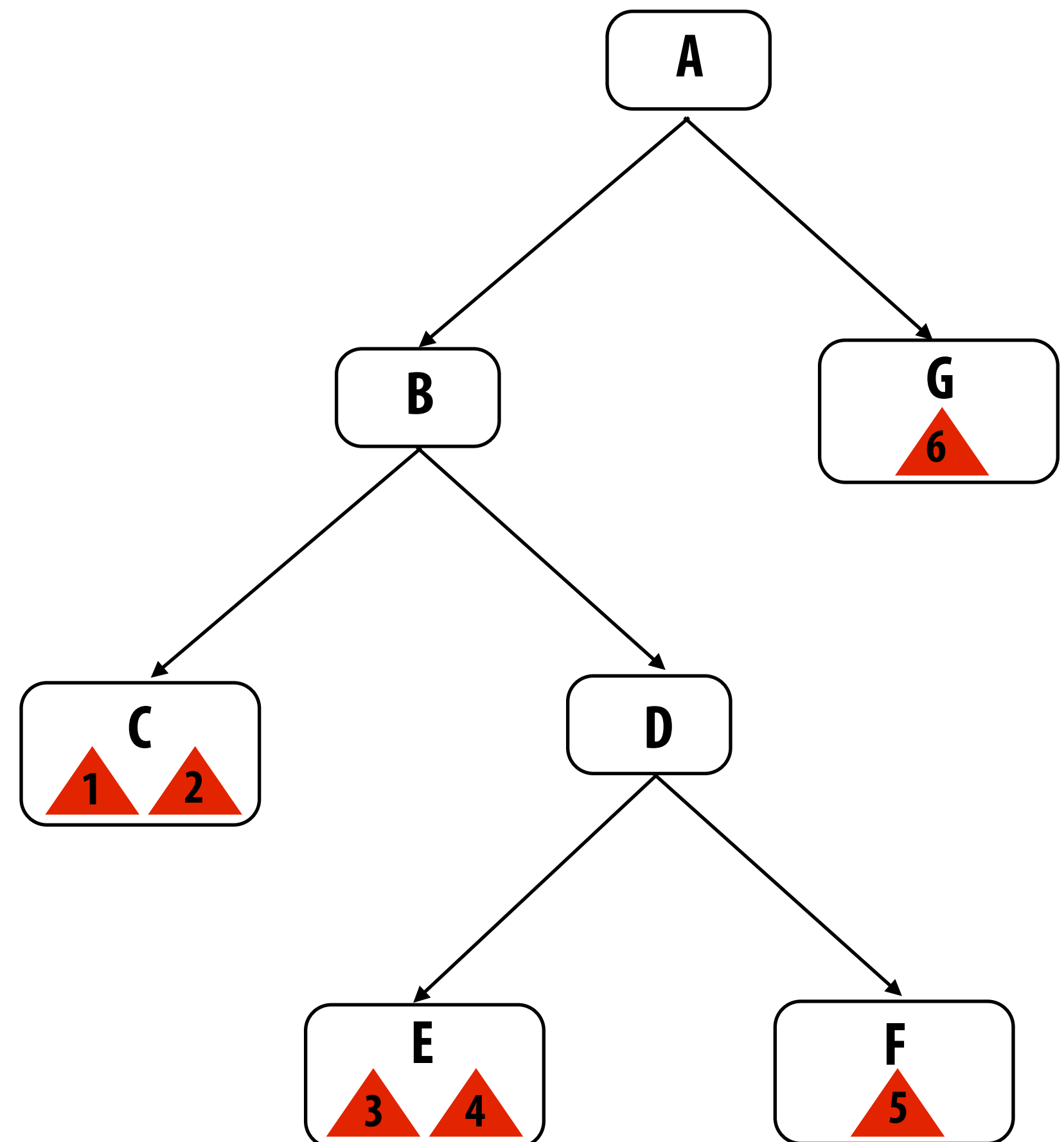
Idea: group objects with spatial proximity (like quad-tree in Barnes-Hut)

- Hierarchical grouping adapts to non-uniform density of scene objects

Scene objects (in 2D)



Bounding Volume Hierarchy (BVH)  
(Binary tree organizing the scene)



# Parallelize across rays

- **Simultaneously intersect multiple rays with scene**
- **Different cores trace different rays in parallel**
  - Trivial “embarrassingly parallel” implementation
- **But how to leverage SIMD parallelism within a core?**
- **Today: we’ll discuss one approach: ray packets**
  - Code is explicitly written to trace N rays at a time, not 1 ray



