Lecture 25:

Parallel Deep Neural Networks

Parallel Computer Architecture and Programming
CMU 15-418/15-618, Fall 2020

Training/evaluating deep neural networks

Technique leading to many high-profile AI advances in recent years

Speech recognition/natural language processing

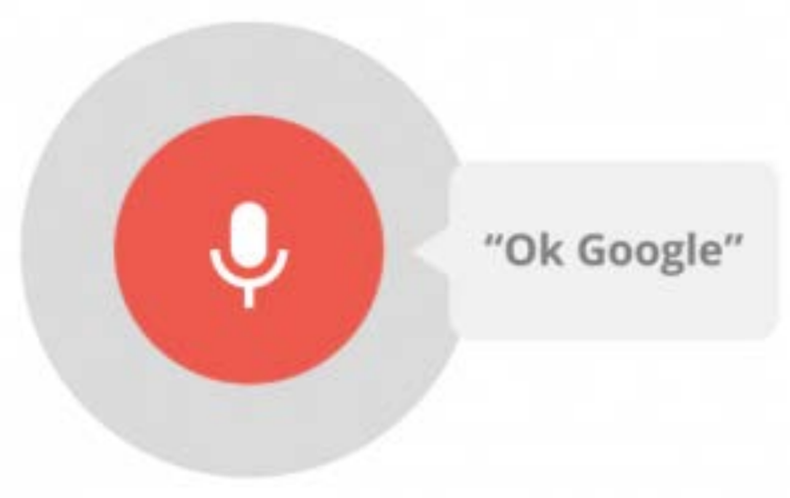
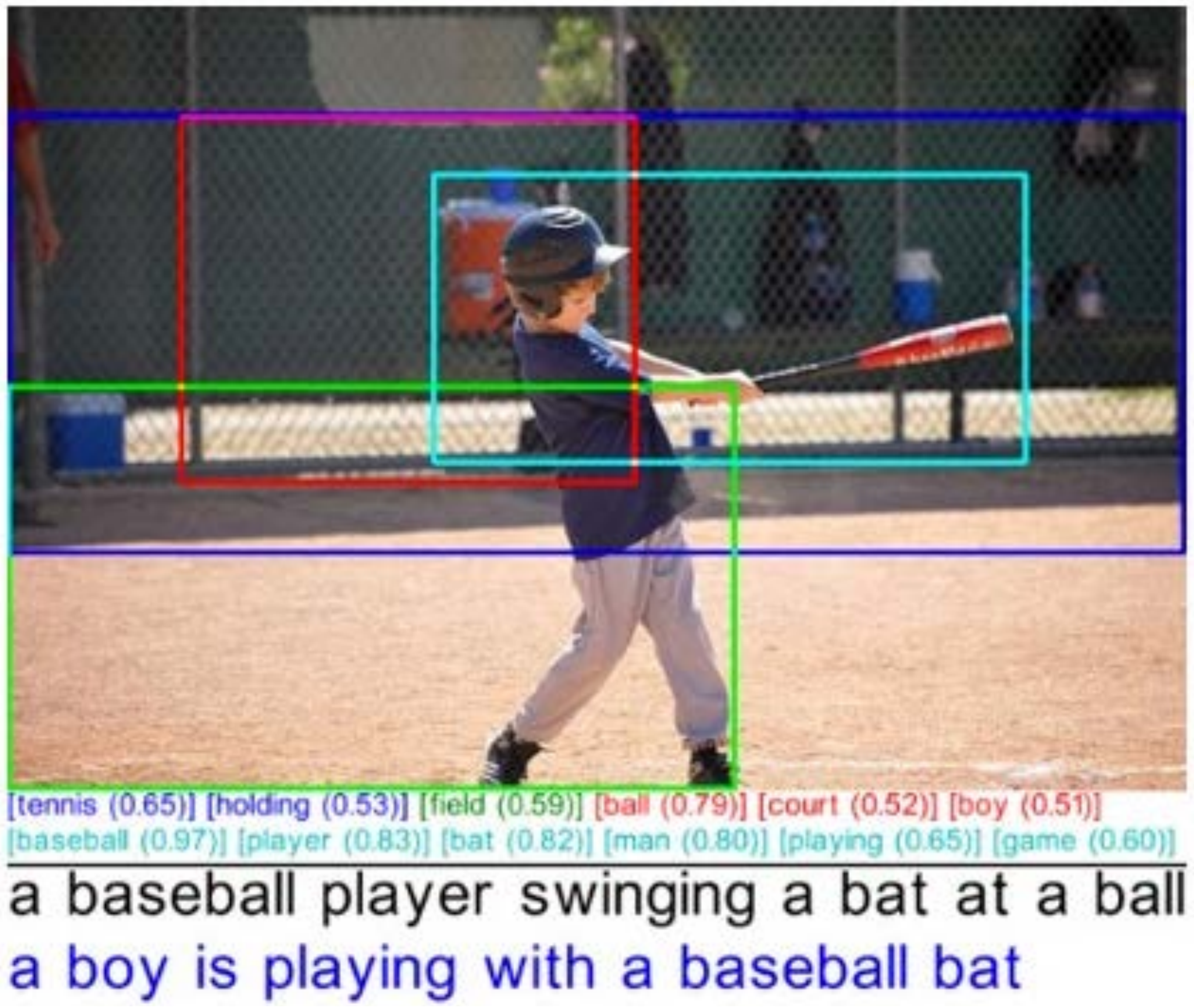
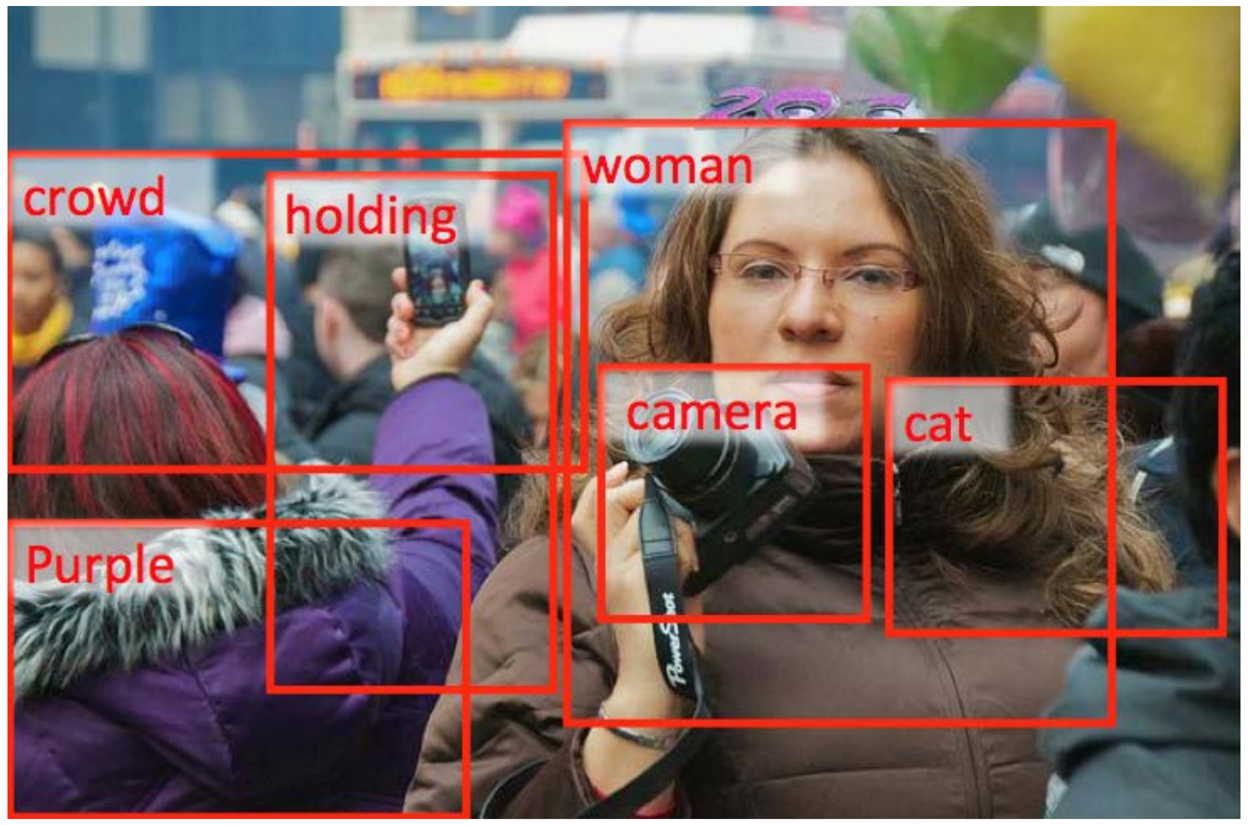


Image interpretation and understanding

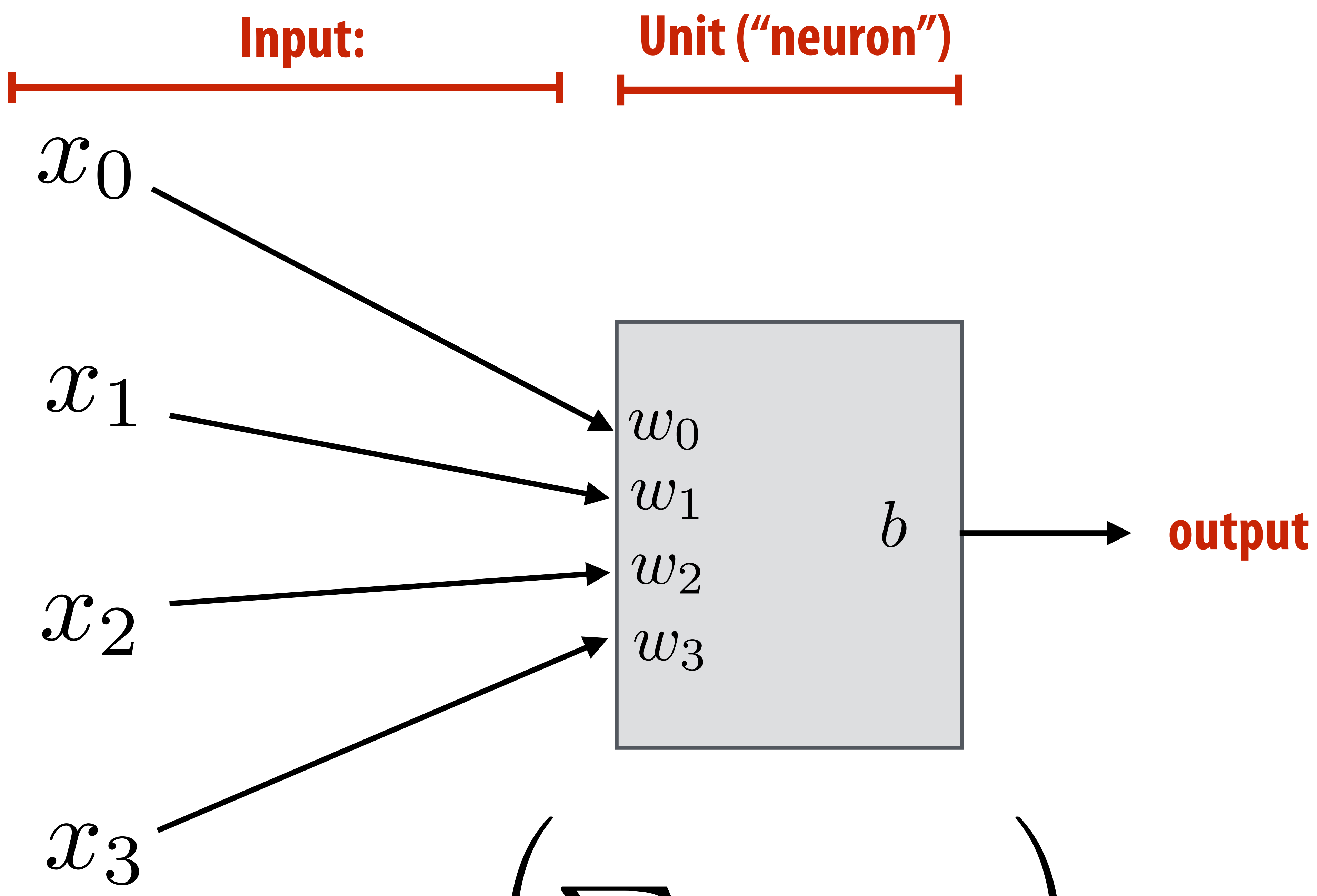

[tennis (0.65)] [holding (0.53)] [field (0.59)] [ball (0.79)] [court (0.52)] [boy (0.51)]
[baseball (0.97)] [player (0.83)] [bat (0.82)] [man (0.80)] [playing (0.65)] [game (0.60)]
a baseball player swinging a bat at a ball
a boy is playing with a baseball bat



What is a deep neural network?

A basic unit:

Unit with *n* inputs described by *n+1* parameters (weights + bias)



Input:

Unit (“neuron”)

output

$$f\left(\sum_i x_i w_i + b\right)$$

Example: rectified linear unit (ReLU)

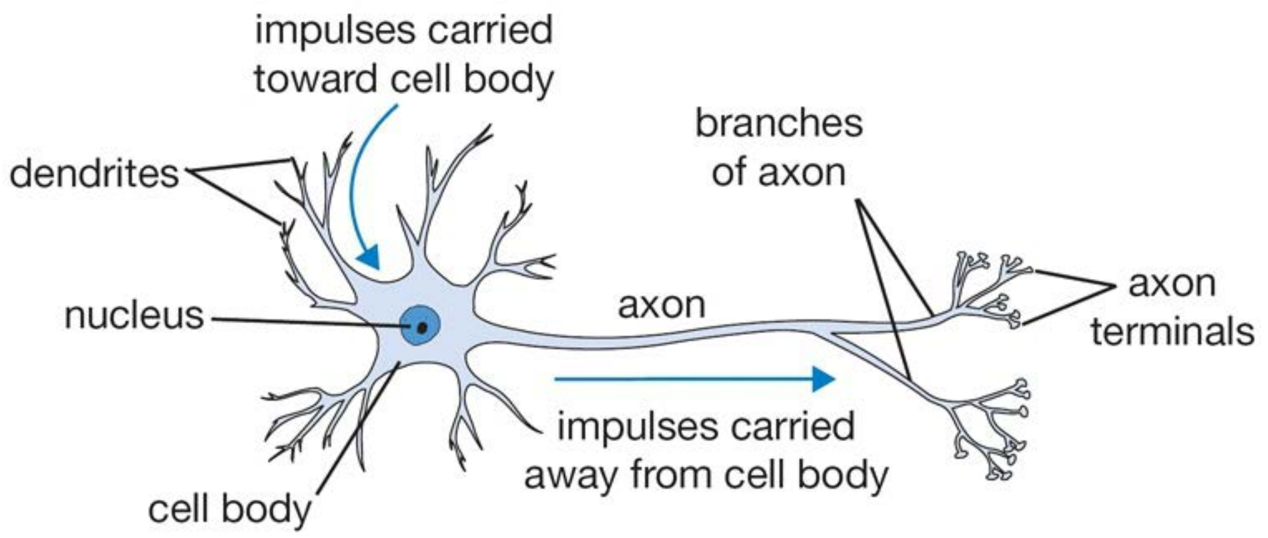
$f(x) = max(0, x)$

Basic computational interpretation:

It’s just a circuit!

Biological inspiration:

unit output corresponds loosely to activation of neuron

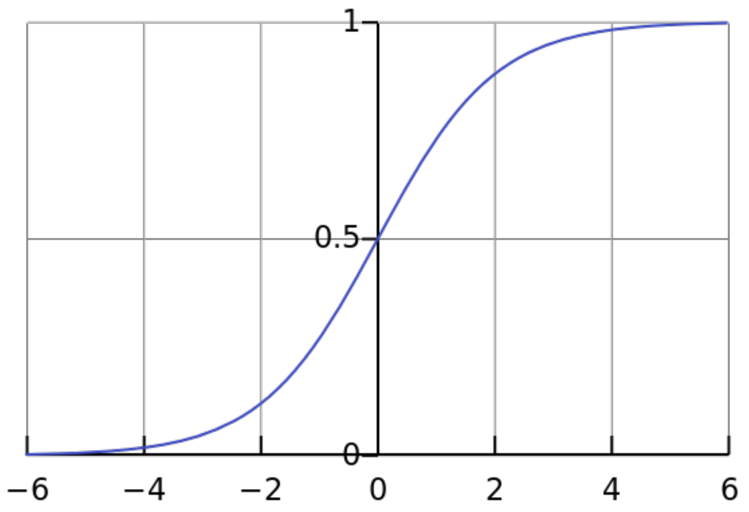


impulses carried toward cell body
dendrites
branches of axon
nucleus
axon
axon terminals
impulses carried away from cell body
cell body

Machine learning interpretation:

binary classifier: interpret output as the probability of one class

$$f(x) = \frac{1}{1 + e^{-x}}$$



Two Distinct Issues with Deep Networks

- **Evaluation**
  - often takes milliseconds
- **Training**
  - often takes hours, days, weeks

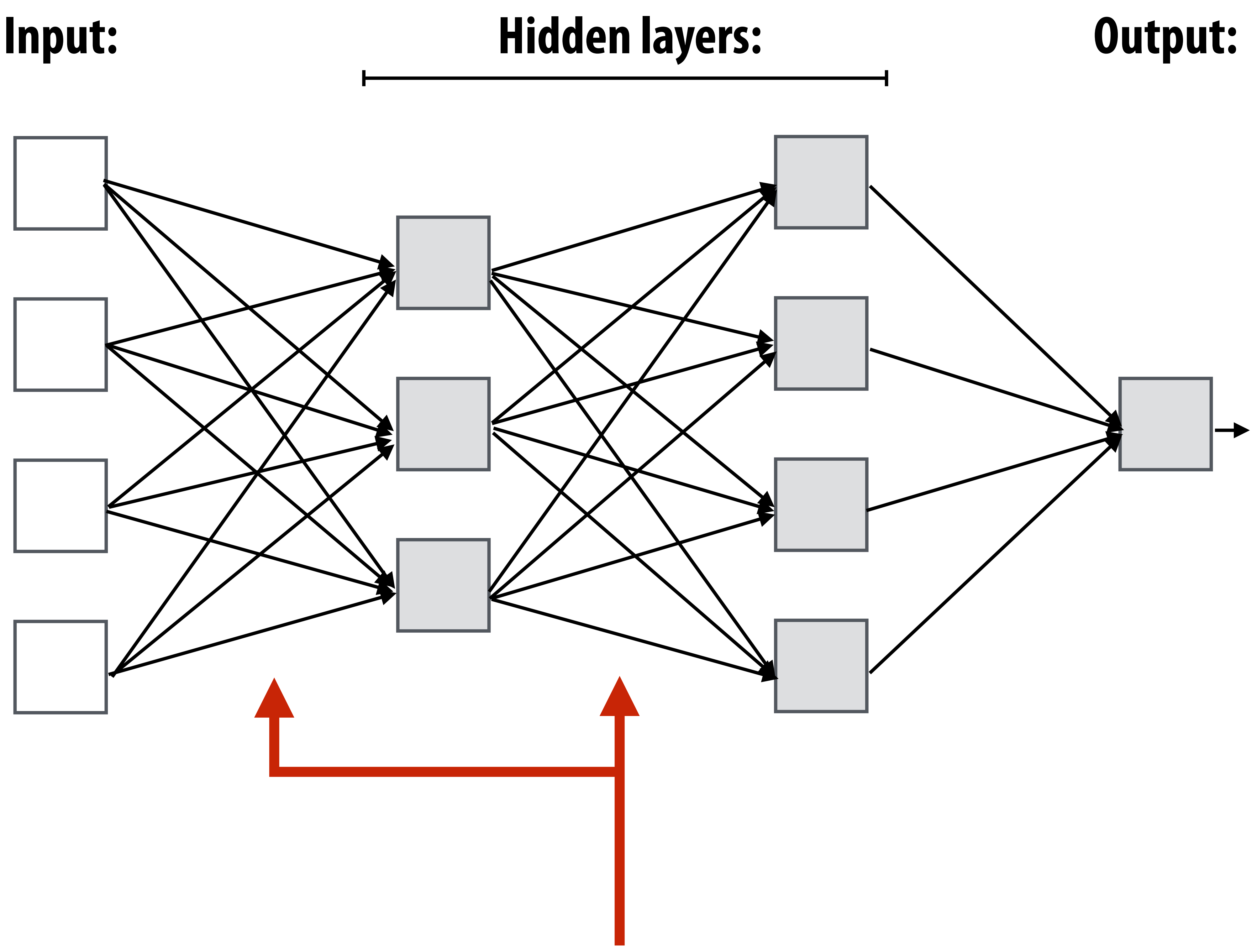
What is a deep neural network? topology

This network has: 4 inputs, 1 output, 7 hidden units

"Deep" = at least one hidden layer

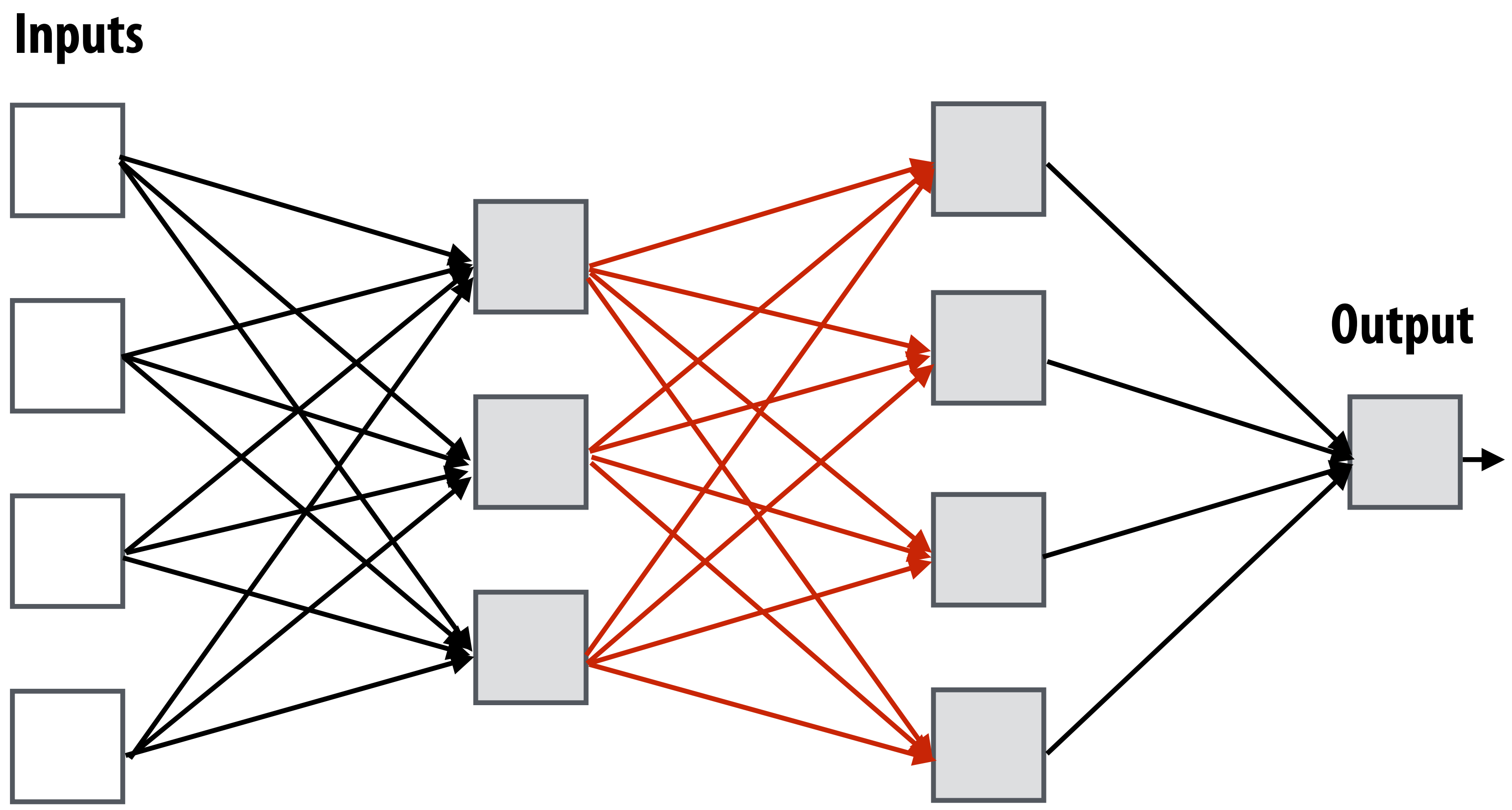
Hidden layer 1: 3 units x (4 weights + 1 bias) = 15 parameters

Hidden layer 2: 4 units x (3 weights + 1 bias) = 16 parameters

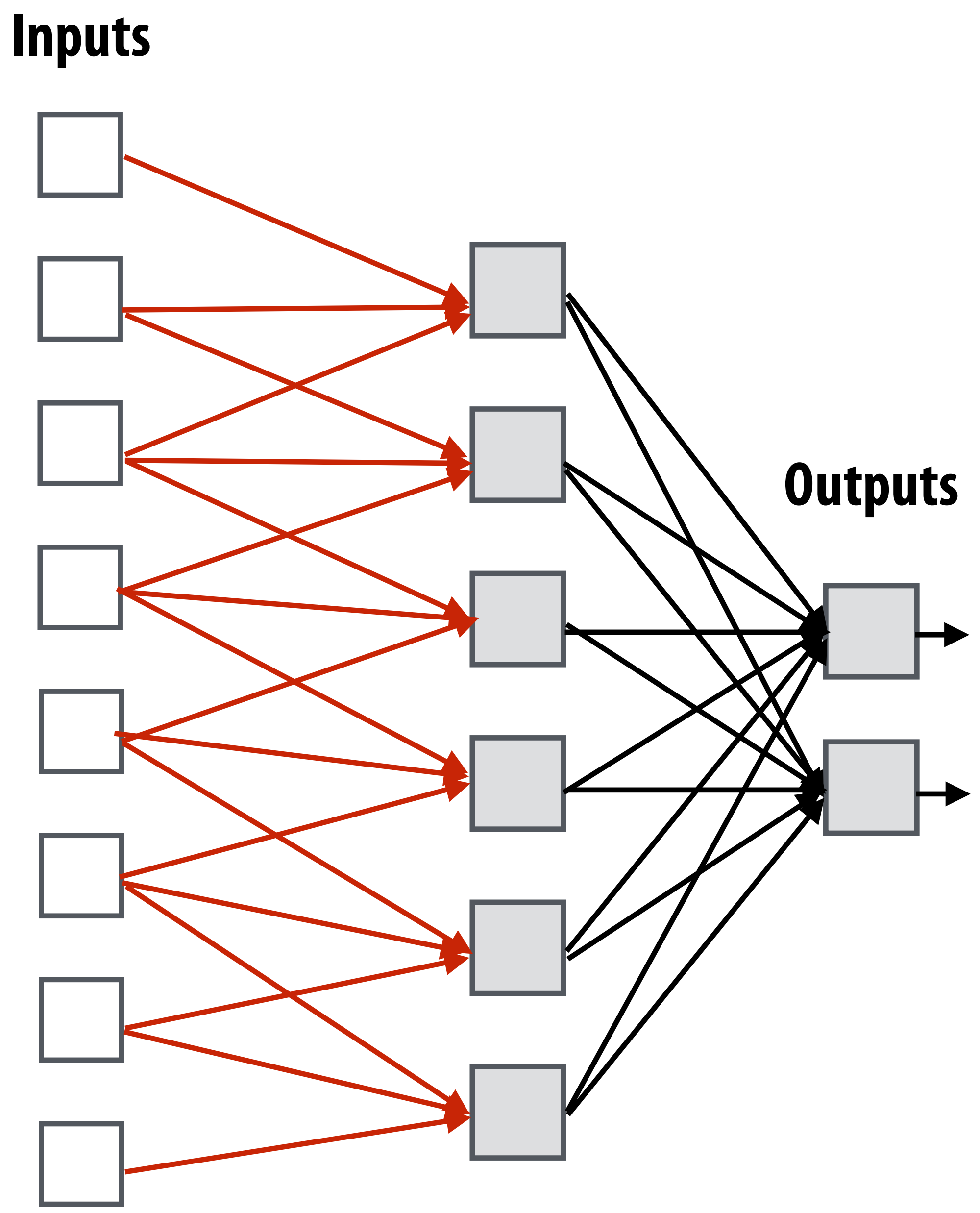


Note fully-connected topology in this example

What is a deep neural network? topology



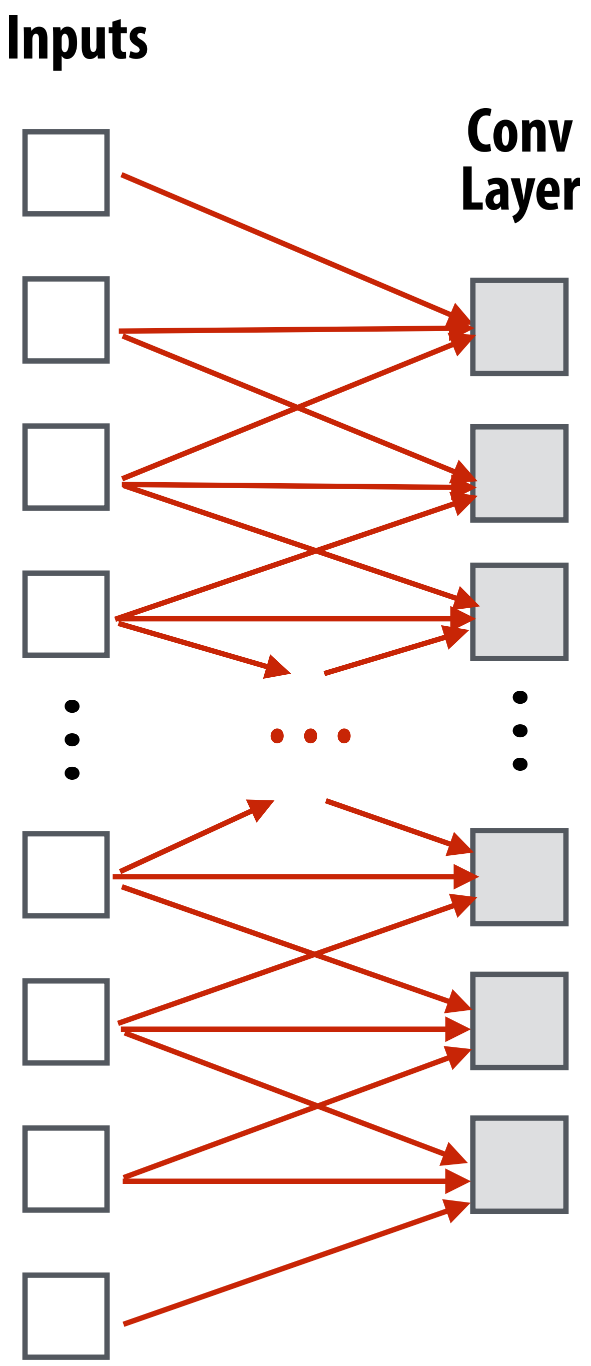
Fully connected layer

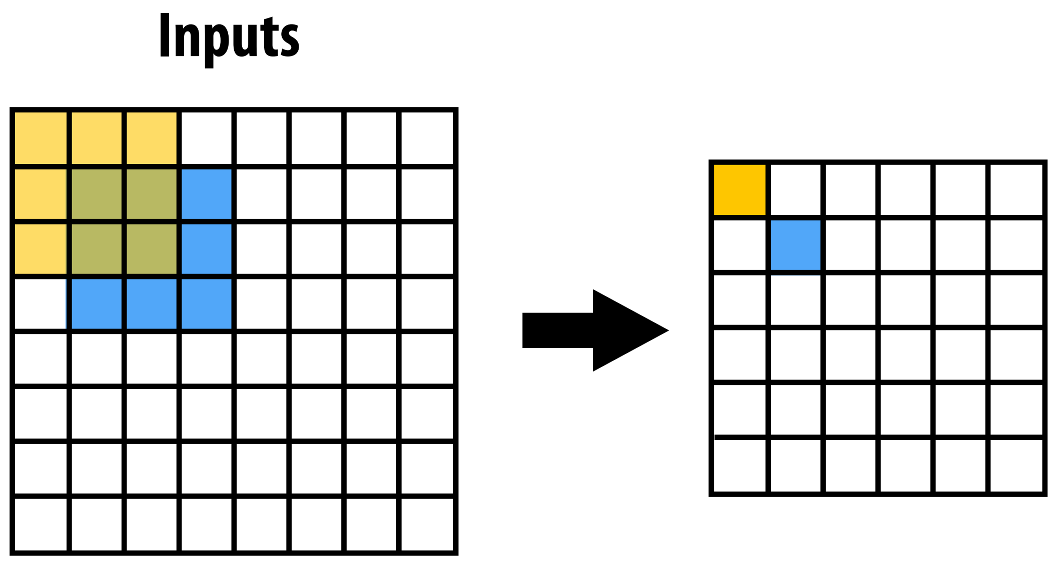


Sparsely (locally) connected

Recall image convolution (3x3 conv)

```
int WIDTH = 1024;
int HEIGHT = 1024;
float input[(WIDTH+2) * (HEIGHT+2)];
float output[WIDTH * HEIGHT];


float weights[] = {1.0/9, 1.0/9, 1.0/9,
                   1.0/9, 1.0/9, 1.0/9,
                   1.0/9, 1.0/9, 1.0/9};


for (int j=0; j<HEIGHT; j++) {
  for (int i=0; i<WIDTH; i++) {
    float tmp = 0.f;
    for (int jj=0; jj<3; jj++)
      for (int ii=0; ii<3; ii++)
        tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
    output[j*WIDTH + i] = tmp;
  }
}
```

