Lecture 24:

Parallel Deep Neural Networks

Parallel Computer Architecture and Programming CMU 15-418/15-618, Fall 2019

Training/evaluating deep neural networks

Technique leading to many high-profile Al advances in recent years

Speech recognition/natural language processing

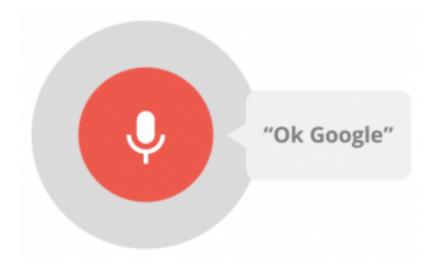
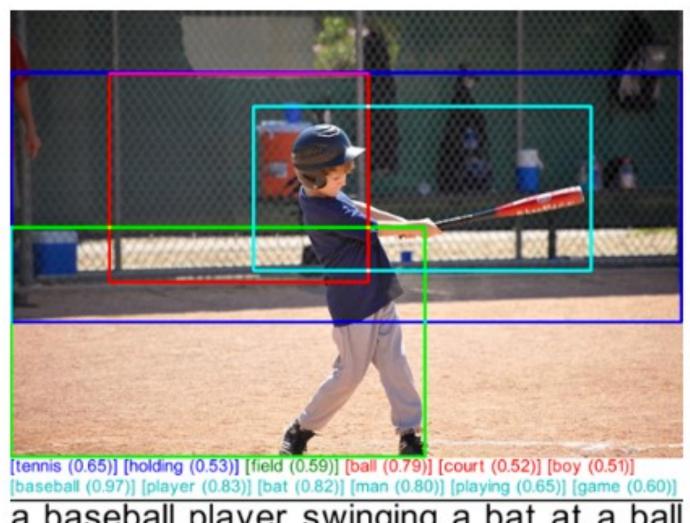
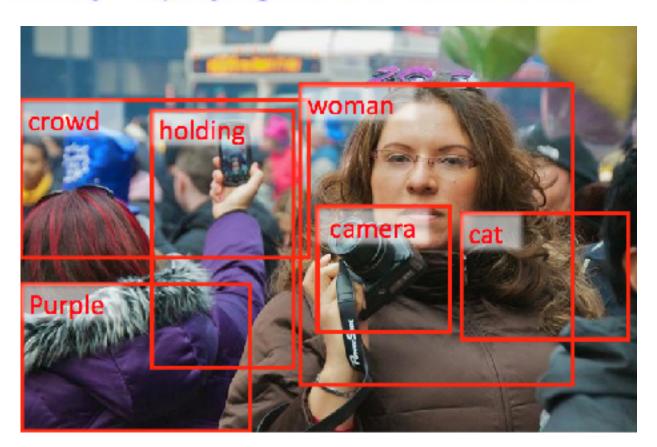


Image interpretation and understanding





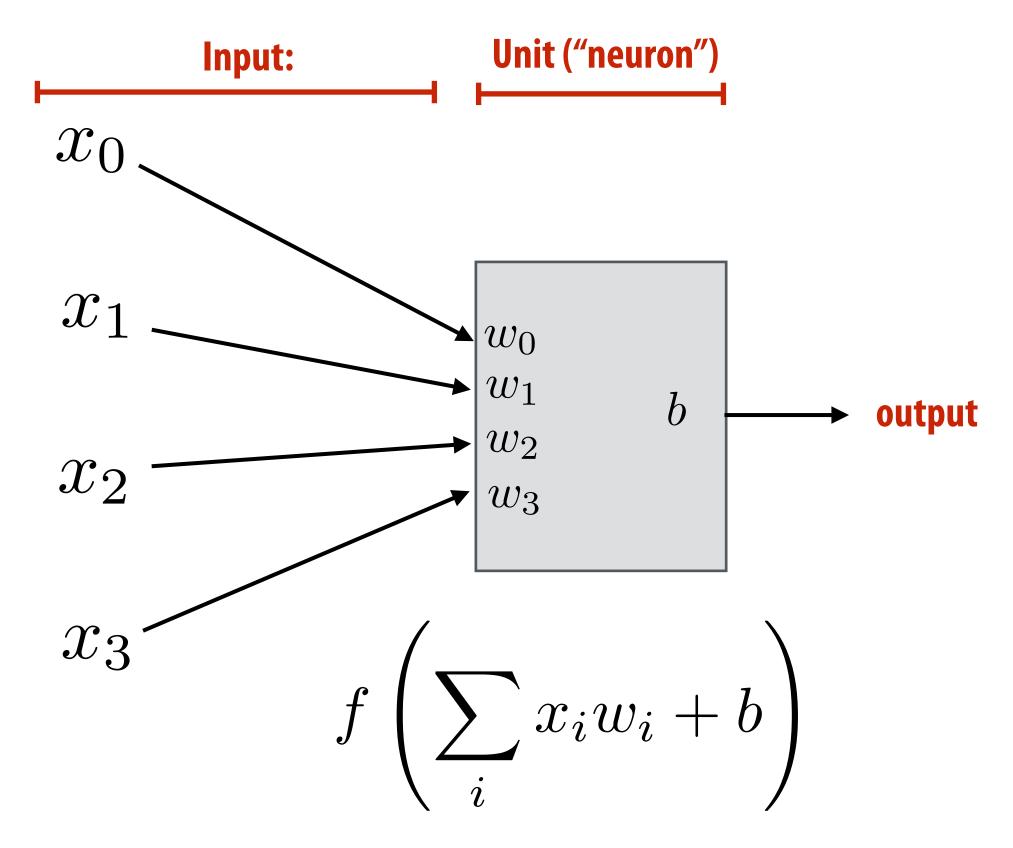
a baseball player swinging a bat at a ball a boy is playing with a baseball bat



What is a deep neural network?

A basic unit:

Unit with n inputs described by n+1 parameters (weights + bias)



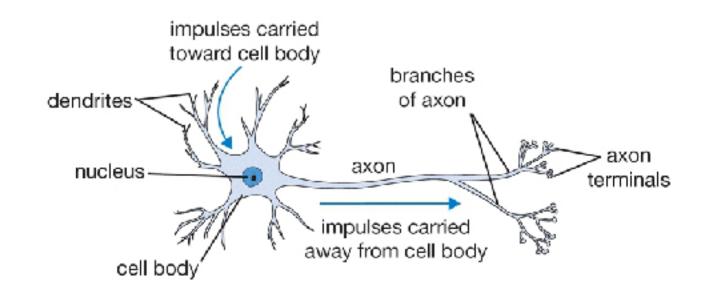
Example: rectified linear unit (ReLU)

$$f(x) = max(0, x)$$

Basic computational interpretation: It's just a circuit!

Biological inspiration:

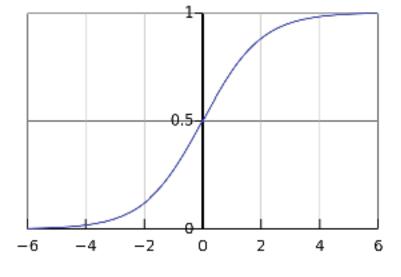
unit output corresponds loosely to activation of neuron



Machine learning interpretation:

binary classifier: interpret output as the probability of one class

$$f(x) = \frac{1}{1 + e^{-x}}$$



Two Distinct Issues with Deep Networks

- Evaluation
 - often takes milliseconds
- Training
 - often takes hours, days, weeks

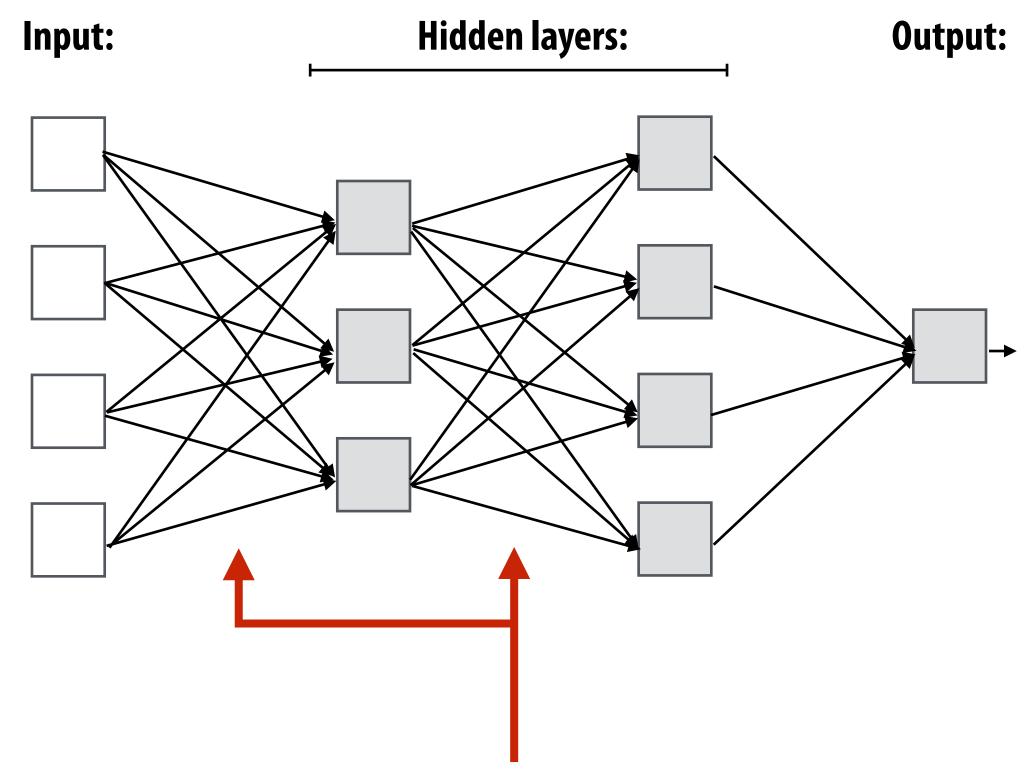
What is a deep neural network? topology

This network has: 4 inputs, 1 output, 7 hidden units

"Deep" = at least one hidden layer

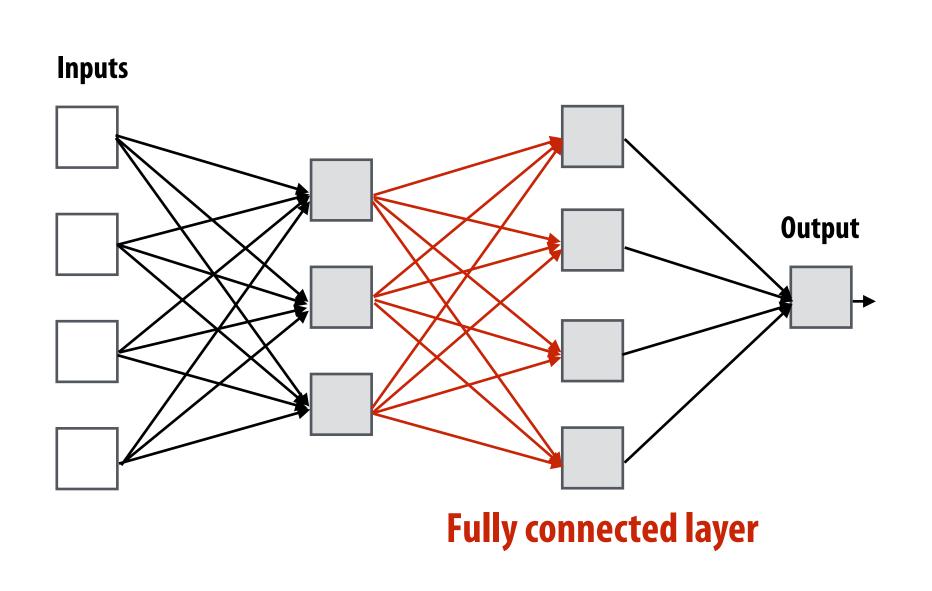
Hidden layer 1: 3 units x (4 weights + 1 bias) = 15 parameters

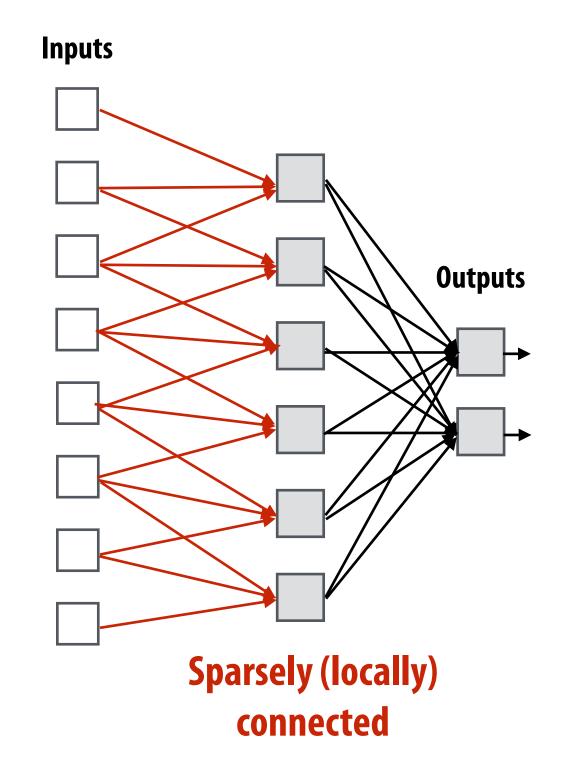
Hidden layer 2: 4 units x (3 weights + 1 bias) = 16 parameters



Note fully-connected topology in this example

What is a deep neural network? topology





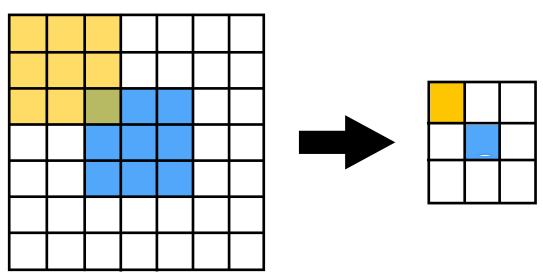
Recall image convolution (3x3 conv)

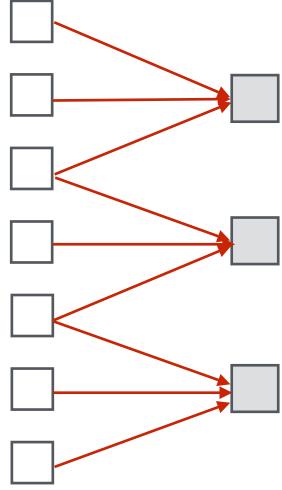
```
Inputs
int WIDTH = 1024;
                                                                                                                                                                                                                 Inputs
                                                                                                                                                                                                                                                                           Conv
int HEIGHT = 1024;
                                                                                                                                                                                                                                                                          Layer
float input[(WIDTH+2) * (HEIGHT+2)];
float output[WIDTH * HEIGHT];
float weights[] = \{1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.
                                                                                         1.0/9, 1.0/9, 1.0/9,
                                                                                          1.0/9, 1.0/9, 1.0/9};
for (int j=0; j<HEIGHT; j++) {</pre>
         for (int i=0; i<WIDTH; i++) {
                                                                                                                                                                                                   Convolutional layer: locally connected AND all units in layer share
                  float tmp = 0.f;
                                                                                                                                                                                                   the same parameters (same weights + same bias):
                   for (int jj=0; jj<3; jj++)
                                                                                                                                                                                                   (note: network diagram only shows links due to one iteration of ii loop)
                            for (int ii=0; ii<3; ii++)
                                     tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
                  output[j*WIDTH + i] = tmp;
```

Strided 3x3 convolution

```
int WIDTH = 1024;
int HEIGHT = 1024;
int STRIDE = 2;
float input[(WIDTH+2) * (HEIGHT+2)];
float output[(WIDTH/STRIDE) * (HEIGHT/STRIDE)];
                                                                                                                                                                                                                                                                                                                                                Inputs
float weights[] = \{1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.0/9, 1.
                                                                                                     1.0/9, 1.0/9, 1.0/9,
                                                                                                     1.0/9, 1.0/9, 1.0/9};
for (int j=0; j<HEIGHT; j+=STRIDE) {</pre>
          for (int i=0; i<WIDTH; i+=STRIDE) {</pre>
                     float tmp = 0.f;
                     for (int jj=0; jj<3; jj++)
                               for (int ii=0; ii<3; ii++) {
                                                tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
                                output[(j/STRIDE)*WIDTH + (i/STRIDE)] = tmp;
```

Inputs





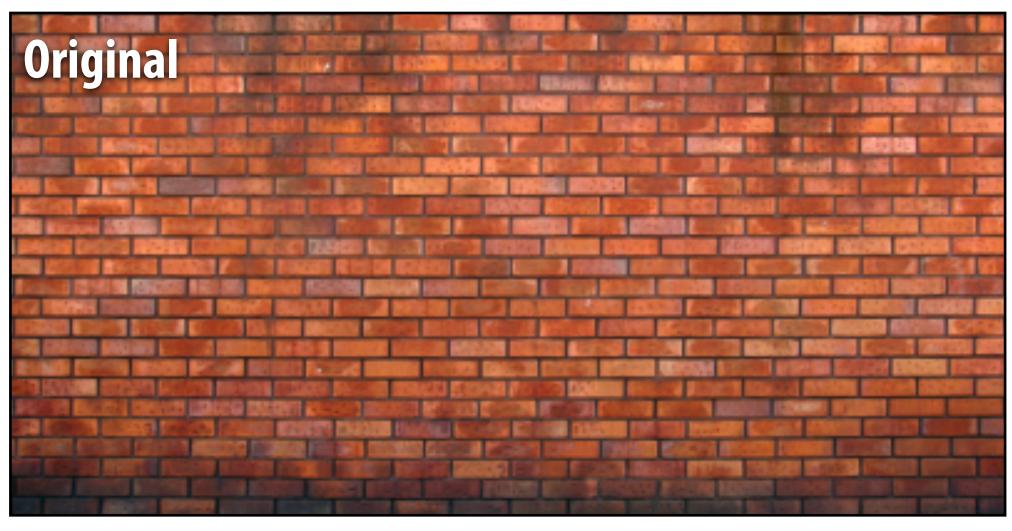
Convolutional layer with stride 2

What does convolution using these filter

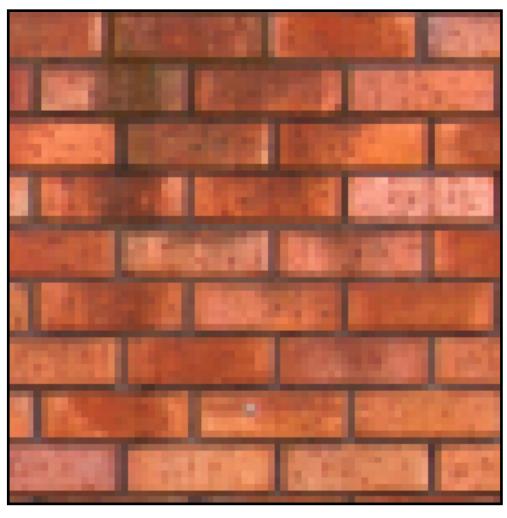
weights do? [.075 .124 .075]
.124 .204 .124