# Simple ray tracer (using a BVH)

```
// stores information about closest hit found so far
struct ClosestHitInfo {
    Primitive primitive;
    float distance;
};

trace(Ray ray, BVHNode node, ClosestHitInfo hitInfo)
{
    if (!intersect(ray, node.bbox) || (closest point on box is farther than hitInfo.distance))
        return;

    if (node.leaf) {
        for (each primitive in node) {
            (hit, distance) = intersect(ray, primitive);
            if (hit && distance < hitInfo.distance) {
                hitInfo.primitive = primitive;
                hitInfo.distance = distance;
            }
        }
    } else {
        trace(ray, node.leftChild, hitInfo);
        trace(ray, node.rightChild, hitInfo);
    }
}
```

# Ray packet tracing

[Wald et al. 2001]

Program explicitly intersects a collection of rays against BVH at once

```
RayPacket
```

```
{  
    Ray rays[PACKET_SIZE];  
    bool active[PACKET_SIZE];  
};
```

```
trace(RayPacket rays, BVHNode node, ClosestHitInfo packetHitInfo)
```

```
{  
    if (!ANY_ACTIVE_intersect(rays, node.bbox) ||  
        (closest point on box (for all active rays) is farther than hitInfo.distance))  
        return;
```

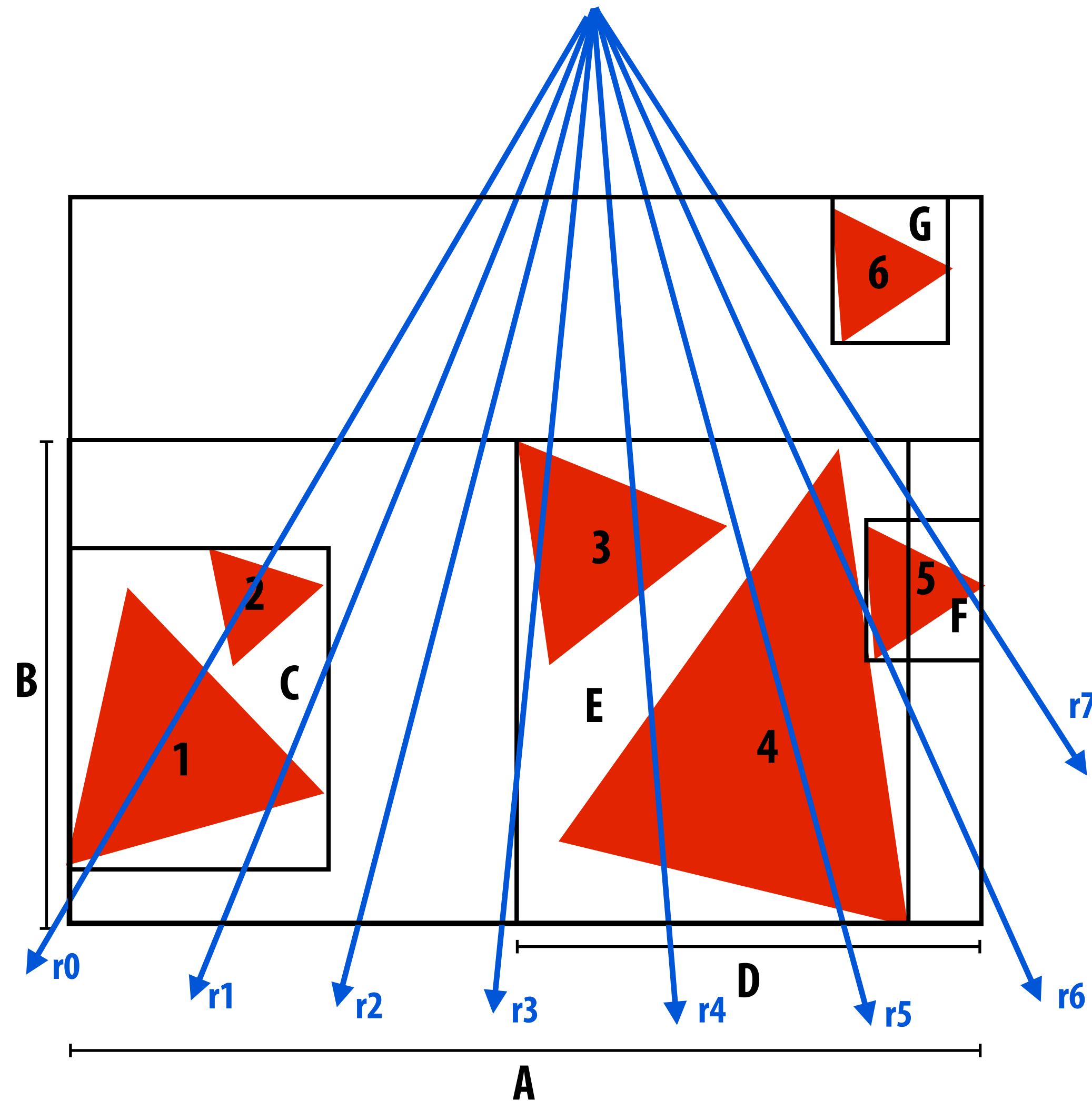
```
    update packet active mask
```

```
    if (node.leaf) {  
        for (each primitive in node) {  
            for (each ACTIVE ray r in packet) {  
                (hit, distance) = intersect(ray, primitive);  
                if (hit && distance < hitInfo.distance) {  
                    hitInfo[r].primitive = primitive;  
                    hitInfo[r].distance = distance;  
                }  
            }  
        }  
    }
```

