15-462: Computer Graphics

Math for Computer Graphics

Transformations

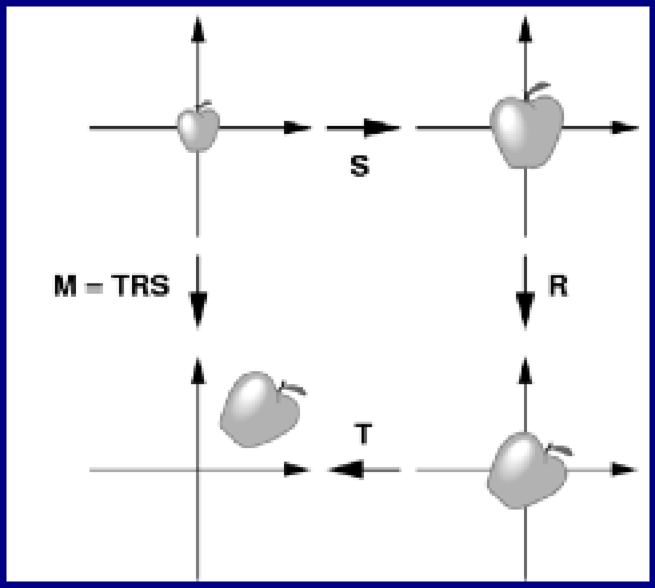
Translation, rotation, scaling
- 2D
- 3D

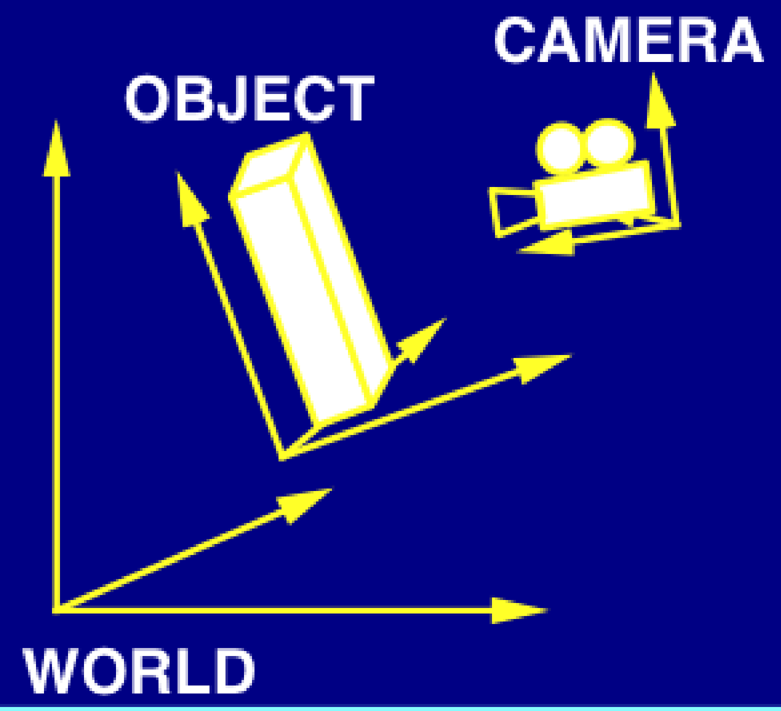
Homogeneous coordinates
Transforming normals
Examples