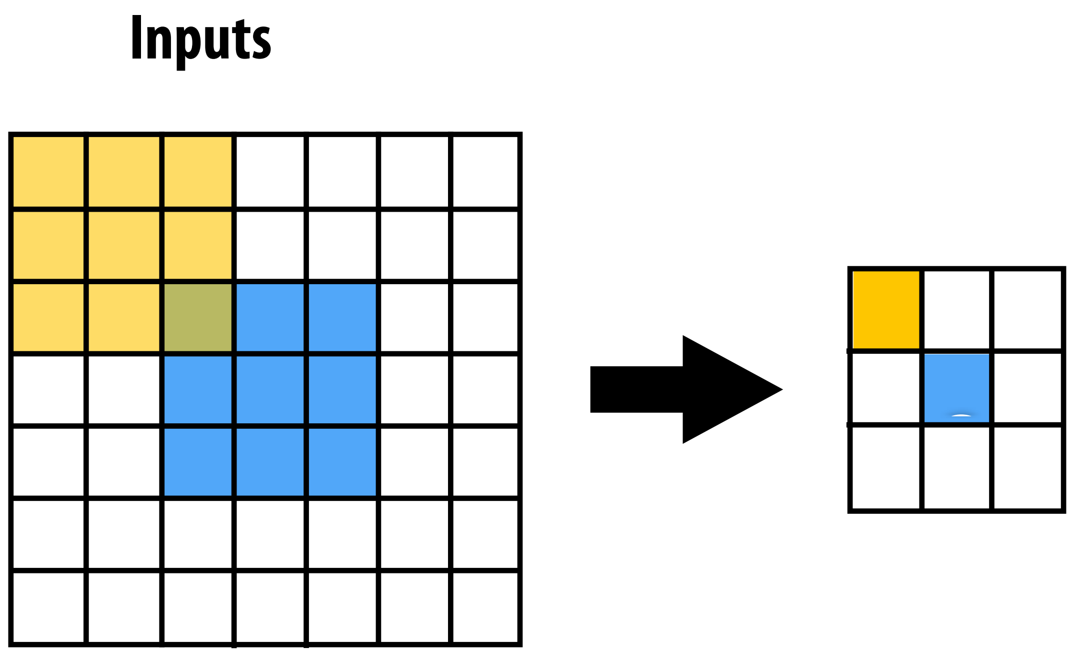


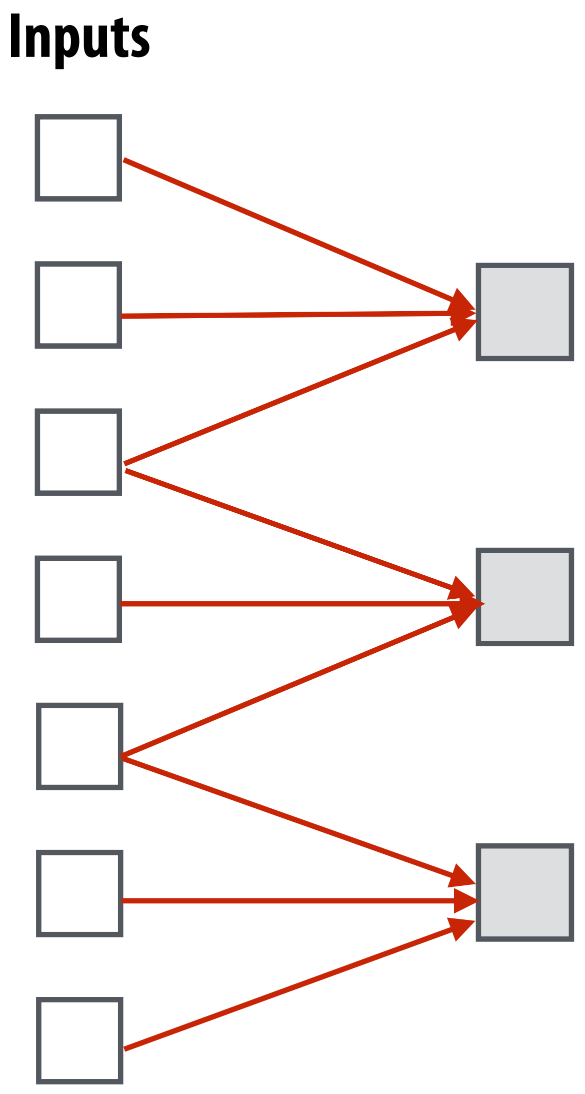


Convolutional layer: locally connected AND all units in layer share the same parameters (same weights + same bias):
(note: network diagram only shows links due to one iteration of `ii` loop)

Strided 3x3 convolution

```
int WIDTH = 1024;
int HEIGHT = 1024;
int STRIDE = 2;
float input[(WIDTH+2) * (HEIGHT+2)];
float output[(WIDTH/STRIDE) * (HEIGHT/STRIDE)];


float weights[] = {1.0/9, 1.0/9, 1.0/9,
                   1.0/9, 1.0/9, 1.0/9,
                   1.0/9, 1.0/9, 1.0/9};


for (int j=0; j<HEIGHT; j+=STRIDE) {
  for (int i=0; i<WIDTH; i+=STRIDE) {
    float tmp = 0.f;
    for (int jj=0; jj<3; jj++)
      for (int ii=0; ii<3; ii++) {
         tmp += input[(j+jj)*(WIDTH+2) + (i+ii)] * weights[jj*3 + ii];
      output[(j/STRIDE)*WIDTH + (i/STRIDE)] = tmp;
  }
}
```

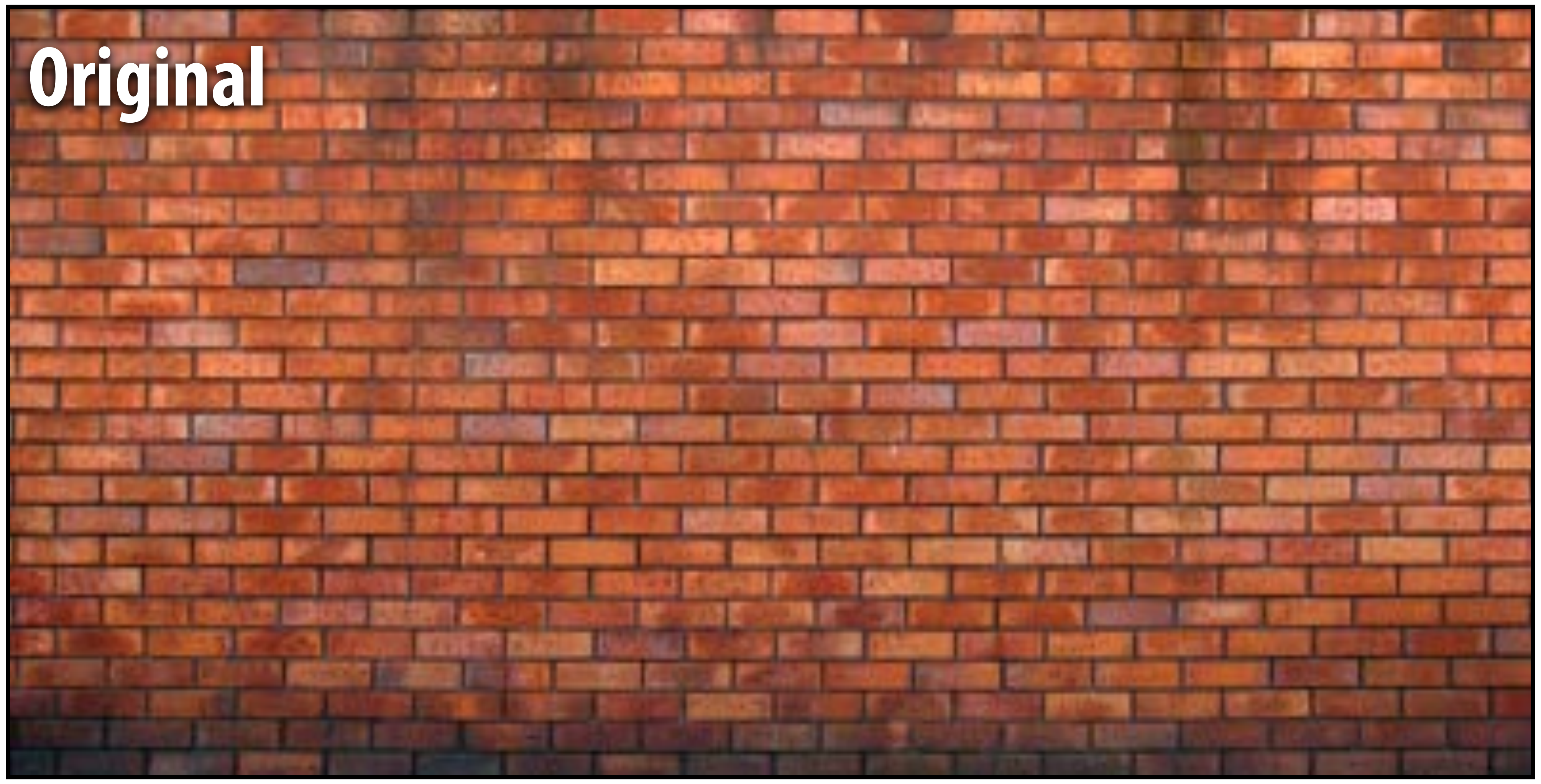


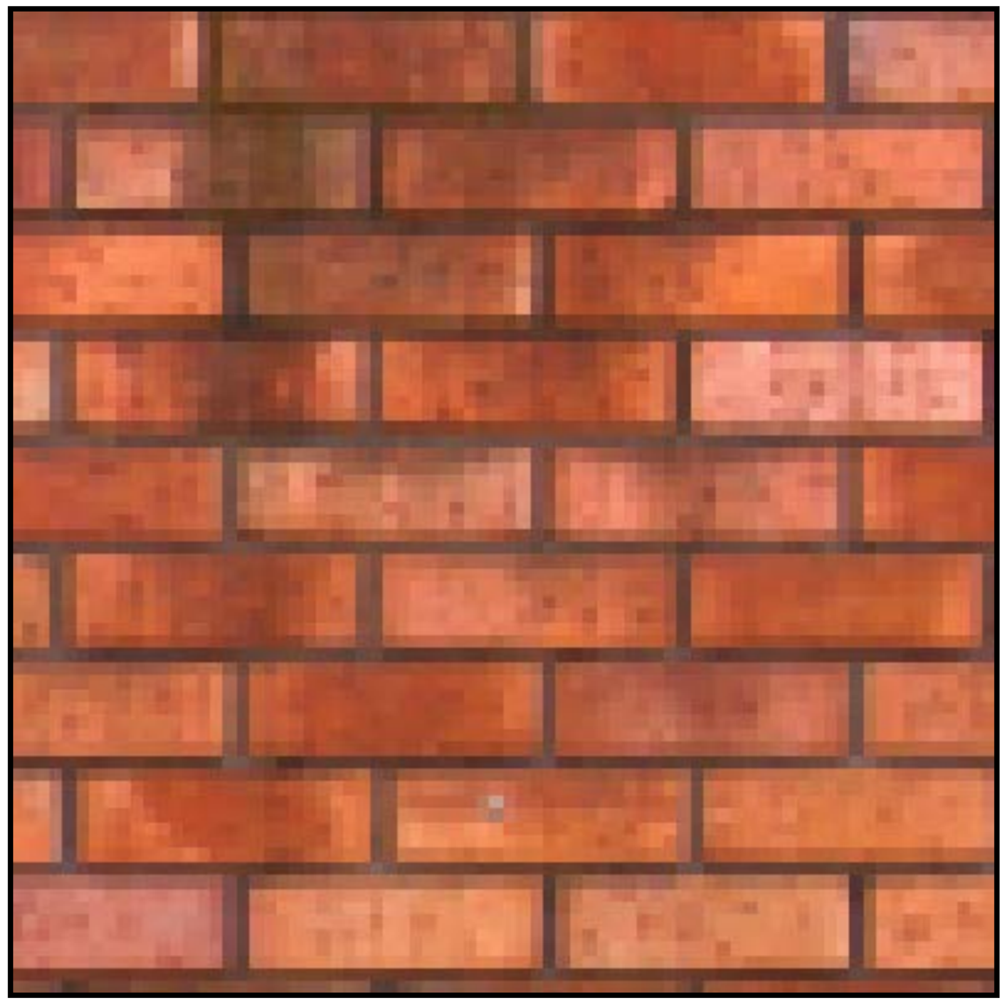


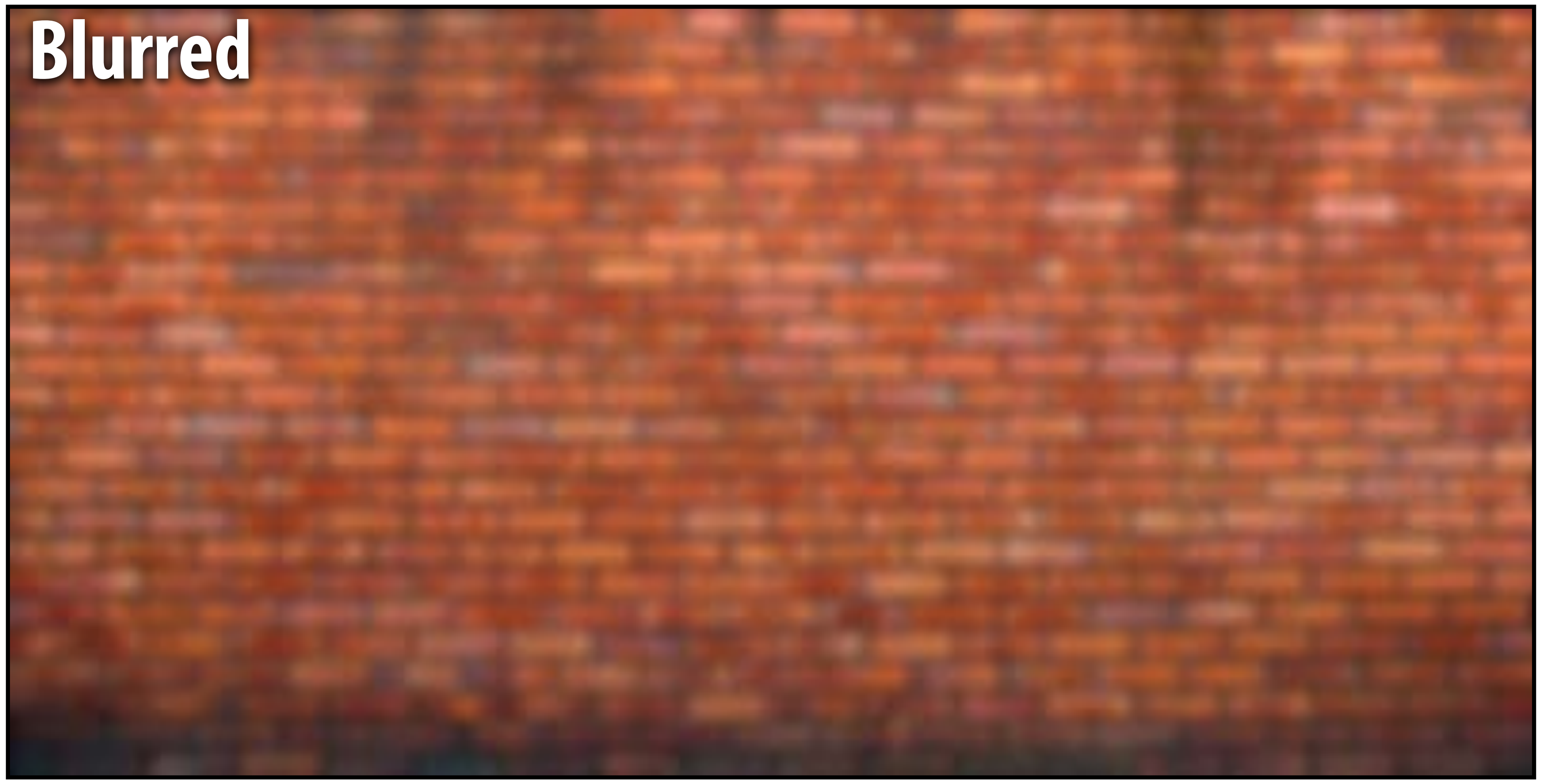
Convolutional layer with stride 2

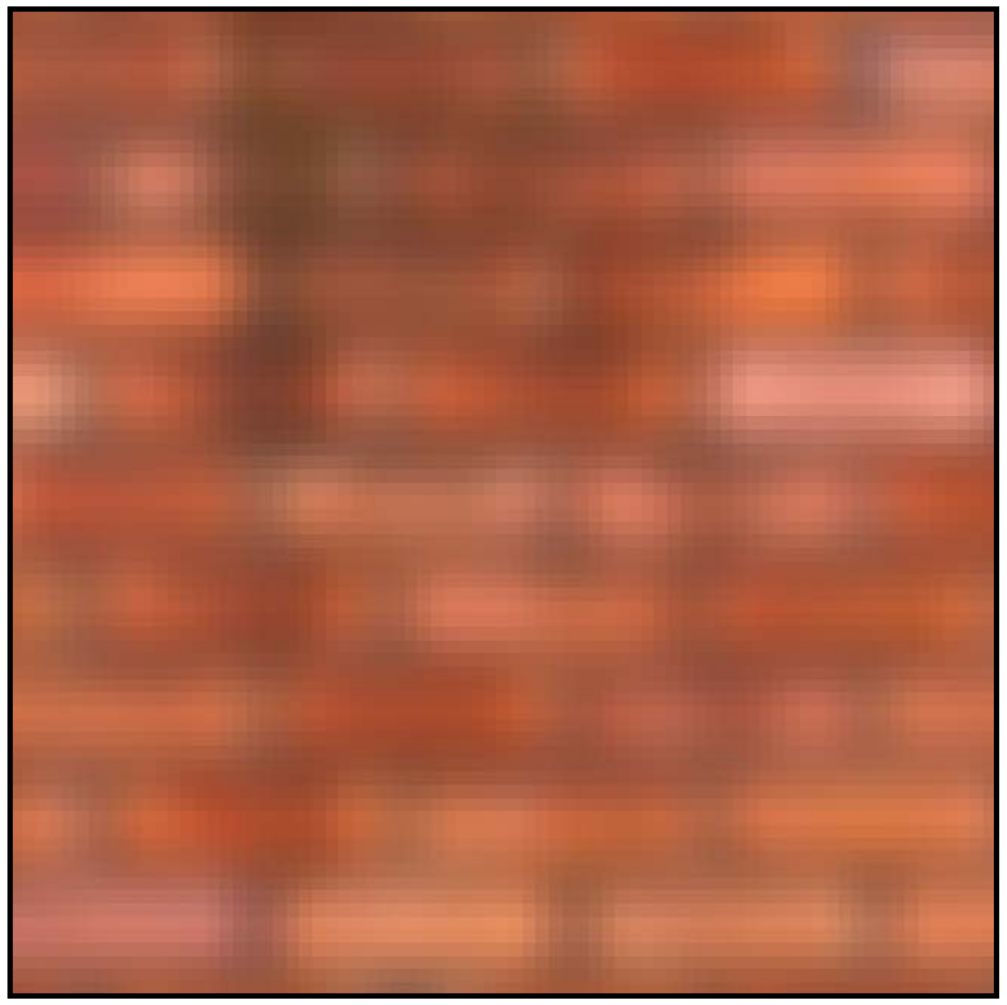
What does convolution using these filter weights do? $\begin{bmatrix} .075 & .124 & .075 \\ .124 & .204 & .124 \\ .075 & .124 & .075 \end{bmatrix}$

“Gaussian Blur”









What does convolution with these filters do?

$$\begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

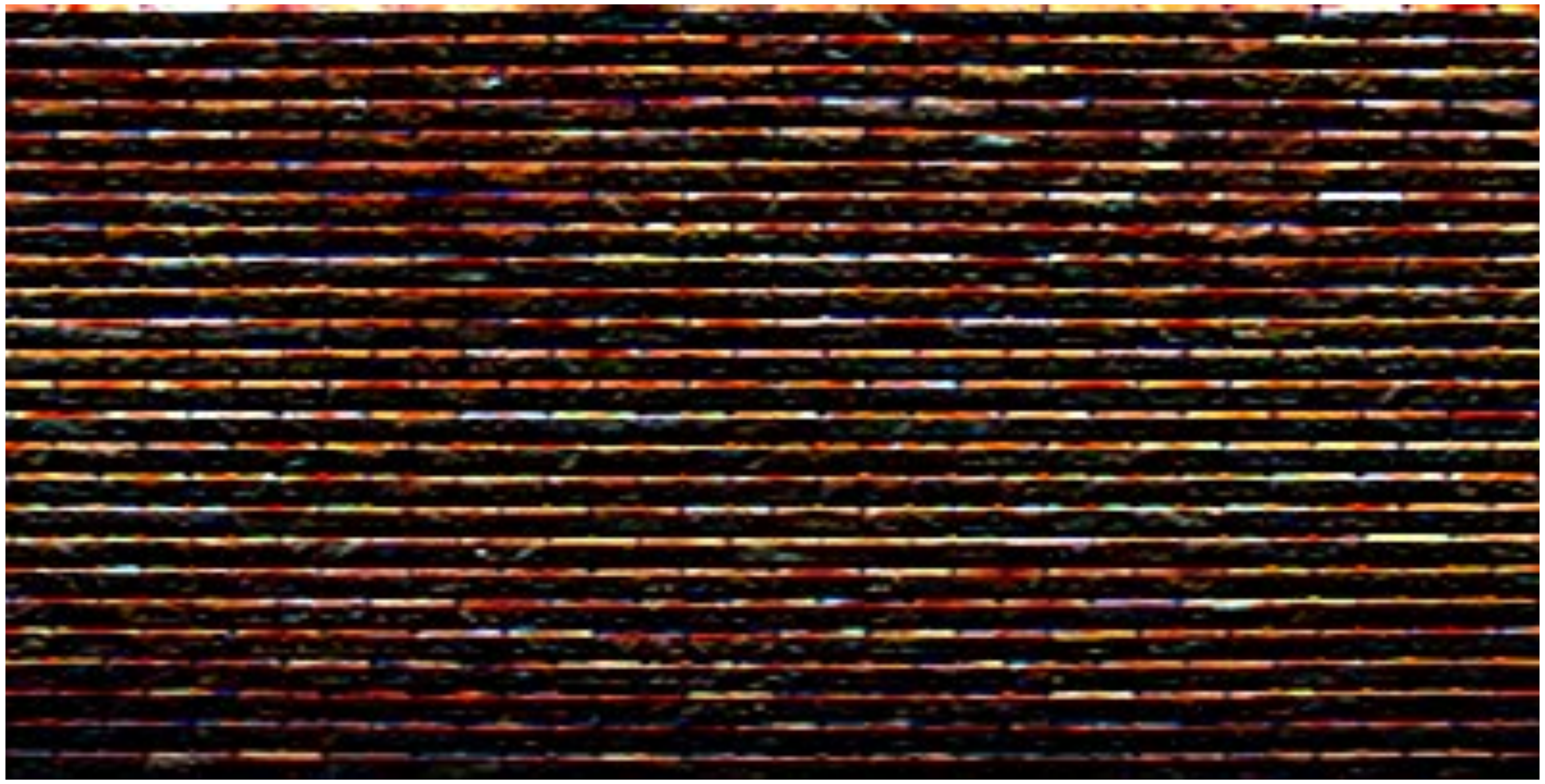
$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Extracts horizontal gradients