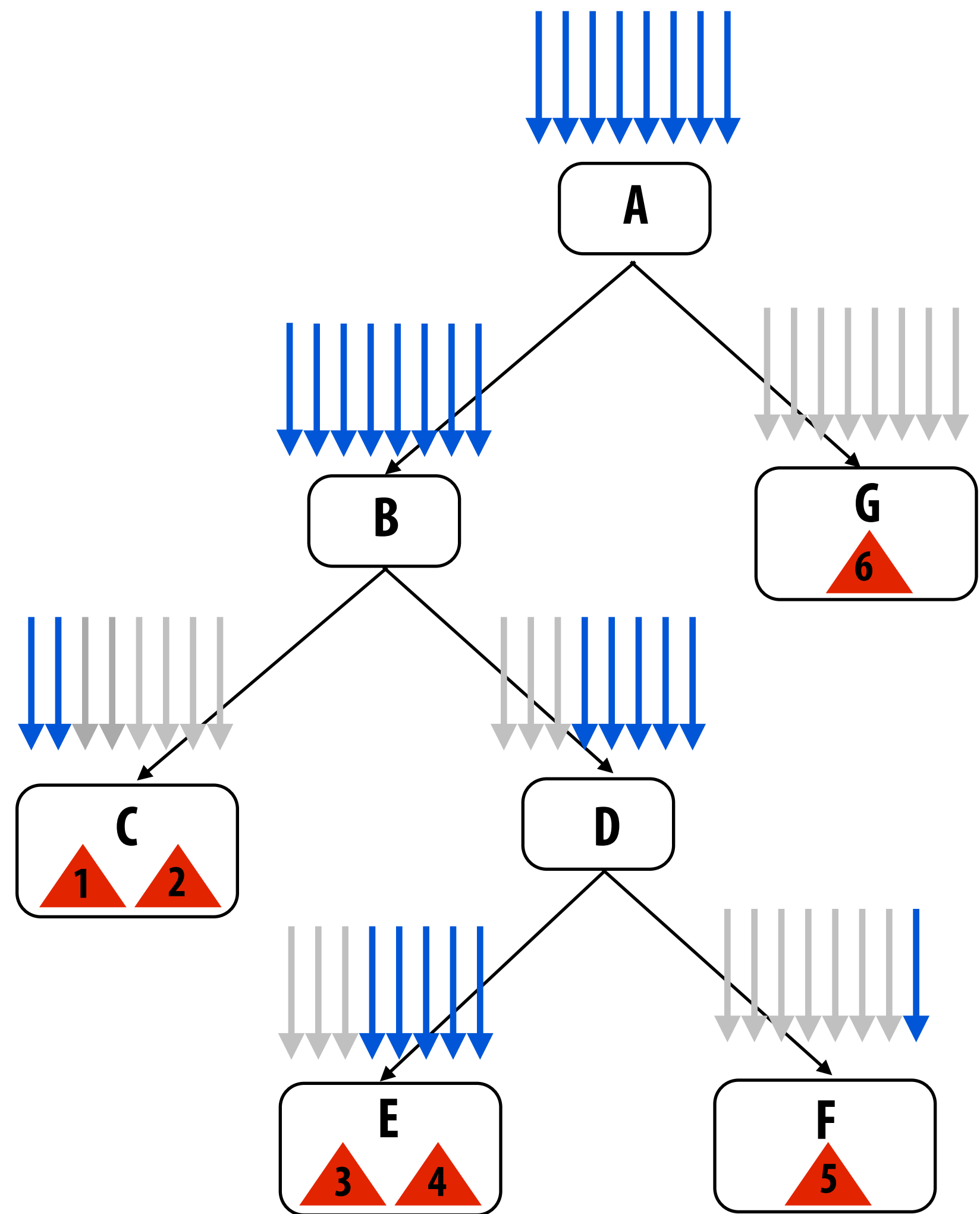
```
    } else {  
        trace(rays, node.leftChild, hitInfo);  
        trace(rays, node.rightChild, hitInfo);  
    }
```

```
}
```