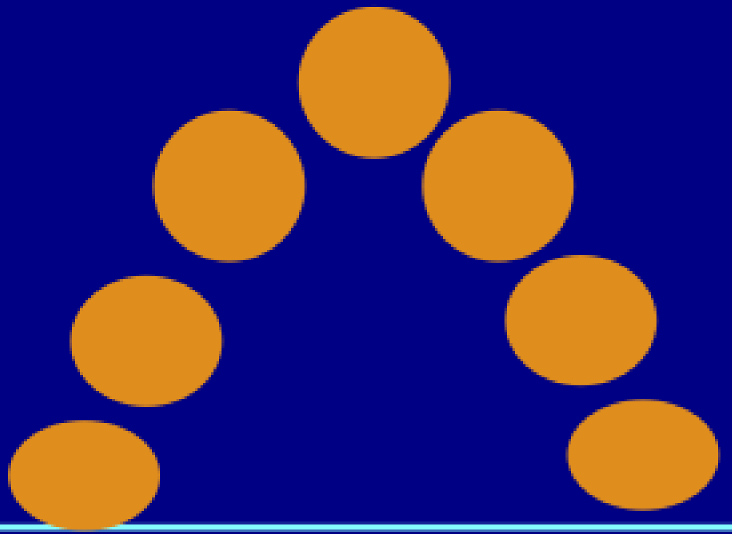
Shirley Chapter 6

Uses of Transformations

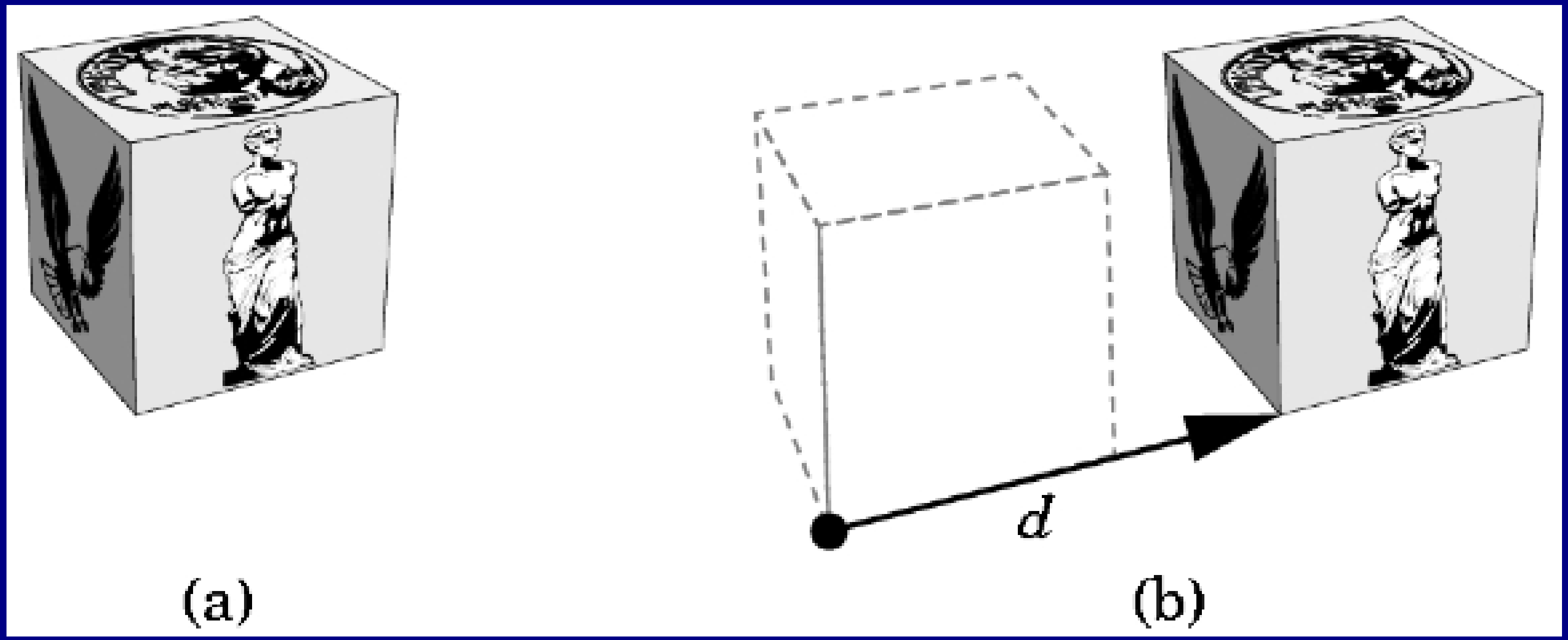
- Modeling
  - build complex models by positioning simple components
  - transform from object coordinates to world coordinates
- Viewing
  - placing the virtual camera in the world
  - specifying transformation from world coordinates to camera coordinates
- Animation
  - vary transformations over time to create motion

