Final Examination, Algorithms, Spring 1996

Question 1 (25 points = 7 + 9 + 9) For each of the following questions, give a brief (at most 10 lines) answer :

a) Show either that the following problem is NP-Hard or that is solvable in polynomial time : Given n integers $a_1, a_2, \ldots a_n$ and a target integer b, is there a subset S of $\{1, 2, \ldots n\}$ **of cardinality at most 8** such that $\sum_{i \in S} a_i = b$?

b) Is the following statement true or false ? Explain : Merging n sorted lists of 10 elements each into one sorted list using only comparisons takes time $\Omega(n \log n)$.

c) A bank wants to maintain a database of its n (n large) customers. For each customer, it will keep the name, social security number and some other information. It wants to be able to **add** new customers, **delete** a customer (when the account is closed). It also wants to be able to **find** information about an account given **either** the name **or** the social security number. It wants to perform each operation quickly. Design suitable data structures for the bank.

Question 2 (20 points) You are given an undirected graph $G(V, E)$ and a specific vertex s of the graph. You are to determine whether there is a cycle in the graph involving the vertex s. Describe an $O(|E| + |V|)$ time algorithm to do this. Argue the correctness of your algorithm.

Question 3 (20 points) You are given a tree on vertices $\{1, 2, \ldots n\}$. The tree is described by an array PARENT[1..n] which gives for each node the parent of the node. (Except, the PARENT of the root is set to 0.) Describe a parallel algorithm that computes for each node its depth (the length of the path from the root to the node) using n processors and time $O(\log n)$.

Question 4 (15 points) You are given n points on the plane. Describe an $O(n \log n)$ time algorithm to find a pair of these points with the minimum Manhattan distance between them. (The Manhattan distance between two points $p = (x_p, y_p)$ and $q = (x_q, y_q)$ is $|x_p - x_q| + |y_p - y_q|$.)

Question 5 (20 points) Show that the following decision problem is NP-Complete :

There are n jobs to be processed on two identical machines. You are given the processing times of the n jobs - $a_1, a_2, \ldots a_n$ and a "desired" completion time t. The problem is to decide whether the jobs can all be completed by time t. (So, each job may be processed either on machine 1 or on machine 2; the only decisions to be made in the problem are on which of the two machines each job is to processed.)