# Ray packet tracing



Blue = active rays after node box test



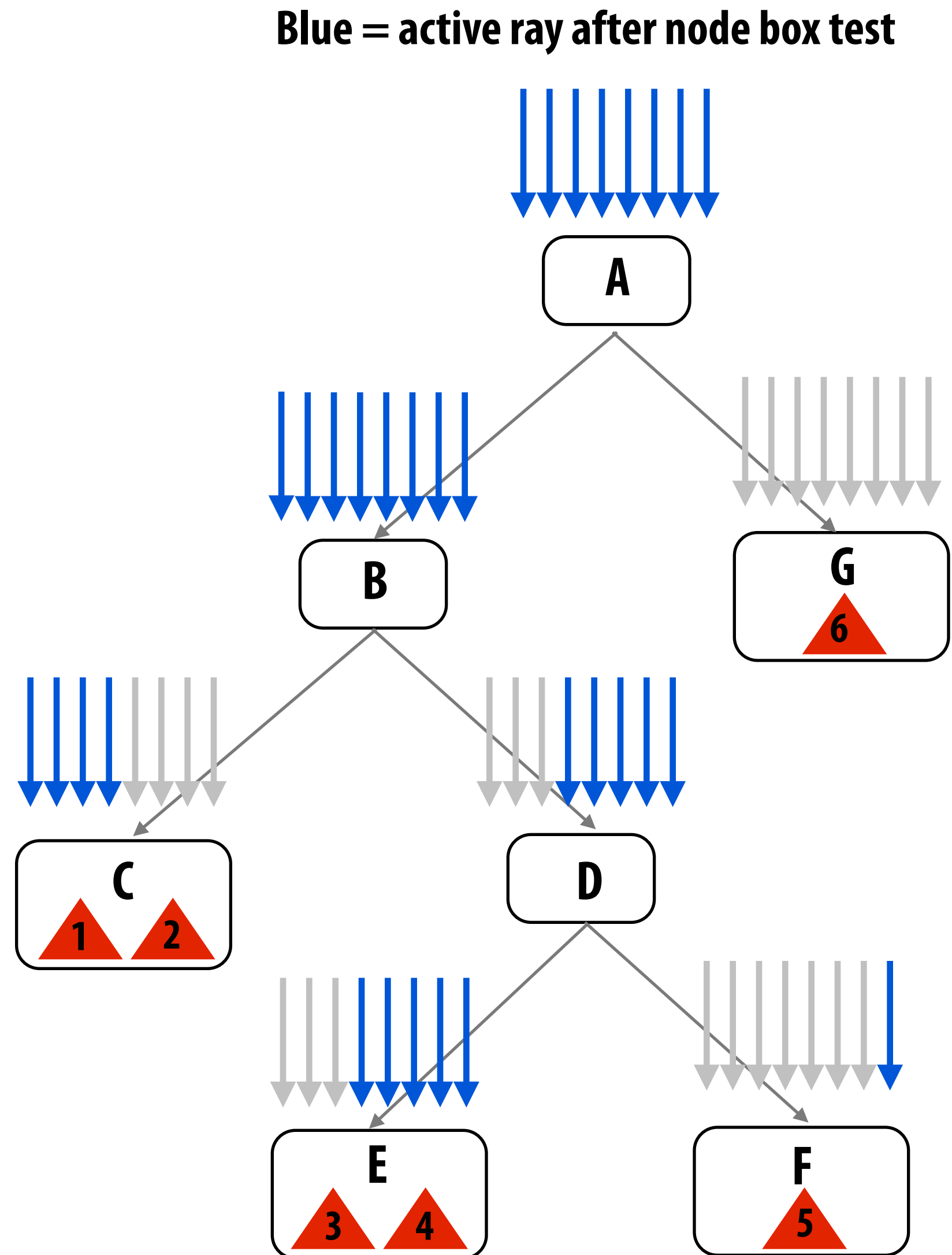
Note:  $r_6$  does not pass node F box test due to closest-so-far check, and thus does not visit F

# Advantages of packets

- **Map packet operations to wide SIMD execution**
  - One vector lane per ray
- **Amortize BVH data fetch: all rays in packet visit node at same time**
  - Load BVH node once for all rays in packet (not once per ray)
  - **Note: there is value to making packets bigger than SIMD width! (e.g., size = 64)**
- **Amortize work (packets are hierarchies over rays)**
  - Use interval arithmetic to conservatively test entire set of rays against node bbox (e.g., think of a packet as a beam)
  - Further arithmetic optimizations possible when all rays share origin
  - **Note: there is value to making packets much bigger than SIMD width!**