.124 .075

"Gaussian Blur"









What does convolution with these filters do?

$$egin{bmatrix} -1 & 0 & 1 \ -2 & 0 & 2 \ -1 & 0 & 1 \ \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

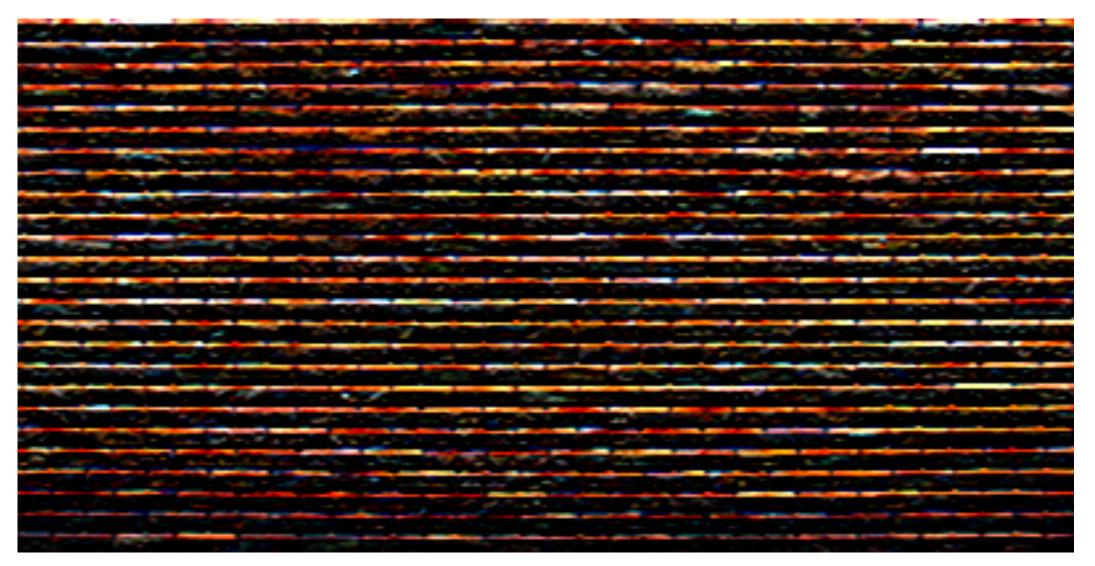
Extracts horizontal gradients

Extracts vertical gradients

Gradient detection filters



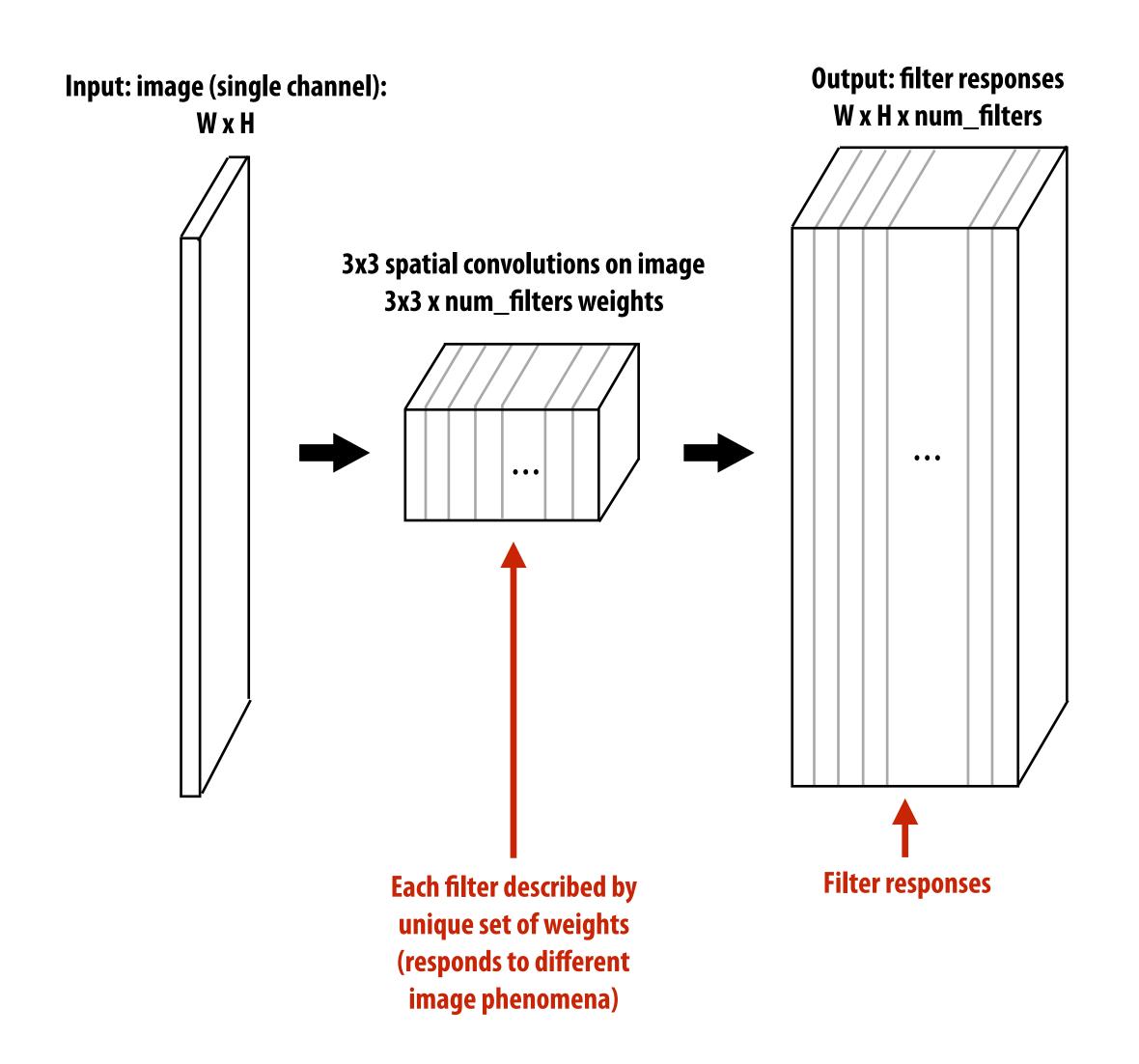
Horizontal gradients



Vertical gradients

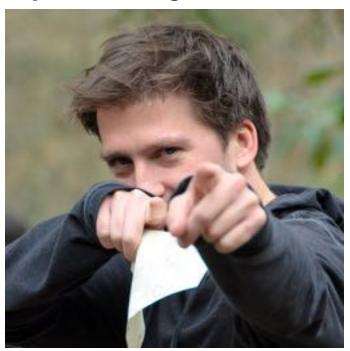
Note: you can think of a filter as a "detector" of a pattern, and the magnitude of a pixel in the output image as the "response" of the filter to the region surrounding each pixel in the input image

Applying many filters to an image at once

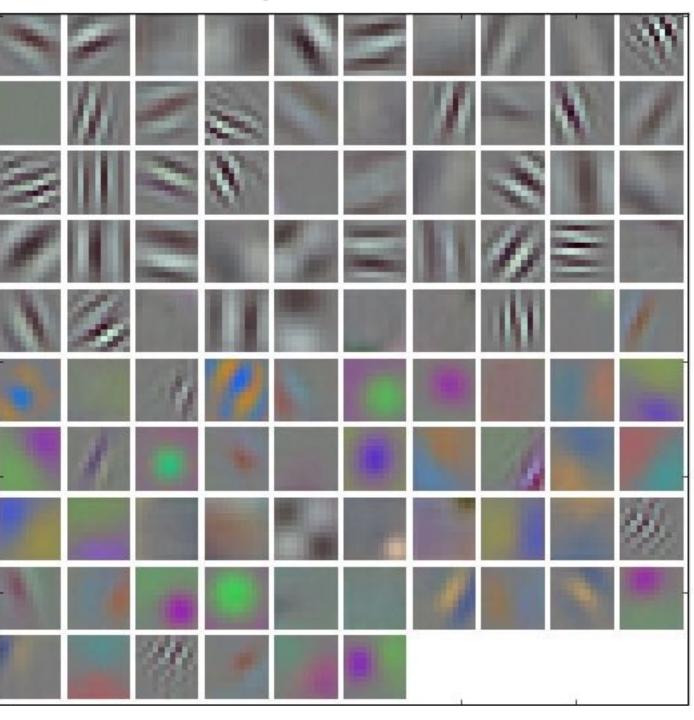


Applying many filters to an image at once

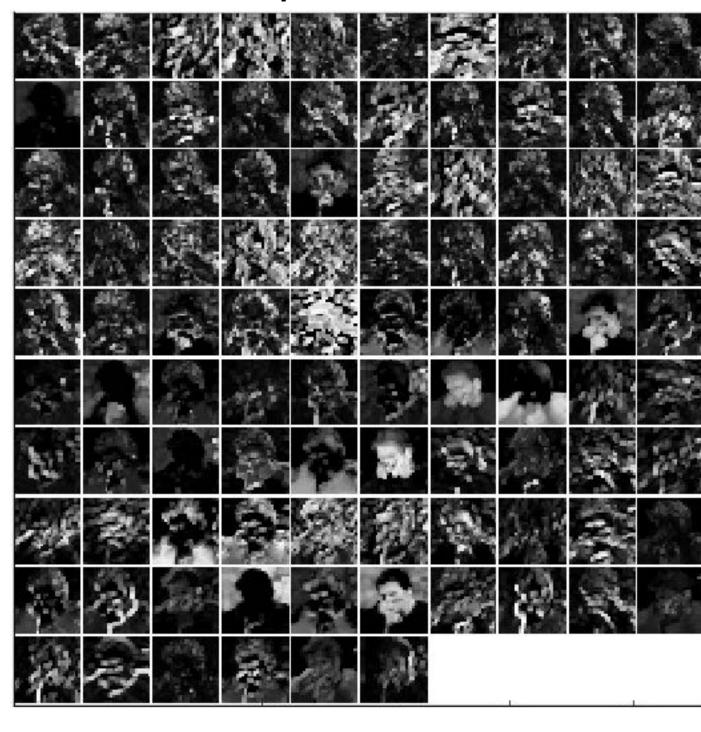
Input RGB image (W x H x 3)



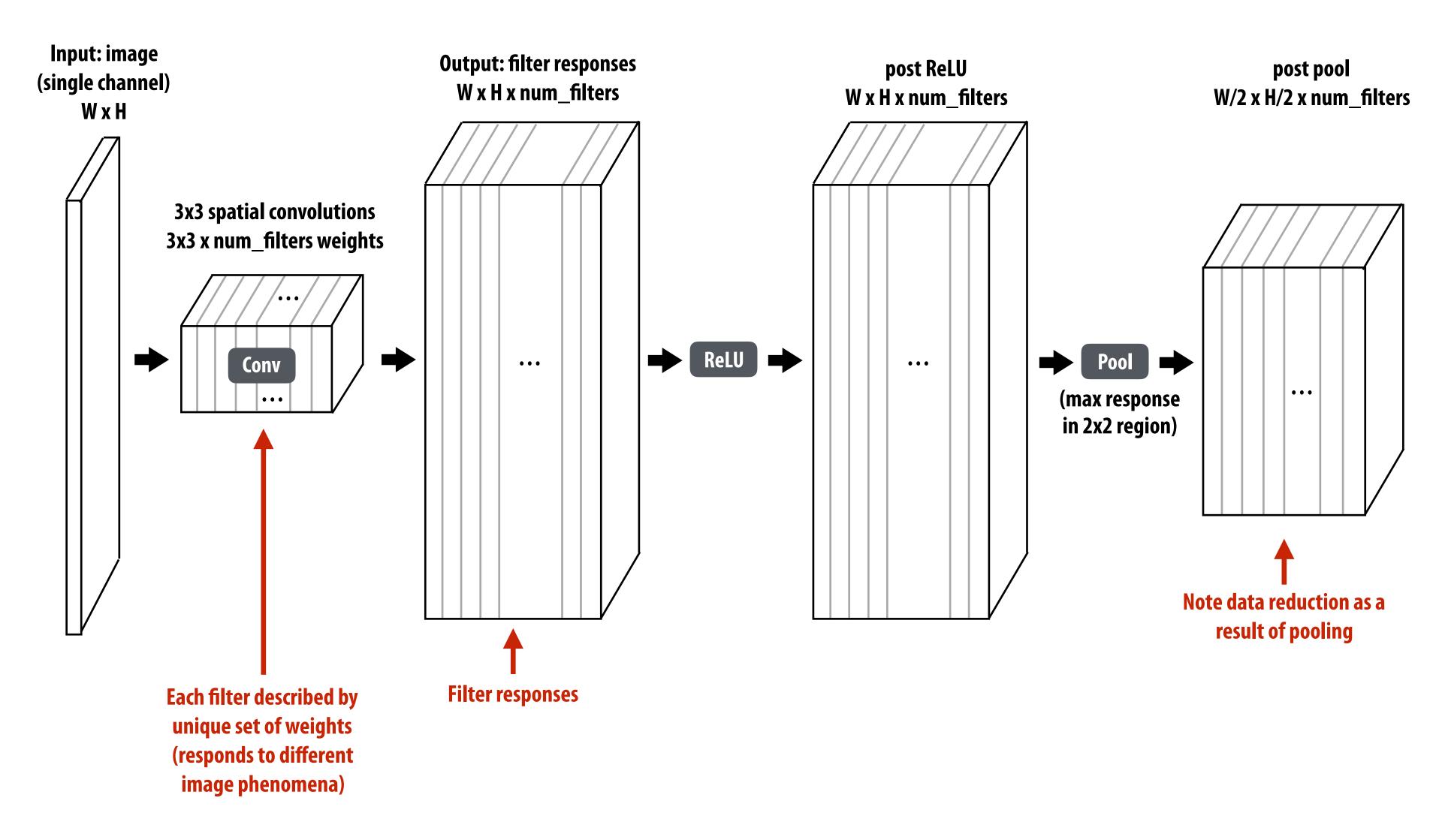
96 11x11x3 filters (operate on RGB)



96 responses (normalized)



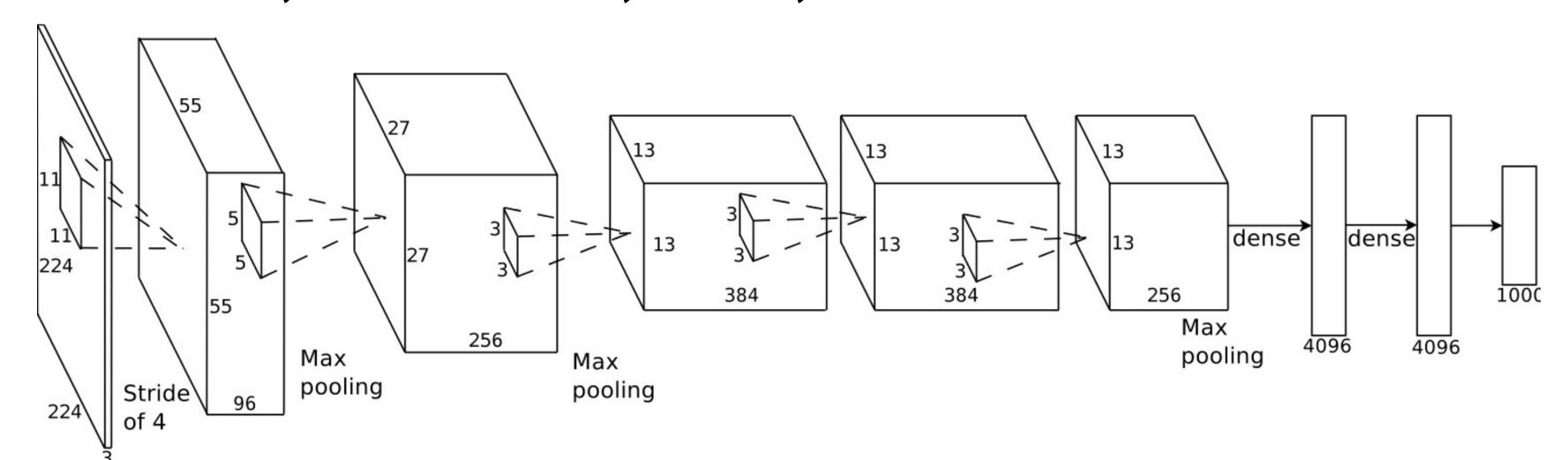
Adding additional layers



Modern object detection networks

Sequences of conv + reLU + (optional) pool layers

AlexNet [Krizhevsky12]: 5 convolutional layers + 3 fully connected



VGG-16 [Simonyan15]: 13 convolutional layers

maxpool maxpool maxpool

conv/reLU: 3x3x64x128 conv/reLU: 3x3x256x512 fully-connected 4096

conv/reLU: 3x3x128x128 conv/reLU: 3x3x512x512 fully-connected 4096 maxpool conv/reLU: 3x3x512x512 fully-connected 1000

maxpool soft-max

Efficiently implementing convolution layers

Direct implementation of conv layer

```
float input[INPUT_HEIGHT][INPUT_WIDTH][INPUT_DEPTH];
float output[INPUT_HEIGHT][INPUT_WIDTH][LAYER_NUM_FILTERS];
float layer_weights[LAYER_CONVY, LAYER_CONVX, INPUT_DEPTH];
// assumes convolution stride is 1
for (int img=0; img<IMAGE_BATCH_SIZE; img++)</pre>
   for (int j=0; j<INPUT_HEIGHT; j++)</pre>
      for (int i=0; i<INPUT_WIDTH; i++)</pre>
         for (int f=0; f<LAYER_NUM_FILTERS; f++) {</pre>
            output[j][i][f] = 0.f;
            for (int kk=0; kk<INPUT_DEPTH; kk++) // sum over filter responses of input channels
               for (int jj=0; jj<LAYER_CONVY; jj++) // spatial convolution</pre>
                   for (int ii=0; ii<LAYER_CONVX; ii+) // spatial convolution</pre>
                       output[j][i][f] += layer_weights[f][jj][ii][kk] * input[j+jj][i+ii][kk];
          }
```

Seven loops with significant input data reuse: reuse of filter weights (during convolution), and reuse of input values (across different filters)

But must roll your own highly optimized implementation of a complicated loop nest.

