Algorithms, Spring 1997

Assignement 5, Due in 1 week

1) (The time table problem) : You wish to get from a city s to a city t; there are n cities which may serve as intermediate points on the route. There are buses running between some pairs of cities; you are given a time table of all buses - i.e., you are given the origin and destination of each bus, and the departure and arrival times. You are given the time at which you are ready to leave s. Devise an $O(n^2)$ time algorithm to figure out the earliest time you can arrive at t. [Assume that for any two cities A, B, there is at most one bus that goes from A to B.]

2) A company is planning its production over the next 12 months. You are given for each of the 12 months the maximum amount of the product it can produce and the demand (which it must meet in full); these are nonnegative integers. The company has a warehouse; for each i, j with $1 \leq i < j \leq 12$, you are given the cost c_{ij} of storing one unit of the product from month i to month j. (This cost is linear in the number of units stored.) You wish to answer the question of (i) whether it is possible to meet all demands and (ii) the least cost way of doing it.

Show how to set up first problem as a network flow problem. Show how to set up the second problem as a minimum cost flow problem. [The minimum cost flow problem is : given a network where each edge has a capacity and a unit cost per flow, and a desired flow value from s to t, find the least cost $s - t$ flow of that value.]

3) Let us consider two related problems :

a) Given a CNF Boolean formula, determine whether it is satisfiable.

b) Given a CNF Boolean formula, determine whether it is satisfiable and if so, find a satisfying assignment.

Suppose I give you a subroutine to solve the decision problem a). Show how we may solve the problem b) with n calls to the subroutine, where n is the number of varaibles.