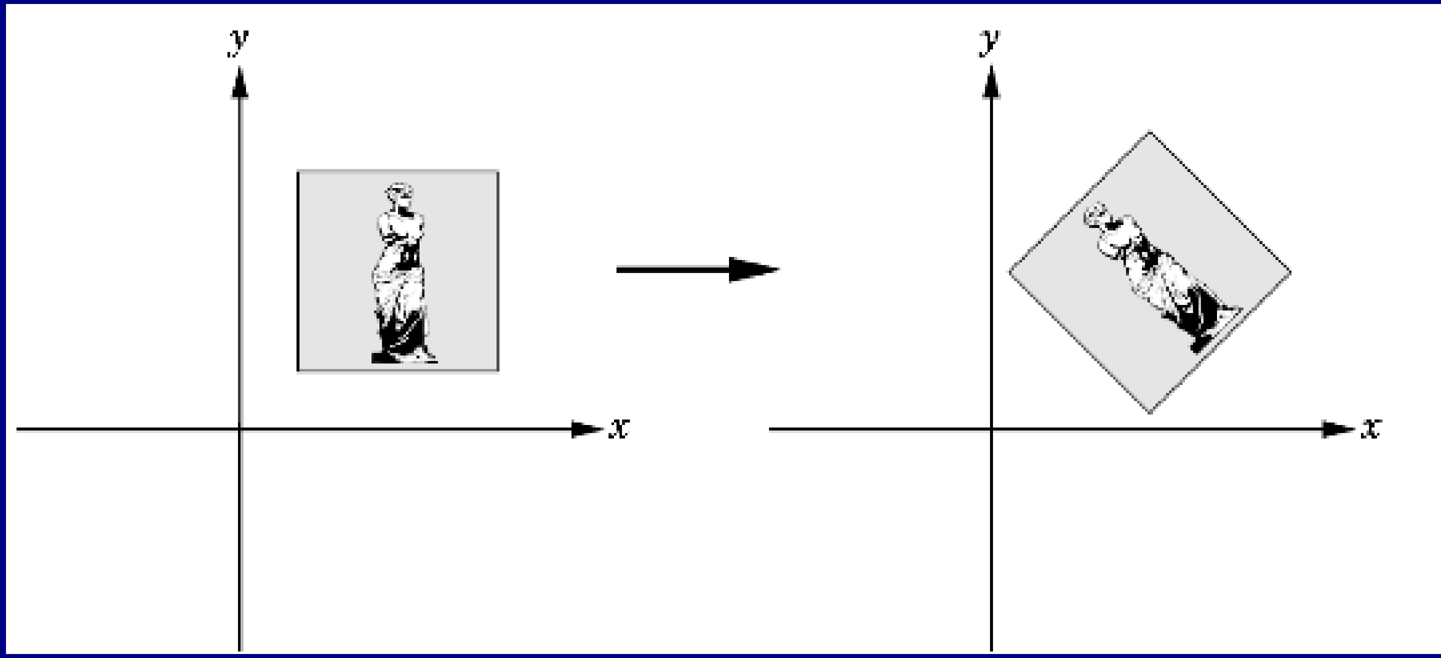


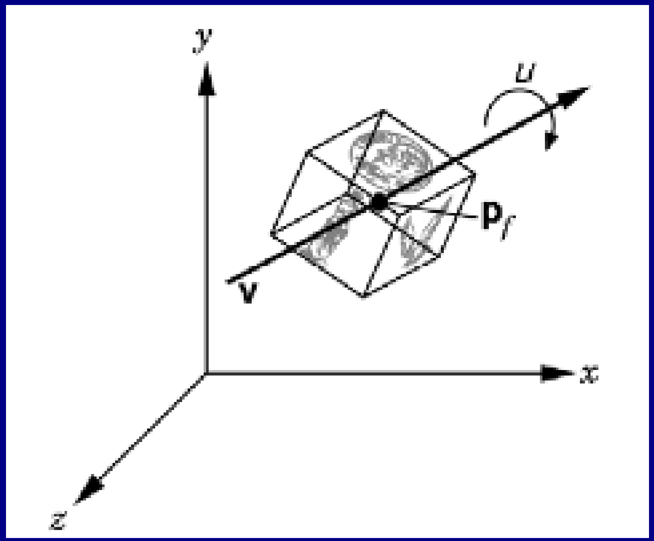




Rigid Body Transformations

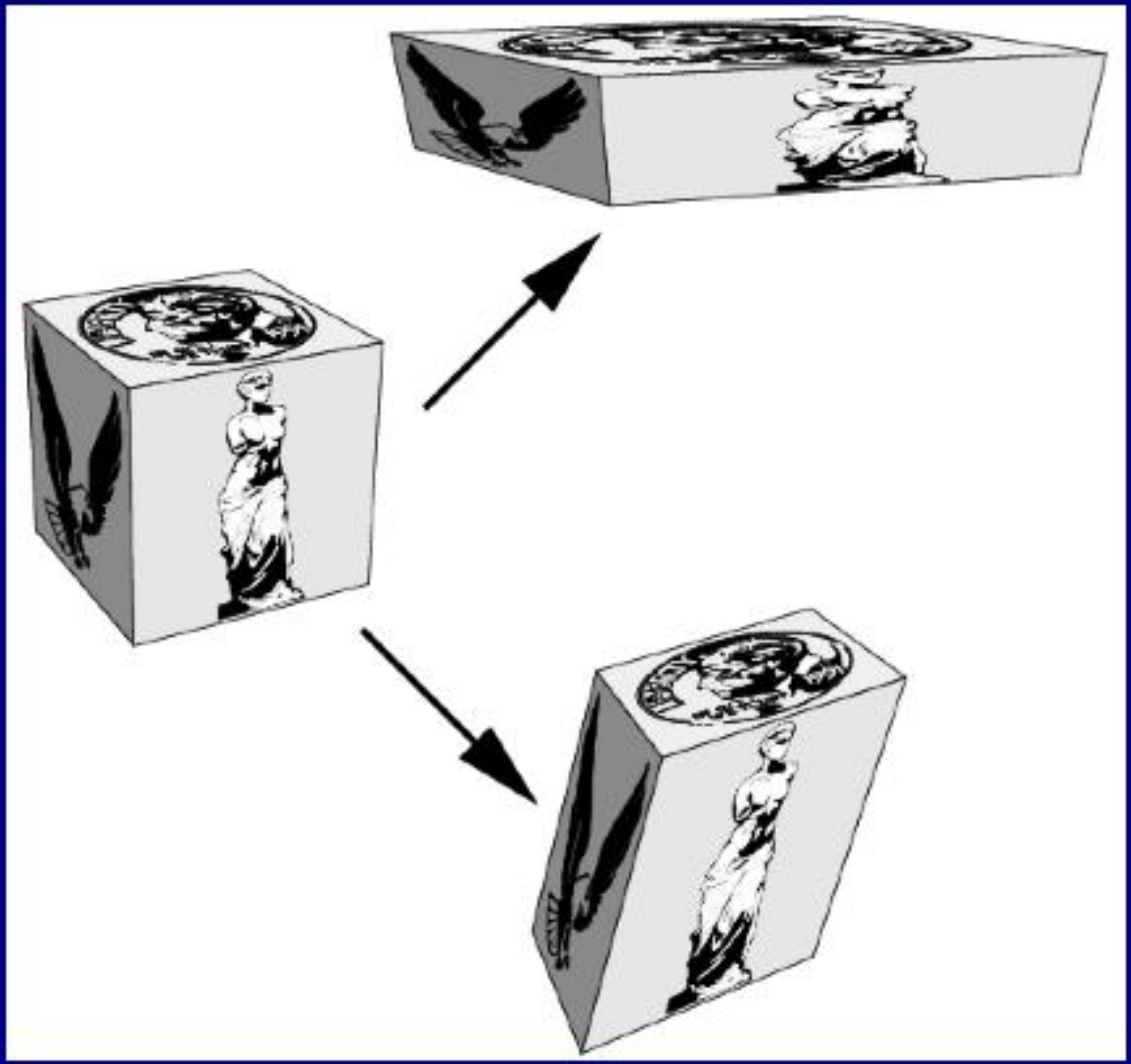