Dense matrix multiplication

```
float A[M][K];
float B[K][N];
float C[M][N];

// compute C += A * B
#pragma omp parallel for
for (int j=0; j<M; j++)
    for (int k=0; i<N; i++)
        for (int k=0; k<K; k++)
        C[j][i] += A[j][k] * B[k][i];</pre>
```

What is the problem with this implementation?

Low arithmetic intensity (does not exploit temporal locality in access to A and B)

Blocked dense matrix multiplication

```
N
float A[M][K];
float B[K][N];
float C[M][N];
   compute C += A * B
#pragma omp parallel for
for (int jblock=0; jblock<M; jblock+=BLOCKSIZE_J)</pre>
  for (int iblock=0; iblock<N; iblock+=BLOCKSIZE_I)</pre>
     for (int kblock=0; kblock<K; kblock+=BLOCKSIZE_K)</pre>
        for (int j=0; j<BLOCKSIZE_J; j++)</pre>
            for (int i=0; i<BLOCKSIZE_I; i++)</pre>
               for (int k=0; k<BLOCKSIZE_K; k++)</pre>
                  C[jblock+j][iblock+i] += A[jblock+j][kblock+k] * B[kblock+k][iblock+i];
```

Idea: compute partial result for block of C while required blocks of A and B remain in cache (Assumes BLOCKSIZE chosen to allow block of A, B, and C to remain resident)

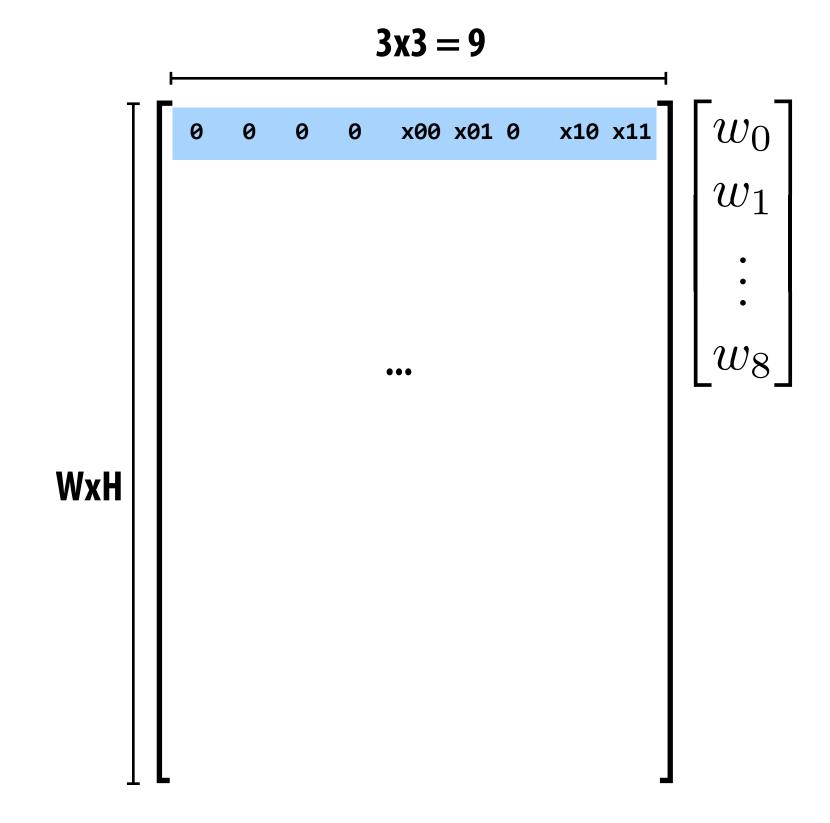
Self check: do you want as big a BLOCKSIZE as possible? Why?

Convolution as matrix-vector product

Construct matrix from elements of input image

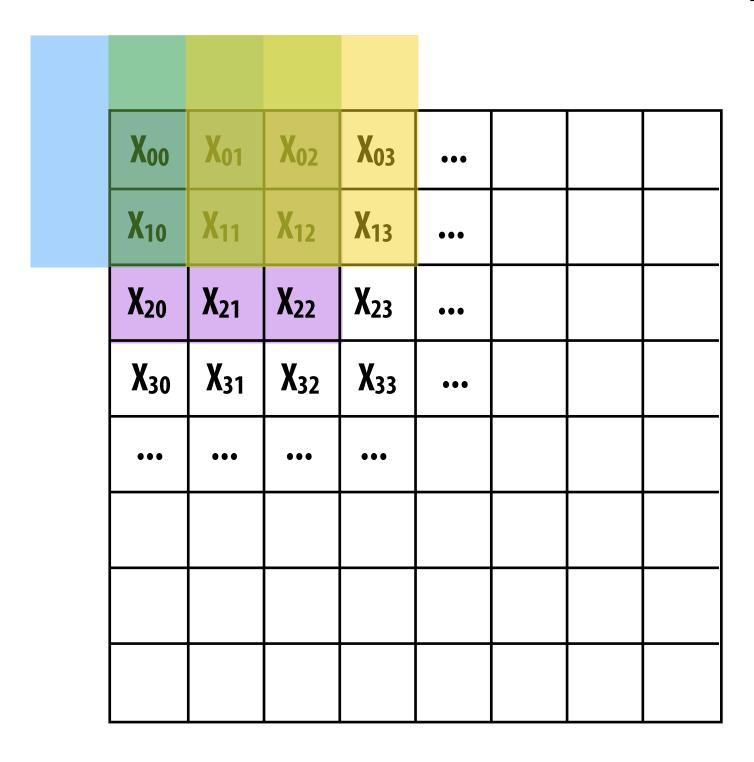
X ₀₀	X ₀₁	X ₀₂	X ₀₃	•••			
X ₁₀	X ₁₁	X ₁₂	X ₁₃	•••			
X ₂₀	X ₂₁	X ₂₂	X ₂₃	•••			
X ₃₀	X ₃₁	X ₃₂	X ₃₃	•••			
•••	•••	•••	•••				
	X ₁₀ X ₂₀ X ₃₀	X10 X11 X20 X21 X30 X31	X10 X11 X12 X20 X21 X22 X30 X31 X32	X10 X11 X12 X13 X20 X21 X22 X23 X30 X31 X32 X33	X10 X11 X12 X13 X20 X21 X22 X23 X30 X31 X32 X33	X10 X11 X12 X13 X20 X21 X22 X23 X30 X31 X32 X33	X10 X11 X12 X13 X20 X21 X22 X23 X30 X31 X32 X33

O(N) storage overhead for filter with N elements Must construct input data matrix

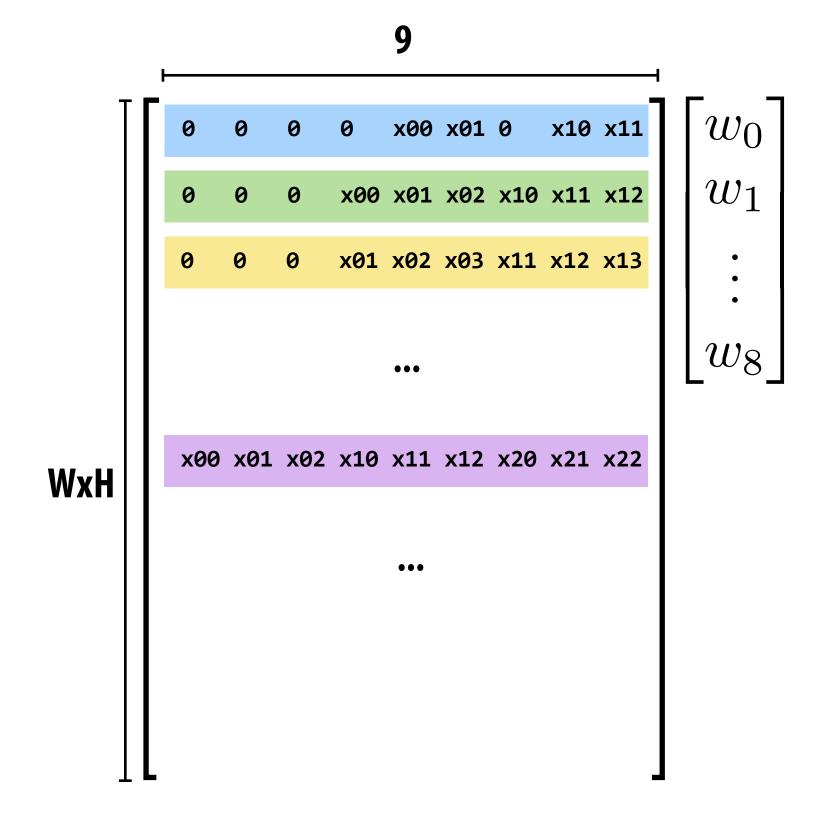


3x3 convolution as matrix-vector product

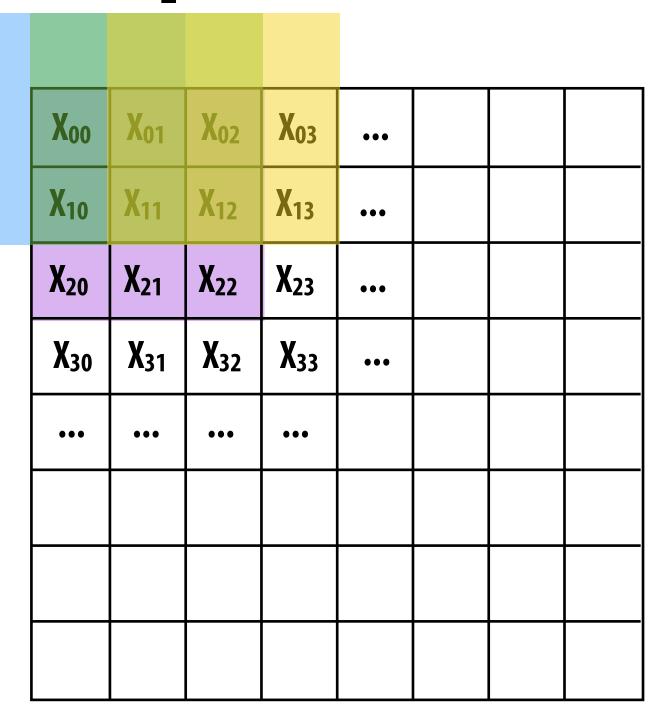
Construct matrix from elements of input image



O(N) storage overhead for filter with N elements Must construct input data matrix

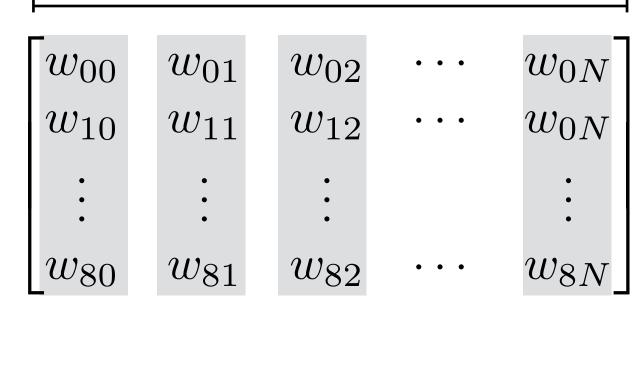


Multiple convolutions as matrix-matrix mult



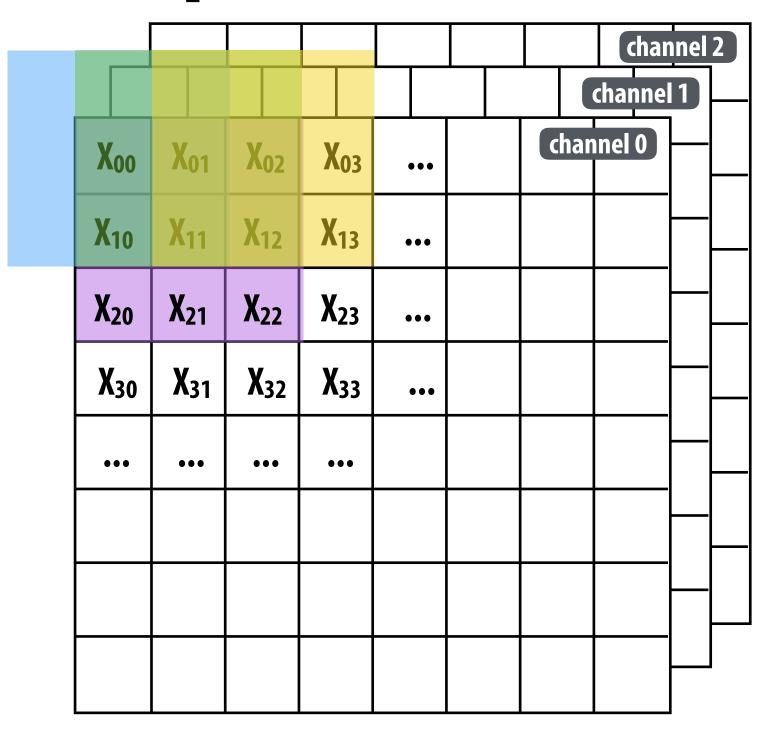
9
| 0 0 0 0 x00 x01 0 x10 x11 | 0 0 0 x00 x01 x02 x10 x11 x12 | 0 0 0 x01 x02 x03 x11 x12 x13 | \text{WxH}

x00 x01 x02 x10 x11 x12 x20 x21 x22



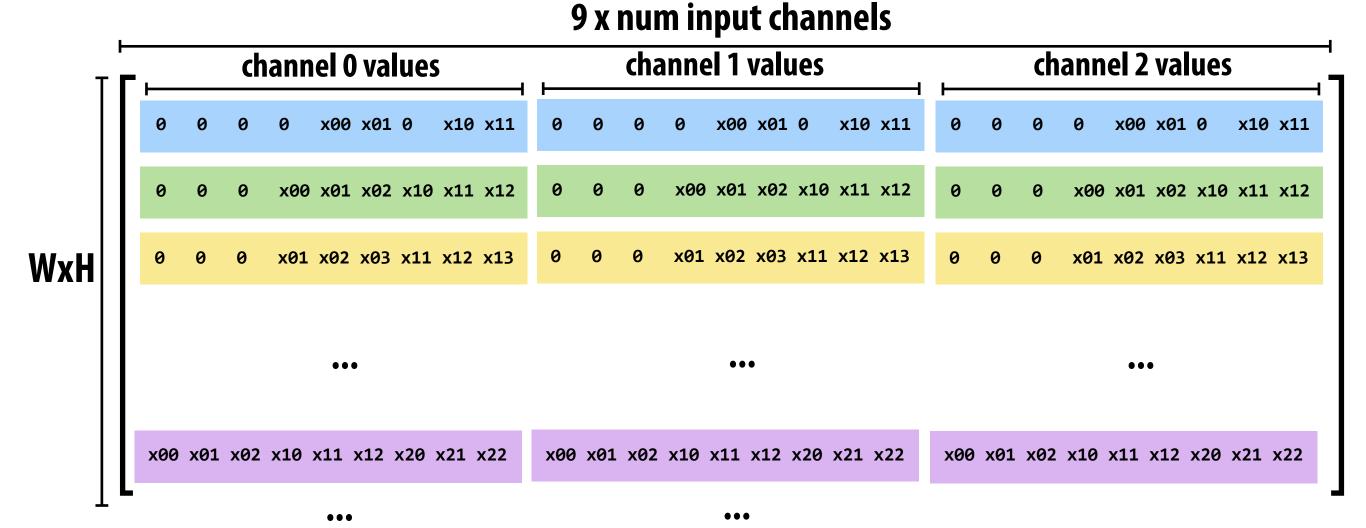
num filters

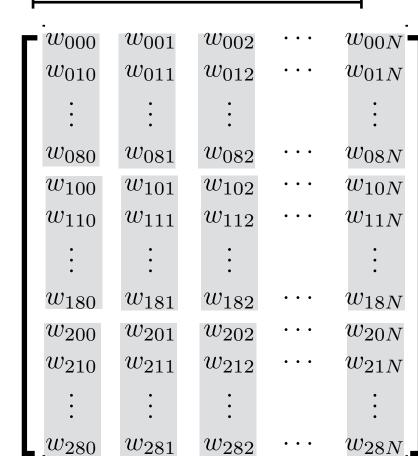
Multiple convolutions on multiple input channels



For each filter, sum responses over input channels

Equivalent to (3 x 3 x num_channels) convolution on (W x H x num_channels) input data





num filters

VGG memory footprint

Calculations assume 32-bit values (image batch size = 1)

(per image) weights mem: input: 224 x 224 RGB image 224x224x3 conv: (3x3x3) x 64 6.5 KB 224x224x64 conv: (3x3x64) x 64 **144 KB** 224x224x64 112x112x64 maxpool conv: (3x3x64) x 128 **228 KB** 112x112x128 112x112x128 conv: (3x3x128) x 128 **576 KB** 56x56x128 maxpool 56x56x256 conv: (3x3x128) x 256 1.1 MB conv: (3x3x256) x 256 2.3 MB 56x56x256 conv: (3x3x256) x 256 2.3 MB 56x56x256 28x28x256 maxpool conv: (3x3x256) x 512 4.5 MB 28x28x512 **9 MB** conv: (3x3x512) x 512 28x28x512 **9 MB** conv: (3x3x512) x 512 28x28x512 14x14x512 maxpool 14x14x512 conv: (3x3x512) x 512 9 MB **9 MB** 14x14x512 conv: (3x3x512) x 512 conv: (3x3x512) x 512 9 MB 14x14x512 7x7x512 maxpool fully-connected 4096 392 MB 4096 **64 MB** fully-connected 4096 4096 fully-connected 1000 15.6 MB 1000 soft-max 1000

inputs/outputs get multiplied by image batch size

output size

multiply by next layer's conv window size to form input matrix to next conv layer!!! (for VGG, this is a 9x data amplification)

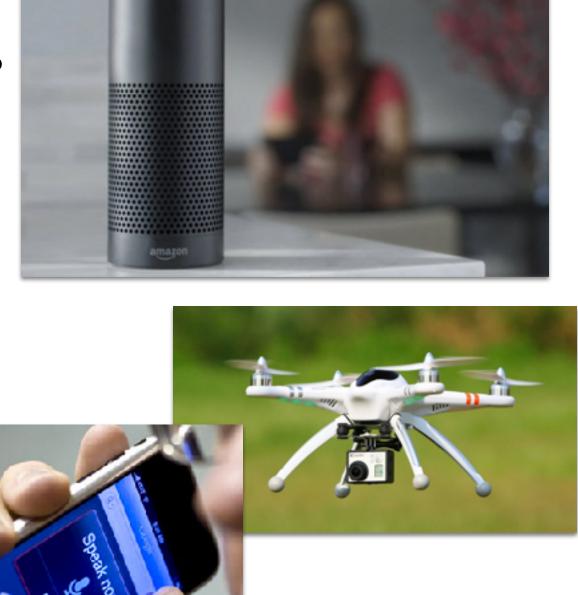
(mem)	u
150K	
12.3 MB	
12.3 MB	
3.1 MB	
6.2 MB	
6.2 MB	
1.5 MB	
3.1 MB	
3.1 MB	
3.1 MB	
766 KB	
1.5 MB	
1.5 MB	
1.5 MB	
383 KB	
383 KB	
383 KB	
383 KB	
98 KB	
16 KB	
16 KB	
4 KB	
4 KB	

Reducing network footprint

- Large storage cost for model parameters
 - AlexNet model: ~200 MB
 - VGG-16 model: ~500 MB
 - This doesn't even account for intermediates during evaluation
- **■** Footprint: cumbersome to store, download, etc.
 - 500 MB app downloads make users unhappy!