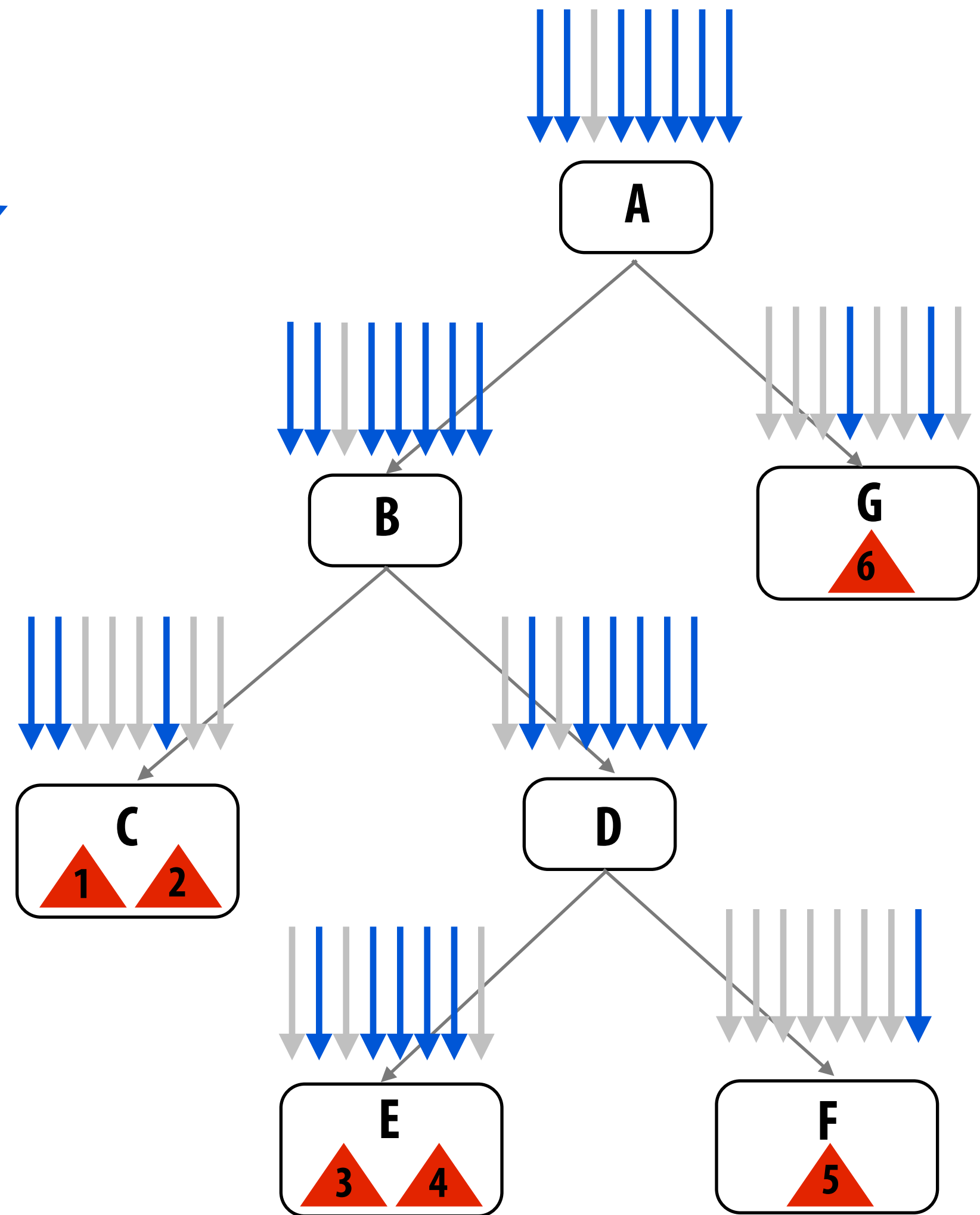
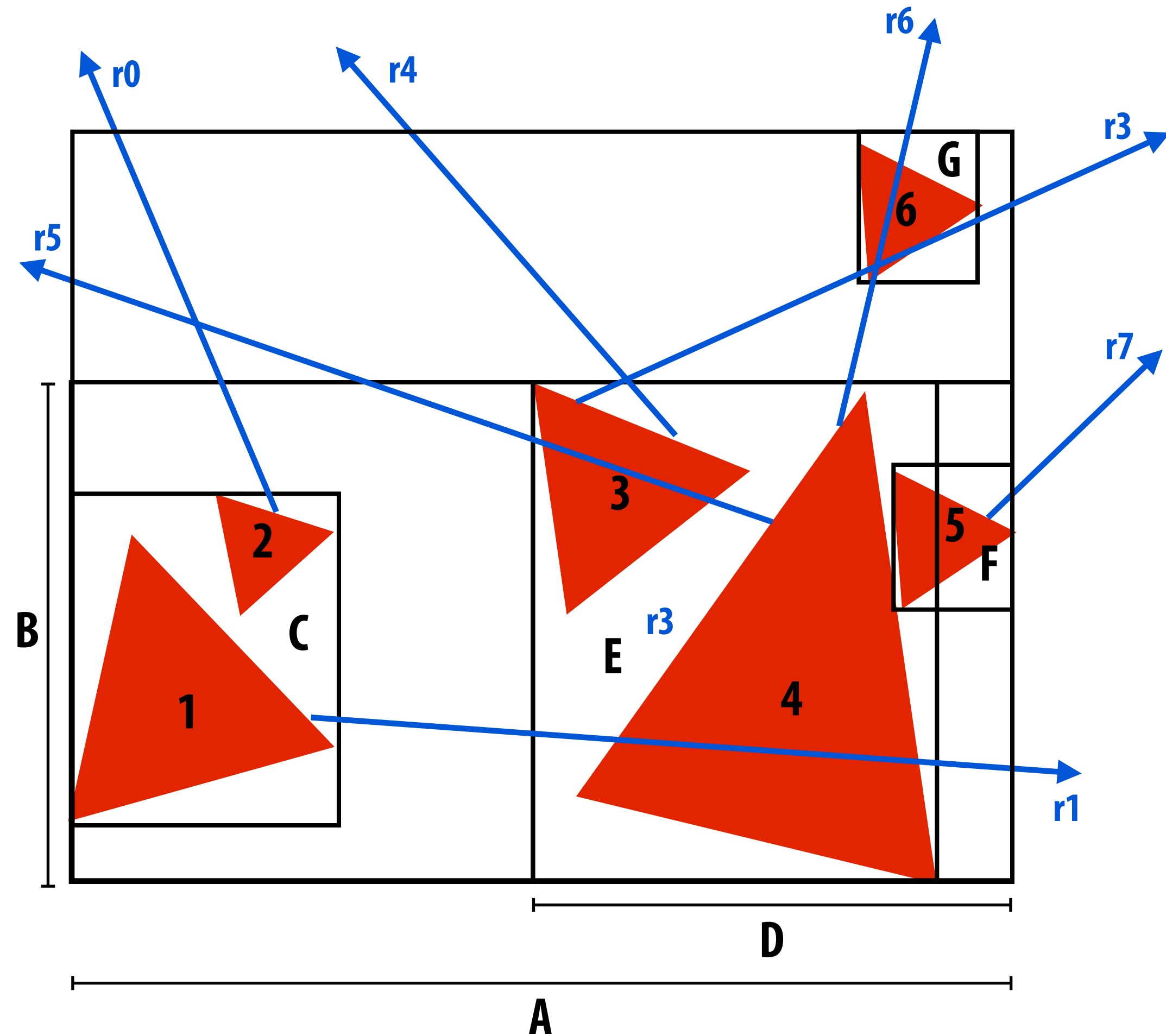
# Disadvantages of packets