Extracts vertical gradients

Gradient detection filters

Horizontal gradients



Vertical gradients

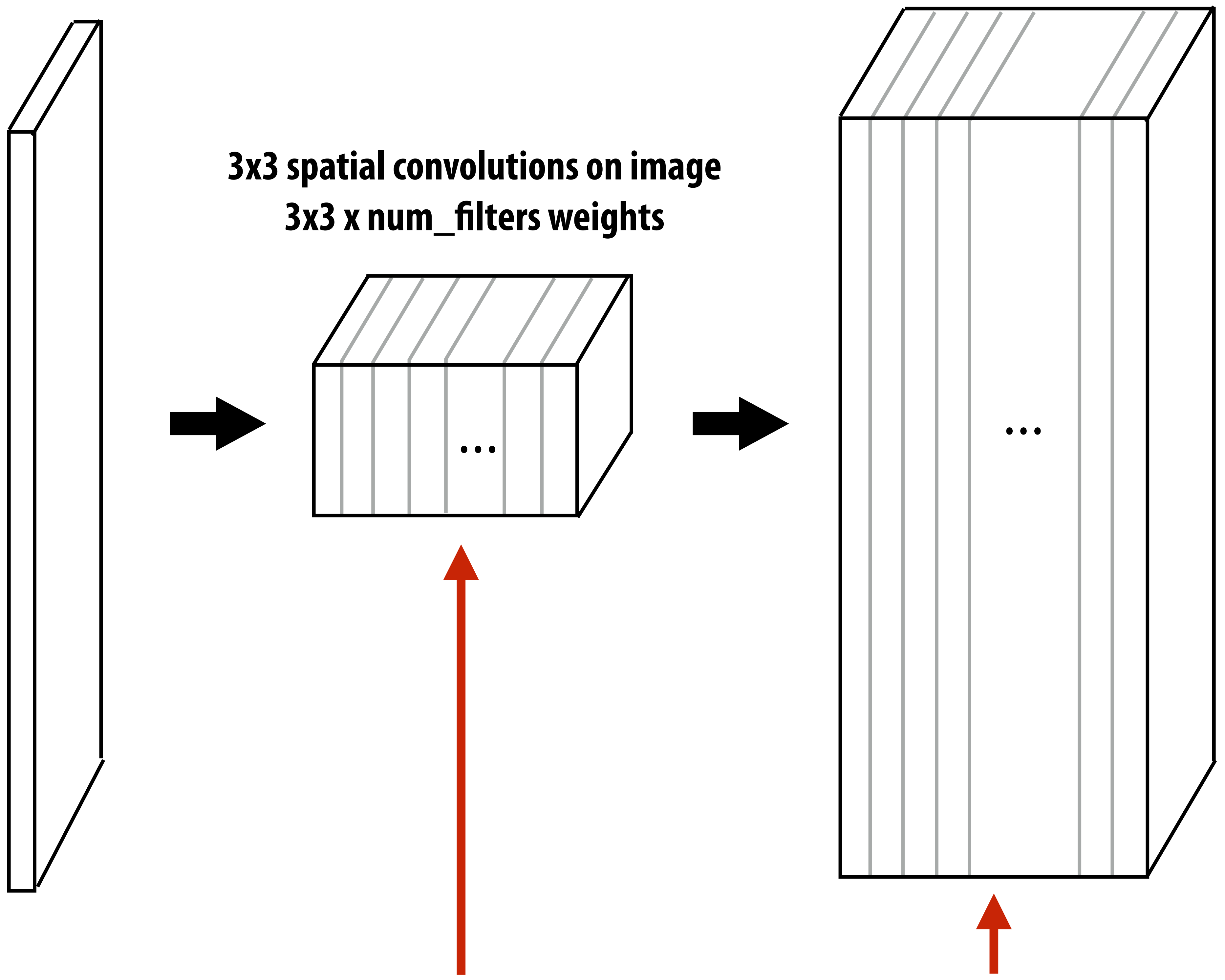
Note: you can think of a filter as a "detector" of a pattern, and the magnitude of a pixel in the output image as the "response" of the filter to the region surrounding each pixel in the input image

Applying many filters to an image at once

Input: image (single channel):
W x H

3x3 spatial convolutions on image
3x3 x num_filters weights

Output: filter responses
W x H x num_filters



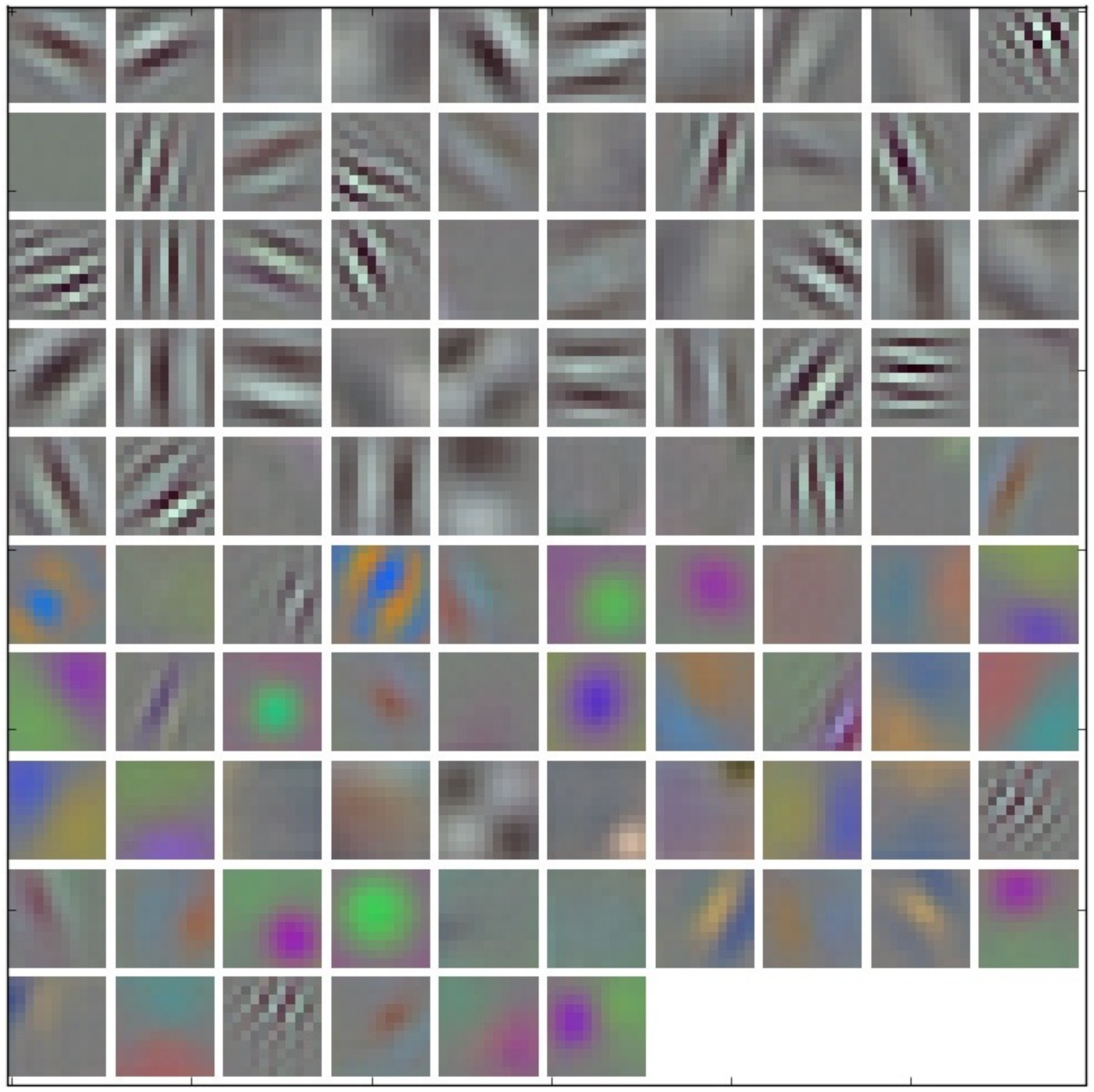
Each filter described by
unique set of weights
(responds to different
image phenomena)

Filter responses

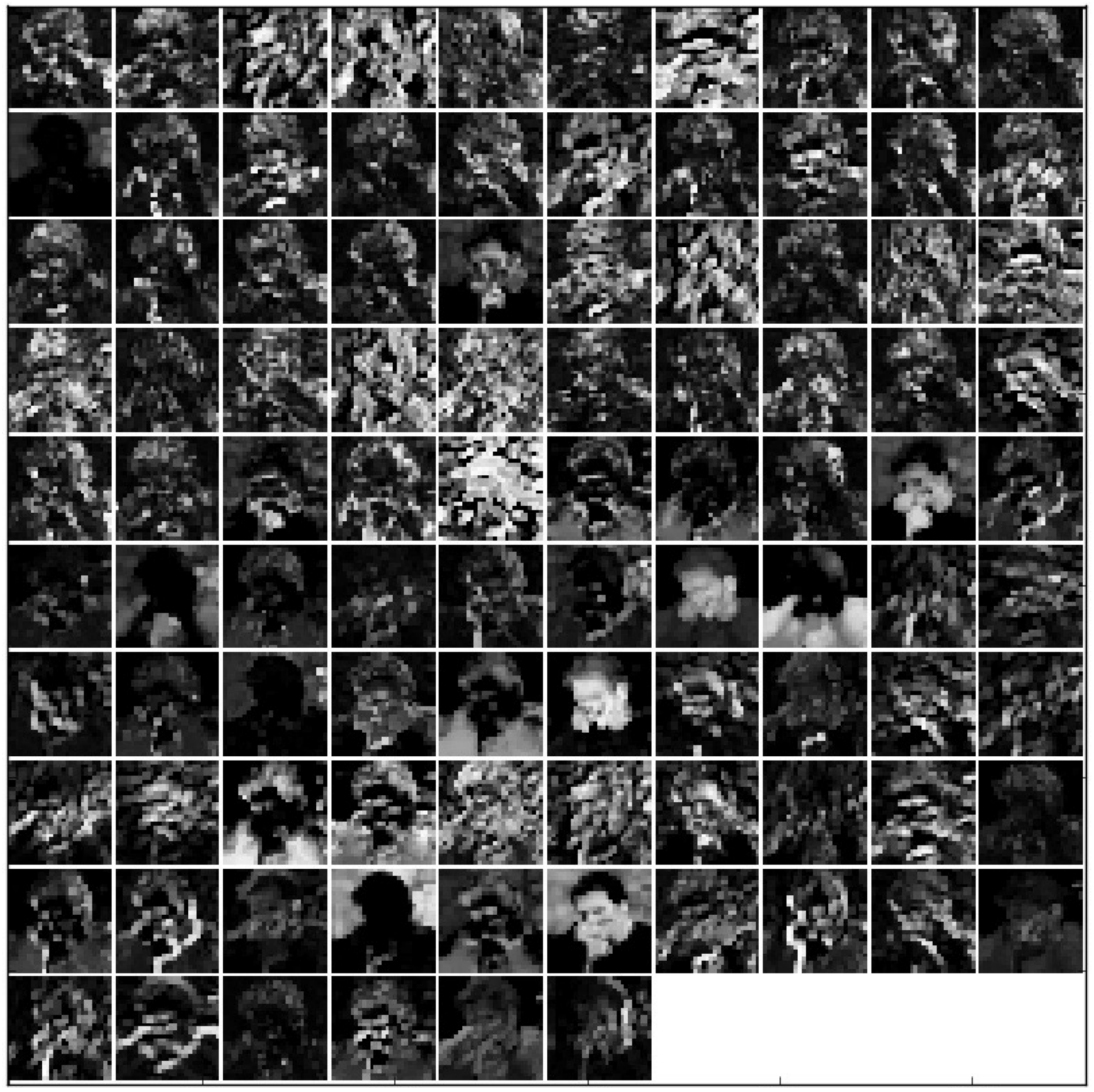
Applying many filters to an image at once

Input RGB image (W x H x 3)

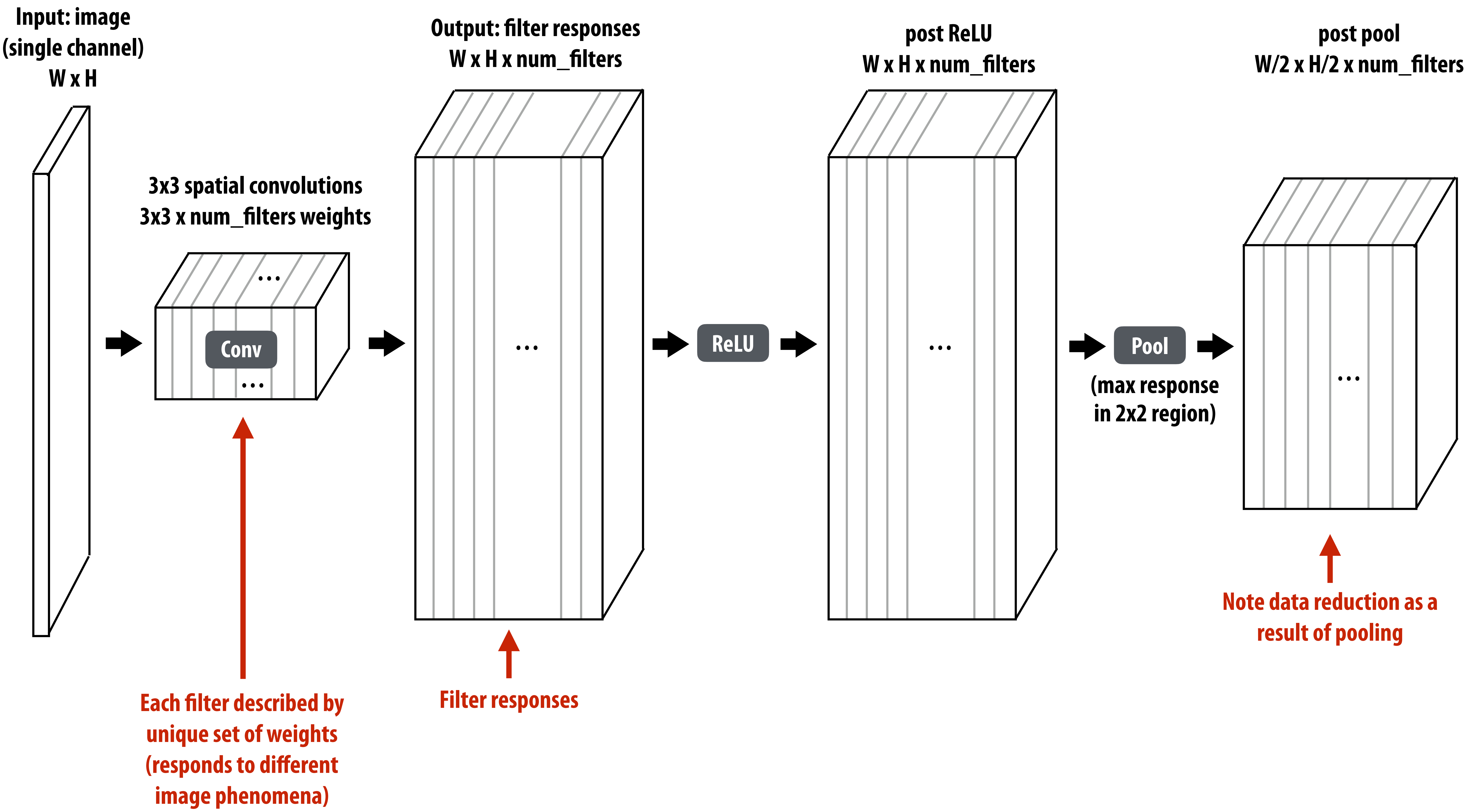
96 11x11x3 filters
(operate on RGB)



96 responses (normalized)



Adding additional layers



Input: image
(single channel)
W x H

3x3 spatial convolutions
3x3 x num_filters weights

Output: filter responses
W x H x num_filters

post ReLU
W x H x num_filters

post pool
W/2 x H/2 x num_filters

(max response
in 2x2 region)

Each filter described by
unique set of weights
(responds to different
image phenomena)

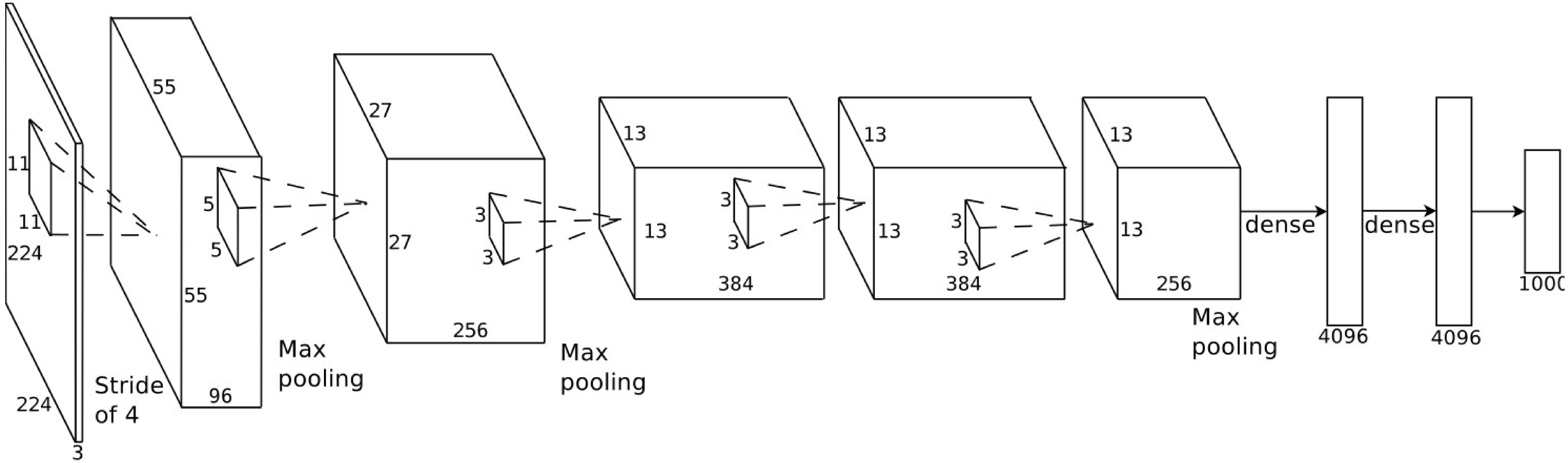
Filter responses

Note data reduction as a
result of pooling

Modern object detection networks

Sequences of conv + reLU + (optional) pool layers

AlexNet [Krizhevsky12]: 5 convolutional layers + 3 fully connected



VGG-16 [Simonyan15]: 13 convolutional layers

input: 224 x 224 RGB
conv/reLU: 3x3x3x64
conv/reLU: 3x3x64x64
maxpool
conv/reLU: 3x3x64x128
conv/reLU: 3x3x128x128
maxpool

conv/reLU: 3x3x128x256
conv/reLU: 3x3x256x256
conv/reLU: 3x3x256x256
maxpool
conv/reLU: 3x3x256x512
conv/reLU: 3x3x512x512
conv/reLU: 3x3x512x512
maxpool

conv/reLU: 3x3x512x512
conv/reLU: 3x3x512x512
conv/reLU: 3x3x512x512
maxpool
fully-connected 4096
fully-connected 4096
fully-connected 1000
soft-max

[VGG illustration credit: Yang et al.]

Efficiently implementing convolution layers

Direct implementation of conv layer

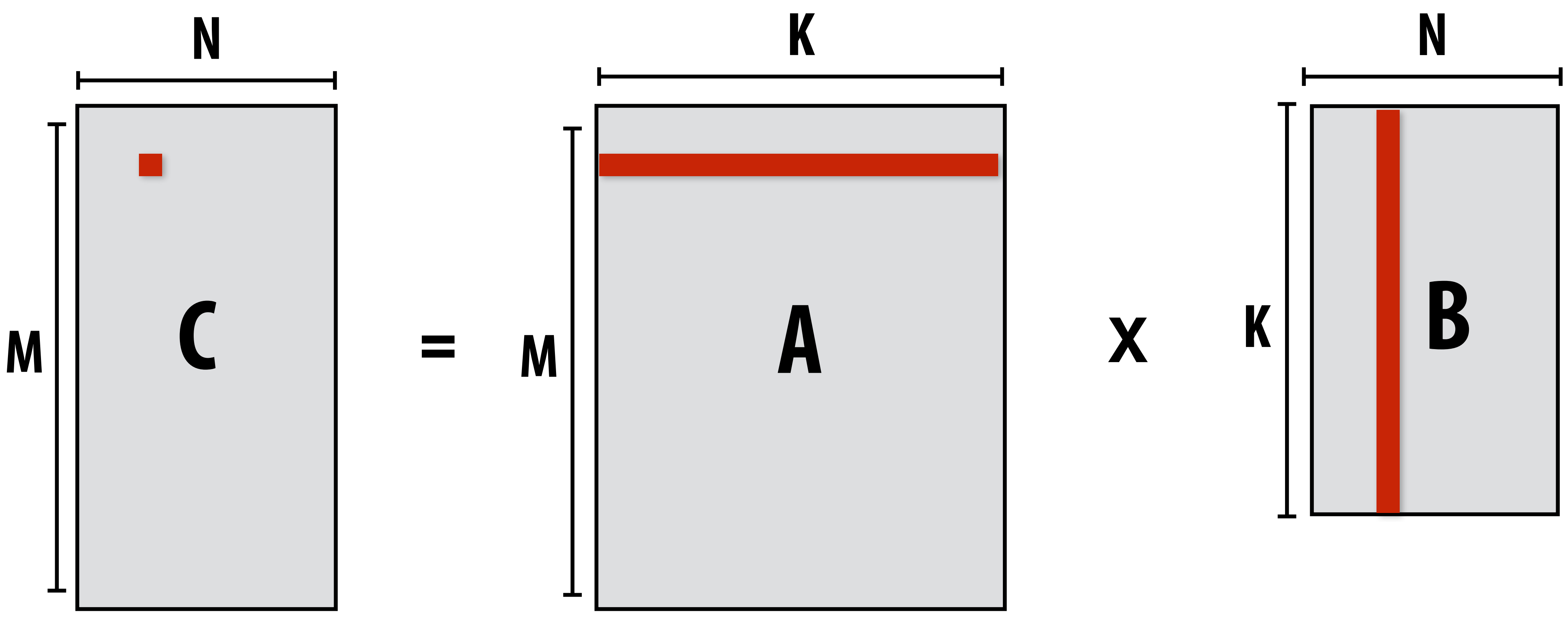
```
float input[INPUT_HEIGHT][INPUT_WIDTH][INPUT_DEPTH];
float output[INPUT_HEIGHT][INPUT_WIDTH][LAYER_NUM_FILTERS];
float layer_weights[LAYER_CONVY, LAYER_CONVX, INPUT_DEPTH];

// assumes convolution stride is 1
for (int img=0; img<IMAGE_BATCH_SIZE; img++)
   for (int j=0; j<INPUT_HEIGHT; j++)
      for (int i=0; i<INPUT_WIDTH; i++)
         for (int f=0; f<LAYER_NUM_FILTERS; f++) {
            output[j][i][f] = 0.f;
            for (int kk=0; kk<INPUT_DEPTH; kk++)  // sum over filter responses of input channels
               for (int jj=0; jj<LAYER_CONVY; jj++) // spatial convolution
                  for (int ii=0; ii<LAYER_CONVX; ii+)  // spatial convolution
                      output[j][i][f] += layer_weights[f][jj][ii][kk] * input[j+jj][i+ii][kk];
          }
```

Seven loops with significant input data reuse: reuse of filter weights (during convolution), and reuse of input values (across different filters)

But must roll your own highly optimized implementation of a complicated loop nest.

Dense matrix multiplication

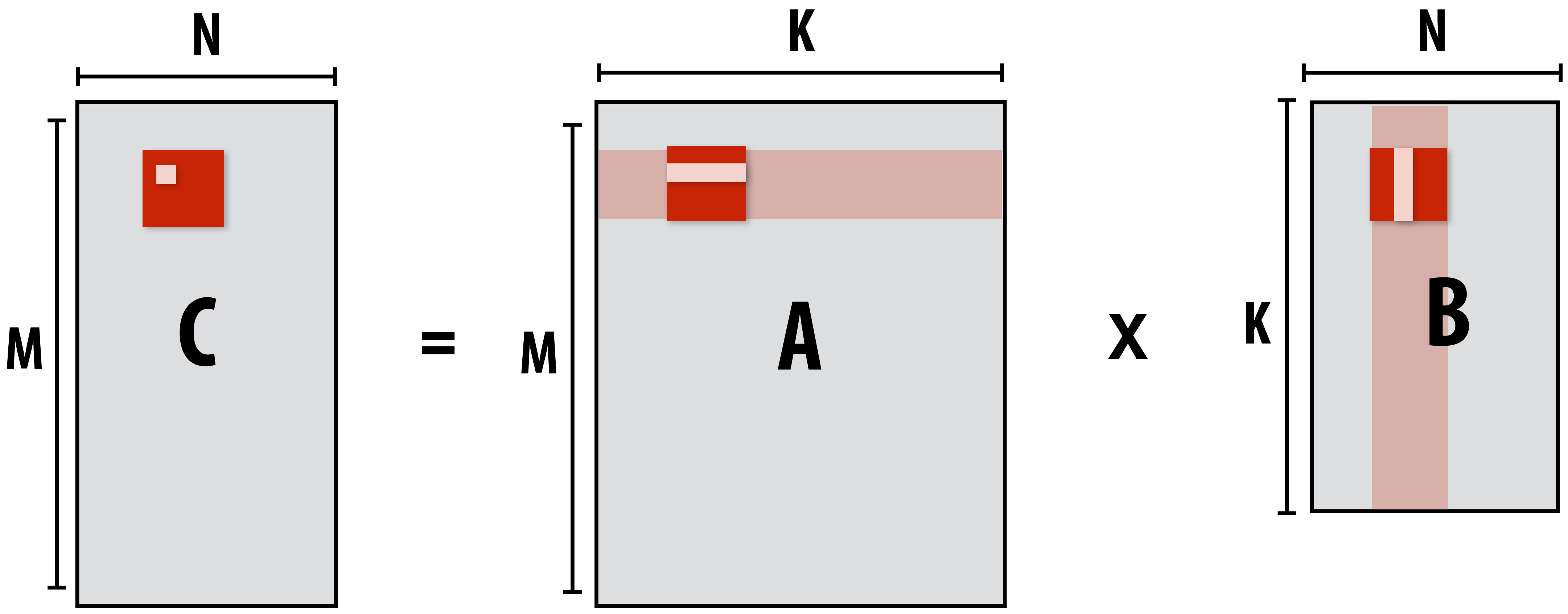
```
float A[M][K];
float B[K][N];
float C[M][N];

// compute C += A * B
#pragma omp parallel for
for (int j=0; j<M; j++)
  for (int i=0; i<N; i++)
     for (int k=0; k<K; k++)
         C[j][i] += A[j][k] * B[k][i];
```



What is the problem with this implementation?

Low arithmetic intensity (does not exploit temporal locality in access to A and B)

Blocked dense matrix multiplication



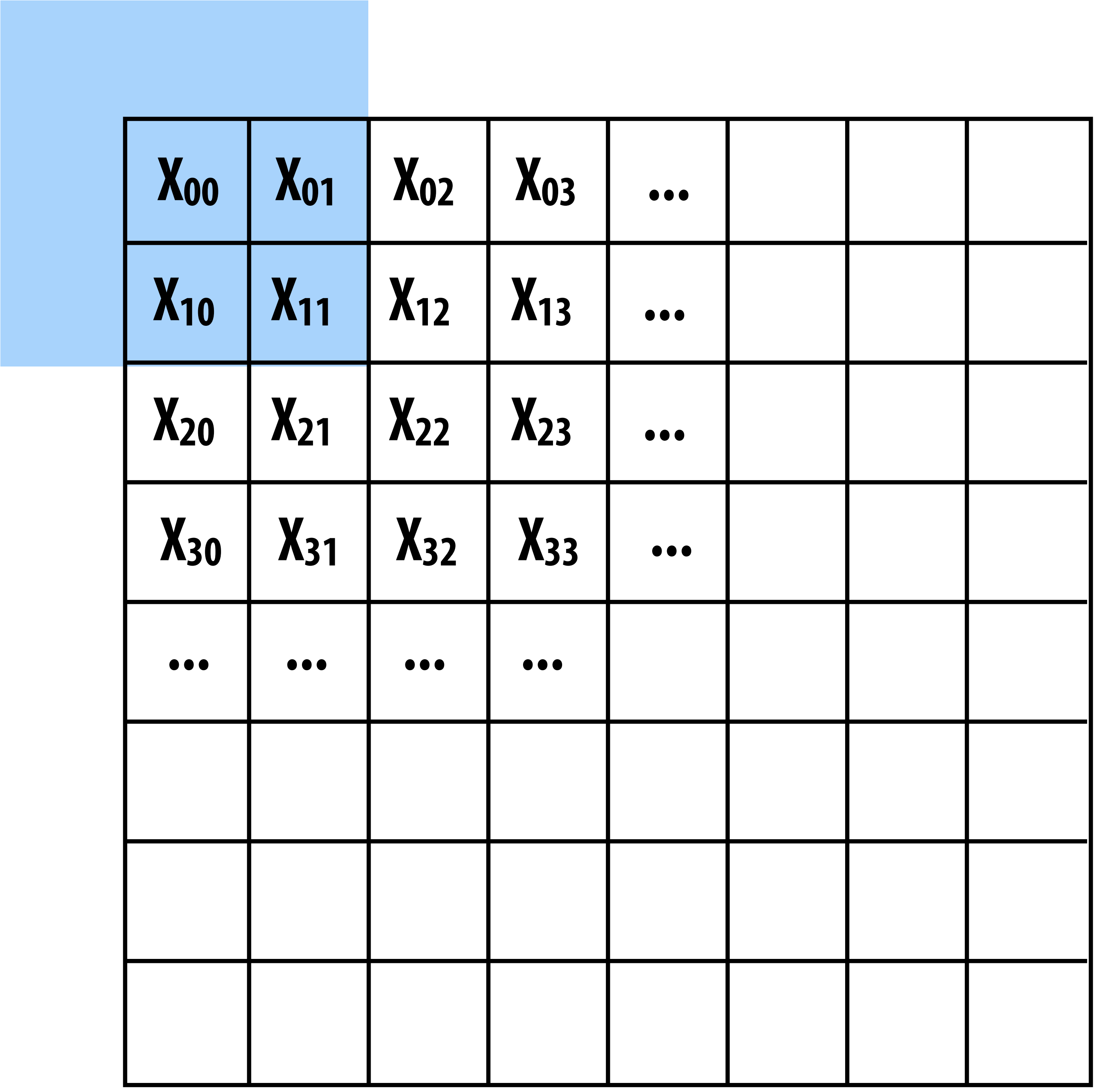
```
float A[M][K];
float B[K][N];
float C[M][N];

// compute C += A * B
#pragma omp parallel for
for (int jblock=0; jblock<M; jblock+=BLOCKSIZE_J)
  for (int iblock=0; iblock<N; iblock+=BLOCKSIZE_I)
     for (int kblock=0; kblock<K; kblock+=BLOCKSIZE_K)
        for (int j=0; j<BLOCKSIZE_J; j++)
           for (int i=0; i<BLOCKSIZE_I; i++)
              for (int k=0; k<BLOCKSIZE_K; k++)
                 C[jblock+j][iblock+i] += A[jblock+j][kblock+k] * B[kblock+k][iblock+i];
```

Idea: compute partial result for block of C while required blocks of A and B remain in cache (Assumes BLOCKSIZE chosen to allow block of A, B, and C to remain resident)

Self check: do you want as big a BLOCKSIZE as possible? Why?