- Running on input stream at 20 Hz
- 640 pJ per 32-bit DRAM access
- (20 x 1B x 640pJ) = 12.8W for DRAM access (more than power budget of any modern smartphone)



Compressing a network

Step 1: prune low-weight links (iteratively retrain network, then prune)

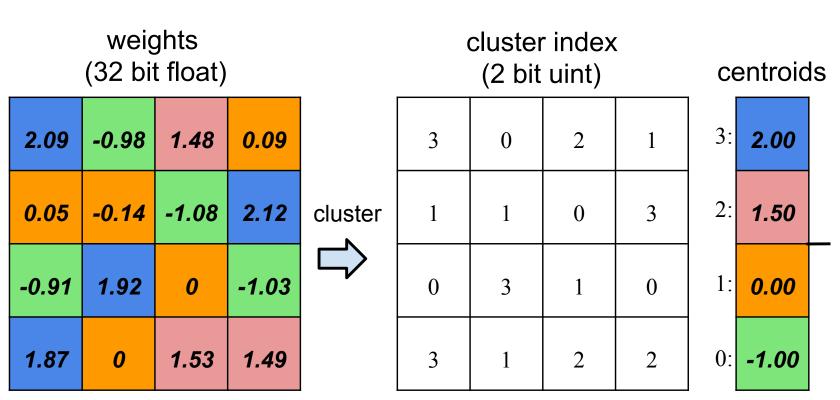
- Over 90% of weights can be removed without significant loss of accuracy
- Store weight matrices in compressed sparse row (CSR) format

0	1.8	0	0	0.5	0	0	0	0	2.1	• • •	
---	-----	---	---	-----	---	---	---	---	-----	-------	--

Step 2: weight sharing: make surviving connects share a small set of weights

- Cluster weights via k-means clustering (irregular ("learned") quantization)
- Compress weights by only storing cluster index (lg(k) bits)
- Retrain network to improve quality of cluster centroids

Step 3: Huffman encode quantized weights and CSR indices



VGG-16 compression

Substantial savings due to combination of pruning, quantization, Huffman encoding

		Waights%	Weigh	Weight	Index	Index	Compress	Compress
Layer	#Weights	Weights%	bits	bits	bits	bits	rate	rate
		(P)	(P+Q)	(P+Q+H)	(P+Q)	(P+Q+H)	(P+Q)	(P+Q+H)
conv1_1	2K	58%	8	6.8	5	1.7	40.0%	29.97%
$conv1_2$	37K	22%	8	6.5	5	2.6	9.8%	6.99%
$conv2_1$	74K	34%	8	5.6	5	2.4	14.3%	8.91%
$conv2_2$	148K	36%	8	5.9	5	2.3	14.7%	9.31%
conv3_1	295K	53%	8	4.8	5	1.8	21.7%	11.15%
conv3_2	590K	24%	8	4.6	5	2.9	9.7%	5.67%
conv3_3	590K	42%	8	4.6	5	2.2	17.0%	8.96%
conv4_1	1 M	32%	8	4.6	5	2.6	13.1%	7.29%
conv4_2	2M	27%	8	4.2	5	2.9	10.9%	5.93%
conv4_3	2M	34%	8	4.4	5	2.5	14.0%	7.47%
$conv5_1$	2M	35%	8	4.7	5	2.5	14.3%	8.00%
conv5_2	2M	29%	8	4.6	5	2.7	11.7%	6.52%
conv5_3	2M	36%	8	4.6	5	2.3	14.8%	7.79%
fc6	103M	4%	5	3.6	5	3.5	1.6%	1.10%
fc7	17M	4%	5	4	5	4.3	1.5%	1.25%
fc8	4M	23%	5	4	5	3.4	7.1%	5.24%
Total	138M	$7.5\%(13\times)$	6.4	4.1	5	3.1	3.2% (31 ×)	2.05% (49×)

P = connection pruning (prune low weight connections)

Q = quantize surviving weights (using shared weights)

H = **Huffman** encode

ImageNet Image Classification Performance

	Top-1 Error	Top-5 Error	Model size		
VGG-16 Ref	31.50%	11.32%	552 MB		
VGG-16 Compressed	31.17%	10.91%	11.3 MB	$oxed{49} imes$	

Deep neural networks on GPUs

- Today, best performing DNN implementations target GPUs
 - High arithmetic intensity computations (computational characteristics similar to dense matrix-matrix multiplication)
 - Benefit from flop-rich architectures
 - Highly-optimized library of kernels exist for GPUs (cuDNN)

Most CPU-based implementations use basic matrix-multiplication-based formulation (good implementations could run faster!)





Facebook's Big Sur

Summary: Efficiently Evaluating DNNs

- Computational structure
 - Convlayers: high arithmetic intensity, significant portion of cost of evaluating a network
 - Similar data access patterns to dense-matrix multiplication (exploiting temporal reuse is key)
 - But straight reduction to matrix-matrix multiplication is often sub-optimal
 - Work-efficient techniques for convolutional layers (FFT-based, Wingrad convolutions)
- Large numbers of parameters: significant interest in reducing size of networks for both training and evaluation
 - Pruning: remove least important network links
 - Quantization: low-precision parameter representations often suffice
- Many ongoing studies of specialized hardware architectures for efficient evaluation
 - Future CPUs/GPUs, ASICs, FPGS,
 - Specialization will be important to achieving "always on" applications

Two Distinct Issues with Deep Networks

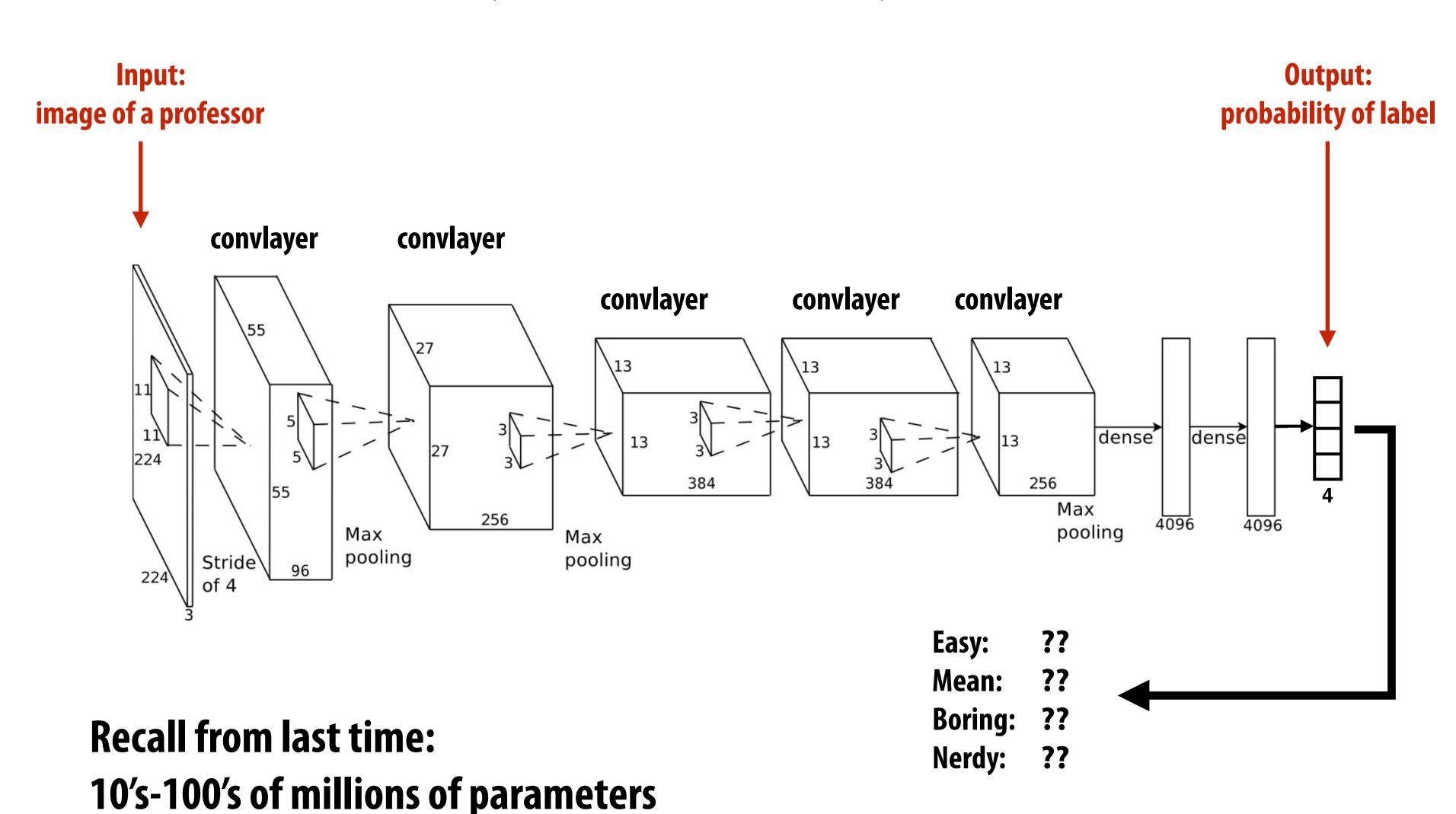
- Evaluation
 - often takes milliseconds
- Training
 - often takes hours, days, weeks

"Training a network"

- Training a network is the process of learning the value of network parameters so that output of the network provides the desired result for a task
 - [Krizhevsky12] task = object classification
 - input 224 x 224 x 3 RGB image
 - output probability of 1000 ImageNet object classes: "dog", "cat", etc...
 - ∼ 60M weights

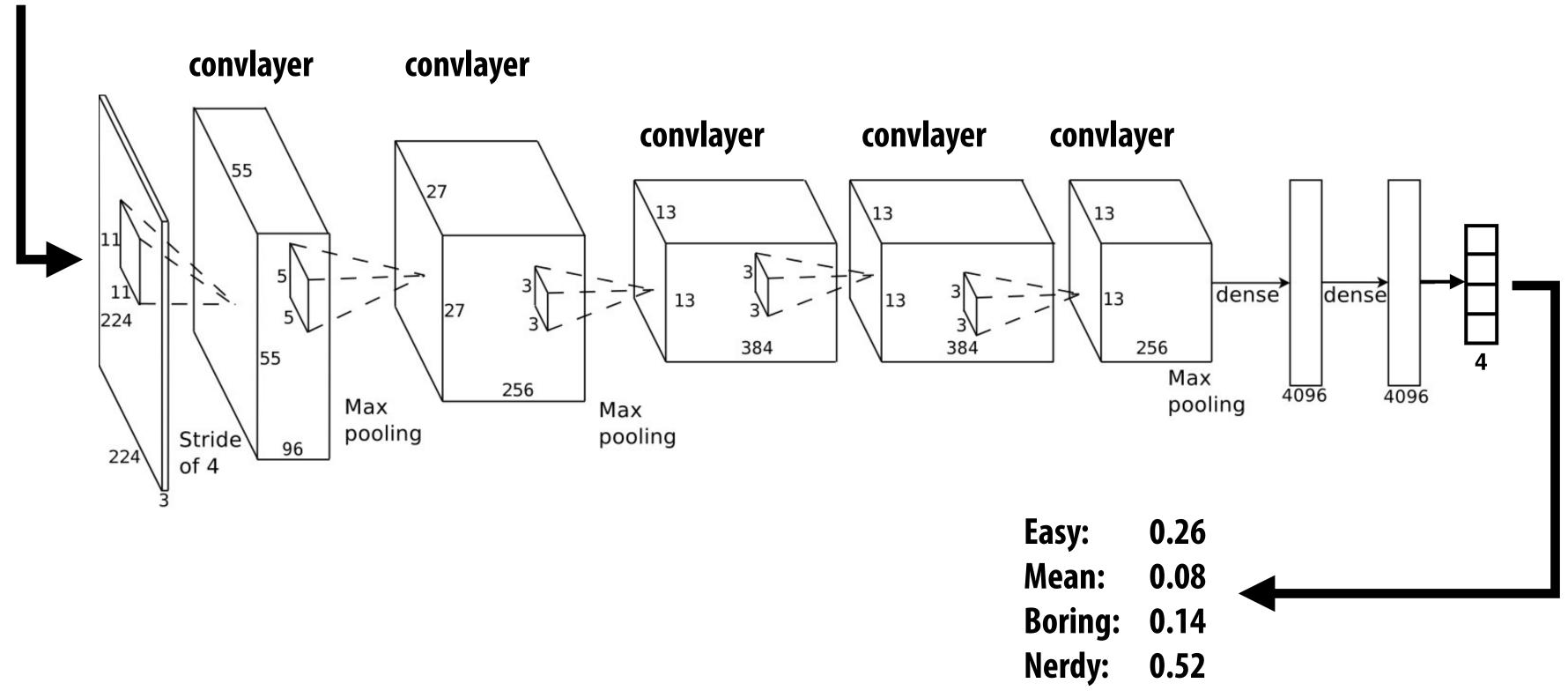
Professor classification network

Classifies professors as easy, mean, boring, or nerdy based on their appearance.



Professor classification network





Where did the parameters come from?