- If any ray must visit a node, it drags all rays in the packet along with it)
- Loss of efficiency: node traversal, intersection, etc. amortized over less than a packet's worth of rays
- Not all SIMD lanes doing useful work



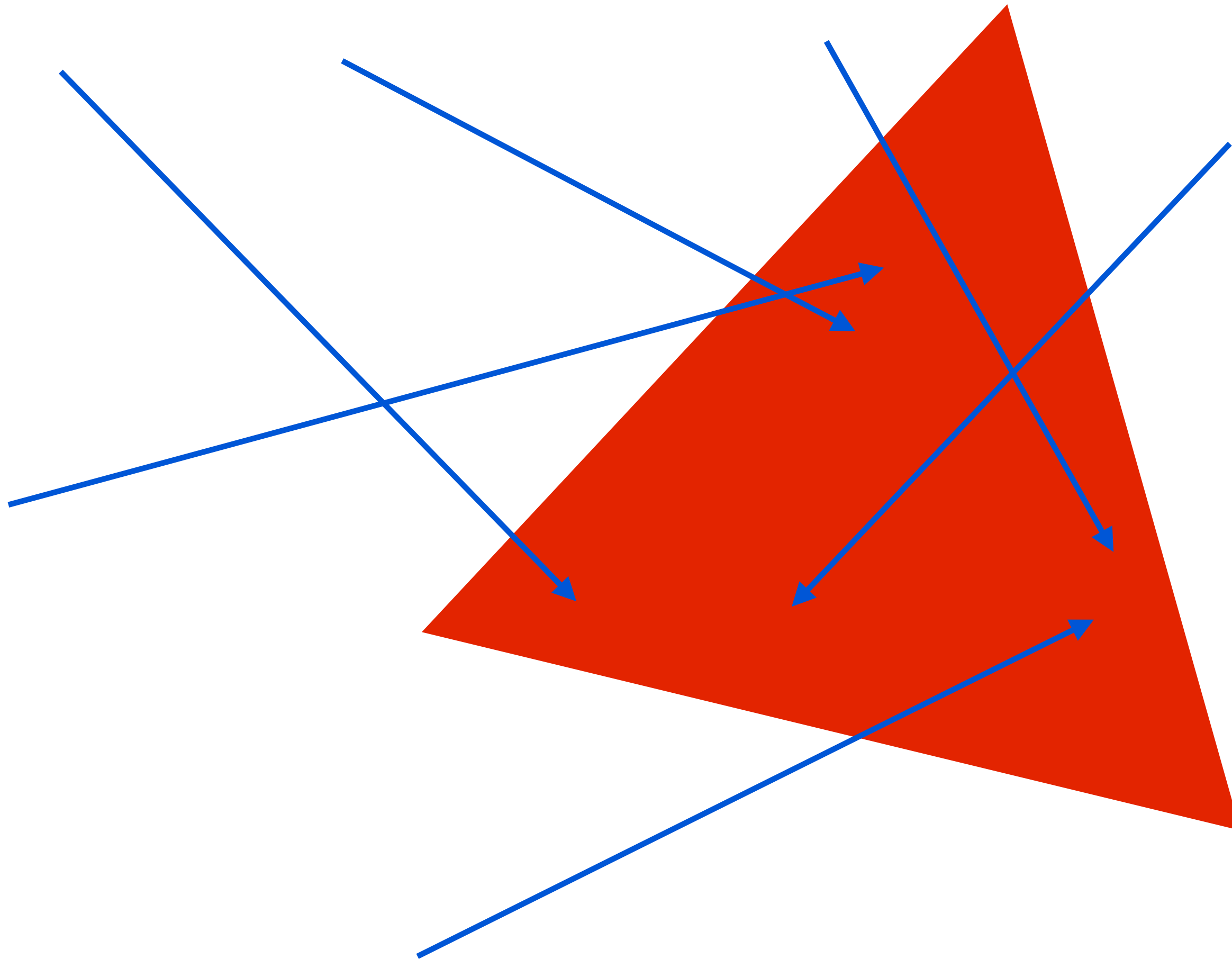