Convolution as matrix-vector product

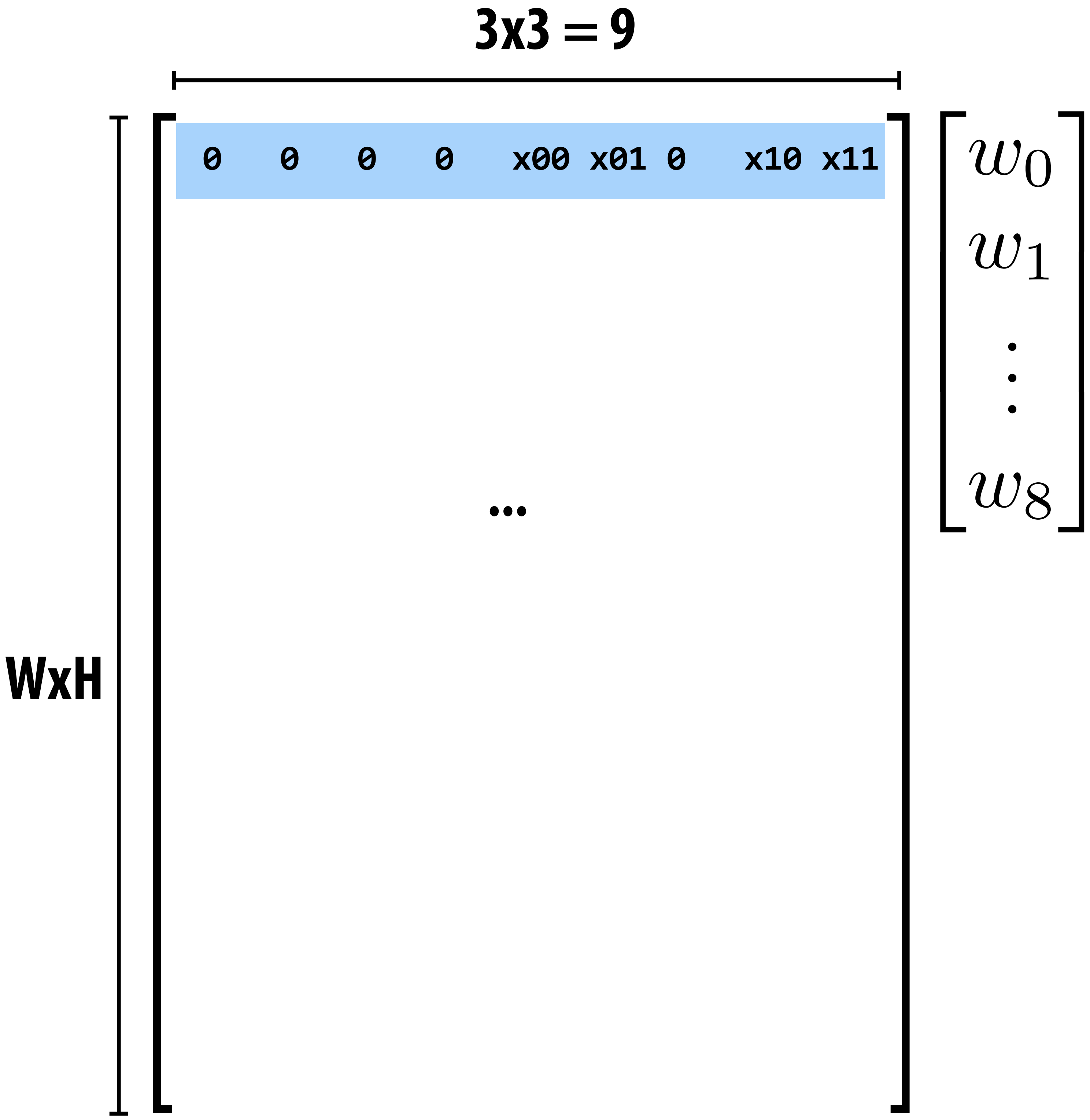
Construct matrix from elements of input image



X_{00}	X_{01}	X_{02}	X_{03}	...			
X_{10}	X_{11}	X_{12}	X_{13}	...			
X_{20}	X_{21}	X_{22}	X_{23}	...			
X_{30}	X_{31}	X_{32}	X_{33}	...			
...	...	...	...				

O(N) storage overhead for filter with N elements

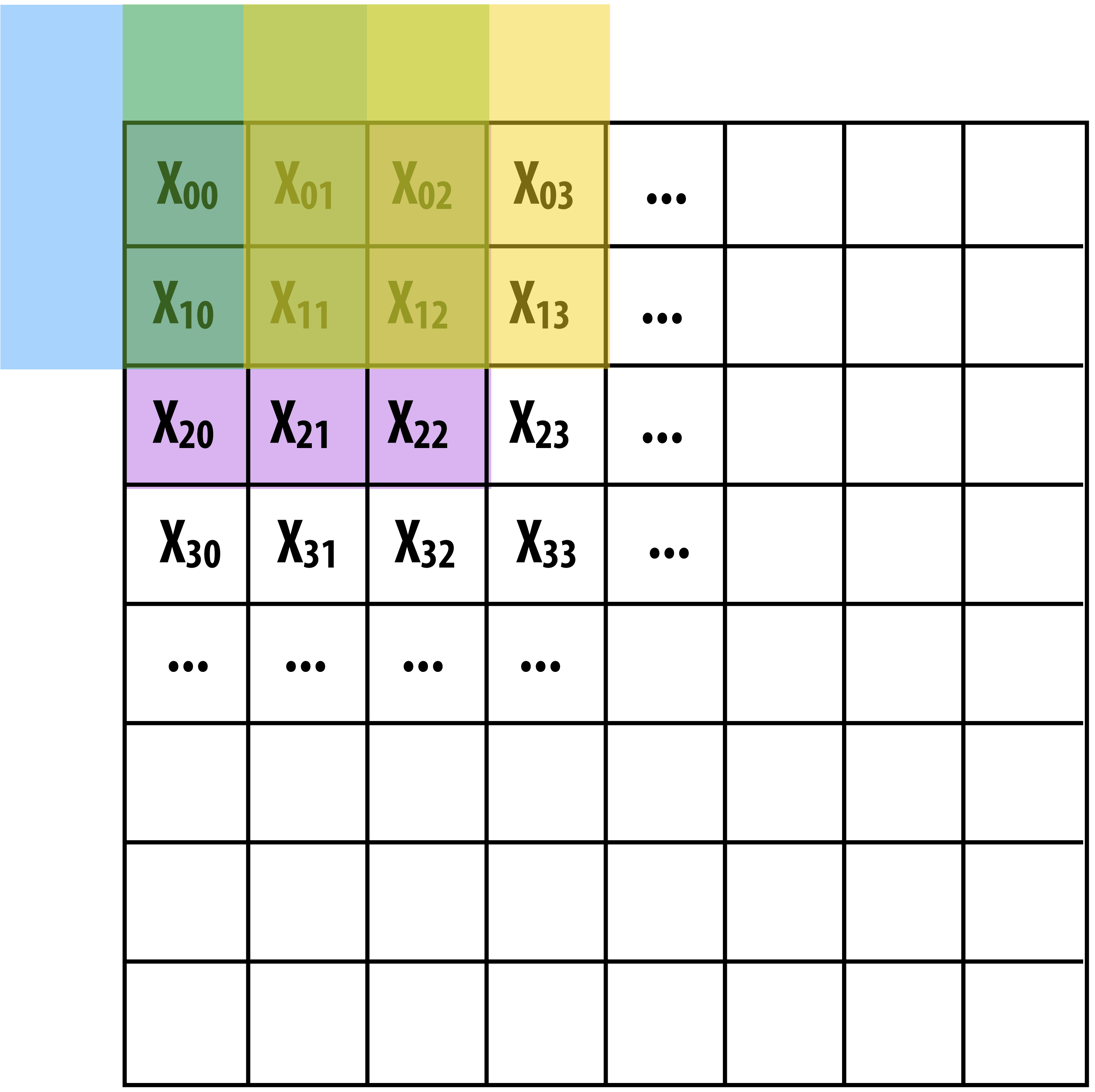
Must construct input data matrix



Note: 0-pad matrix

3x3 convolution as matrix-vector product

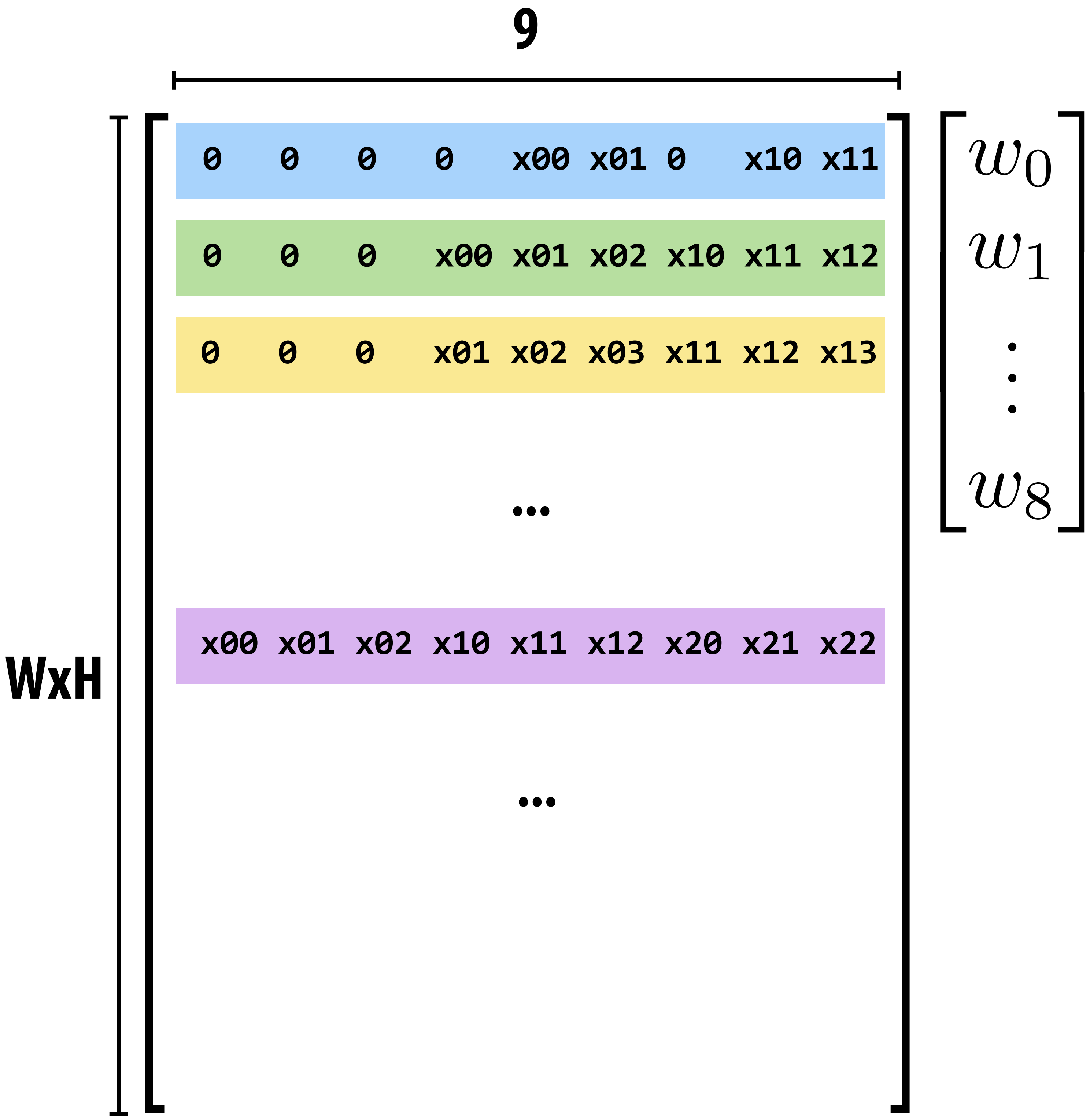
Construct matrix from elements of input image



X_{00}	X_{01}	X_{02}	X_{03}	...			
X_{10}	X_{11}	X_{12}	X_{13}	...			
X_{20}	X_{21}	X_{22}	X_{23}	...			
X_{30}	X_{31}	X_{32}	X_{33}	...			
...	...	...	...				

O(N) storage overhead for filter with N elements

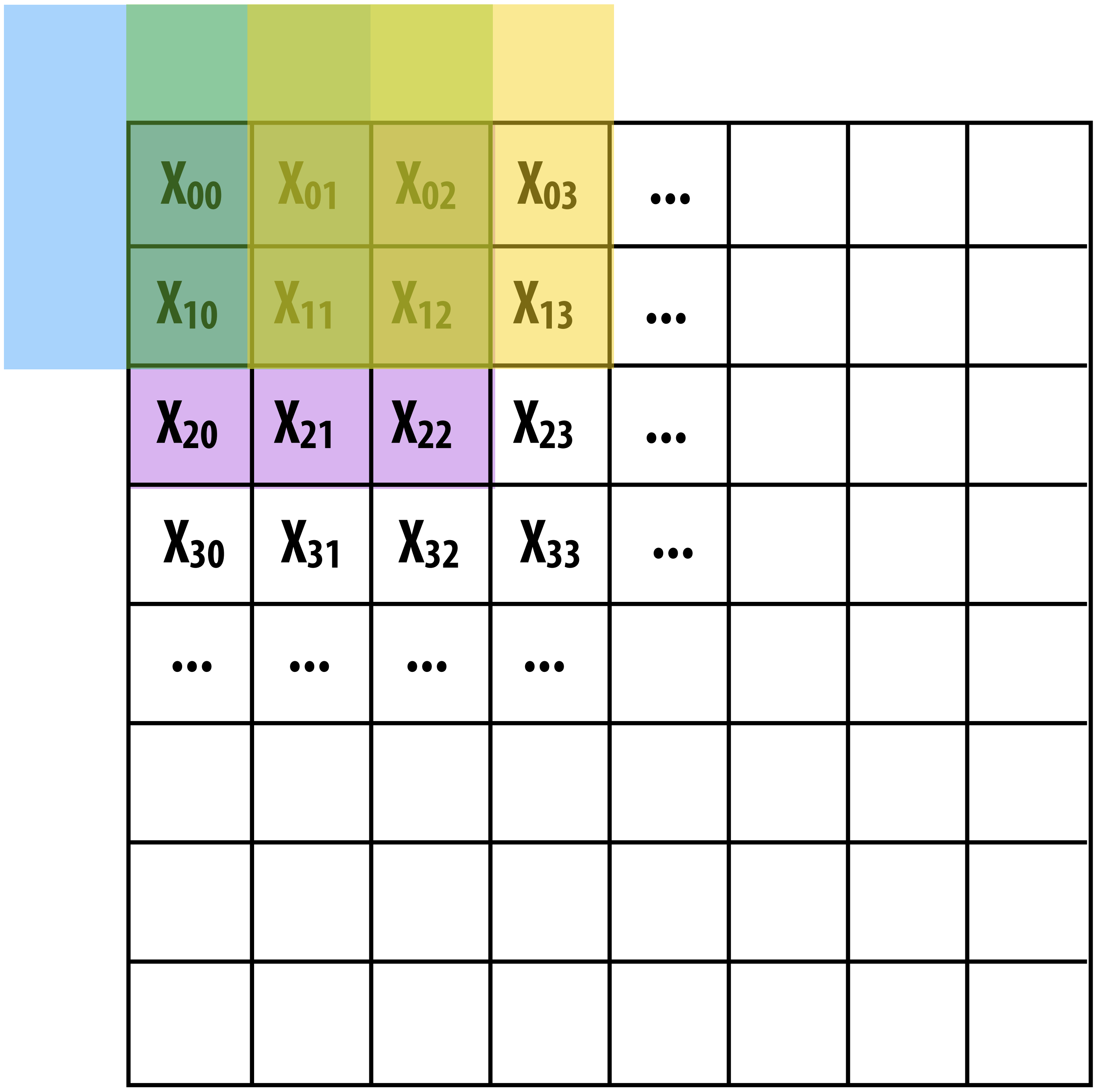
Must construct input data matrix



```
0   0   0   0   x00 x01 0   x10 x11
0   0   0   x00 x01 x02 x10 x11 x12
0   0   0   x01 x02 x03 x11 x12 x13
...
x00 x01 x02 x10 x11 x12 x20 x21 x22
...
```

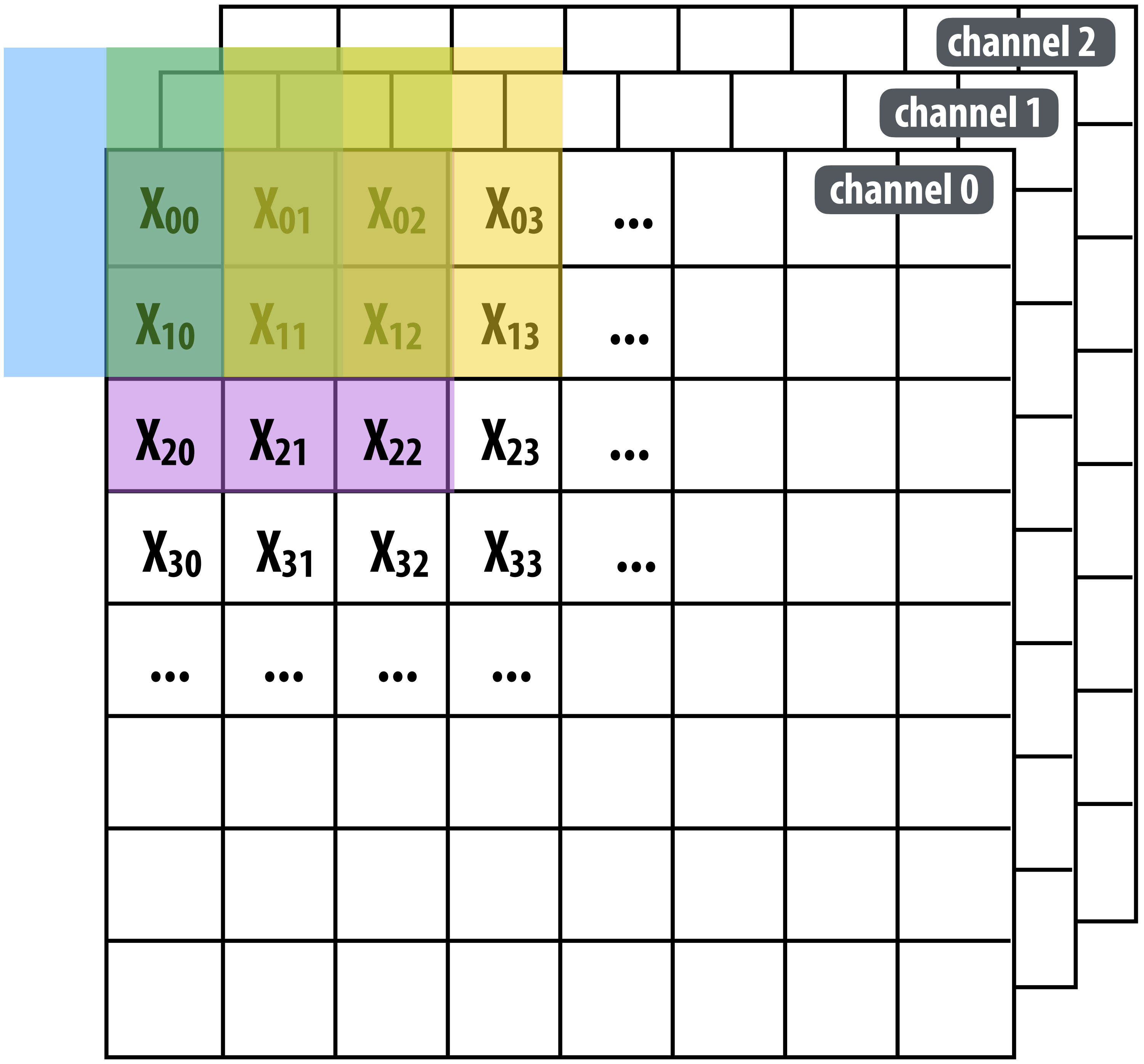
Note: 0-pad matrix

Multiple convolutions as matrix-matrix mult



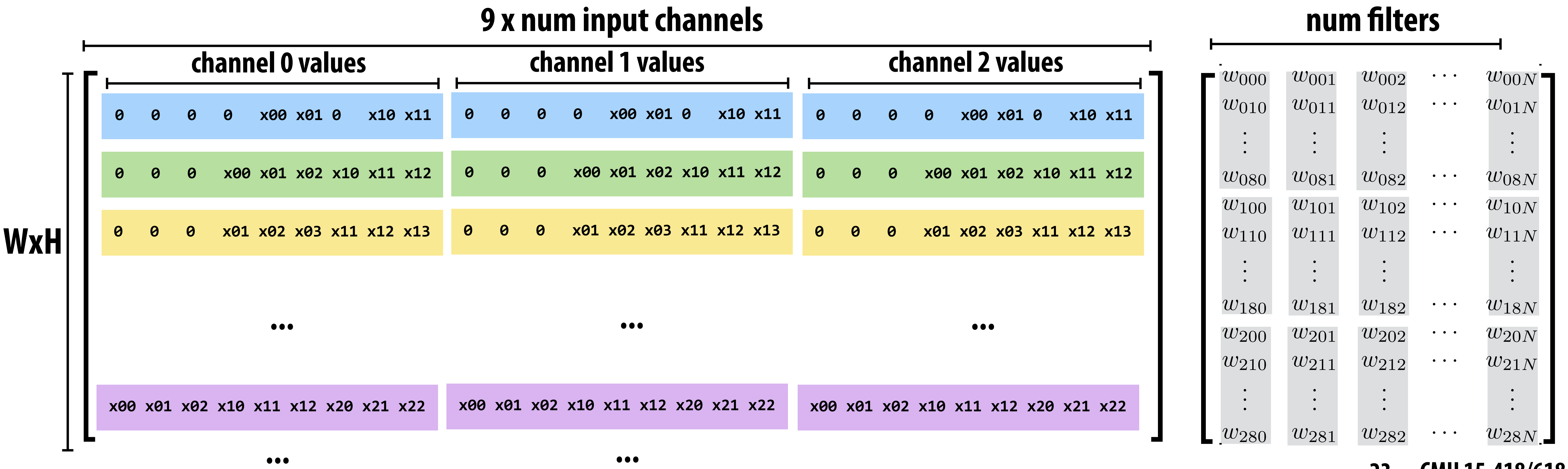
$$\text{WxH}\ \overbrace{\begin{bmatrix} \texttt{0} & \texttt{0} & \texttt{0} & \texttt{0} & \texttt{x00} & \texttt{x01} & \texttt{0} & \texttt{x10} & \texttt{x11} \\ \texttt{0} & \texttt{0} & \texttt{0} & \texttt{x00} & \texttt{x01} & \texttt{x02} & \texttt{x10} & \texttt{x11} & \texttt{x12} \\ \texttt{0} & \texttt{0} & \texttt{0} & \texttt{x01} & \texttt{x02} & \texttt{x03} & \texttt{x11} & \texttt{x12} & \texttt{x13} \\ & & & & \dots & & & & \\ \texttt{x00} & \texttt{x01} & \texttt{x02} & \texttt{x10} & \texttt{x11} & \texttt{x12} & \texttt{x20} & \texttt{x21} & \texttt{x22} \\ & & & & \dots & & & & \end{bmatrix}}^{9}
\quad
\overbrace{\begin{bmatrix} w_{00} & w_{01} & w_{02} & \cdots & w_{0N} \\ w_{10} & w_{11} & w_{12} & \cdots & w_{0N} \\ \vdots & \vdots & \vdots & & \vdots \\ w_{80} & w_{81} & w_{82} & \cdots & w_{8N} \end{bmatrix}}^{\text{num filters}}$$

Multiple convolutions on multiple input channels



For each filter, sum responses over input channels

Equivalent to (3 x 3 x num_channels) convolution on (W x H x num_channels) input data



channel 0 values	channel 1 values	channel 2 values
0 0 0 0 x00 x01 0 x10 x11	0 0 0 0 x00 x01 0 x10 x11	0 0 0 0 x00 x01 0 x10 x11
0 0 0 x00 x01 x02 x10 x11 x12	0 0 0 x00 x01 x02 x10 x11 x12	0 0 0 x00 x01 x02 x10 x11 x12
0 0 0 x01 x02 x03 x11 x12 x13	0 0 0 x01 x02 x03 x11 x12 x13	0 0 0 x01 x02 x03 x11 x12 x13
...	...	...
x00 x01 x02 x10 x11 x12 x20 x21 x22	x00 x01 x02 x10 x11 x12 x20 x21 x22	x00 x01 x02 x10 x11 x12 x20 x21 x22
...	...	

$$\begin{bmatrix} w_{000} & w_{001} & w_{002} & \cdots & w_{00N} \\ w_{010} & w_{011} & w_{012} & \cdots & w_{01N} \\ \vdots & \vdots & \vdots & & \vdots \\ w_{080} & w_{081} & w_{082} & \cdots & w_{08N} \\ w_{100} & w_{101} & w_{102} & \cdots & w_{10N} \\ w_{110} & w_{111} & w_{112} & \cdots & w_{11N} \\ \vdots & \vdots & \vdots & & \vdots \\ w_{180} & w_{181} & w_{182} & \cdots & w_{18N} \\ w_{200} & w_{201} & w_{202} & \cdots & w_{20N} \\ w_{210} & w_{211} & w_{212} & \cdots & w_{21N} \\ \vdots & \vdots & \vdots & & \vdots \\ w_{280} & w_{281} & w_{282} & \cdots & w_{28N} \end{bmatrix}$$

VGG memory footprint

Calculations assume 32-bit values (image batch size = 1)

inputs/outputs get multiplied by image batch size

multiply by next layer's conv window size to form input matrix to next conv layer!!! (for VGG, this is a 9x data amplification)

	weights mem:	output size (per image)	(mem)
input: 224 x 224 RGB image	—	224x224x3	150K
conv: (3x3x3) x 64	6.5 KB	224x224x64	12.3 MB
conv: (3x3x64) x 64	144 KB	224x224x64	12.3 MB
maxpool	—	112x112x64	3.1 MB
conv: (3x3x64) x 128	228 KB	112x112x128	6.2 MB
conv: (3x3x128) x 128	576 KB	112x112x128	6.2 MB
maxpool	—	56x56x128	1.5 MB
conv: (3x3x128) x 256	1.1 MB	56x56x256	3.1 MB
conv: (3x3x256) x 256	2.3 MB	56x56x256	3.1 MB
conv: (3x3x256) x 256	2.3 MB	56x56x256	3.1 MB
maxpool	—	28x28x256	766 KB
conv: (3x3x256) x 512	4.5 MB	28x28x512	1.5 MB
conv: (3x3x512) x 512	9 MB	28x28x512	1.5 MB
conv: (3x3x512) x 512	9 MB	28x28x512	1.5 MB
maxpool	—	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
maxpool	—	7x7x512	98 KB
fully-connected 4096	392 MB	4096	16 KB
fully-connected 4096	64 MB	4096	16 KB
fully-connected 1000	15.6 MB	1000	4 KB
soft-max		1000	4 KB

Reducing network footprint

- Large storage cost for model parameters
  - AlexNet model: ~200 MB
  - VGG-16 model: ~500 MB
  - This doesn’t even account for intermediates during evaluation

- Footprint: cumbersome to store, download, etc.
  - 500 MB app downloads make users unhappy!