Training data (ground truth answers)







[label omitted]



[label omitted]



Nerdy



[label omitted] [label omitted]





[label omitted]



[label omitted]



[label omitted]



Nerdy



[label omitted]



[label omitted]



Nerdy



[label omitted]



[label omitted]



[label omitted]



Nerdy



[label omitted]



[label omitted]



[label omitted]

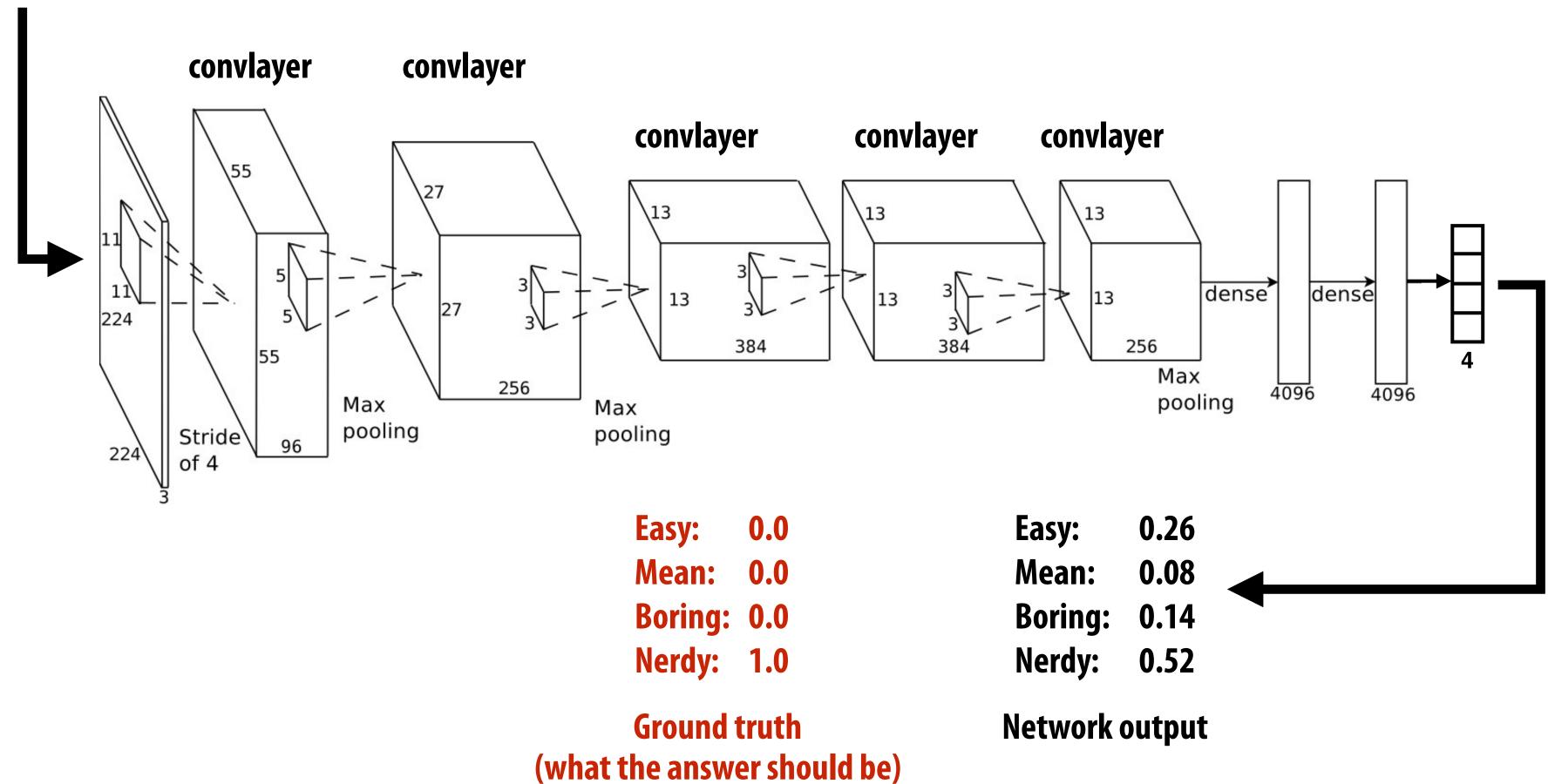


Nerdy

Professor classification network



New image of Kayvon (not in training set)



Error (loss)

Ground truth:

(what the answer should be)

Easy: 0.0

Mean: 0.0

Boring: 0.0

Nerdy: 1.0

Network output: *

Easy: 0.26

Mean: 0.08

Boring: 0.14

Nerdy: 0.52

Common example: softmax loss:

$$L = -log\left(\frac{e^{f_c}}{\sum_{j} e^{f_j}}\right)$$
 Output of network for all categories

^{*} In practice a network using a softmax classifier outputs unnormalized, log probabilities (f_j), but I'm showing a probability distribution above for clarity

Training

<u>Goal of training</u>: <u>learning good values of network parameters</u> so that network outputs the correct classification result for any input image

<u>Idea</u>: minimize loss for all the training examples (for which the correct answer is known)

$$L = \sum_i L_i$$
 (total loss for entire training set is sum of losses L_i for each training example x_i)

<u>Intuition</u>: if the network gets the answer correct for a wide range of training examples, then hopefully it has learned parameter values that yield the correct answer for future images as well.

Intuition: gradient descent

Say you had a function f that contained a hidden parameters p_1 and p_2 : $f(x_i)$

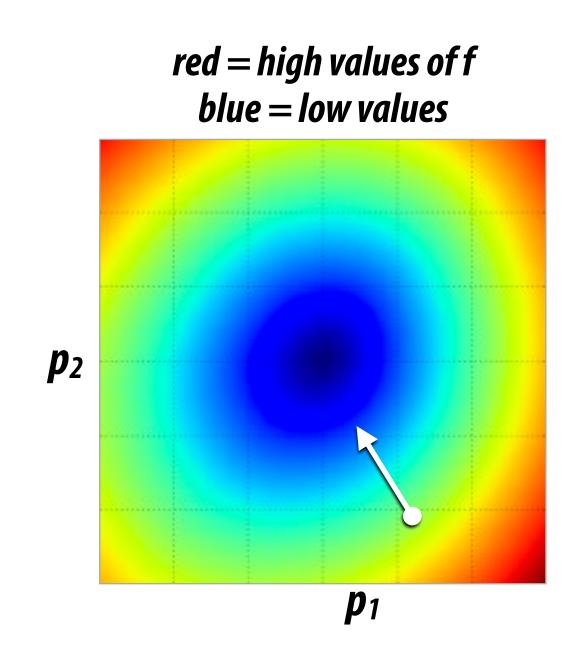
And for some input x_i , your training data says the function should output 0.

But for the current values of p_1 and p_2 , it currently outputs 10.

$$f(x_i, p_1, p_2) = 10$$

And say I also gave you expressions for the derivative of f with respect to p_1 and p_2 so you could compute their value at x_i .

$$\frac{df}{dp_1} = 2 \quad \frac{df}{dp_2} = -5 \qquad \nabla f = [2, -5]$$



How might you adjust the values p_1 and p_2 to reduce the error for this training example?

Basic gradient descent

```
while (loss too high):
    for each item x_i in training set:
        grad += evaluate_loss_gradient(f, loss_func, params, x_i)
    params += -grad * step_size;
```

Mini-batch stochastic gradient descent (mini-batch SGD): choose a random (small) subset of the training examples to compute gradient in each iteration of the while loop

How to compute df/dp for a complex neural network with millions of parameters?

Derivatives using the chain rule

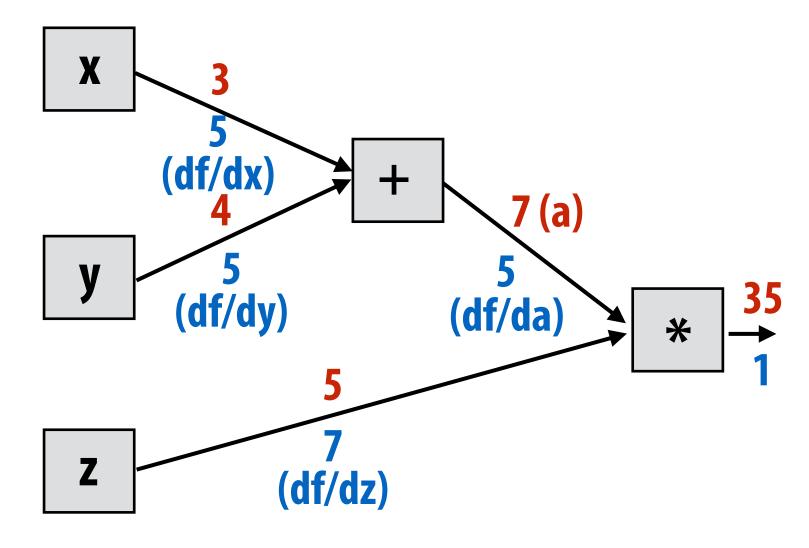
$$f(x, y, z) = (x + y)z = az$$

Where:
$$a = x + y$$

$$\frac{df}{da} = z \quad \frac{da}{dx} = 1 \quad \frac{da}{dy} = 1$$

So, by the derivative chain rule:

$$\frac{df}{dx} = \frac{df}{da}\frac{da}{dx} = z$$



Red = **output** of node

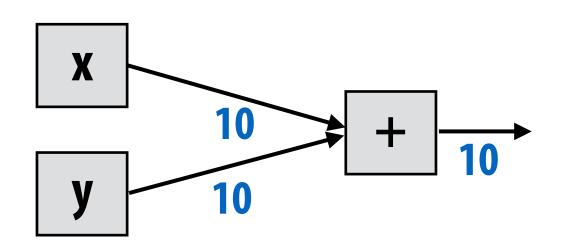
Blue = df/dnode

Backpropagation

Red = **output of node**

Blue = df/dnode

Recall:
$$\frac{df}{dx} = \frac{df}{dg} \frac{dg}{dx}$$



$$g(x,y) = x + y$$

$$\frac{dg}{dx} = 1, \ \frac{dg}{dy} = 1$$

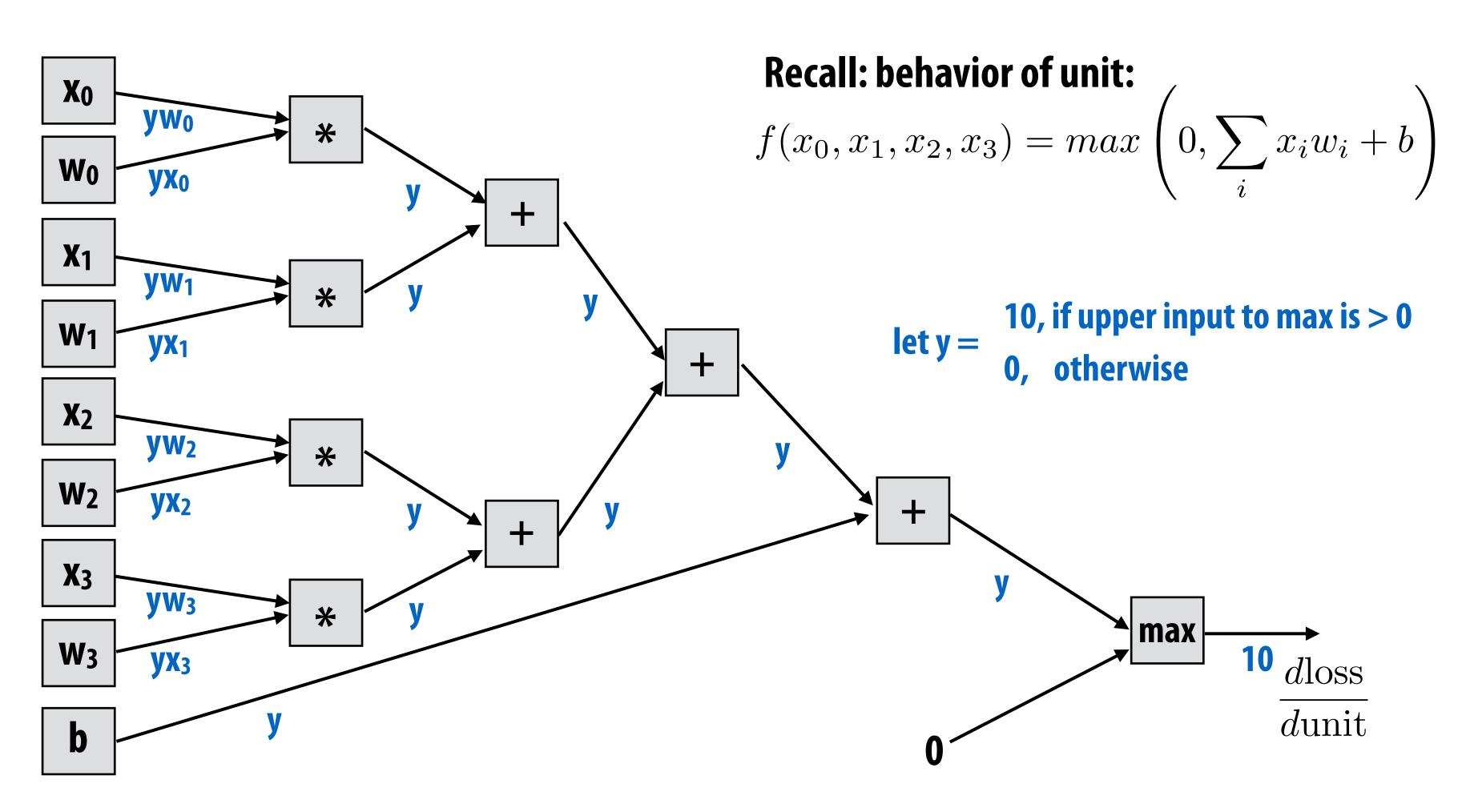
$$g(x,y) = \max(x,y)$$

$$g(x,y) = \max(x,y)$$
 $\frac{dg}{dx} =$ 1, if x > y 0, otherwise

$$g(x,y) = xy$$

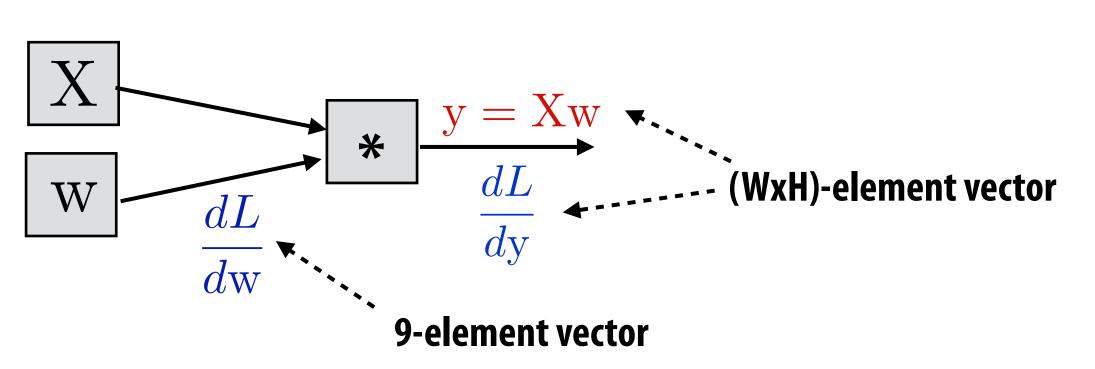
$$\frac{dg}{dx} = y, \ \frac{dg}{dy} = x$$

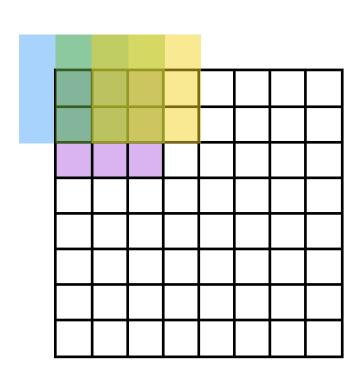
Backpropagating through single unit



Observe: output of prior layer $(x_i's)$ and output of this unit must be retained in order to compute weight gradients for this unit during backprop.

Backpropagation: matrix form





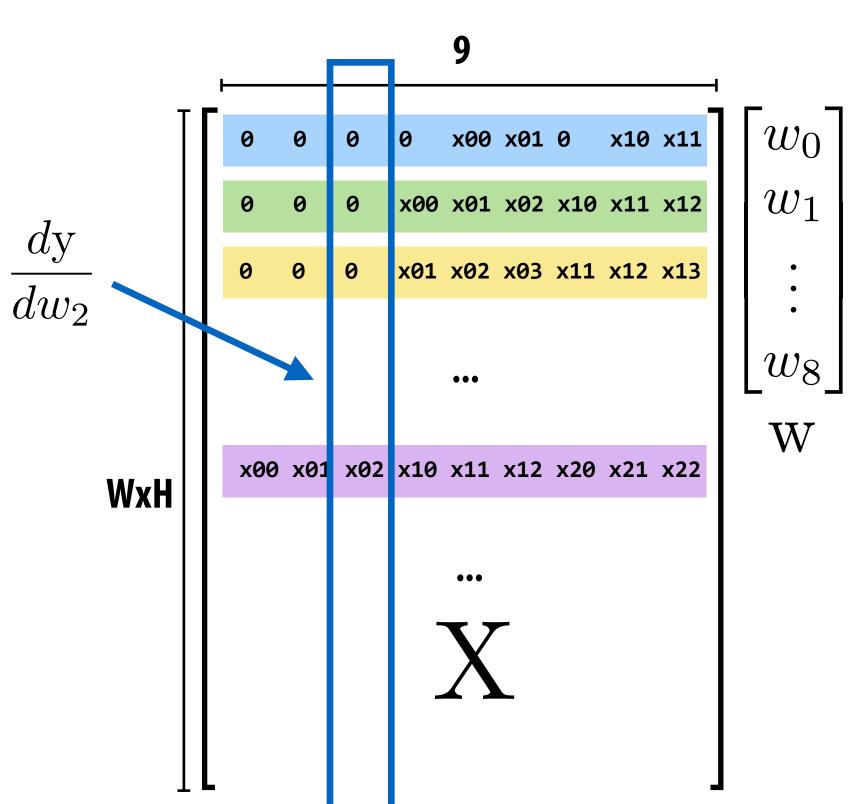
$$\frac{dy_j}{dw_i} = X_{ji}$$

$$\frac{dL}{dw_i} = \sum_{j} \frac{dL}{dy_j} \frac{dy_j}{dw_i}$$

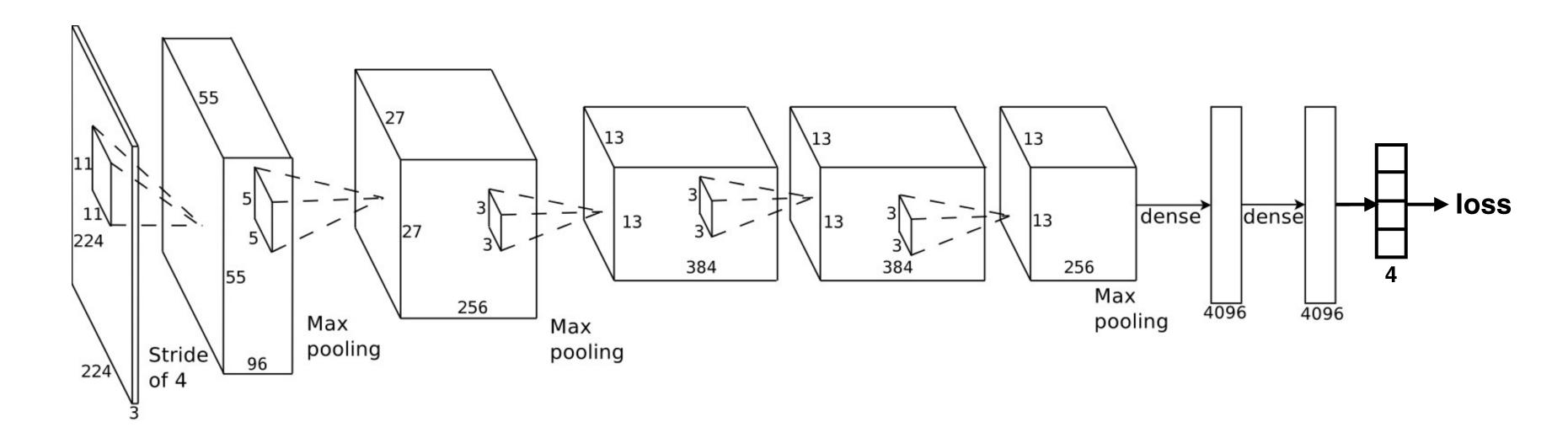
$$= \sum_{j} \frac{dL}{dy_j} X_{ji}$$

Therefore:

$$\frac{dL}{d\mathbf{w}} = \mathbf{X}^T \frac{dL}{d\mathbf{y}}$$



Back-propagation through the entire professor classification network



For each training example x_i in mini-batch:

Perform forward evaluation to compute loss for x_i

Note: must retain all layer outputs + output gradients (needed to compute weight gradients during backpropagation)

Compute gradient of loss w.r.t. final layer's outputs

Backpropagate gradient to compute gradient of loss w.r.t. all network parameters

Accumulate gradients (over all images in batch)

Update all parameter values: wi_new = wi_old - step_size * gradi

VGG memory footprint

Calculations assume 32-bit values (image batch size = 1)

input: 224 x 224 RGB image
conv: (3x3x3) x 64
conv: (3x3x64) x 64
maxpool
conv: (3x3x64) x 128
conv: (3x3x128) x 128
maxpool
conv: (3x3x128) x 256
conv: (3x3x256) x 256
conv: (3x3x256) x 256
maxpool
conv: (3x3x256) x 512
conv: (3x3x512) x 512
conv: (3x3x512) x 512
maxpool
conv: (3x3x512) x 512
conv: (3x3x512) x 512
conv: (3x3x512) x 512
maxpool
fully-connected 4096
fully-connected 4096
fully-connected 1000
soft-max

iliage battii size — 17			
weights mem:			
— 6.5 KB 144 KB	Must also store per- weight gradients		
228 KB 576 KB	Many implementations also store gradient "momentum" as well		
1.1 MB 2.3 MB	(multiply by 3)		
2.3 MB —			
4.5 MB 9 MB			
9 MB — 9 MB			
9 MB 9 MB			
— 392 MB 64 MB			
15.6 MB			

inputs/outputs get
multiplied by mini-
batch size

output size	2.
(per image)	(mem)
224x224x3	150K
224x224x64	12.3 MB
224x224x64	12.3 MB
112x112x64	3.1 MB
112x112x128	6.2 MB
112x112x128	6.2 MB
56x56x128	1.5 MB
56x56x256	3.1 MB
56x56x256	3.1 MB
56x56x256	3.1 MB
28x28x256	766 KB
28x28x512	1.5 MB
28x28x512	1.5 MB
28x28x512	1.5 MB
14x14x512	383 KB
7x7x512	98 KB
4096	16 KB
4096	16 KB
1000	4 KB
1000	4 KB

Unlike forward evaluation:

- 1. must store outputs and gradient of outputs
 - cannot immediately free outputs once consumed by next level of network

SGD workload

Deep network training workload

Huge computational expense

- Must evaluate the network (forward and backward) for millions of training images
- Must iterate for many iterations of gradient descent (100's of thousands)
- Training modern networks takes days

Large memory footprint

- Must maintain network layer outputs from forward pass
- Additional memory to store gradients for each parameter
- Recall parameters for popular VGG-16 network require ~500 MB of memory (training requires GBs of memory for academic networks)
- Scaling to larger networks requires partitioning network across nodes to keep network
 + intermediates in memory

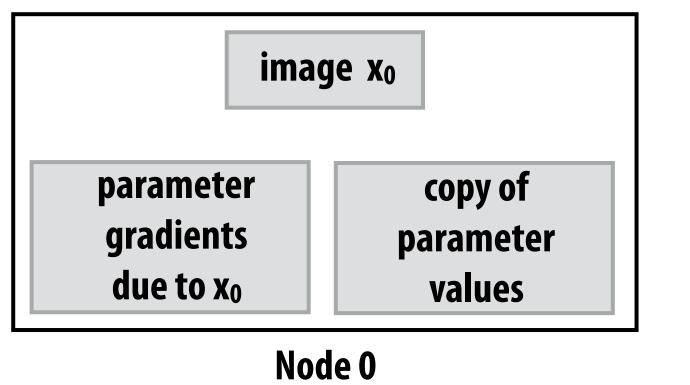
Dependencies /synchronization (not embarrassingly parallel)

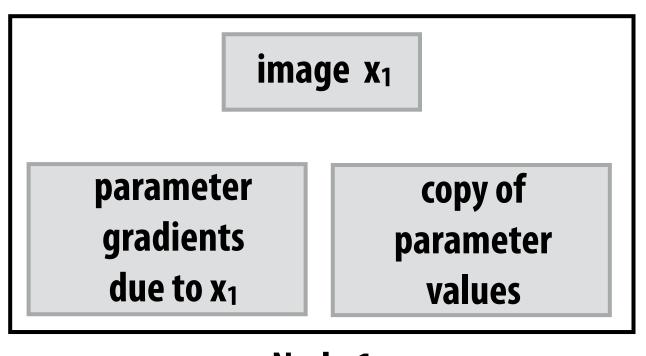
- Each parameter update step depends on previous
- Many units contribute to same parameter gradients (fine-scale reduction)
- Different images in mini batch contribute to same parameter gradients

Data-parallel training (across images)

```
for each item x_i in mini-batch:
    grad += evaluate_loss_gradient(f, loss_func, params, x_i)
params += -grad * step_size;
```

Consider parallelization of the outer for loop across machines in a cluster





Node 1

```
partition mini-batch across nodes
for each item x_i in mini-batch assigned to local node:
    // just like single node training
    grad += evaluate_loss_gradient(f, loss_func, params, x_i)
barrier();
sum reduce gradients, communicate results to all nodes
barrier();
update copy of parameter values
```

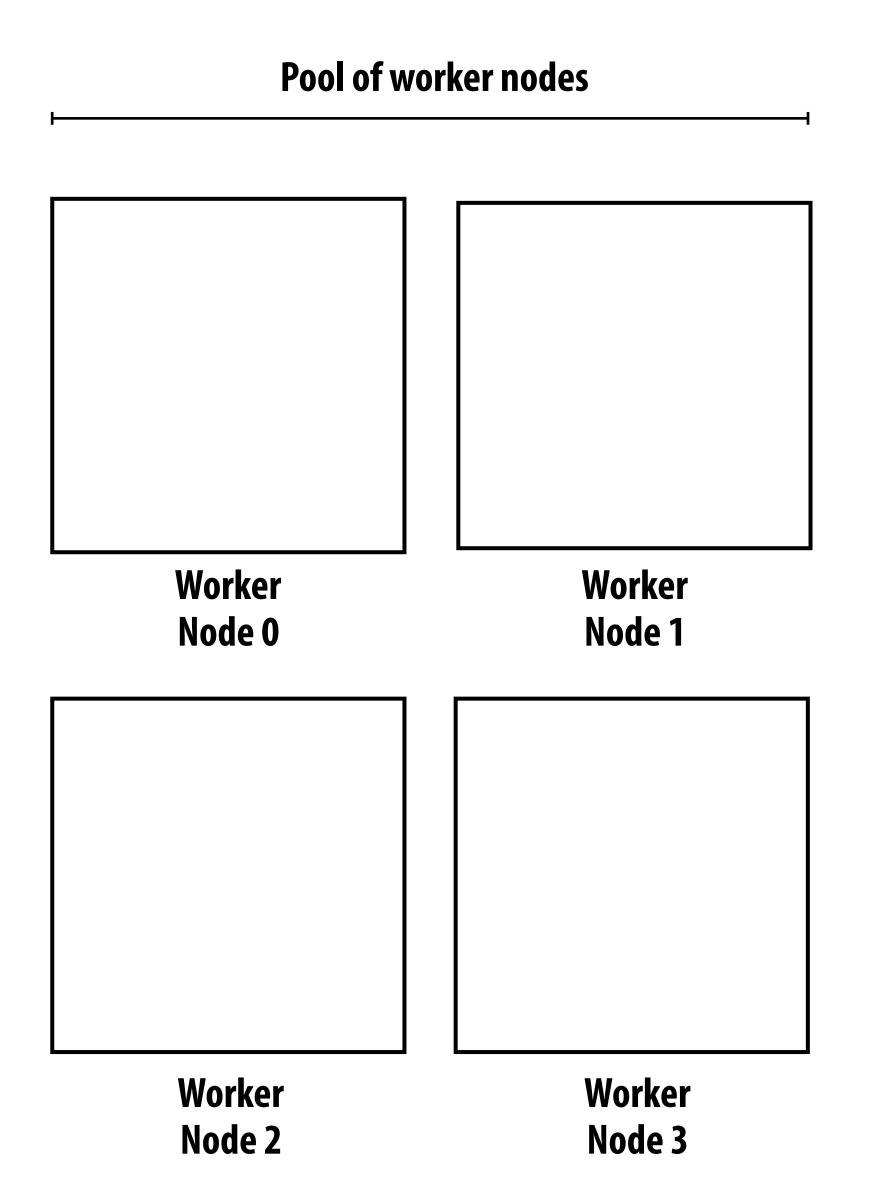
Challenges of computing at cluster scale

- Slow communication between nodes
 - Clusters do not feature high-performance interconnects typical of supercomputers
- Nodes with different performance (even if machines are the same)
 - Workload imbalance at barriers (sync points between nodes)

Modern solution: exploit characteristics of SGD using asynchronous execution!

Parameter server design

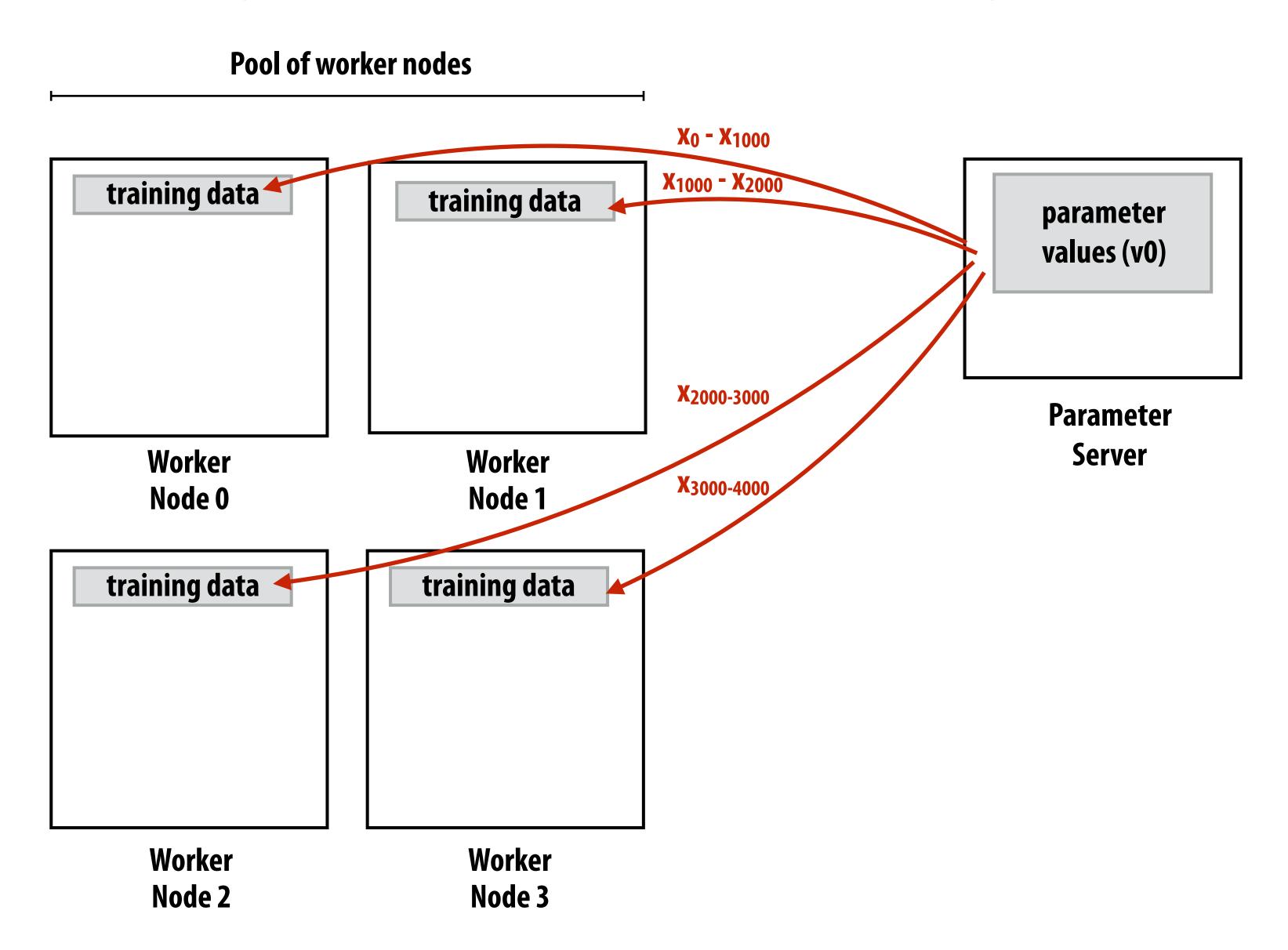
Parameter Server [Li OSDI14]
Google's DistBelief [Dean NIPS12]
Microsoft's Project Adam [Chilimbi OSDI14]



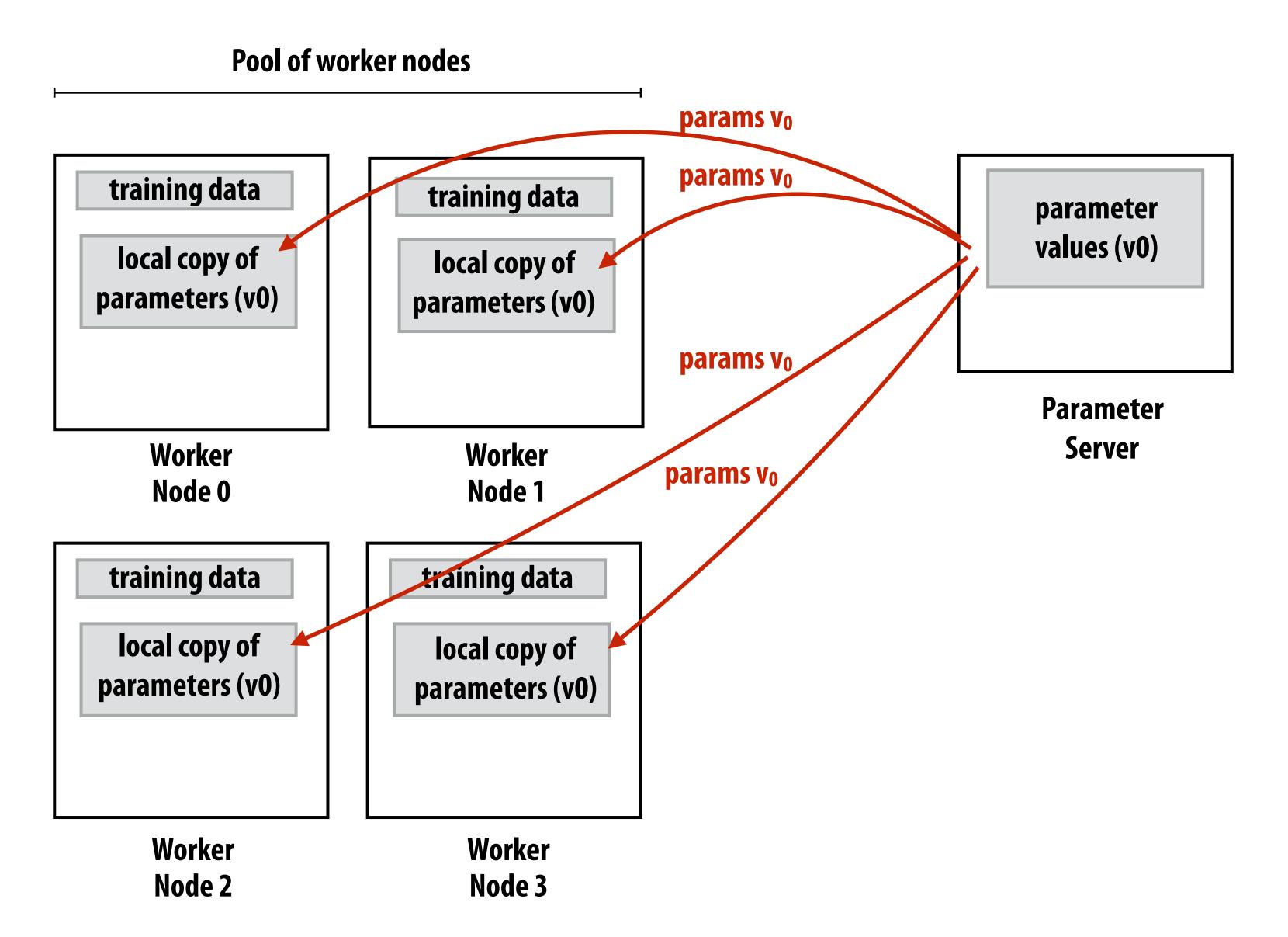
parameter values

Parameter Server

Training data partitioned among workers



Copy of parameters sent to workers



Workers independently compute local "subgradients"

Pool of worker nodes

local copy of parameters (v0)

local subgradients

Worker Node 0

local copy of parameters (v0)

local subgradients

Worker Node 2 local copy of parameters (v0)

local subgradients

Worker Node 1

local copy of parameters (v0)