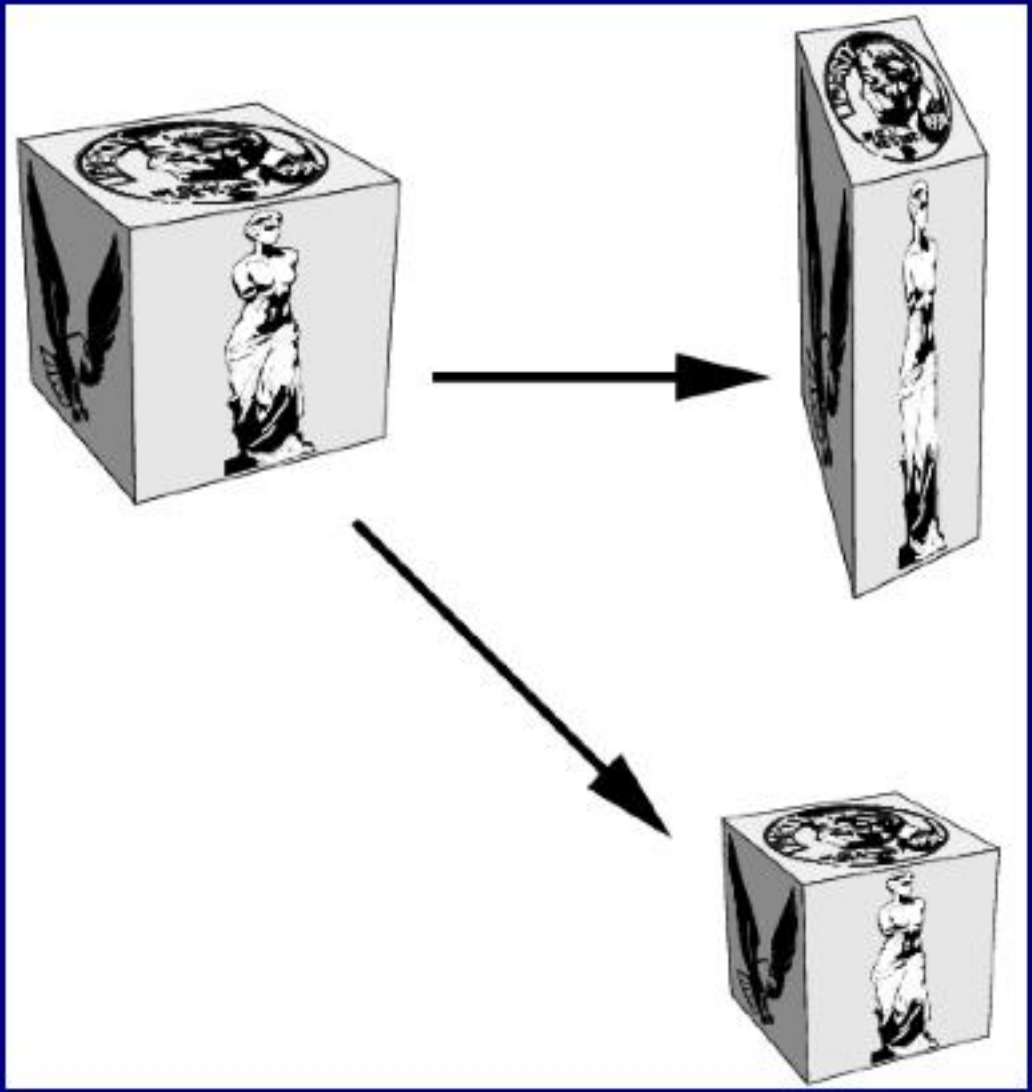




Rotation angle and line about which to rotate

Non-rigid Body Transformations





Distance between points on object do not remain constant