- Consider energy cost of 1B parameter network
  - Running on input stream at 20 Hz
  - 640 pJ per 32-bit DRAM access
  - (20 x 1B x 640pJ) = 12.8W for DRAM access

    (more than power budget of any modern smartphone)

Compressing a network

[Han ICLR16]

Step 1: **prune low-weight links** (iteratively retrain network, then prune)

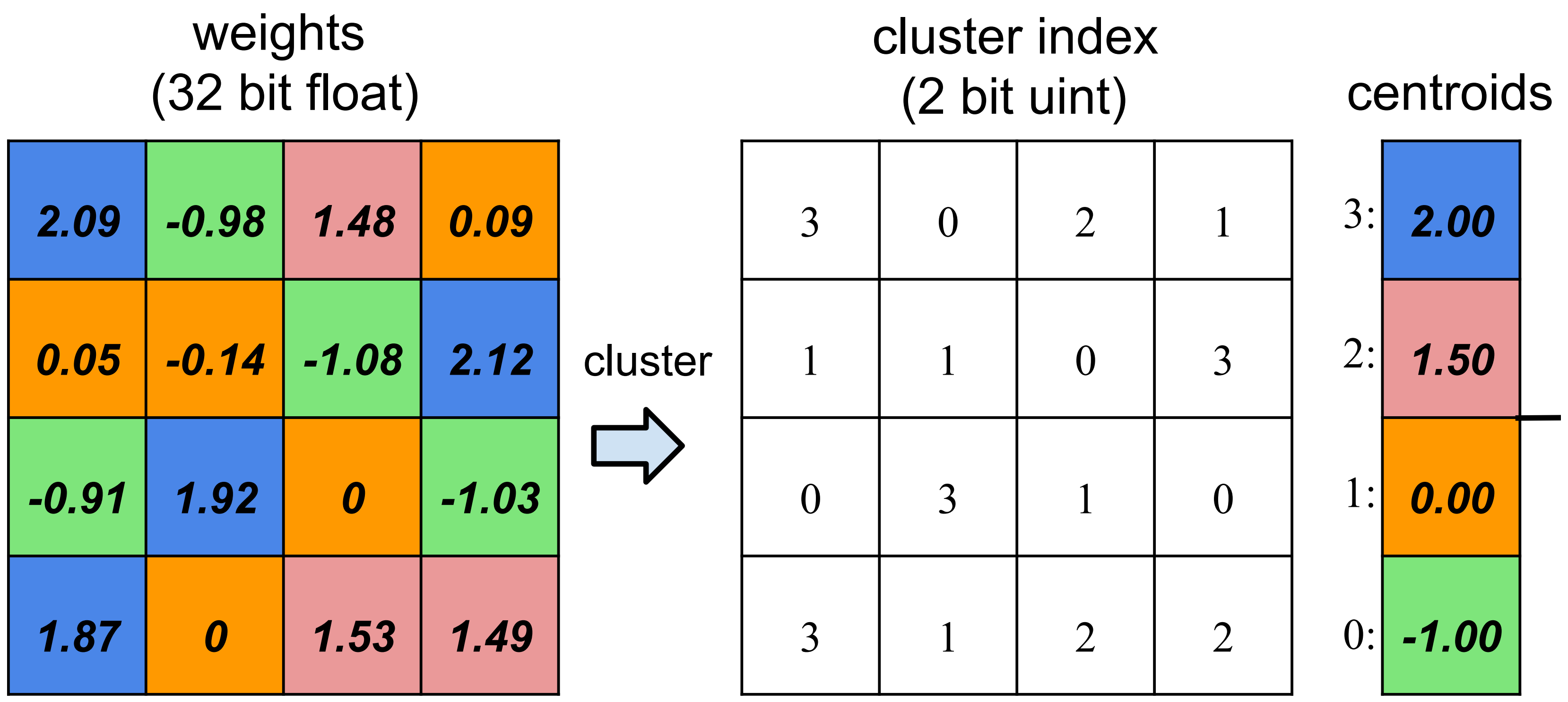
- Over 90% of weights can be removed without significant loss of accuracy
- Store weight matrices in compressed sparse row (CSR) format

Indicies	1	4	9	...
Value	1.8	0.5	2.1	

0	1.8	0	0	0.5	0	0	0	0	2.1	...

Step 2: **weight sharing**: make surviving connects share a small set of weights

- Cluster weights via k-means clustering (irregular ("learned") quantization)
- Compress weights by only storing cluster index (lg(k) bits)
- Retrain network to improve quality of cluster centroids



2.09	-0.98	1.48	0.09
0.05	-0.14	-1.08	2.12
-0.91	1.92	0	-1.03
1.87	0	1.53	1.49

3	0	2	1
1	1	0	3
0	3	1	0
3	1	2	2

3:	2.00
2:	1.50
1:	0.00
0:	-1.00

Step 3: **Huffman encode** quantized weights and CSR indices

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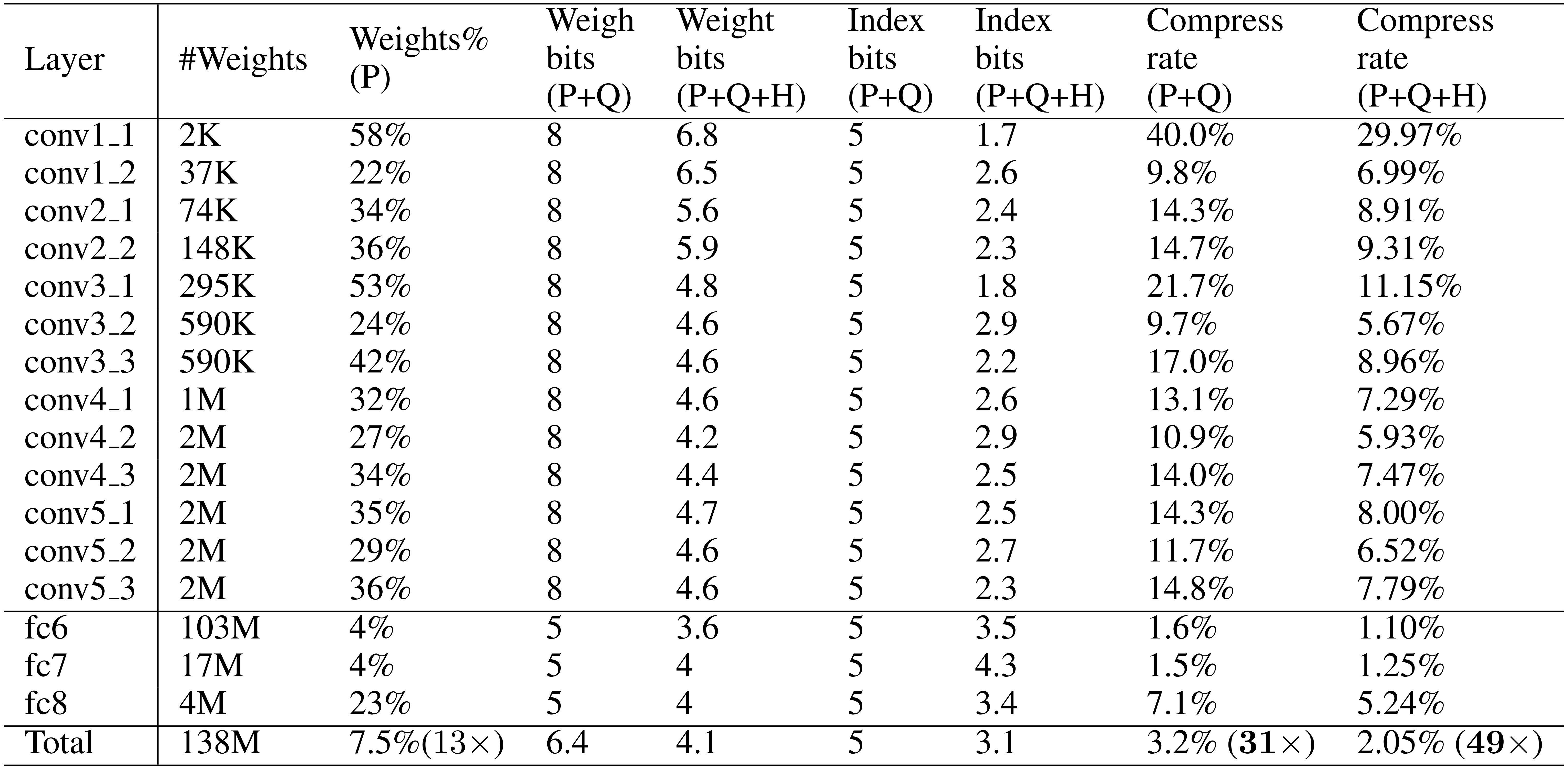
VGG-16 compression

Substantial savings due to combination of pruning, quantization, Huffman encoding



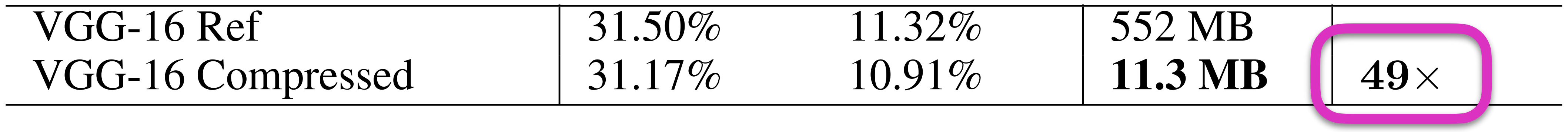
Layer	#Weights	Weights% (P)	Weigh bits (P+Q)	Weight bits (P+Q+H)	Index bits (P+Q)	Index bits (P+Q+H)	Compress rate (P+Q)	Compress rate (P+Q+H)
conv1_1	2K	58%	8	6.8	5	1.7	40.0%	29.97%
conv1_2	37K	22%	8	6.5	5	2.6	9.8%	6.99%
conv2_1	74K	34%	8	5.6	5	2.4	14.3%	8.91%
conv2_2	148K	36%	8	5.9	5	2.3	14.7%	9.31%
conv3_1	295K	53%	8	4.8	5	1.8	21.7%	11.15%
conv3_2	590K	24%	8	4.6	5	2.9	9.7%	5.67%
conv3_3	590K	42%	8	4.6	5	2.2	17.0%	8.96%
conv4_1	1M	32%	8	4.6	5	2.6	13.1%	7.29%
conv4_2	2M	27%	8	4.2	5	2.9	10.9%	5.93%
conv4_3	2M	34%	8	4.4	5	2.5	14.0%	7.47%
conv5_1	2M	35%	8	4.7	5	2.5	14.3%	8.00%
conv5_2	2M	29%	8	4.6	5	2.7	11.7%	6.52%
conv5_3	2M	36%	8	4.6	5	2.3	14.8%	7.79%
fc6	103M	4%	5	3.6	5	3.5	1.6%	1.10%
fc7	17M	4%	5	4	5	4.3	1.5%	1.25%
fc8	4M	23%	5	4	5	3.4	7.1%	5.24%
Total	138M	7.5%(13×)	6.4	4.1	5	3.1	3.2% (**31**×)	2.05% (**49**×)

P = connection pruning (prune low weight connections)

Q = quantize surviving weights (using shared weights)

H = Huffman encode

ImageNet Image Classification Performance



	Top-1 Error	Top-5 Error	Model size	
VGG-16 Ref	31.50%	11.32%	552 MB	
VGG-16 Compressed	31.17%	10.91%	**11.3 MB**	49×

Deep neural networks on GPUs

- Today, best performing DNN implementations target GPUs
  - High arithmetic intensity computations (computational characteristics similar to dense matrix-matrix multiplication)
  - Benefit from flop-rich architectures
  - Highly-optimized library of kernels exist for GPUs (cuDNN)
    - Most CPU-based implementations use basic matrix-multiplication-based formulation (good implementations could run faster!)

Facebook's Big Sur

Summary: Efficiently Evaluating DNNs

- Computational structure
  - Convlayers: high arithmetic intensity, significant portion of cost of evaluating a network
  - Similar data access patterns to dense-matrix multiplication (exploiting temporal reuse is key)
  - But straight reduction to matrix-matrix multiplication is often sub-optimal
  - Work-efficient techniques for convolutional layers (FFT-based, Wingrad convolutions)
- Large numbers of parameters: significant interest in reducing size of networks for both training and evaluation
  - Pruning: remove least important network links
  - Quantization: low-precision parameter representations often suffice
- Many ongoing studies of specialized hardware architectures for efficient evaluation
  - Future CPUs/GPUs, ASICs, FPGS, …
  - Specialization will be important to achieving "always on" applications

Two Distinct Issues with Deep Networks

- Evaluation
  - often takes milliseconds
- Training
  - often takes hours, days, weeks

“Training a network”

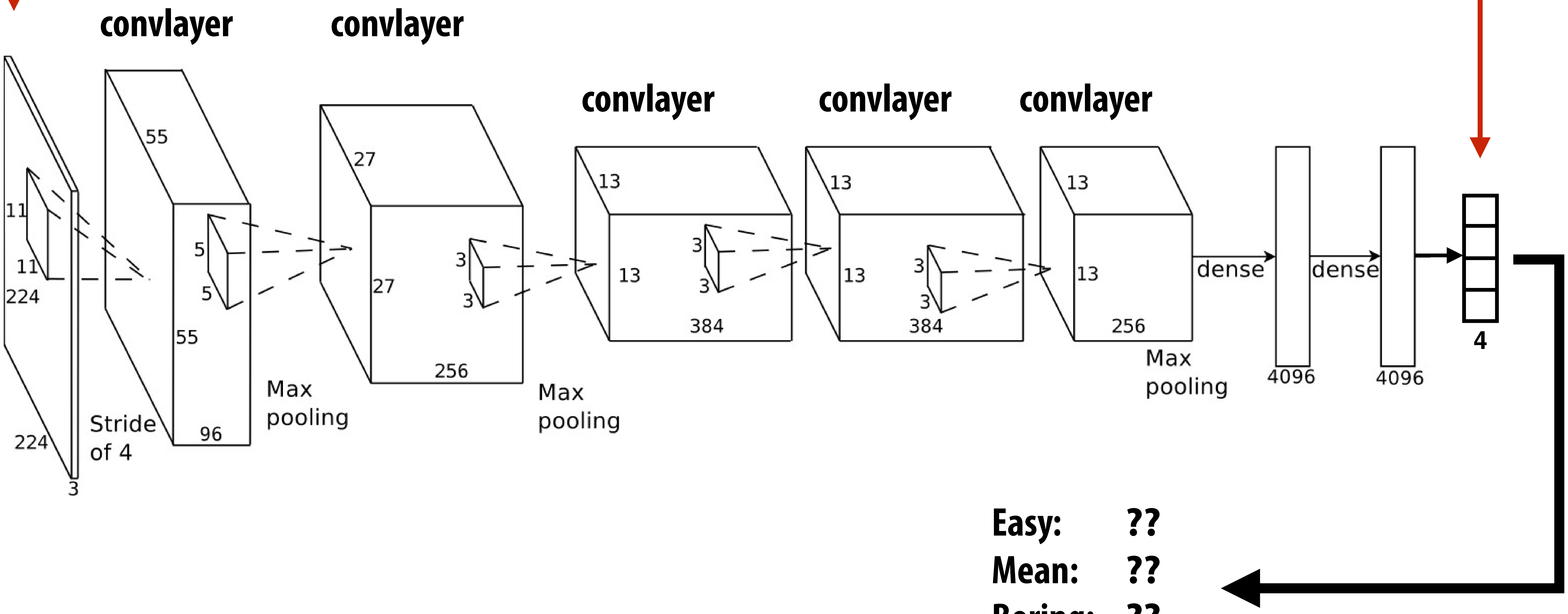
- Training a network is the process of learning the value of network parameters so that output of the network provides the desired result for a task
  - [Krizhevsky12] task = object classification
    - input 224 x 224 x 3 RGB image
    - output probability of 1000 ImageNet object classes: “dog”, “cat”, etc…
    - ~ 60M weights

Professor classification network

Classifies professors as easy, mean, boring, or nerdy based on their appearance.

Input:
image of a professor

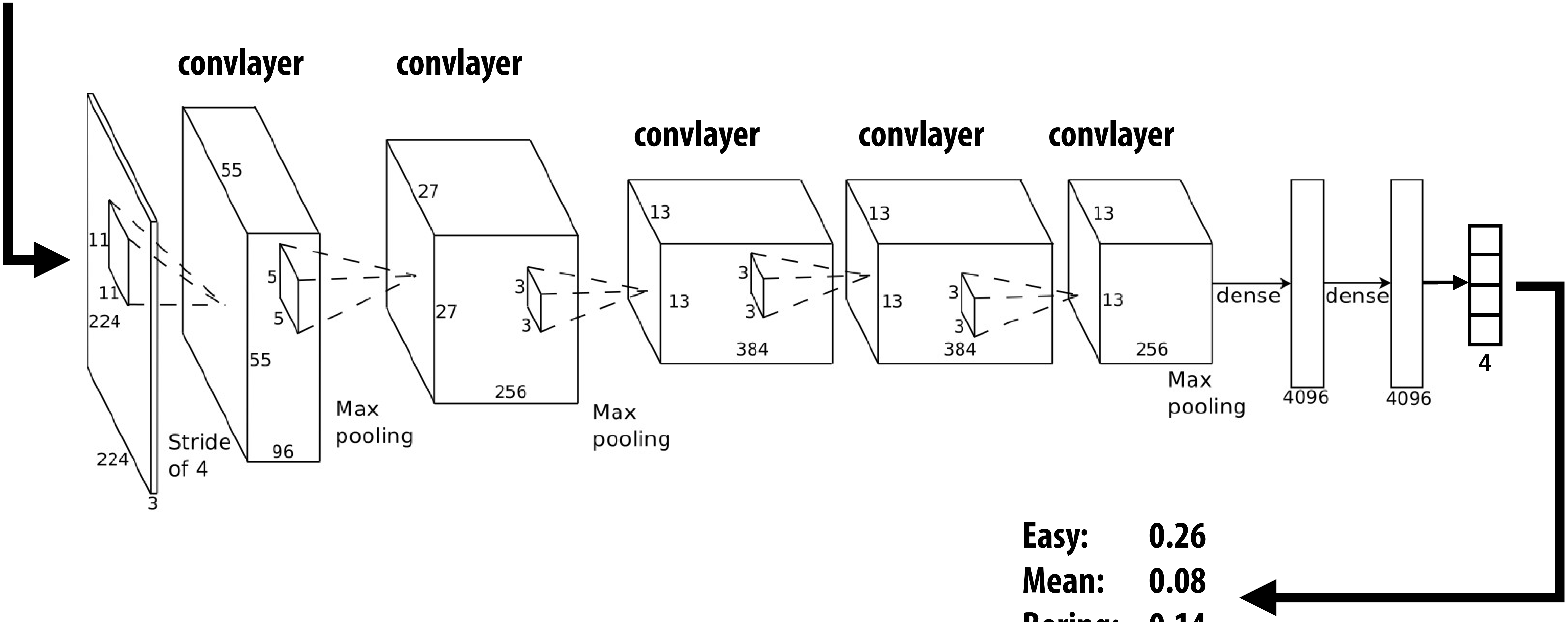
Output:
probability of label



Easy: ??
Mean: ??
Boring: ??
Nerdy: ??

Recall from last time:
10’s-100’s of millions of parameters

Professor classification network



Easy:	**0.26**
Mean:	**0.08**
Boring:	**0.14**
Nerdy:	**0.52**

Where did the parameters come from?

Training data (ground truth answers)

[label omitted]

[label omitted]

[label omitted]

Nerdy

[label omitted]

[label omitted]

[label omitted]

[label omitted]

[label omitted]

Nerdy

[label omitted]

[label omitted]

Nerdy

[label omitted]

[label omitted]

[label omitted]

Nerdy

[label omitted]

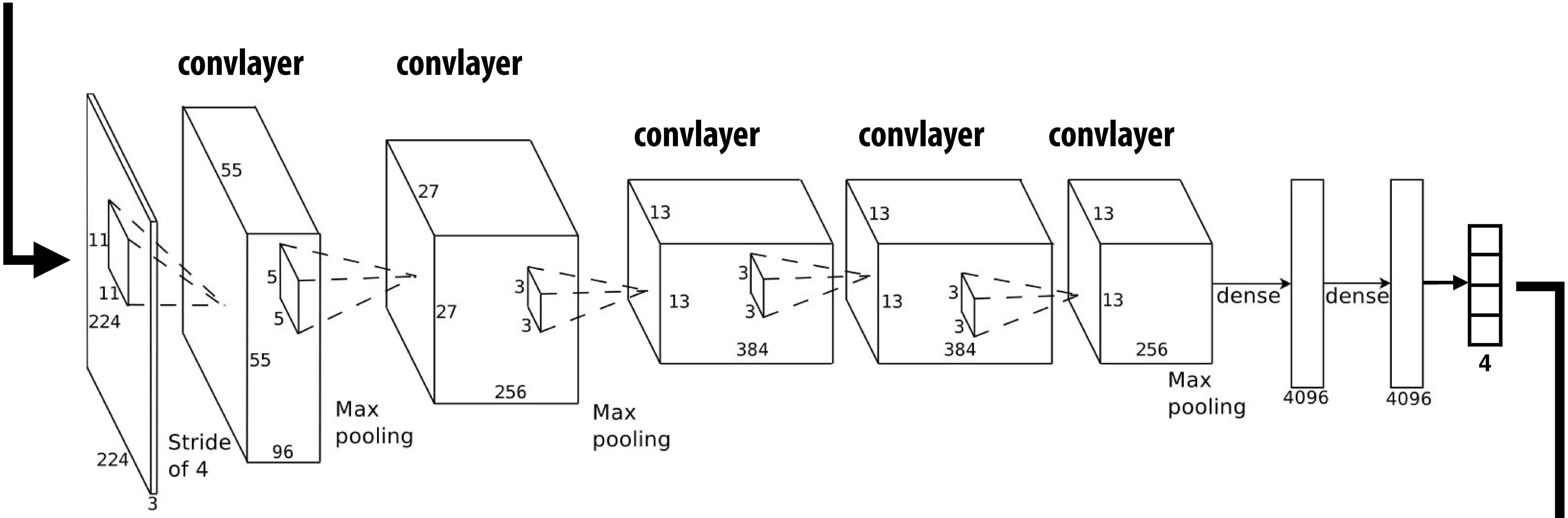
[label omitted]

[label omitted]

Nerdy

Professor classification network

New image of Kayvon
(not in training set)



Easy:	0.0
Mean:	0.0
Boring:	0.0
Nerdy:	1.0

Ground truth
(what the answer should be)

Easy:	0.26
Mean:	0.08
Boring:	0.14
Nerdy:	0.52

Network output

Error (loss)

Ground truth:
(what the answer should be)

Easy:	0.0
Mean:	0.0
Boring:	0.0
Nerdy:	1.0

Network output: *

Easy:	0.26
Mean:	0.08
Boring:	0.14
Nerdy:	0.52

Common example: softmax loss:

$$L = -log\left(\frac{e^{f_c}}{\sum_j e^{f_j}}\right)$$

Output of network for correct category

Output of network for all categories

* In practice a network using a softmax classifier outputs unnormalized, log probabilities (f_j), but I'm showing a probability distribution above for clarity

Training

Goal of training: learning good values of network parameters so that network outputs the correct classification result for any input image

Idea: minimize loss for all the training examples (for which the correct answer is known)

$$L = \sum_i L_i$$

(total loss for entire training set is sum of losses L_i for each training example x_i)

Intuition: if the network gets the answer correct for a wide range of training examples, then hopefully it has learned parameter values that yield the correct answer for future images as well.

Intuition: gradient descent

Say you had a function *f* that contained a hidden parameters *p_1 and p_2*: $f(x_i)$

And for some input *x_i*, your training data says the function should output 0.

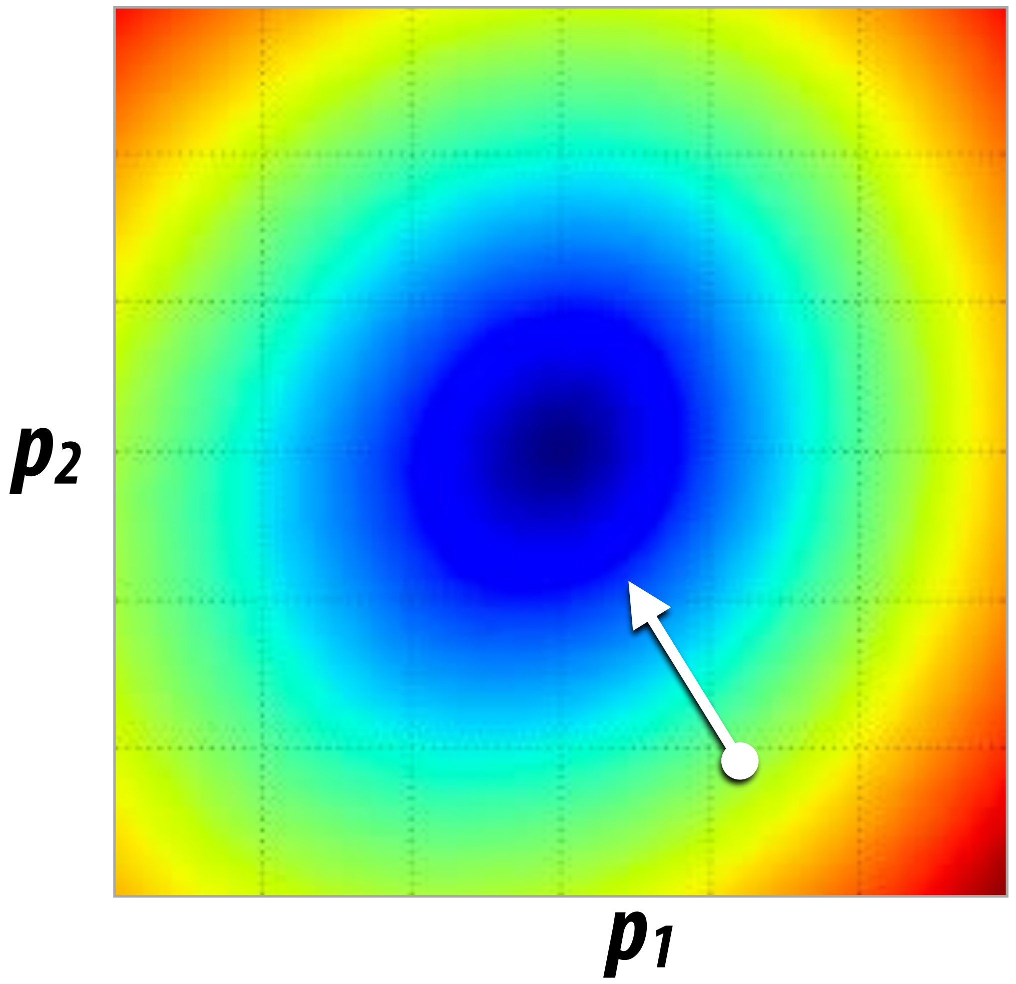
But for the current values of *p_1 and p_2*, it currently outputs 10.

$$f(x_i, p_1, p_2) = 10$$

And say I also gave you expressions for the derivative of *f* with respect to *p_1 and p_2* so you could compute their value at *x_i*.

$$\frac{df}{dp_1} = 2 \quad \frac{df}{dp_2} = -5 \qquad \nabla f = [2, -5]$$

red = high values of f
blue = low values



How might you adjust the values *p_1 and p_2* to reduce the error for this training example?

Basic gradient descent

```
while (loss too high):

   for each item x_i in training set:
      grad += evaluate_loss_gradient(f, loss_func, params, x_i)

   params += -grad * step_size;
```

Mini-batch stochastic gradient descent (mini-batch SGD): choose a random (small) subset of the training examples to compute gradient in each iteration of the while loop

How to compute df/dp for a complex neural network with millions of parameters?