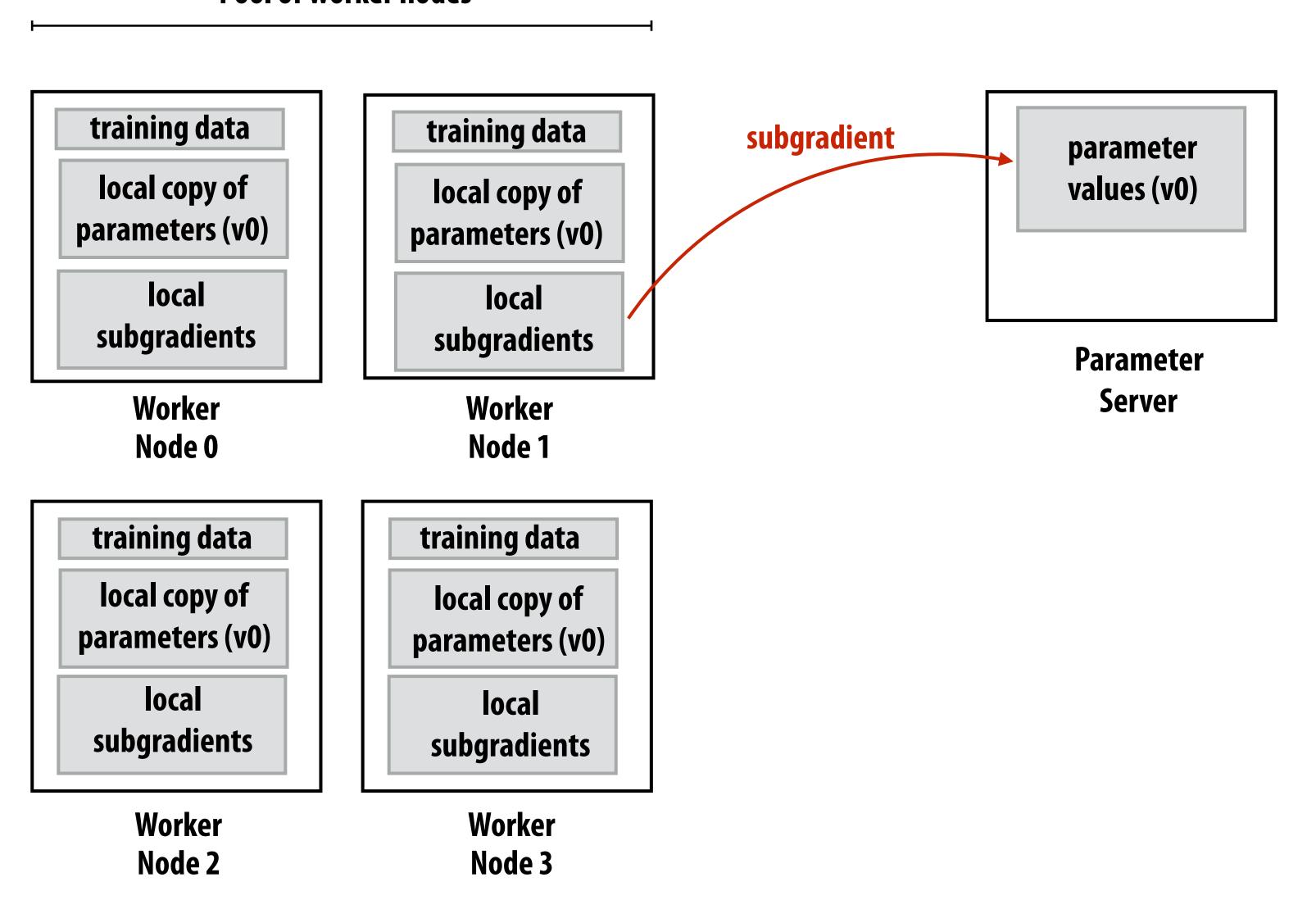
local subgradients

Worker Node 3 parameter values (v0)

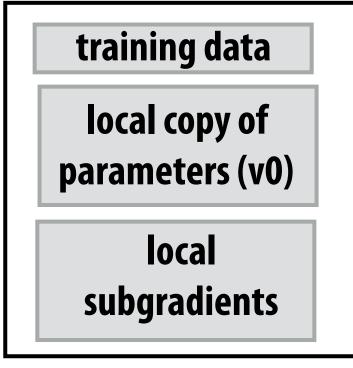
Parameter Server

Worker sends subgradient to parameter server

Pool of worker nodes



Server updates global parameter values based on subgradient



Worker Node 0

local copy of parameters (v0)

local subgradients

Worker Node 2 local copy of parameters (v0)

local subgradients

Worker Node 1

local copy of parameters (v0)

local subgradients

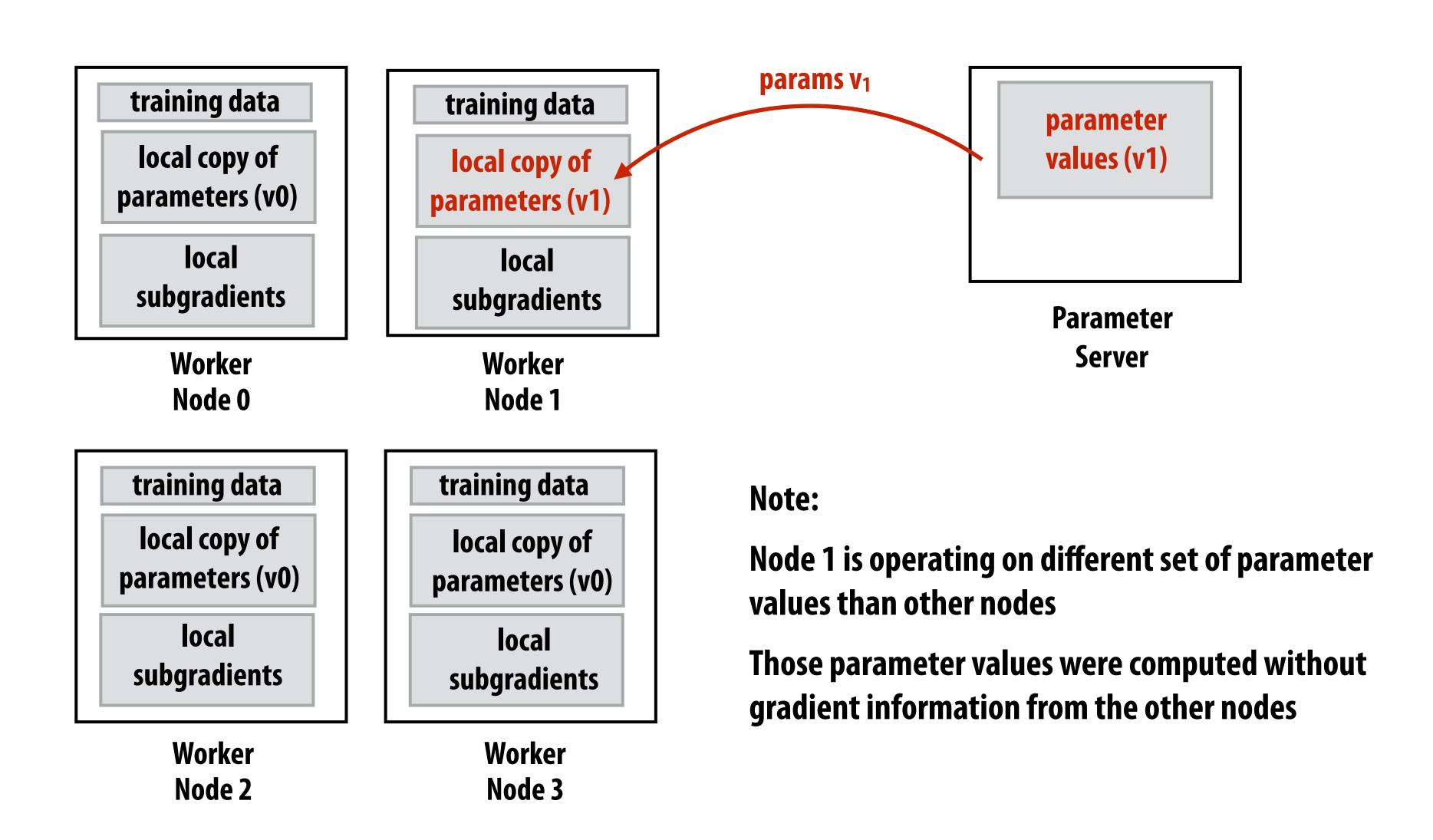
Worker Node 3 parameter values (v1)

Parameter Server

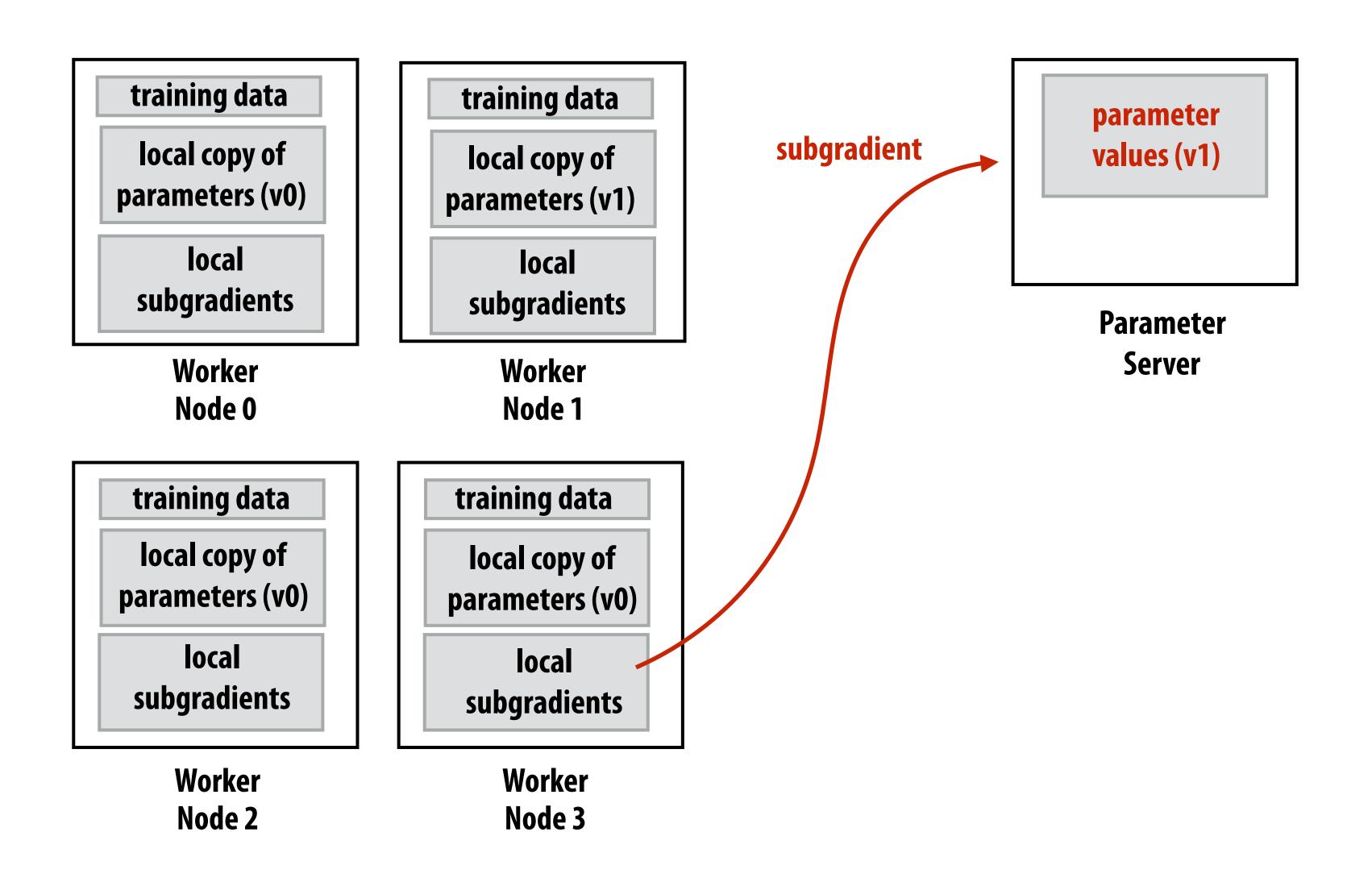
params += -subgrad * step_size;

Updated parameters sent to worker

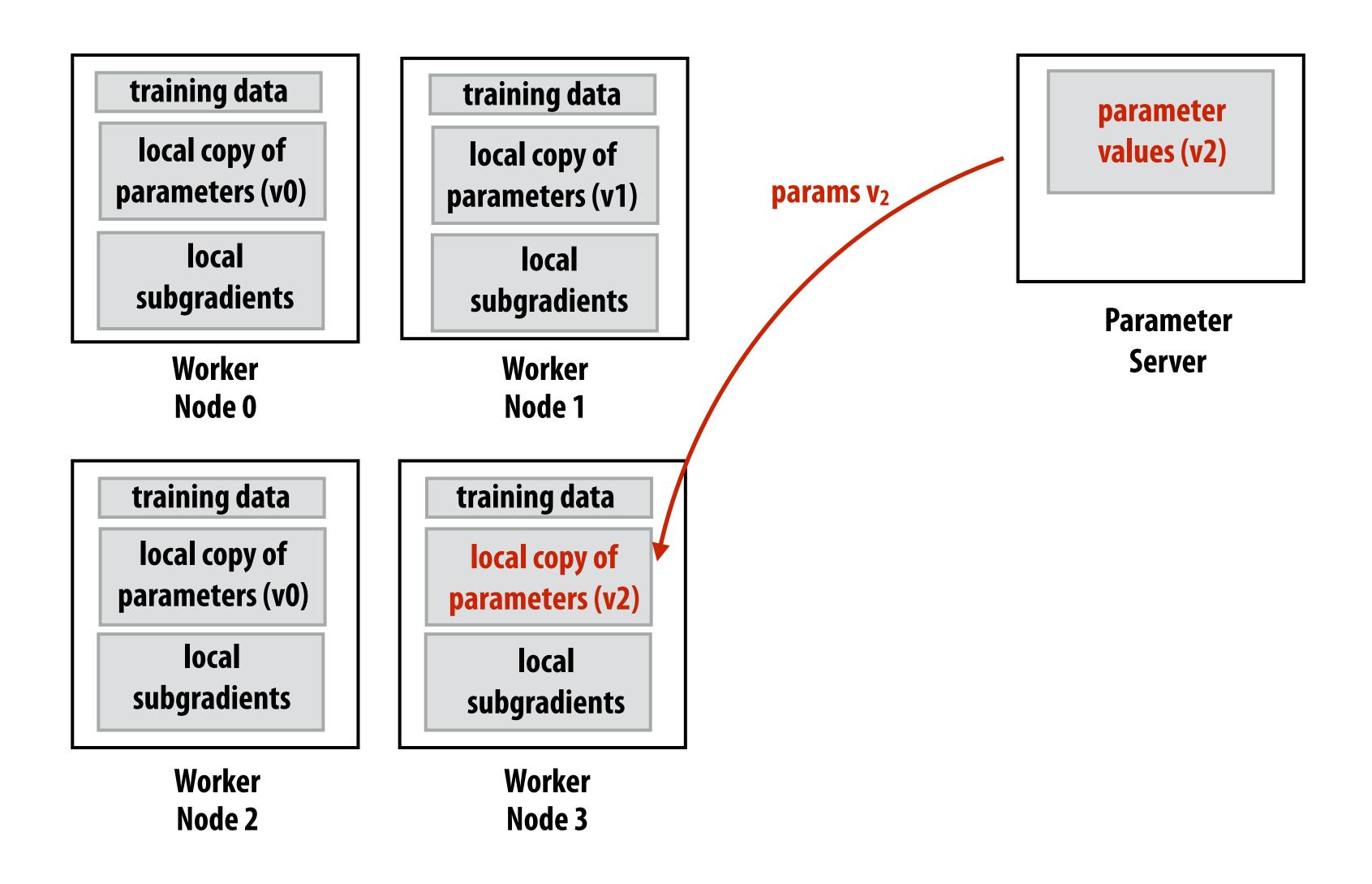
Worker proceeds with another gradient computation step



Updated parameters sent to worker (again)



Worker continues with updated parameters



Summary: asynchronous parameter update

- Idea: avoid global synchronization on all parameter updates between each SGD iteration
 - Design reflects realities of cluster computing:
 - Slow interconnects
 - Unpredictable machine performance
- Solution: asynchronous (and partial) subgradient updates
- Will impact convergence of SGD
 - Node N working on iteration i may not have parameter values that result the results of the i-1 prior SGD iterations

Bottleneck?

What if there is heavy contention for parameter server?

local copy of parameters (v0)

local subgradients

Worker Node 0

local copy of parameters (v0)

local subgradients

Worker Node 2 local copy of parameters (v1)

local subgradients

Worker Node 1

local copy of parameters (v2)

local subgradients

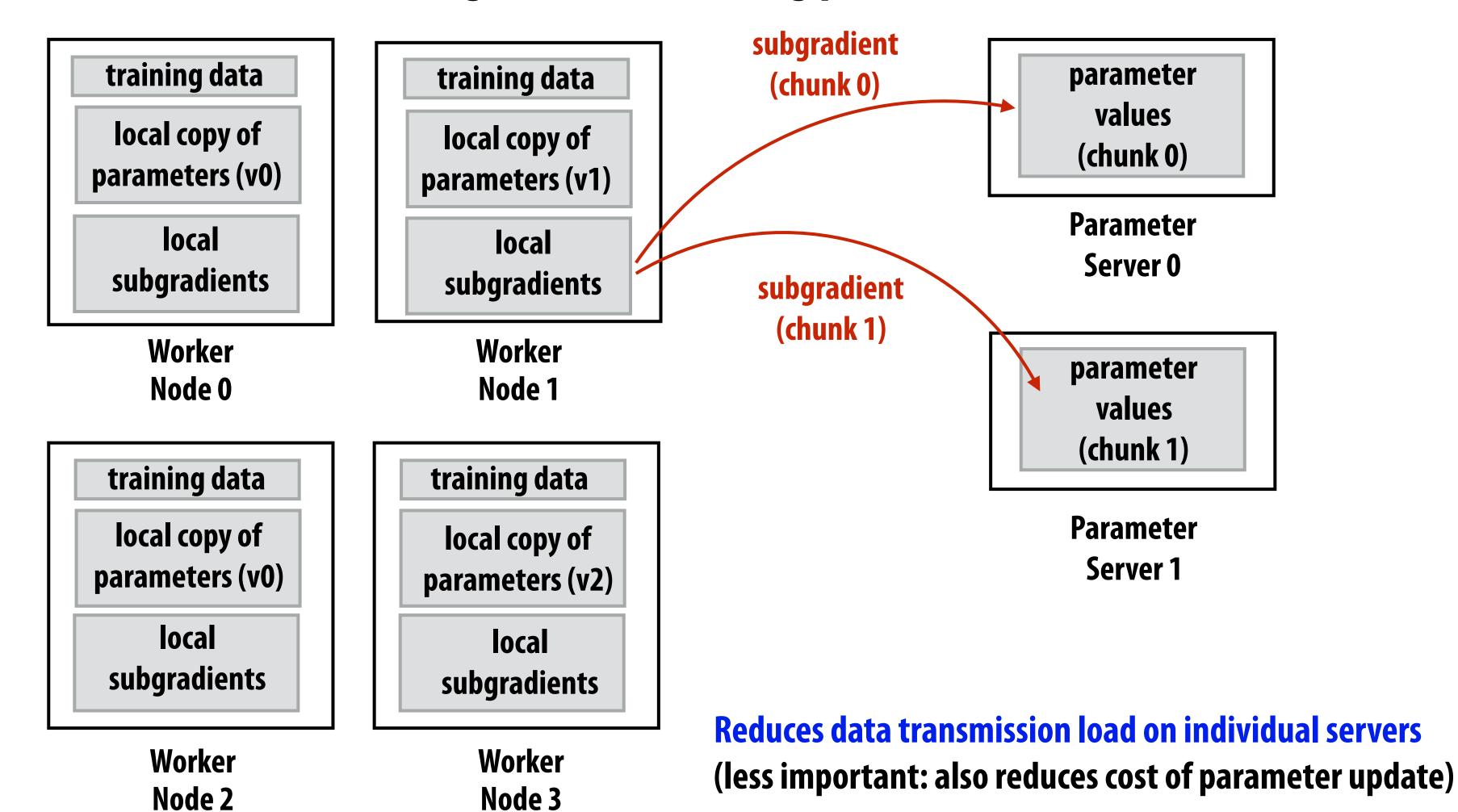
Worker Node 3 parameter values (v2)

Parameter Server

Shard the parameter server

Partition parameters across servers

Worker sends chunk of subgradients to owning parameter server



What if model parameters do not fit on one worker?