Basic 2D Transformations

Scale

Shear

Rotate

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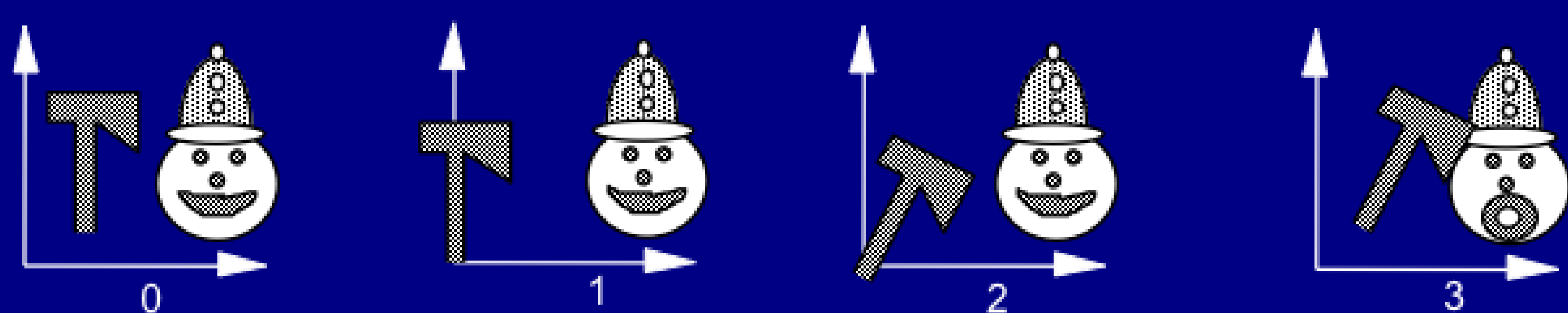
Composition of Transformations