Derivatives using the chain rule

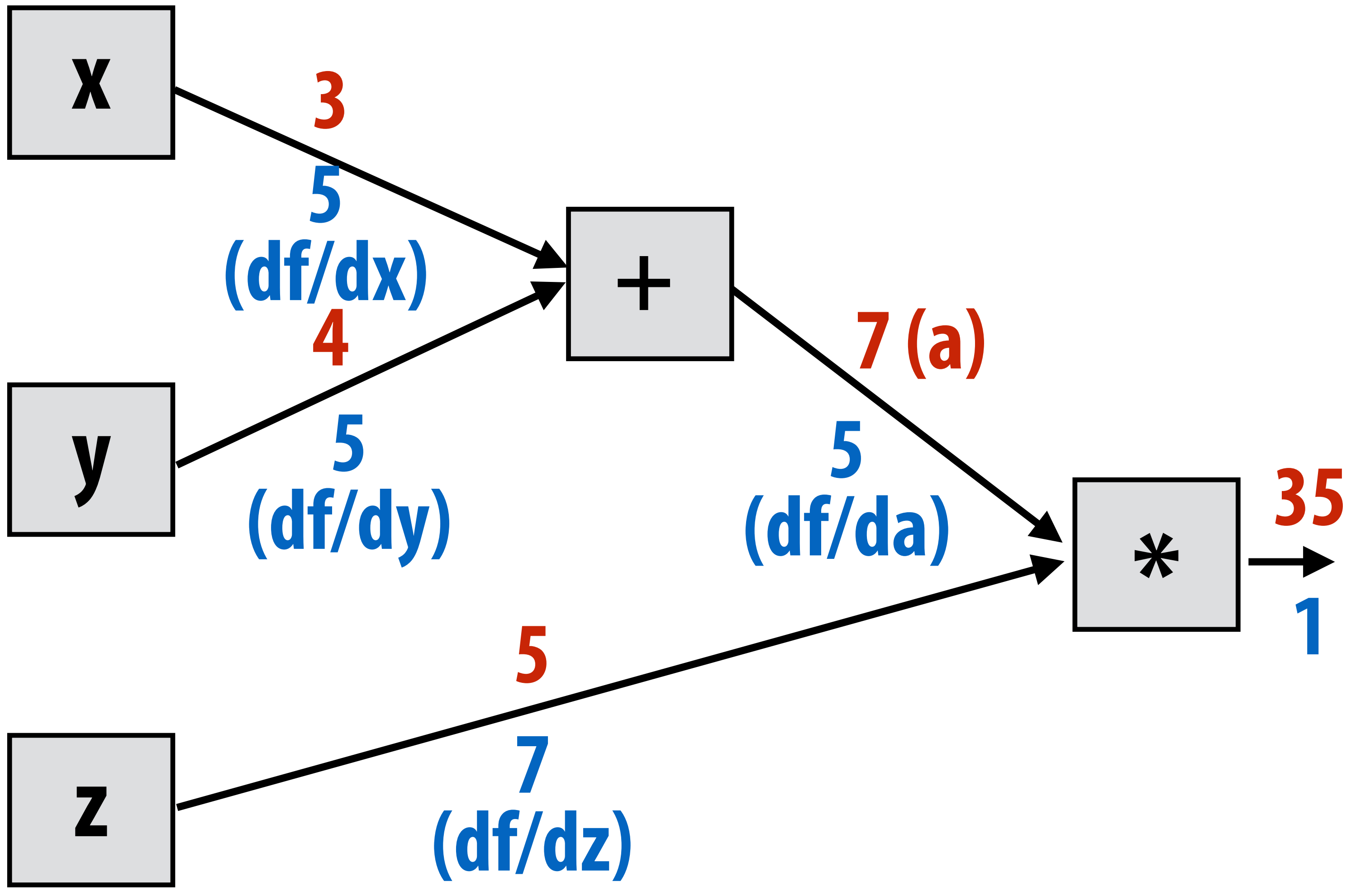
$$f(x,y,z) = (x+y)z = az$$

Where: $a = x + y$

$$\frac{df}{da} = z \qquad \frac{da}{dx} = 1 \qquad \frac{da}{dy} = 1$$

So, by the derivative chain rule:

$$\frac{df}{dx} = \frac{df}{da}\frac{da}{dx} = z$$



Red = output of node

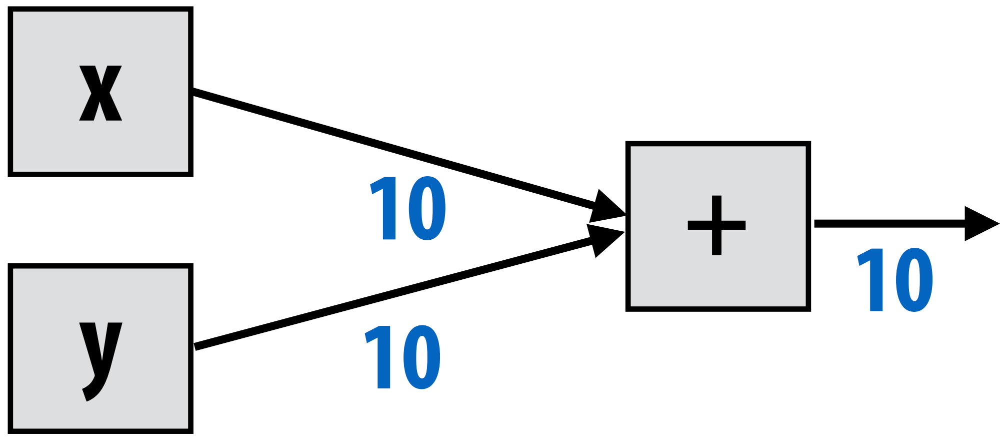
Blue = df/dnode

Backpropagation

Red = output of node

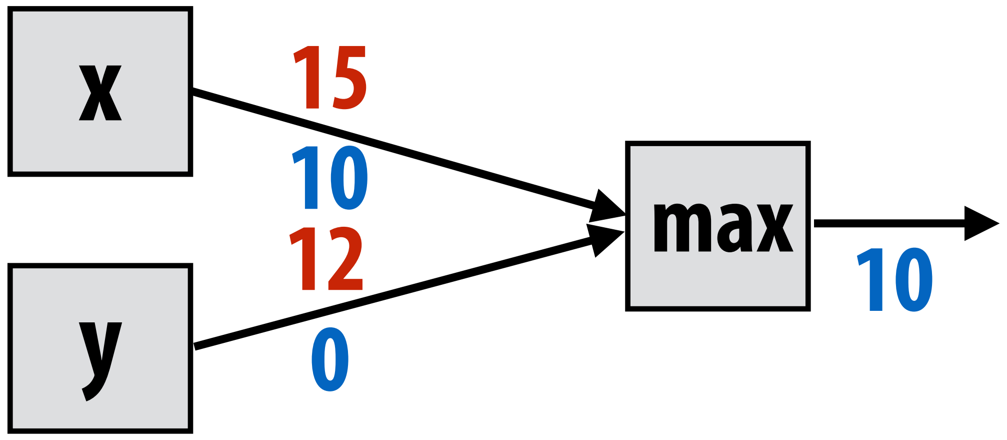
Blue = df/dnode

Recall: $\frac{df}{dx} = \frac{df}{dg}\frac{dg}{dx}$



$g(x, y) = x + y$

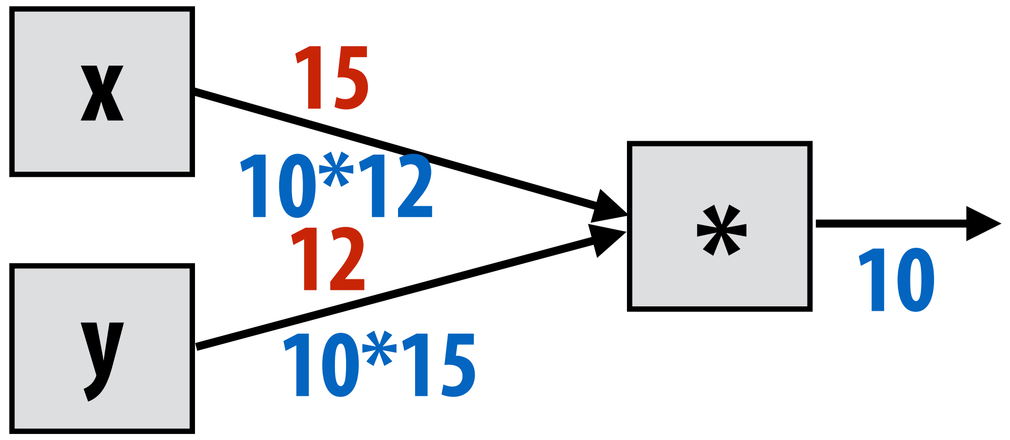
$\frac{dg}{dx} = 1\,, \ \frac{dg}{dy} = 1$



$g(x, y) = \max(x, y)$

$\frac{dg}{dx} =$ **1, if x > y**

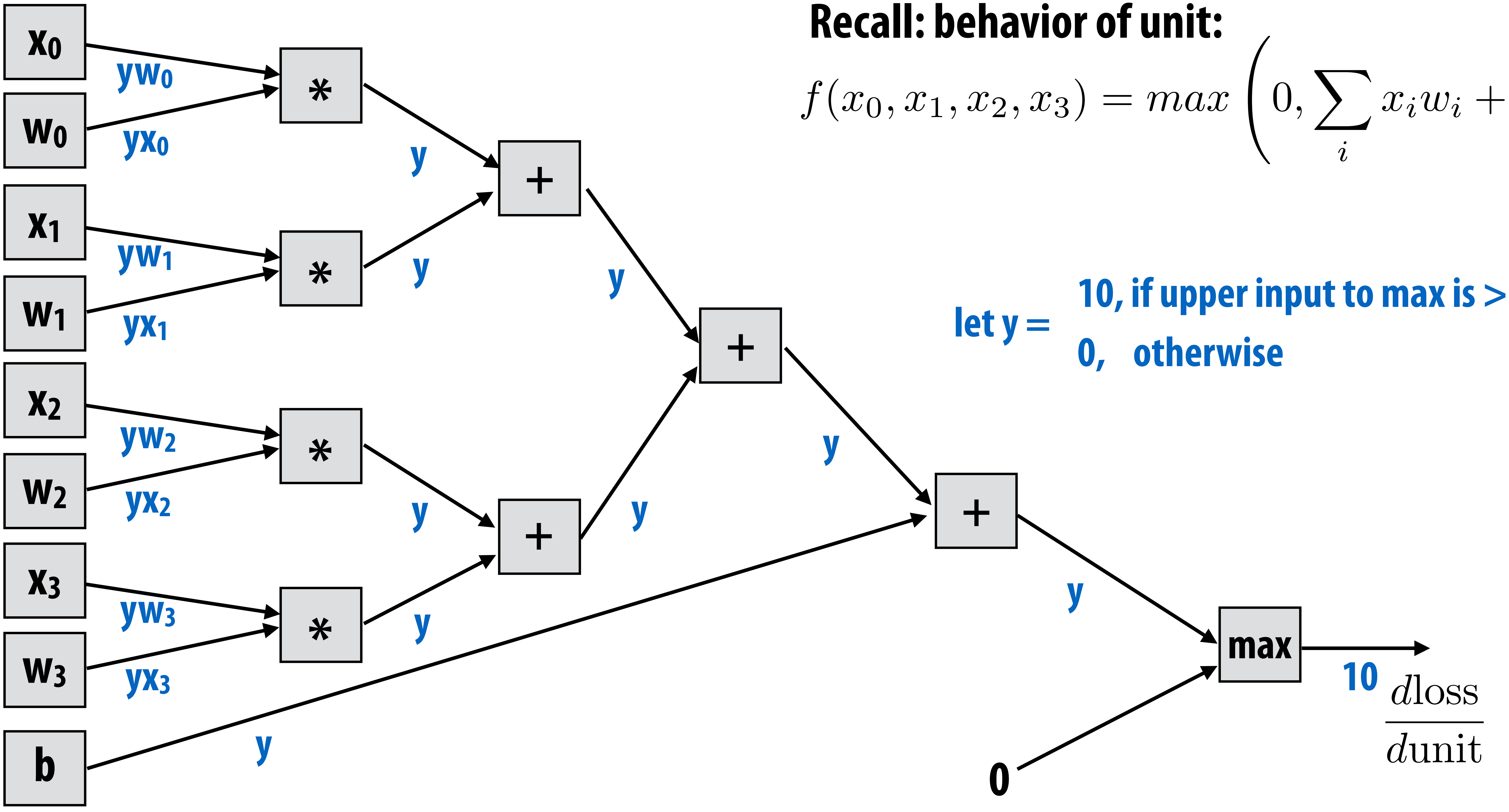
0, otherwise



$g(x, y) = xy$

$\frac{dg}{dx} = y\,, \ \frac{dg}{dy} = x$

Backpropagating through single unit



$$\frac{d\text{loss}}{d\text{unit}}$$

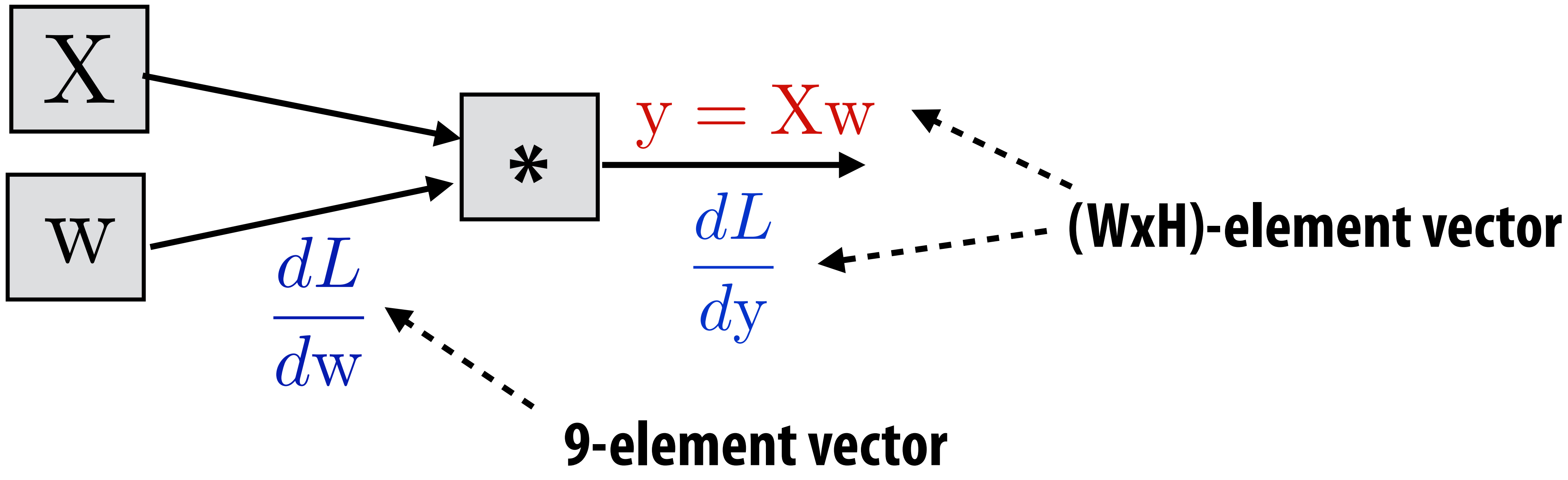
Recall: behavior of unit:

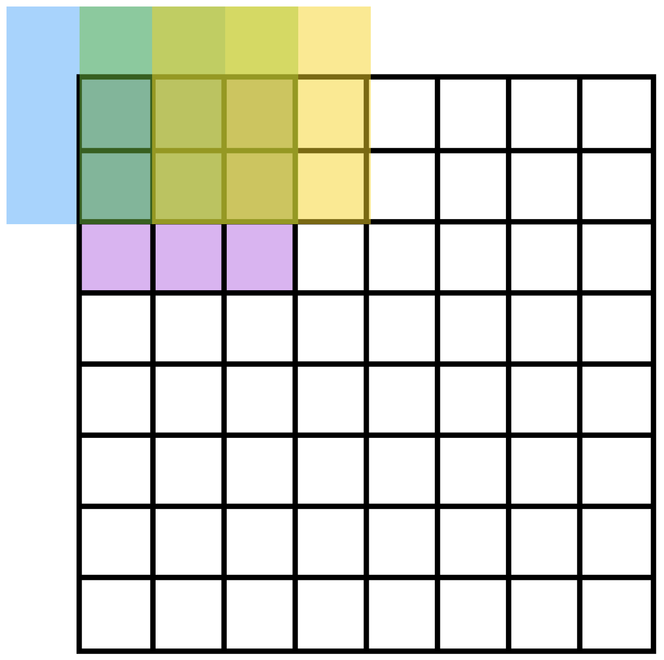
$$f(x_0, x_1, x_2, x_3) = max\left(0, \sum_i x_i w_i + b\right)$$

let y = 10, if upper input to max is > 0
0, otherwise

Observe: output of prior layer (x_i's) and output of this unit must be retained in order to compute weight gradients for this unit during backprop.

Backpropagation: matrix form





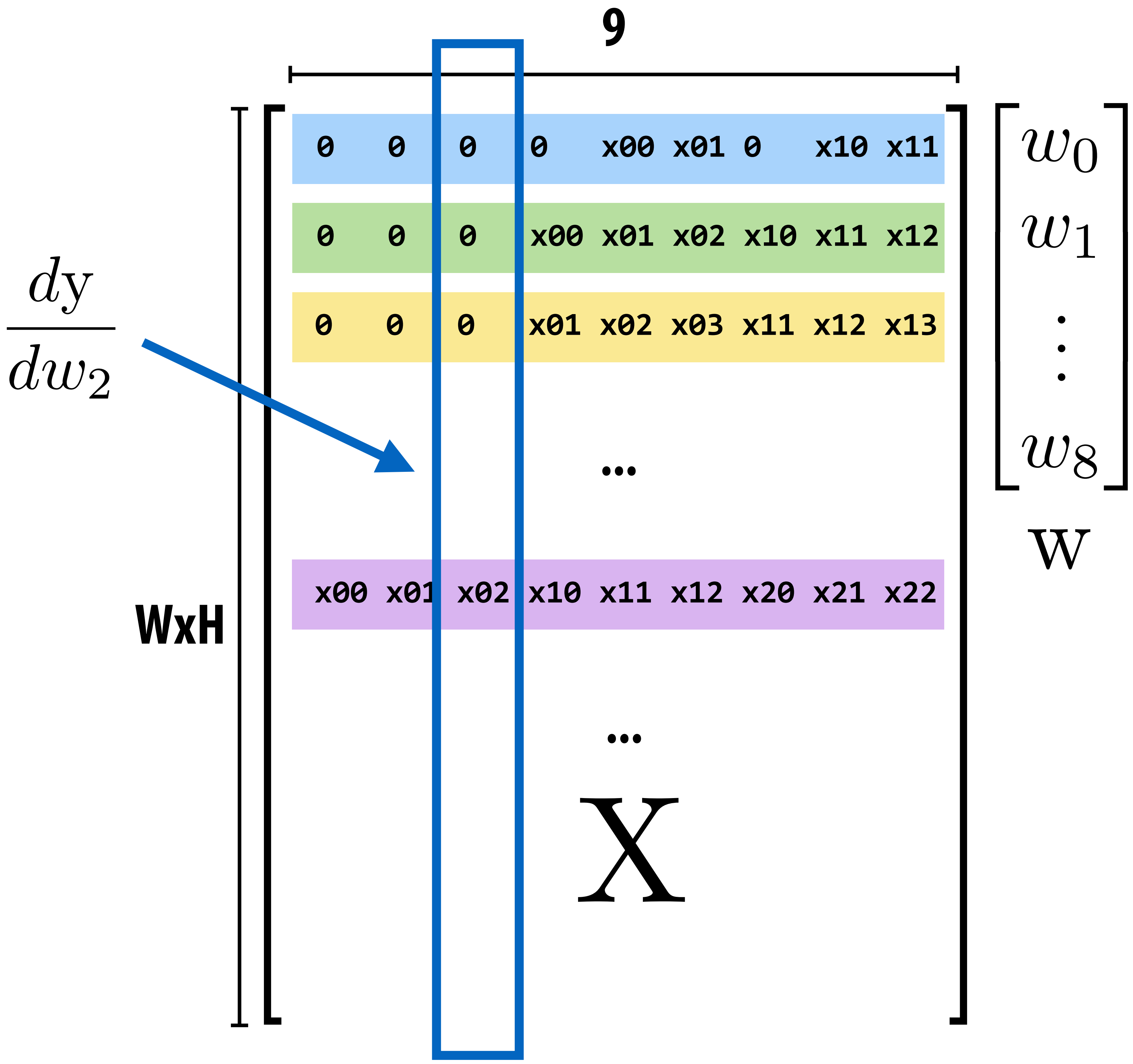
$$\frac{dy_j}{dw_i} = X_{ji}$$

$$\frac{dL}{dw_i} = \sum_j \frac{dL}{dy_j}\frac{dy_j}{dw_i}$$

$$= \sum_j \frac{dL}{dy_j} X_{ji}$$

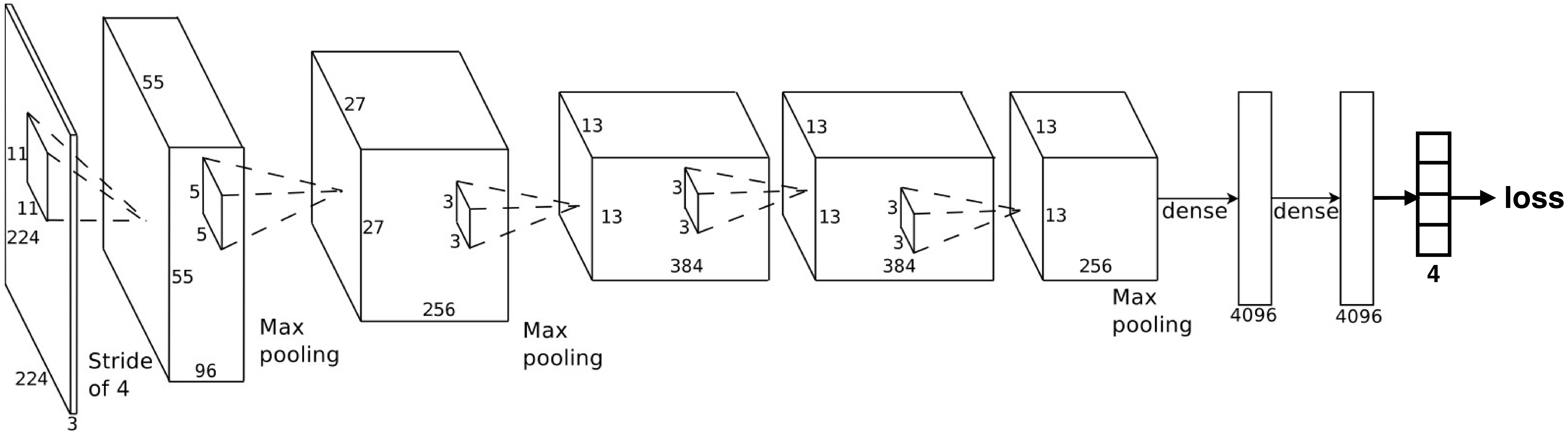
Therefore:

$$\frac{dL}{dw} = X^T \frac{dL}{dy}$$



$$\begin{bmatrix} 0 & 0 & 0 & 0 & x00 & x01 & 0 & x10 & x11 \\ 0 & 0 & 0 & x00 & x01 & x02 & x10 & x11 & x12 \\ 0 & 0 & 0 & x01 & x02 & x03 & x11 & x12 & x13 \\ & & & & \ldots & & & & \\ x00 & x01 & x02 & x10 & x11 & x12 & x20 & x21 & x22 \\ & & & & \ldots & & & & \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_8 \end{bmatrix}$$

Back-propagation through the entire professor classification network



For each training example x_i in mini-batch:

- Perform forward evaluation to compute loss for x_i
  - Note: must retain all layer outputs + output gradients (needed to compute weight gradients during backpropagation)
- Compute gradient of loss w.r.t. final layer's outputs
- Backpropagate gradient to compute gradient of loss w.r.t. all network parameters
- Accumulate gradients (over all images in batch)

Update all parameter values: wi_new = wi_old - step_size * gradi

VGG memory footprint

Calculations assume 32-bit values (image batch size = 1)

inputs/outputs get multiplied by mini-batch size

Unlike forward evaluation:

1. must store outputs and gradient of outputs
2. cannot immediately free outputs once consumed by next level of network

	weights mem:	output size (per image)	(mem)
input: 224 x 224 RGB image	—	224x224x3	150K
conv: (3x3x3) x 64	6.5 KB	224x224x64	12.3 MB
conv: (3x3x64) x 64	144 KB	224x224x64	12.3 MB
maxpool	—	112x112x64	3.1 MB
conv: (3x3x64) x 128	228 KB	112x112x128	6.2 MB
conv: (3x3x128) x 128	576 KB	112x112x128	6.2 MB
maxpool	—	56x56x128	1.5 MB
conv: (3x3x128) x 256	1.1 MB	56x56x256	3.1 MB
conv: (3x3x256) x 256	2.3 MB	56x56x256	3.1 MB
conv: (3x3x256) x 256	2.3 MB	56x56x256	3.1 MB
maxpool	—	28x28x256	766 KB
conv: (3x3x256) x 512	4.5 MB	28x28x512	1.5 MB
conv: (3x3x512) x 512	9 MB	28x28x512	1.5 MB
conv: (3x3x512) x 512	9 MB	28x28x512	1.5 MB
maxpool	—	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
conv: (3x3x512) x 512	9 MB	14x14x512	383 KB
maxpool	—	7x7x512	98 KB
fully-connected 4096	392 MB	4096	16 KB
fully-connected 4096	64 MB	4096	16 KB
fully-connected 1000	15.6 MB	1000	4 KB
soft-max		1000	4 KB

Must also store per-weight gradients

Many implementations also store gradient “momentum” as well (multiply by 3)

SGD workload

```
while (loss too high):

    for each item x_i in mini-batch:
        grad += evaluate_loss_gradient(f, loss_func, params, x_i)

    params += -grad * step_size;
```

At first glance, this loop is sequential (each step of "walking downhill" depends on previous)

Parallel across images

sum reduction

large computation with its own parallelism (but working set may not fit on single machine)

trivial data-parallel over parameters

Deep network training workload

- **Huge computational expense**
  - Must evaluate the network (forward and backward) for millions of training images
  - Must iterate for many iterations of gradient descent (100's of thousands)
  - Training modern networks takes days

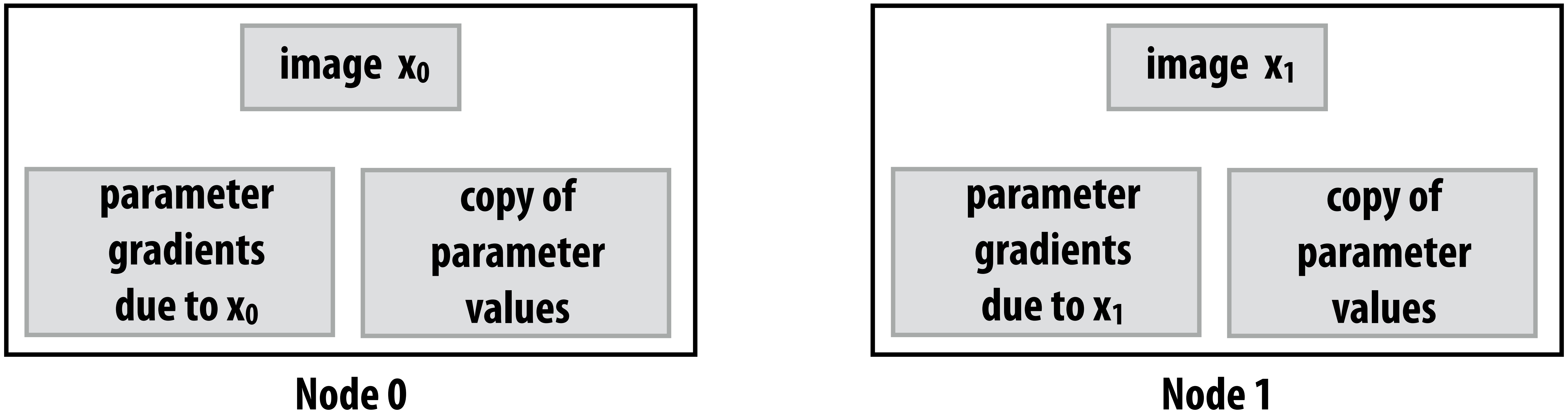
- **Large memory footprint**
  - Must maintain network layer outputs from forward pass
  - Additional memory to store gradients for each parameter
  - Recall parameters for popular VGG-16 network require ~500 MB of memory (training requires GBs of memory for academic networks)
  - Scaling to larger networks requires partitioning network across nodes to keep network + intermediates in memory

- **Dependencies /synchronization (not embarrassingly parallel)**
  - Each parameter update step depends on previous
  - Many units contribute to same parameter gradients (fine-scale reduction)
  - Different images in mini batch contribute to same parameter gradients

Data-parallel training (across images)

```
for each item x_i in mini-batch:
   grad += evaluate_loss_gradient(f, loss_func, params, x_i)
params += -grad * step_size;
```

Consider parallelization of the outer for loop across machines in a cluster



```
partition mini-batch across nodes
for each item x_i in mini-batch assigned to local node:
   // just like single node training
   grad += evaluate_loss_gradient(f, loss_func, params, x_i)
barrier();
sum reduce gradients, communicate results to all nodes
barrier();
update copy of parameter values
```

Challenges of computing at cluster scale

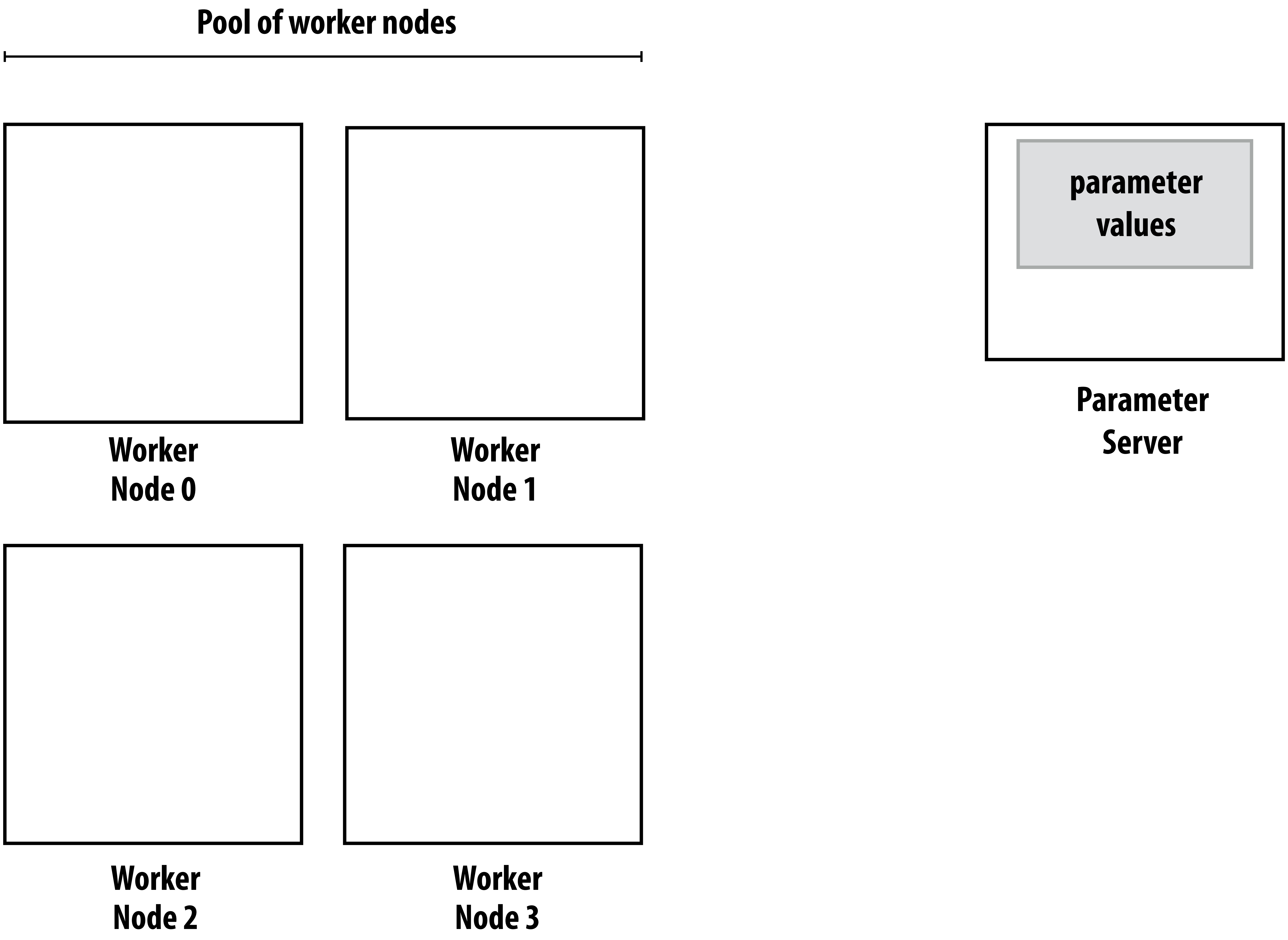
- Slow communication between nodes
  - Clusters do not feature high-performance interconnects typical of supercomputers

- Nodes with different performance (even if machines are the same)
  - Workload imbalance at barriers (sync points between nodes)

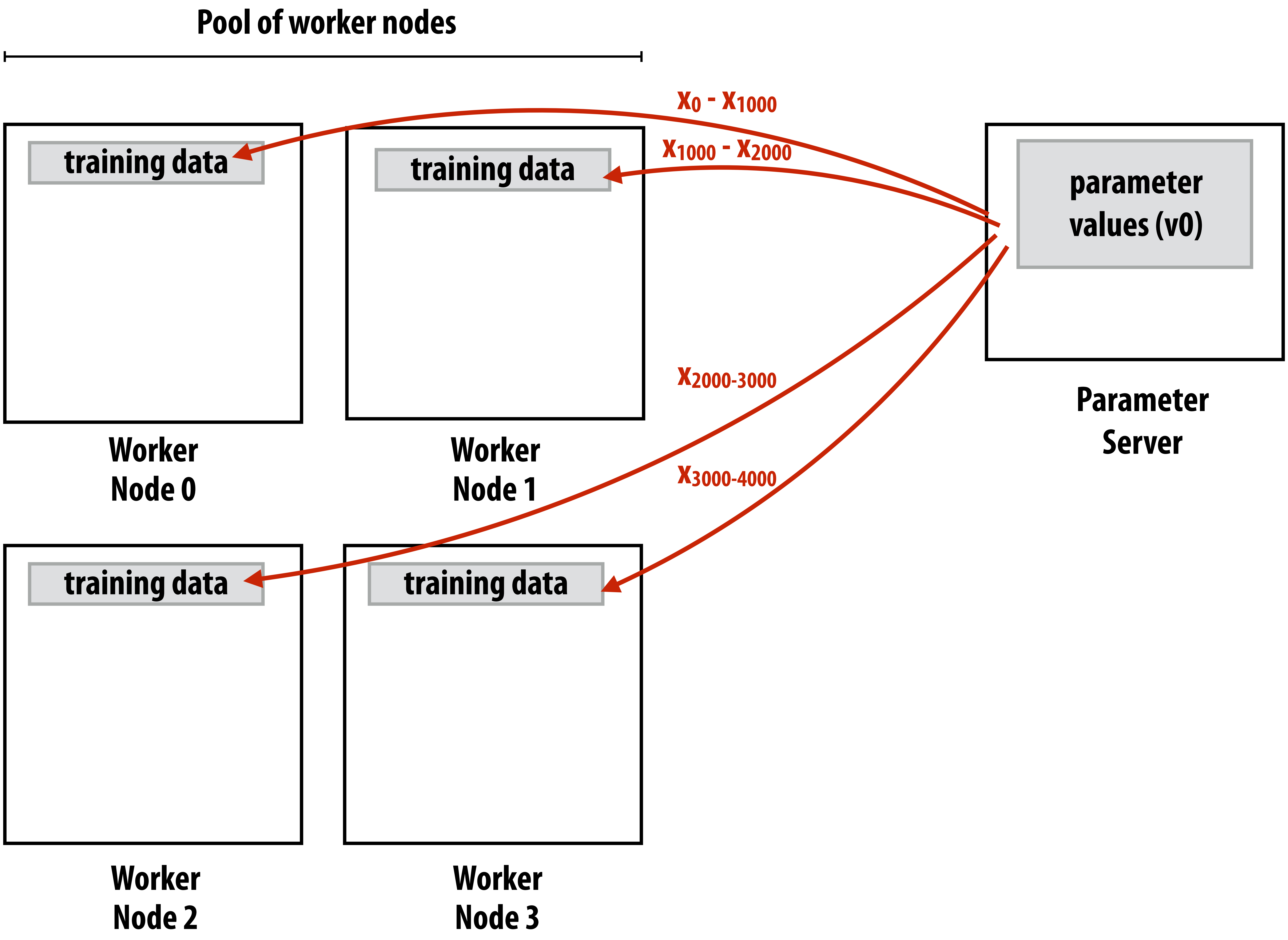
Modern solution: exploit characteristics of SGD using asynchronous execution!