# Ray packet tracing: incoherent rays

Blue = active ray after node box test



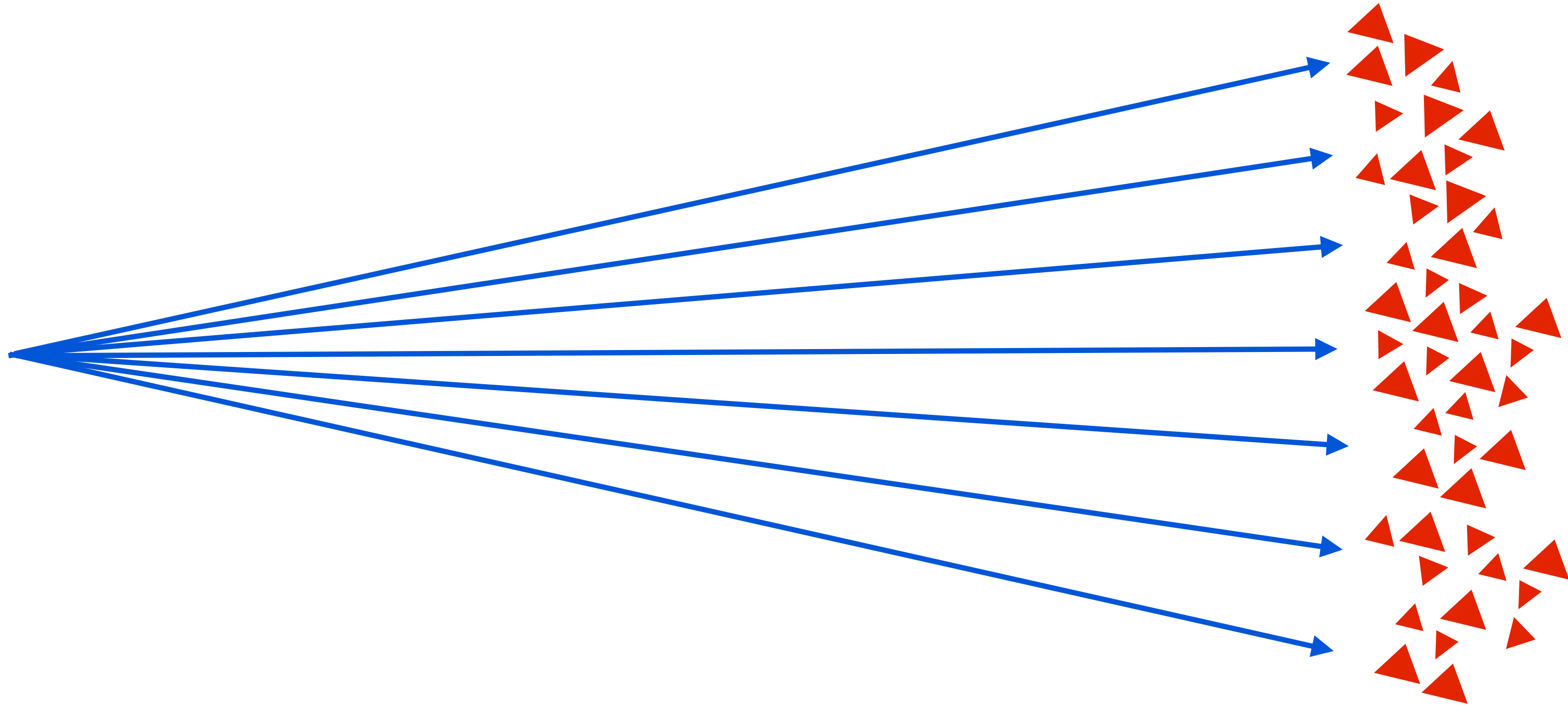
When rays are incoherent, benefit of packets can decrease significantly. This example: packet visits all tree nodes. (So all eight rays visit all tree nodes! No culling benefit!)