Recall high footprint of training large networks (particularly with large mini-batch sizes)

local copy of parameters (v0)

local subgradients

Worker Node 0

local copy of parameters (v0)

local subgradients

Worker Node 2 local copy of parameters (v1)

local subgradients

Worker Node 1

local copy of parameters (v2)

local subgradients

Worker Node 3 parameter values (chunk 0)

Parameter Server 0

parameter values (chunk 1)

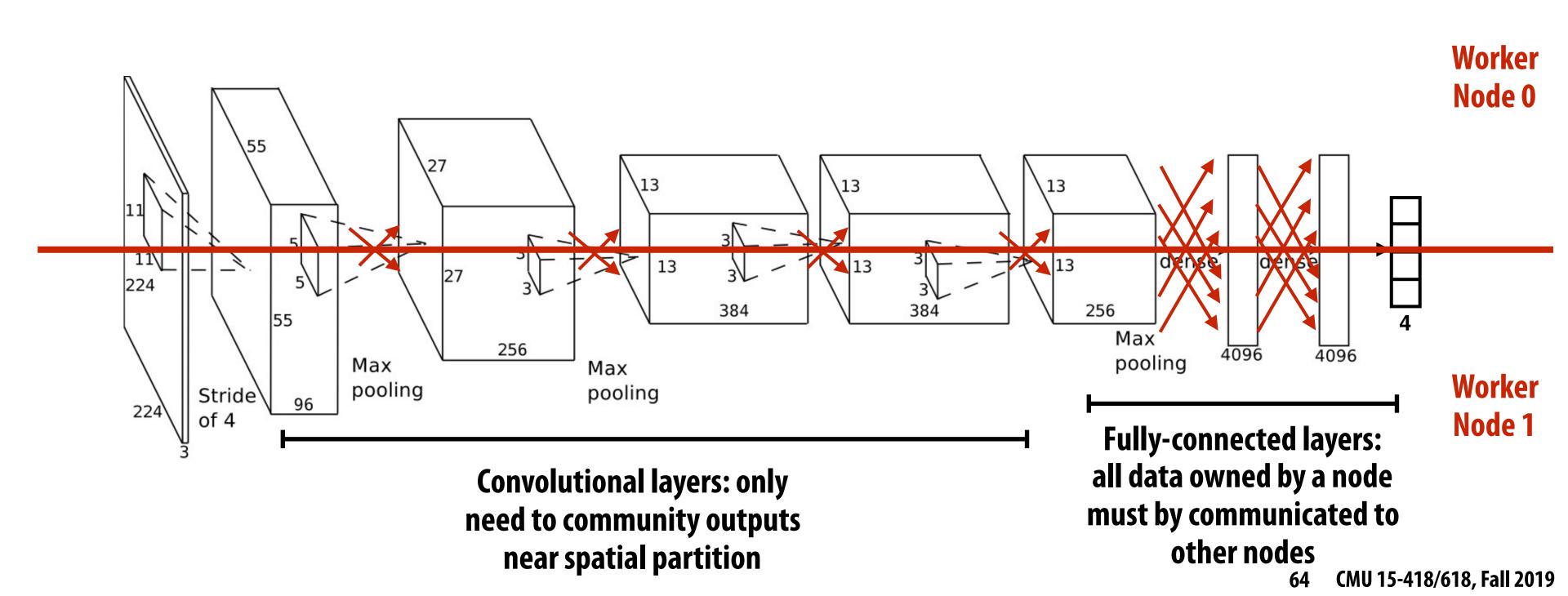
Parameter Server 1

Model parallelism

Partition network parameters across nodes (spatial partitioning to reduce communication)

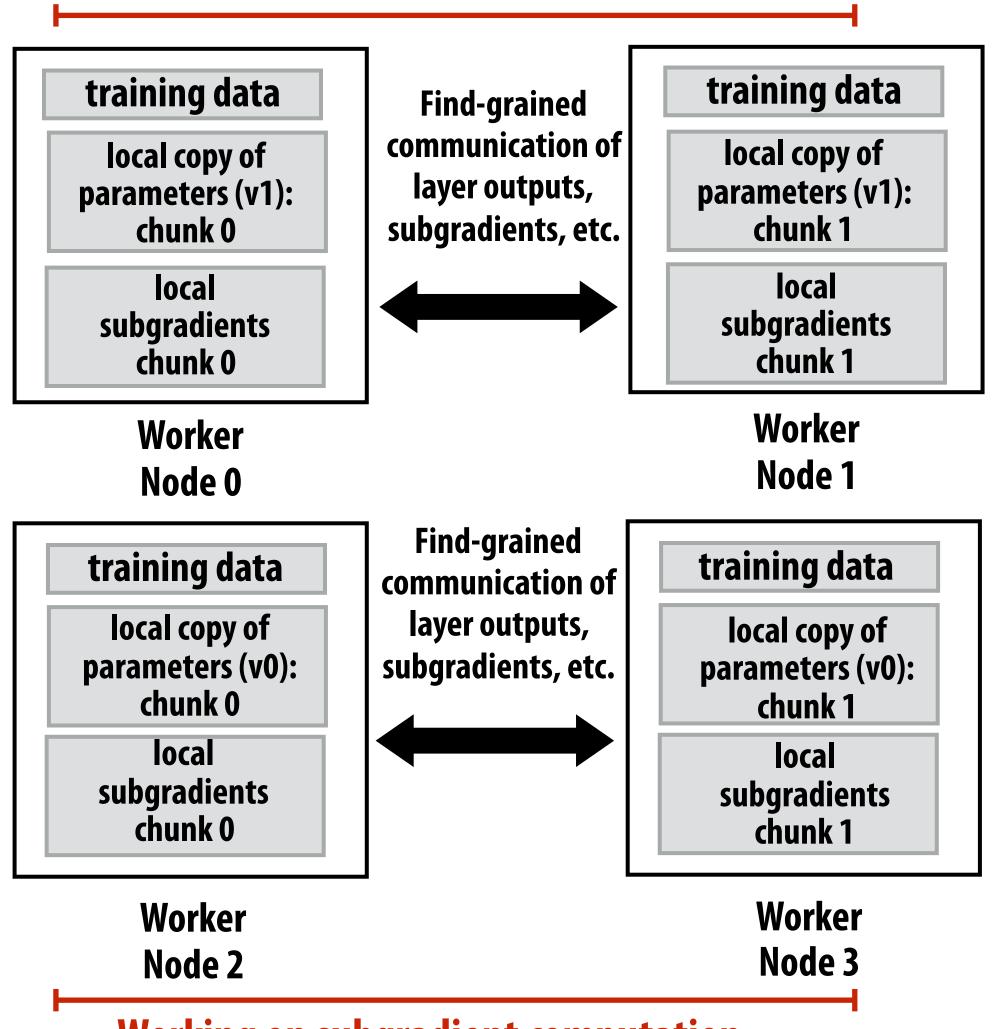
Reduce internode communication through network design:

- Use small spatial convolutions (1x1 convolutions)
- Reduce/shrink fully-connected layers



Training data-parallel and model-parallel execution

Working on subgradient computation for a single copy of the model



parameter values (chunk 0)

Parameter Server 0

parameter
 values
 (chunk 1)

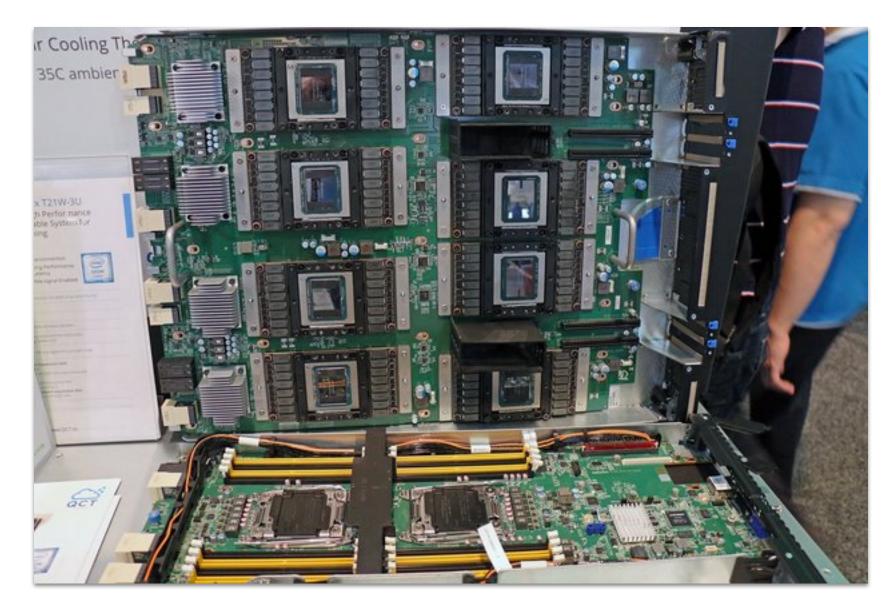
Parameter Server 1

Using supercomputers for training?

- Fast interconnects critical for model-parallel training
 - Fine-grained communication of outputs and gradients
- Fast interconnect diminishes need for async training algorithms
 - Avoid randomness in training due to computation schedule (there remains randomness due to SGD algorithm)



OakRidge Titan Supercomputer

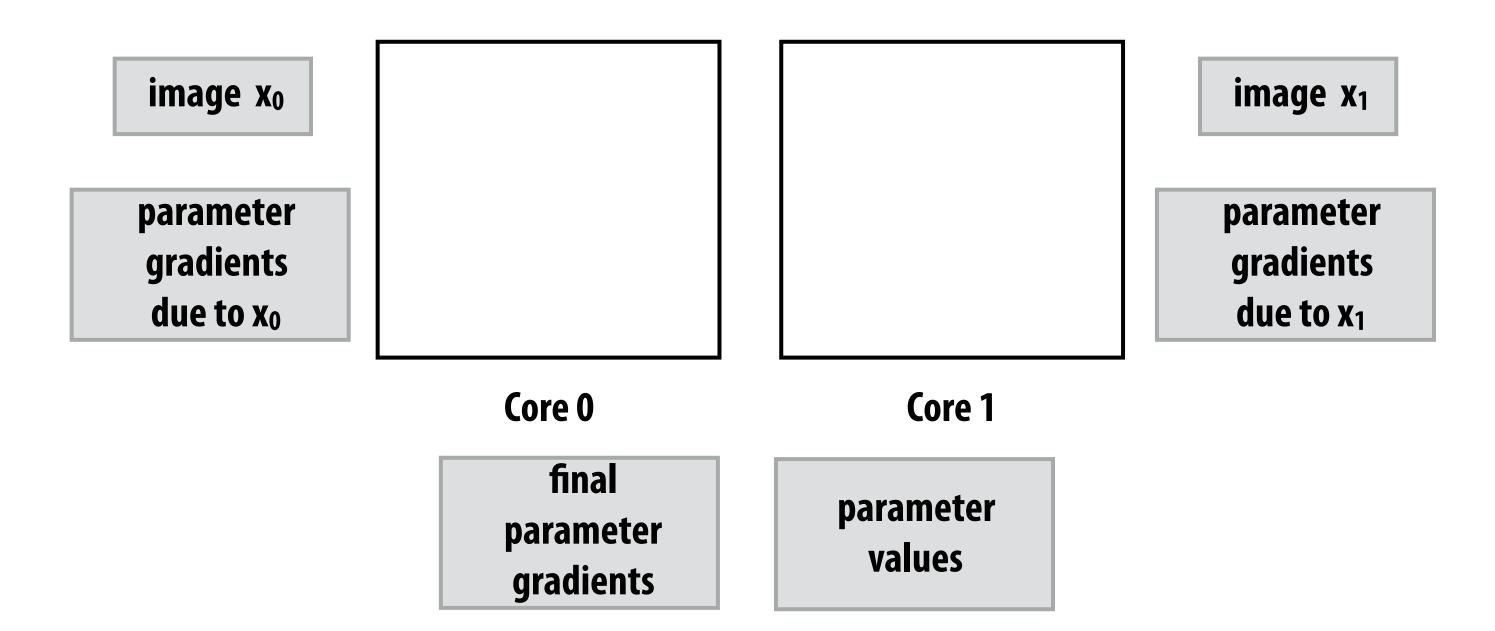


NVIDIA DGX-1: 8 Pascal GPUs connected via high speed NV-Link interconnect

Parallelizing mini-batch on one machine

```
for each item x_i in mini-batch:
    grad += evaluate_loss_gradient(f, loss_func, params, x_i)
params += -grad * step_size;
```

Consider parallelization of the outer for loop across cores



Good: completely independent computations (until gradient reduction)

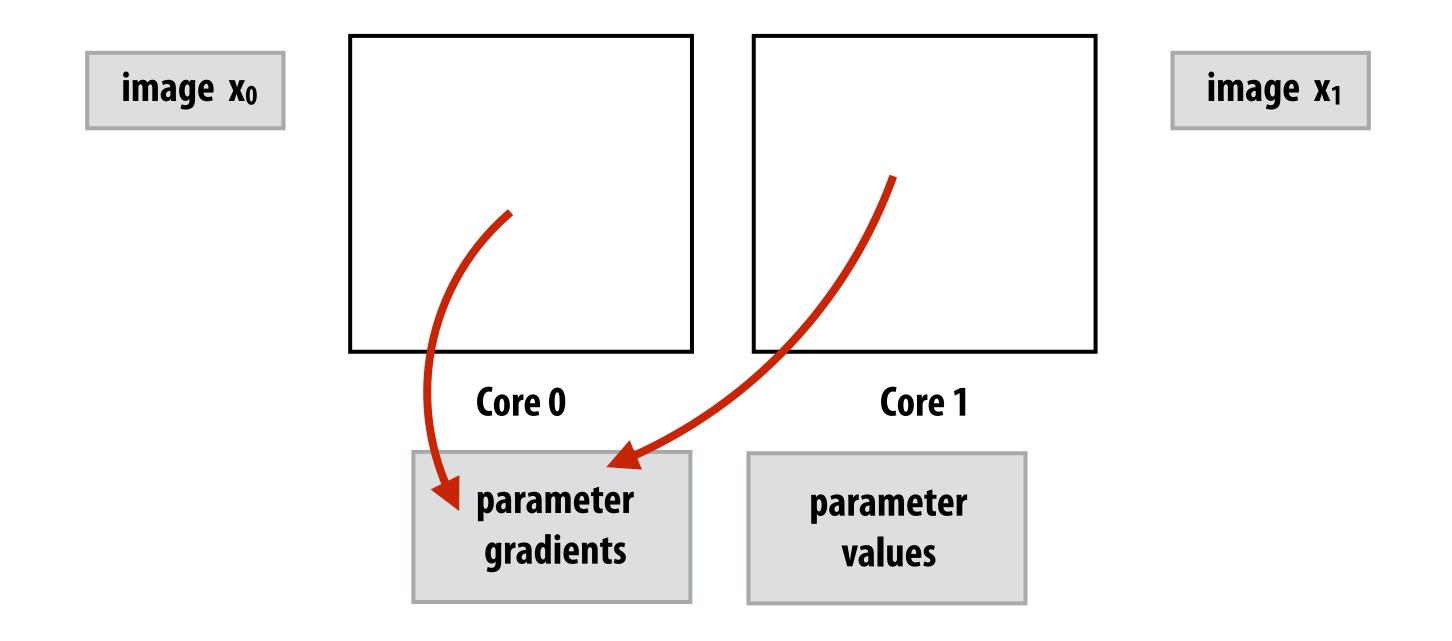
Bad: complete duplication of parameter gradient state (100's MB per core)

Asynchronous update on one node

```
for each item x_i in mini-batch:
    grad += evaluate_loss_gradient(f, loss_func, params, x_i)
params += -grad * step_size;
```

Cores update shared set of gradients.

Skip taking locks / synchronizing across cores: perform "approximate reduction"



Summary: training large networks in parallel

- Most systems rely on asynchronous update to efficiently used clusters of commodity machines
 - Modification of SGD algorithm to meet constraints of modern parallel systems
 - Open question: effects on convergence are problem dependent and not particularly well understood
 - Tighter integration / faster interconnects may provide alternative to these methods (facilitate tightly orchestrated solutions much like supercomputing applications)
- Open question: how big of networks are needed?
 - >90% of connections could be removed without significant impact on quality of network
 - High-performance training of deep networks is an interesting example of constant iteration of algorithm design and parallelization strategy (a key theme of this course! recall the original grid solver example!)