Parameter server design

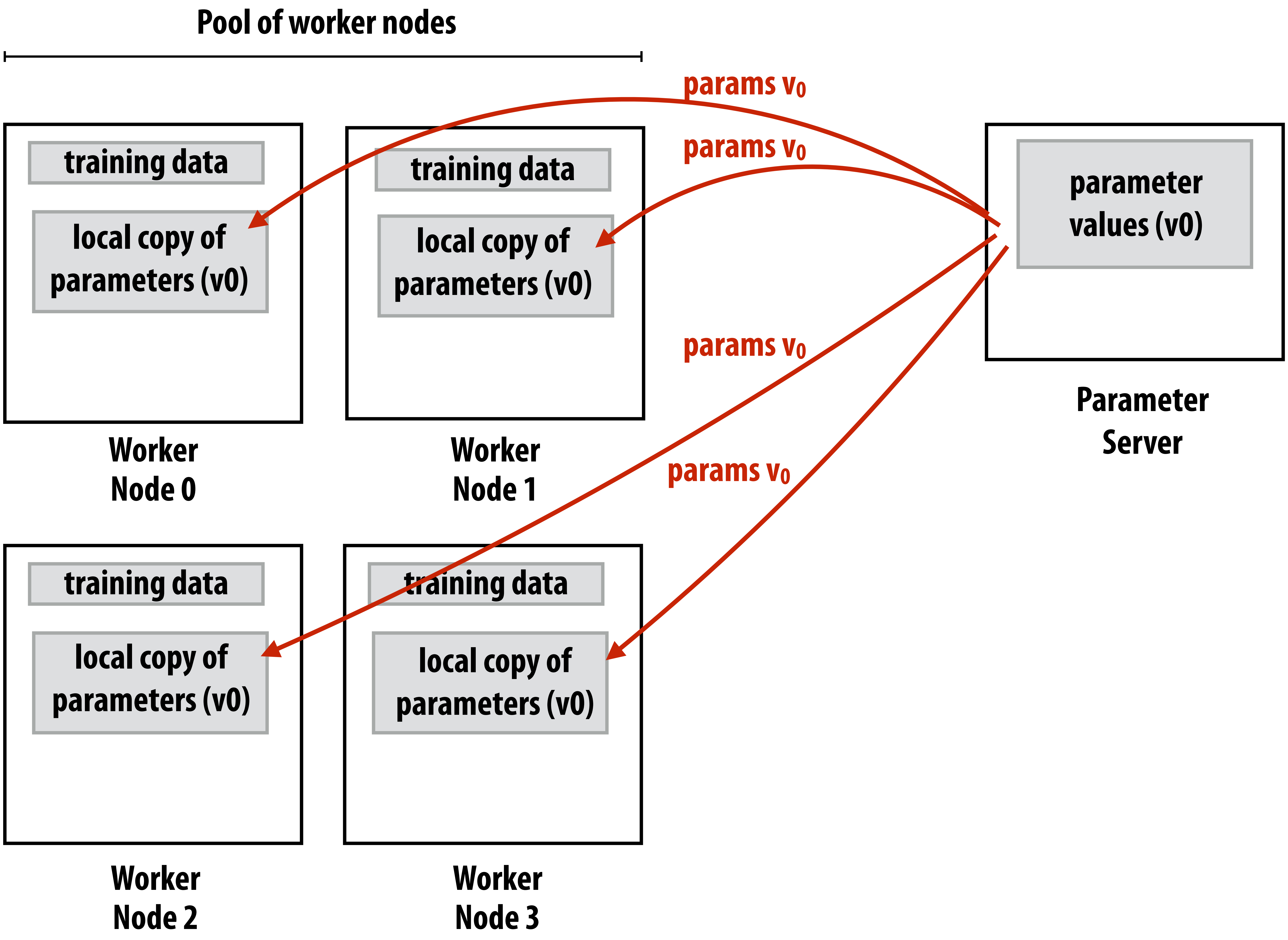
Parameter Server [Li OSDI14]
Google's DistBelief [Dean NIPS12]
Microsoft's Project Adam [Chilimbi OSDI14]



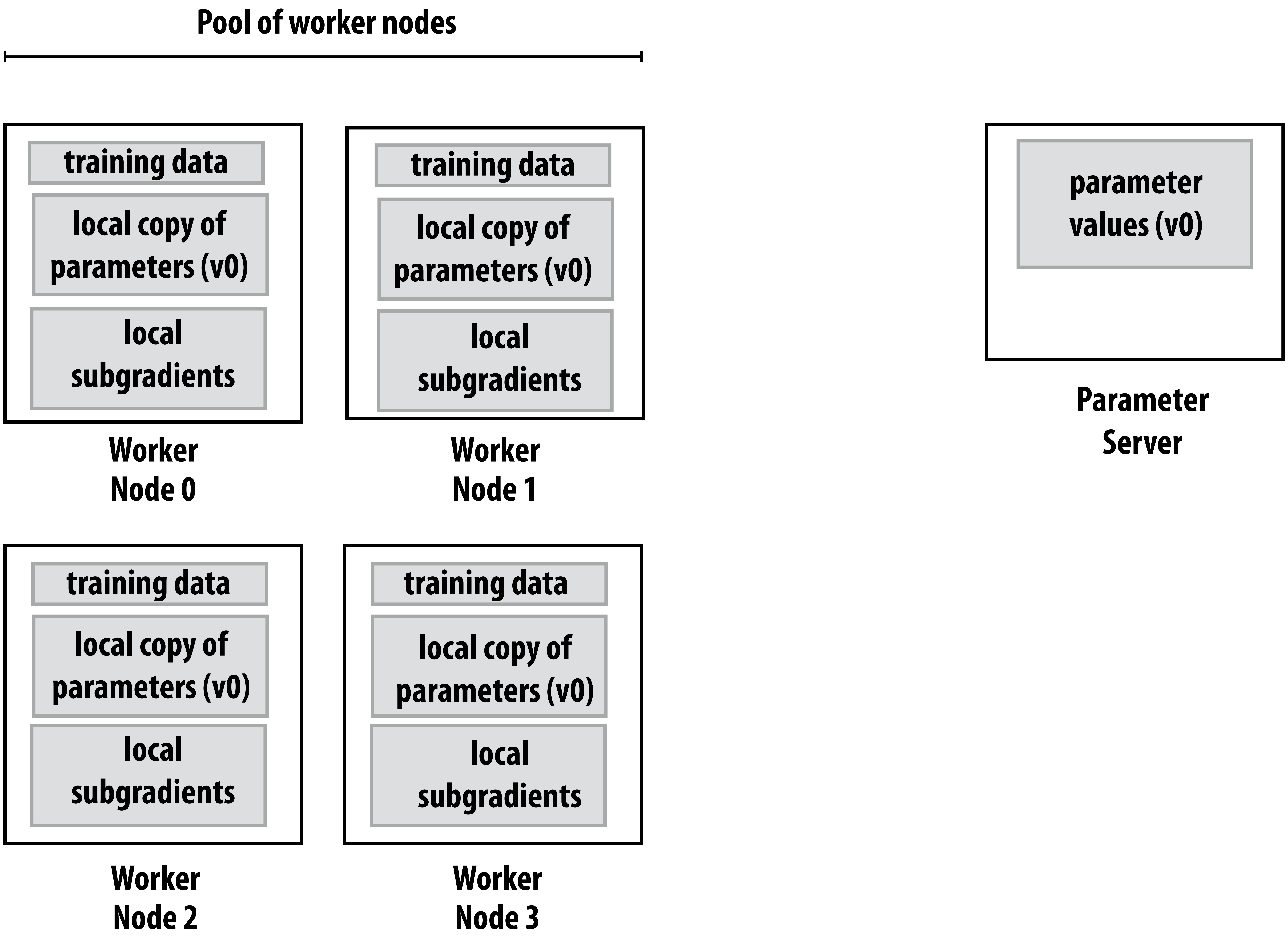
Training data partitioned among workers



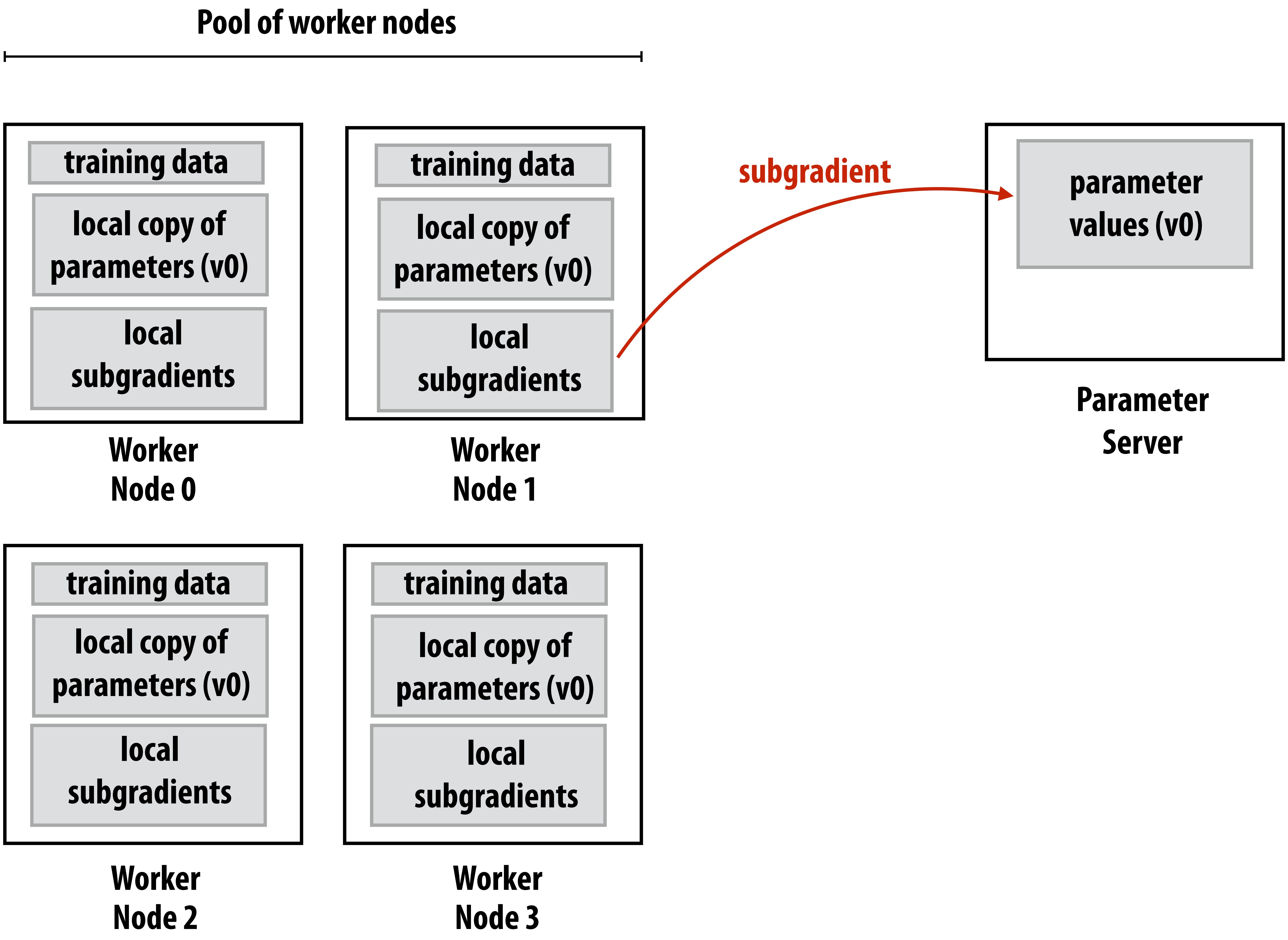
Copy of parameters sent to workers



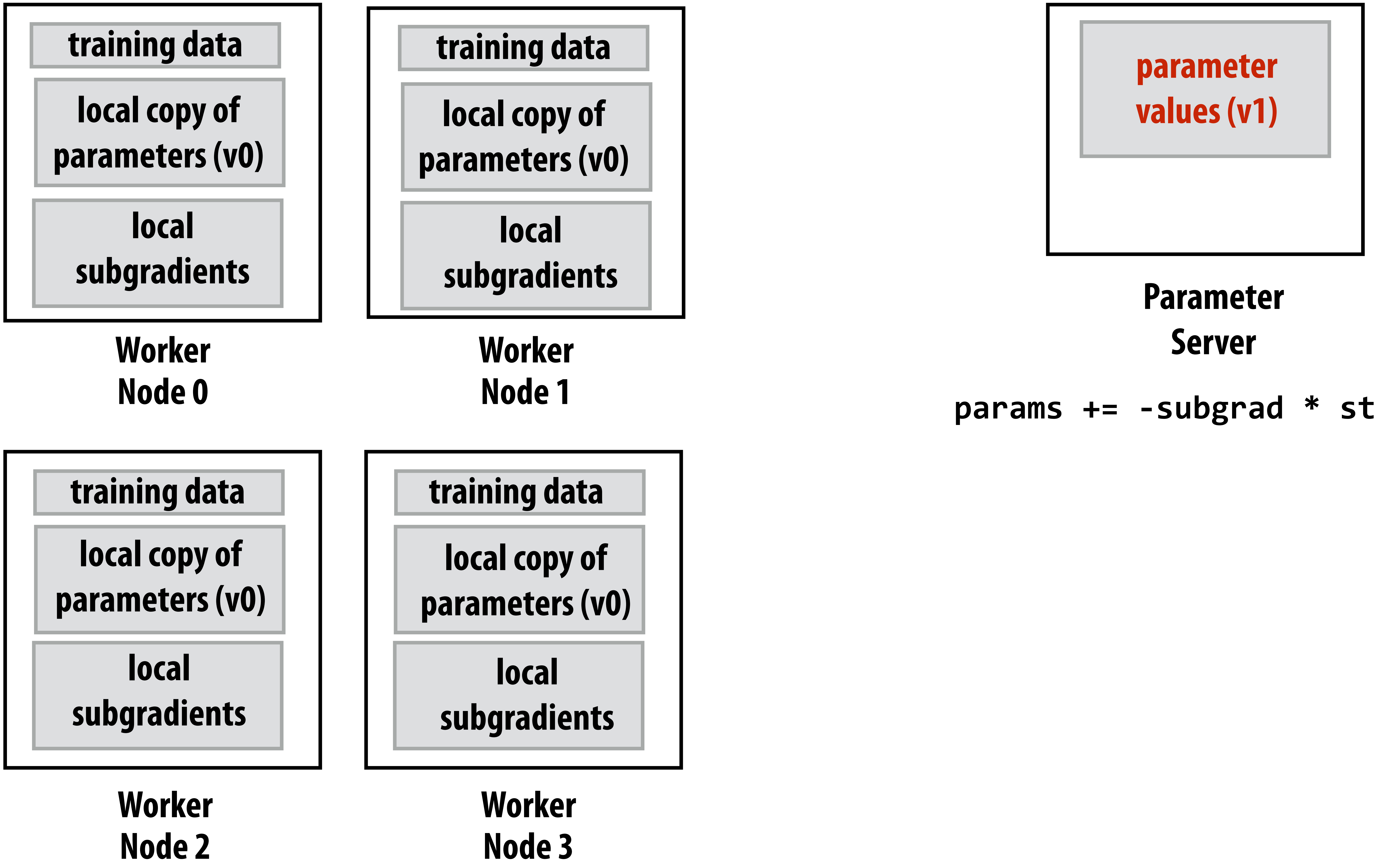
Workers independently compute local “subgradients"



Worker sends subgradient to parameter server



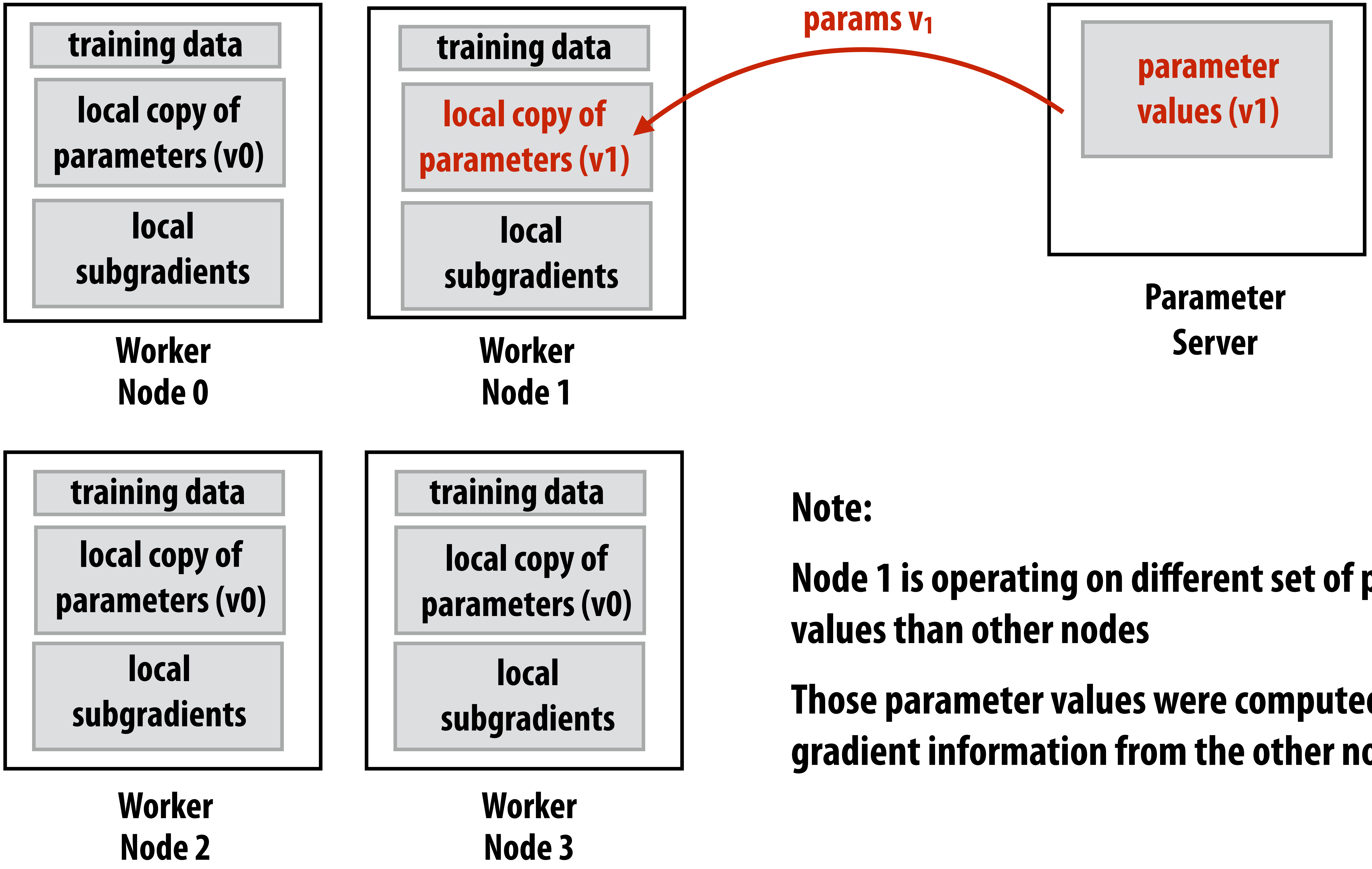
Server updates global parameter values based on subgradient



```
params += -subgrad * step_size;
```

Updated parameters sent to worker

Worker proceeds with another gradient computation step

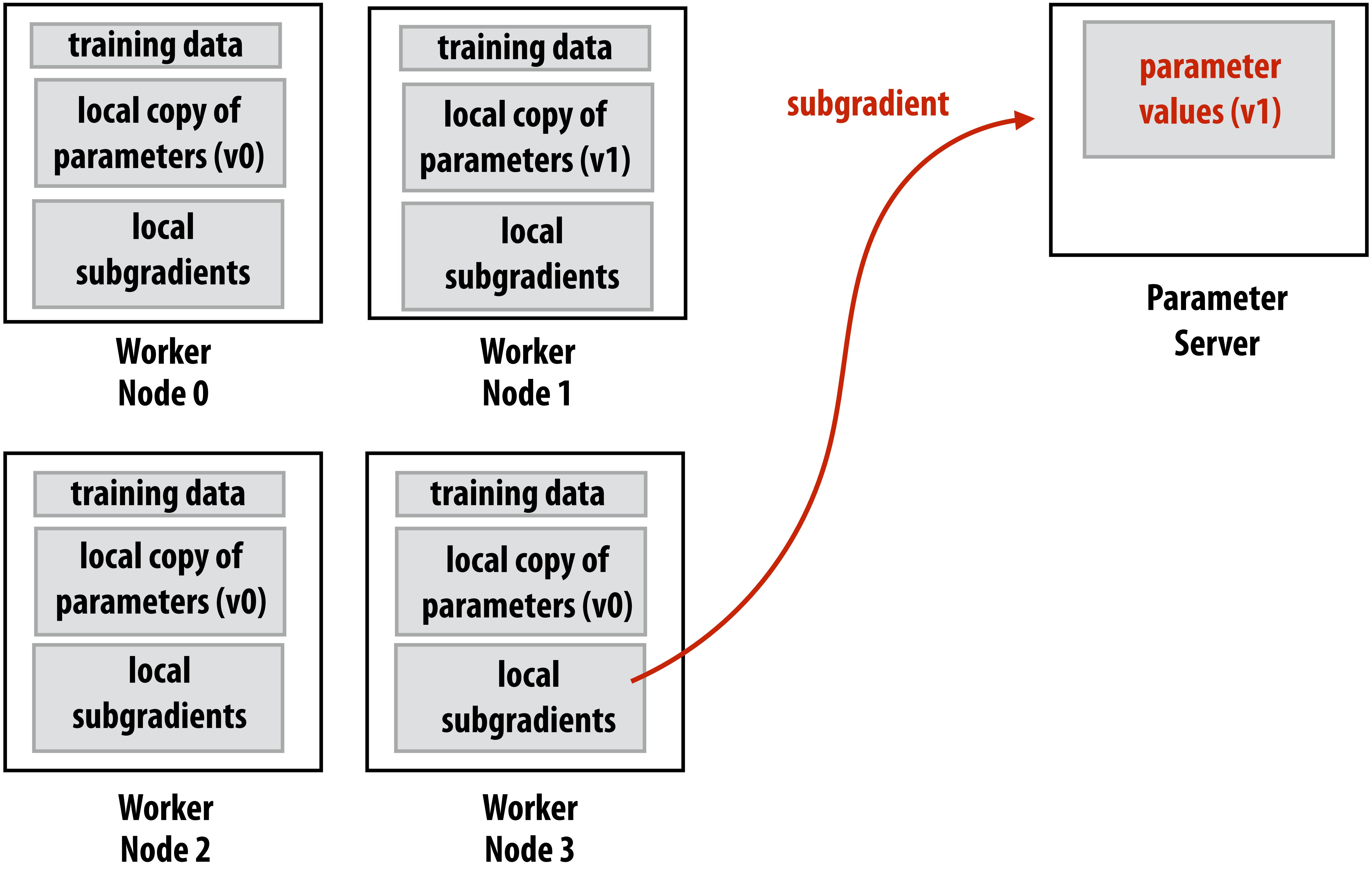


Note:

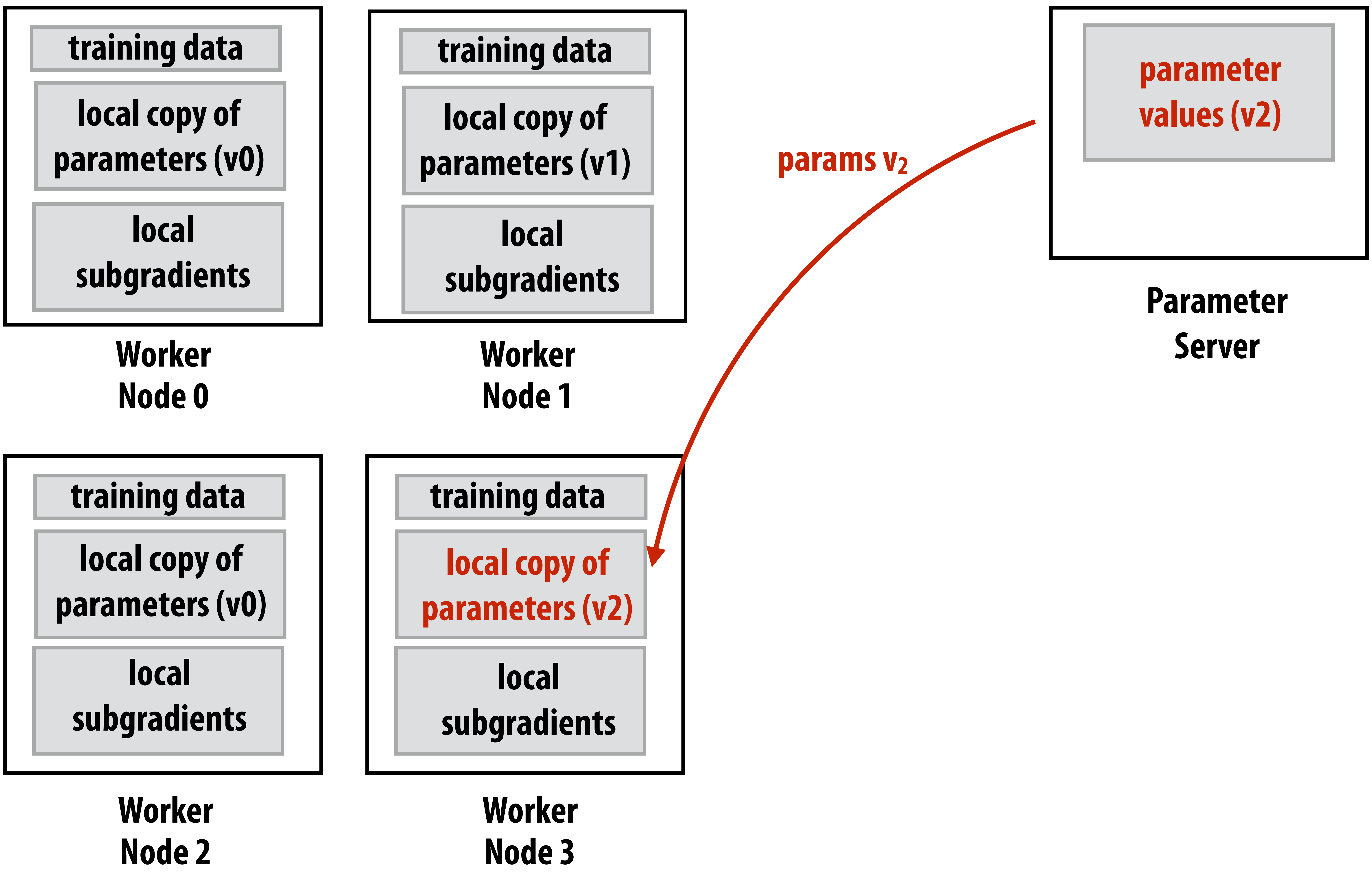
Node 1 is operating on different set of parameter values than other nodes