# Incoherence is a property of both the rays and the scene



**Random rays are “coherent” with respect to the BVH if the scene is one big triangle!**

# Incoherence is a property of both the rays and the scene



**Camera rays become “incoherent” with respect to lower nodes in the BVH if a scene is overly detailed**

**(Side note: this suggests the importance of choosing the right geometric level of detail)**



# Improving packet tracing with ray reordering

[Boulos et al. 2008]

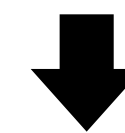
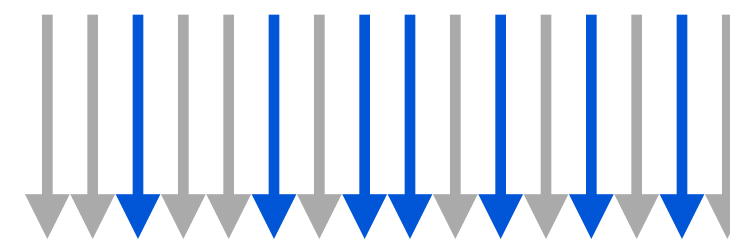
**Idea: when packet utilization drops below threshold, re-sort rays and continue with smaller packet**

- **Increases SIMD utilization**
- **Amortization benefits of smaller packets, but not large packets**

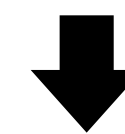
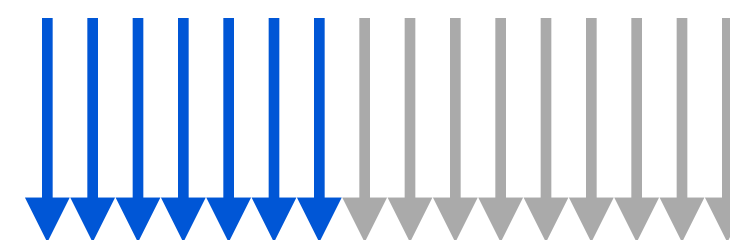
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**Example: consider 8-wide SIMD processor and 16-ray packets  
(2 SIMD instructions required to perform each operation on all rays in packet)**

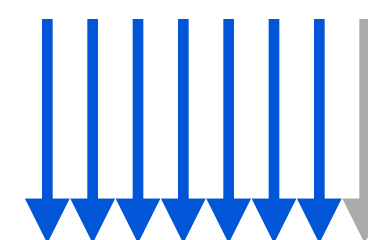
**16-ray packet: 7 of 16 rays active**



**Reorder rays  
Recompute intervals/bounds for active rays**



**Continue tracing with 8-ray packet:  
7 of 8 rays active**



# Giving up on packets

- **Even with reordering, ray coherence during BVH traversal will diminish**
  - **Diffuse bounces result in essentially random ray distribution**
  - **High-resolution geometry encourages incoherence near leaves of tree**
- **In these situations there is little benefit to packets (can even decrease performance compared to single ray code)**

# Packet tracing best practices

- **Use large packets for eye/reflection/point light shadow rays or higher levels of BVH** [Wald et al. 2007]
  - Ray coherence always high at the top of the tree
- **Switch to single ray (intra-ray SIMD) when packet utilization drops below threshold** [Benthin et al. 2011]
  - For wide SIMD machine, a branching-factor-4 BVH works well for both packet traversal and single ray traversal
- **Can use packet reordering to postpone time of switch** [Boulos et al. 2008]
  - Reordering allows packets to provide benefit deeper into tree
  - Not often used in practice due to high implementation complexity

# Summary

- **Today we looked at several different parallel programs**
- **Key questions:**
  - **What are the dependencies?**
  - **What synchronization is necessary?**
  - **How to balance the work?**
  - **How to exploit locality inherent in the problem?**
- **Trends**
  - **Only need enough parallelism to keep all processing elements busy (e.g., data-parallel scan vs. simple multi-core scan)**
  - **Different parallelization strategies may be applicable under different workloads (packets vs. no packets) or different locations in the machine (different implementations of scan internal and external to warp)**