Those parameter values were computed without gradient information from the other nodes

Updated parameters sent to worker (again)



Worker continues with updated parameters

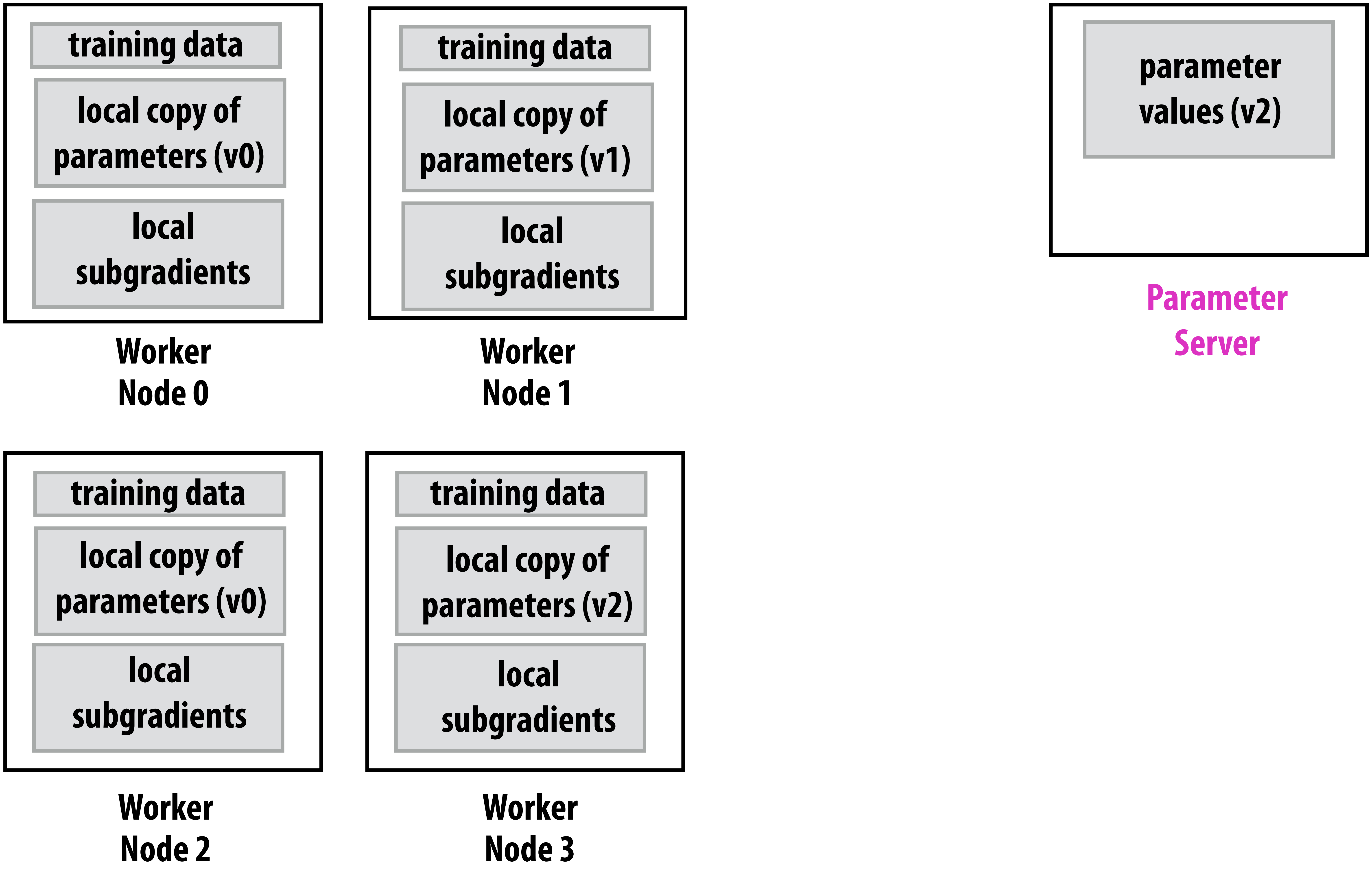


Summary: asynchronous parameter update

- Idea: avoid global synchronization on all parameter updates between each SGD iteration
  - Design reflects realities of cluster computing:
    - Slow interconnects
    - Unpredictable machine performance
- Solution: asynchronous (and partial) subgradient updates
- Will impact convergence of SGD
  - Node N working on iteration i may not have parameter values that result the results of the i-1 prior SGD iterations

Bottleneck?

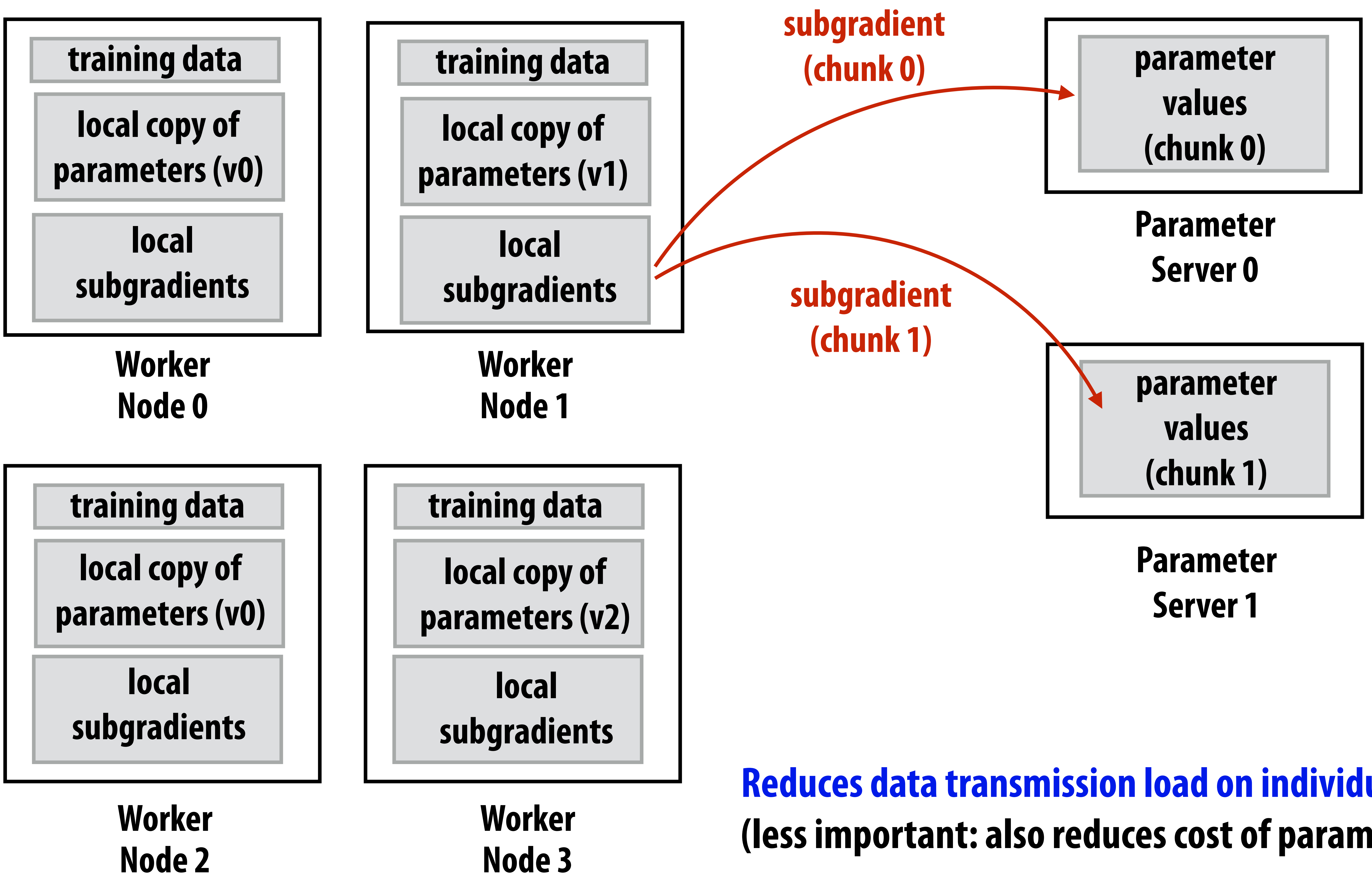
What if there is heavy contention for parameter server?



Shard the parameter server

Partition parameters across servers

Worker sends chunk of subgradients to owning parameter server

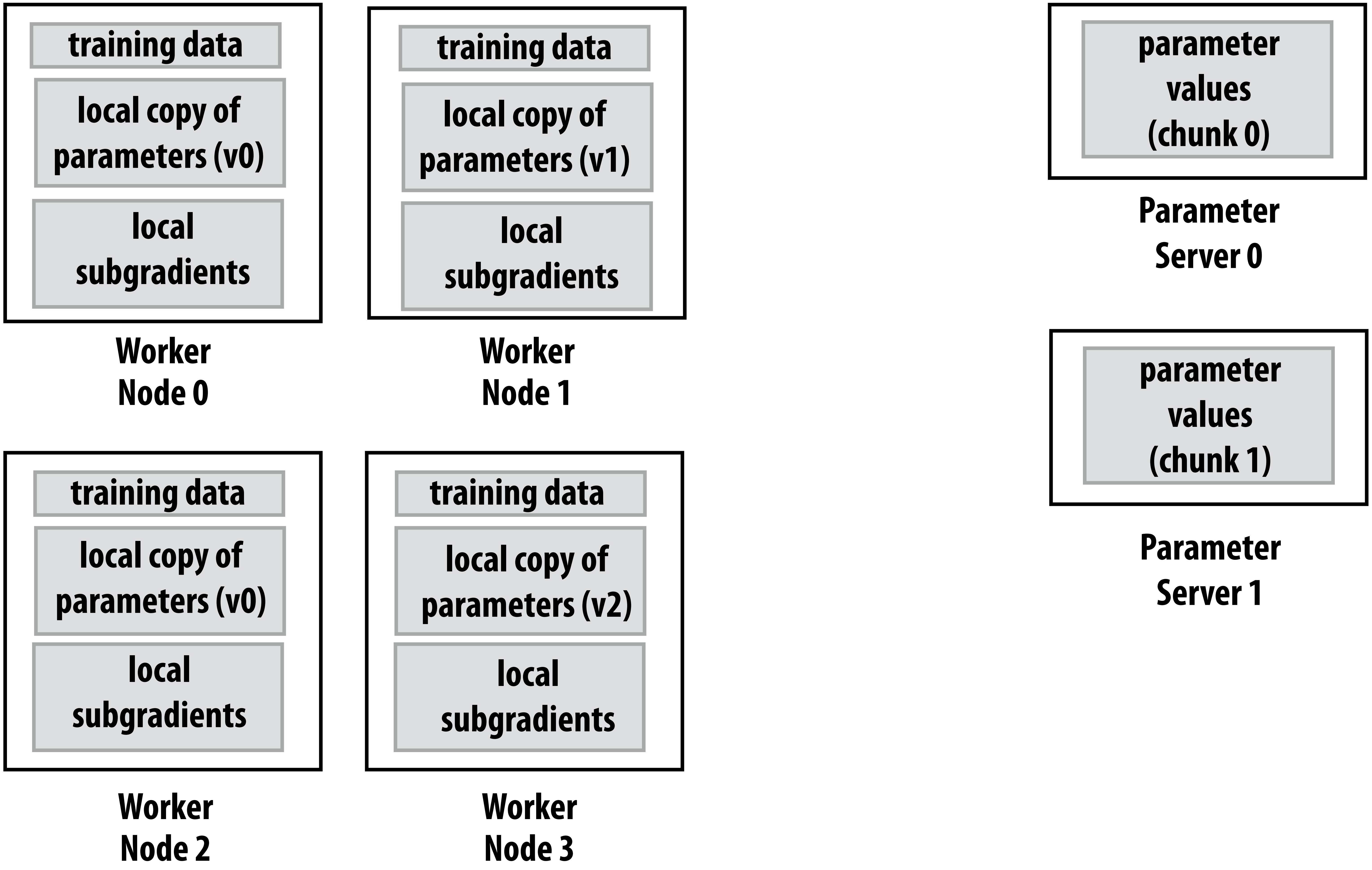


Reduces data transmission load on individual servers

(less important: also reduces cost of parameter update)

What if model parameters do not fit on one worker?

Recall high footprint of training large networks (particularly with large mini-batch sizes)

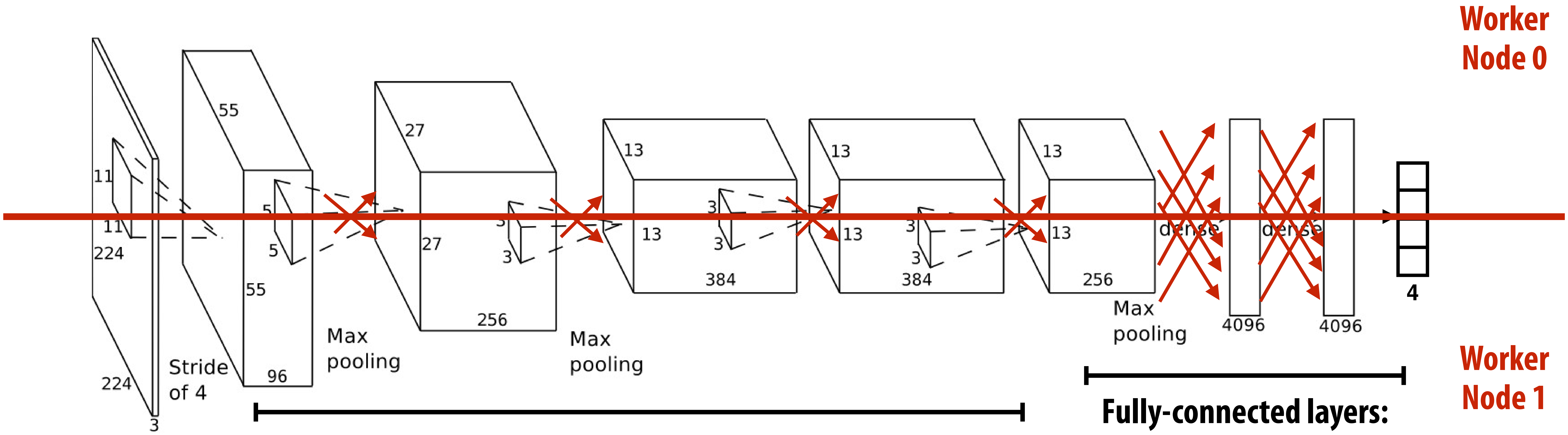


Model parallelism

Partition network parameters across nodes (spatial partitioning to reduce communication)

Reduce internode communication through network design:

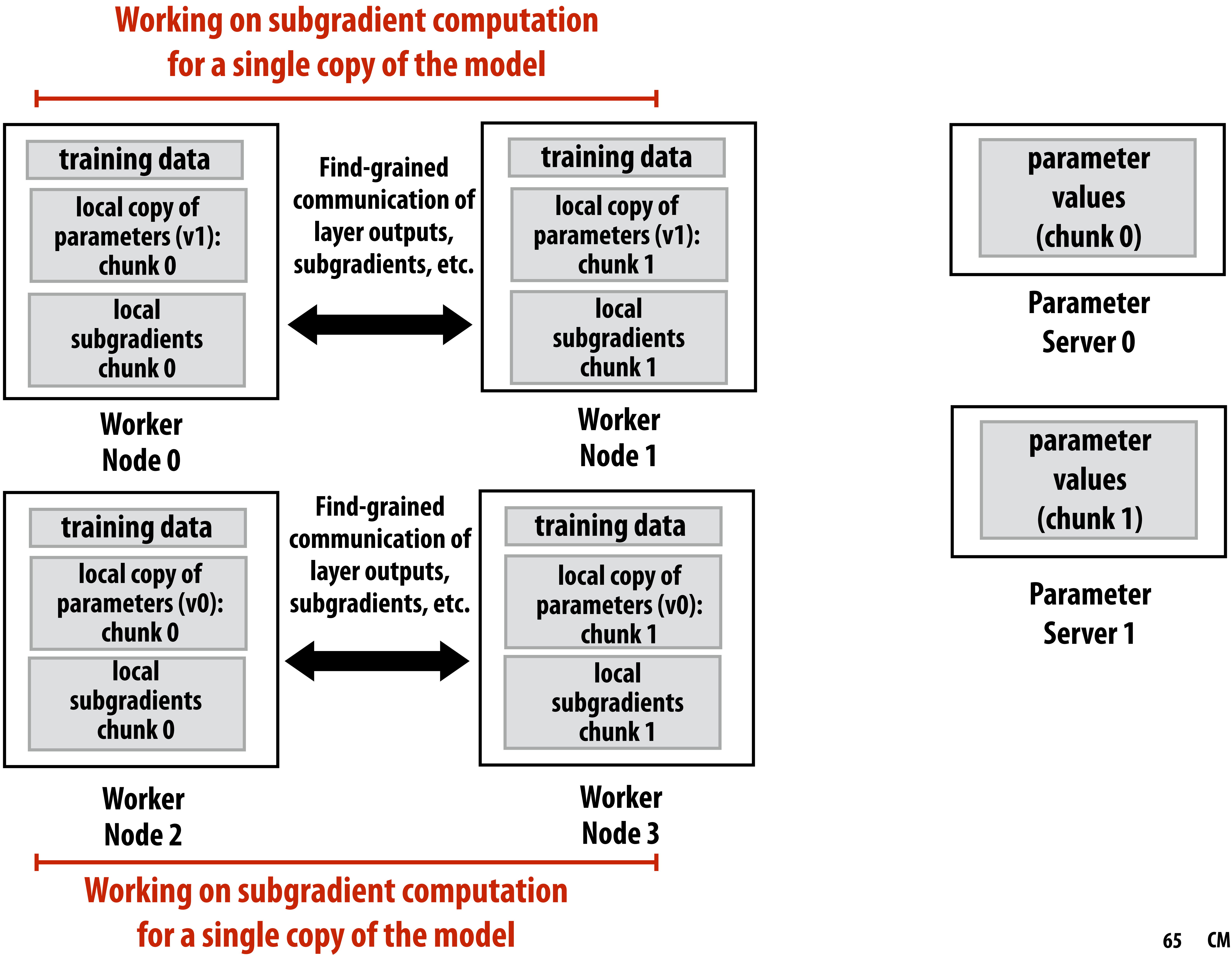
- Use small spatial convolutions (1x1 convolutions)
- Reduce/shrink fully-connected layers



Convolutional layers: only need to community outputs near spatial partition

Fully-connected layers: all data owned by a node must by communicated to other nodes

Training data-parallel and model-parallel execution



Working on subgradient computation for a single copy of the model

training data

local copy of parameters (v1): chunk 0

local subgradients chunk 0

Worker Node 0

Find-grained communication of layer outputs, subgradients, etc.

training data

local copy of parameters (v1): chunk 1

local subgradients chunk 1

Worker Node 1

parameter values (chunk 0)

Parameter Server 0

training data

local copy of parameters (v0): chunk 0

local subgradients chunk 0

Worker Node 2

Find-grained communication of layer outputs, subgradients, etc.

training data

local copy of parameters (v0): chunk 1

local subgradients chunk 1

Worker Node 3

parameter values (chunk 1)

Parameter Server 1

Working on subgradient computation for a single copy of the model

Using supercomputers for training?

- Fast interconnects critical for model-parallel training
  - Fine-grained communication of outputs and gradients
- Fast interconnect diminishes need for async training algorithms
  - Avoid randomness in training due to computation schedule (there remains randomness due to SGD algorithm)

OakRidge Titan Supercomputer

NVIDIA DGX-1: 8 Pascal GPUs connected via high speed NV-Link interconnect

Parallelizing mini-batch on one machine

```
for each item x_i in mini-batch:
   grad += evaluate_loss_gradient(f, loss_func, params, x_i)
params += -grad * step_size;
```

Consider parallelization of the outer for loop across cores

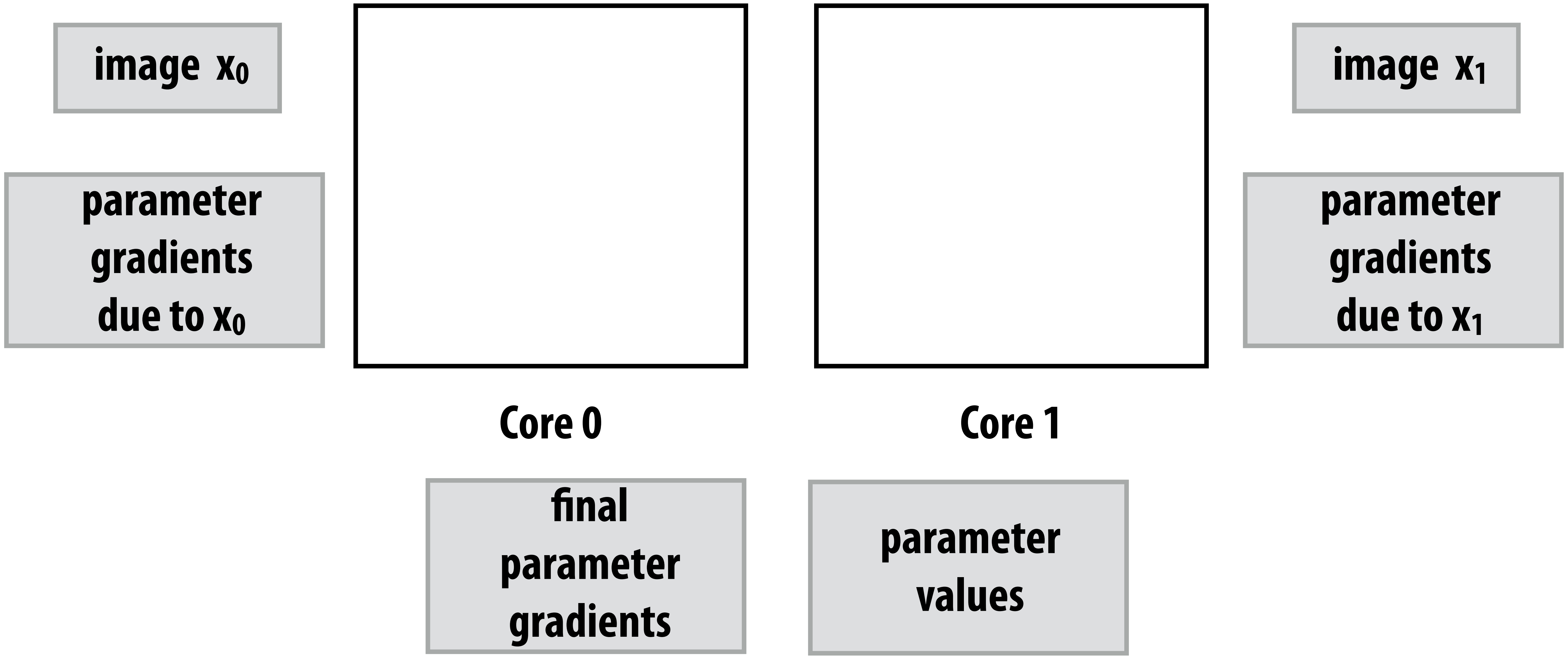


image x_0
parameter gradients due to x_0
Core 0
Core 1
image x_1
parameter gradients due to x_1
final parameter gradients
parameter values

Good: completely independent computations (until gradient reduction)

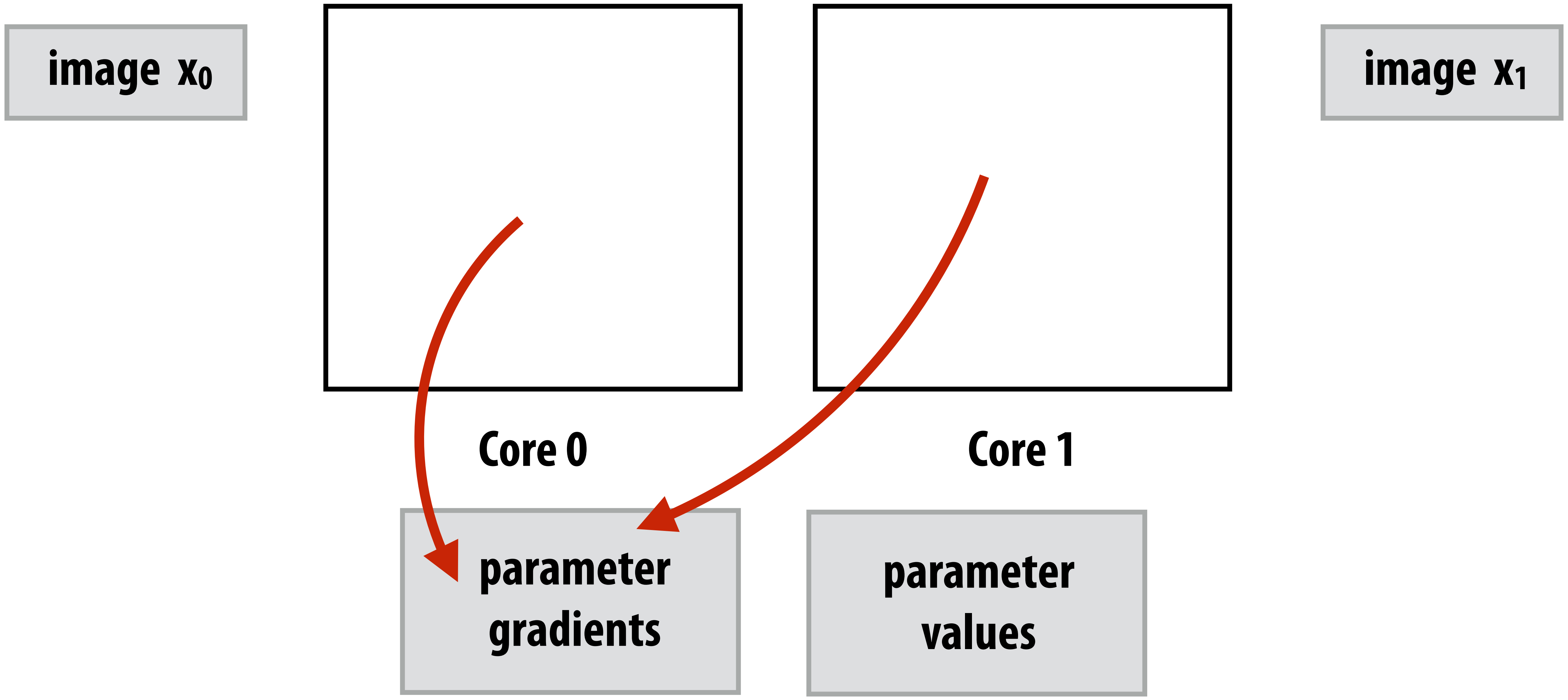
Bad: complete duplication of parameter gradient state (100's MB per core)

Asynchronous update on one node

```
for each item x_i in mini-batch:
   grad += evaluate_loss_gradient(f, loss_func, params, x_i)
params += -grad * step_size;
```

Cores update shared set of gradients.

Skip taking locks / synchronizing across cores: perform “approximate reduction”



Project Adam [Chilimbi OSDI14]

Summary: training large networks in parallel

- Most systems rely on asynchronous update to efficiently used clusters of commodity machines
  - Modification of SGD algorithm to meet constraints of modern parallel systems
  - Open question: effects on convergence are problem dependent and not particularly well understood
  - Tighter integration / faster interconnects may provide alternative to these methods (facilitate tightly orchestrated solutions much like supercomputing applications)
- Open question: how big of networks are needed?
  - >90% of connections could be removed without significant impact on quality of network
  - High-performance training of deep networks is an interesting example of constant iteration of algorithm design and parallelization strategy (a key theme of this course! recall the original grid solver example!)