- Created by stringing basic ones together, e.g.
  - *"translate p to the origin, rotate, then translate back"*

  can also be described as a rotation about p
- Any sequence of linear transformations can be collapsed into a single matrix formed by multiplying the individual matrices together
- Order matters!
- Can apply a whole sequence of transformations at once



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Translate to the origin, rotate, then translate back.

3D Transformations

- 3-D transformations are very similar to the 2-D case
- Scale
- Shear
- Rotation is a bit more complicated in 3-D
  - different rotation axes

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But what about translation?

- Translation is not linear--how to represent as a matrix?

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But what about translation?

- Translation is not linear--how to represent as a matrix?

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- Trick: add extra coordinate to each vector
- This extra coordinate is the *homogeneous* coordinate, or *w*
- When extra coordinate is used, vector is said to be represented in *homogeneous coordinates*
- We call these matrices *Homogeneous* Transformations

W? Where did that come from?

- Practical answer:
  - W is a clever algebraic trick.
  - Don't worry about it too much. The w value will be 1.0 for the time being (until we get to perspective viewing transformations)

- More complete answer:
  - (x,y,w) coordinates form a 3D *projective space.*
  - All nonzero scalar multiples of (x,y,1) form an equivalence class of points that project to the same 2D Cartesian point (x,y).
  - For 3-D graphics, the 4D projective space point (x,y,z,w) maps to the 3D point (x,y,z) in the same way.

Homogeneous 2D Transformations

The basic 2D transformations become

Translate:

$$\begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

Scale:

$$\begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

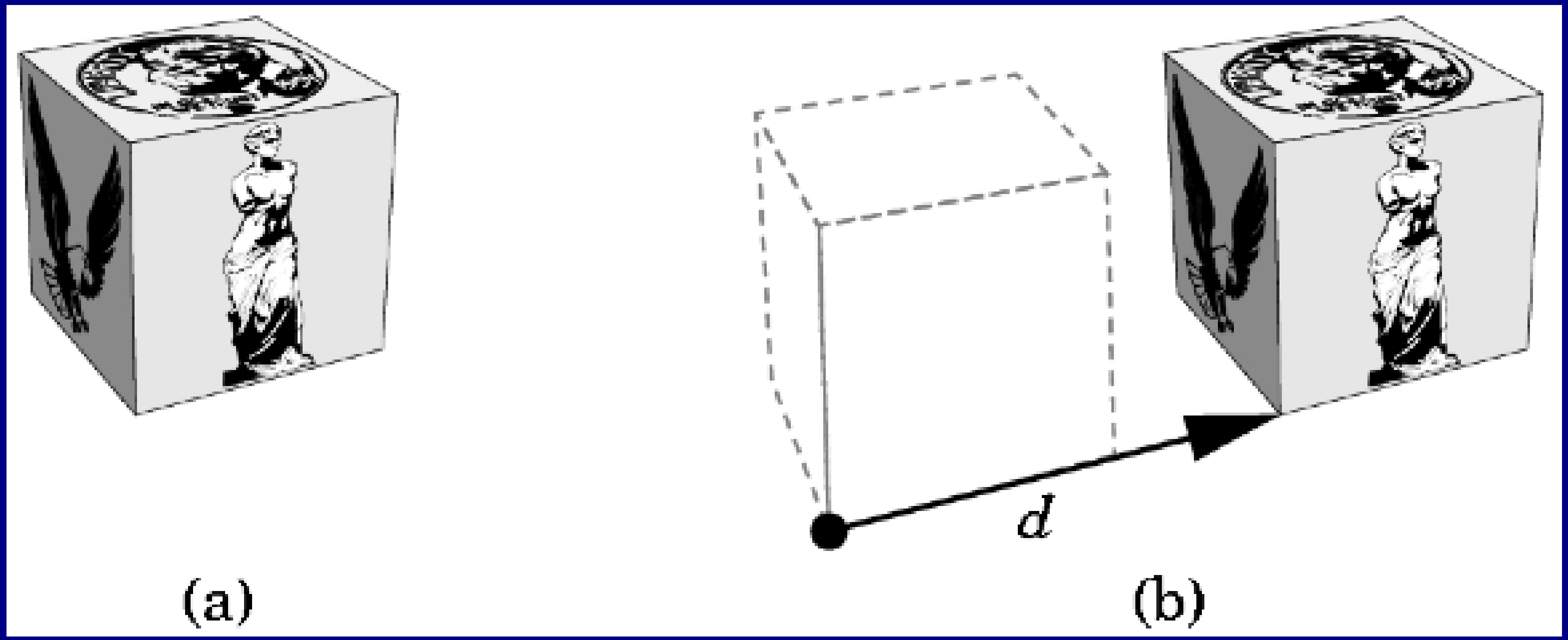
Rotate:

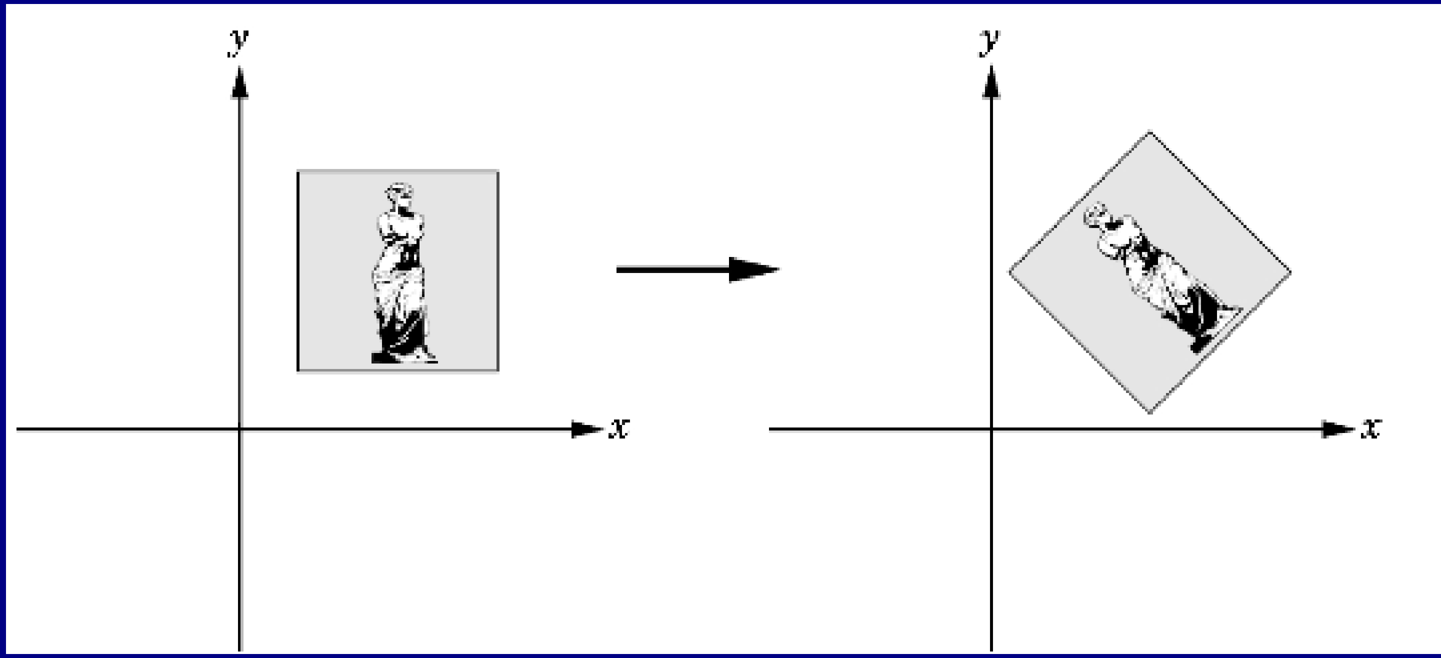
$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

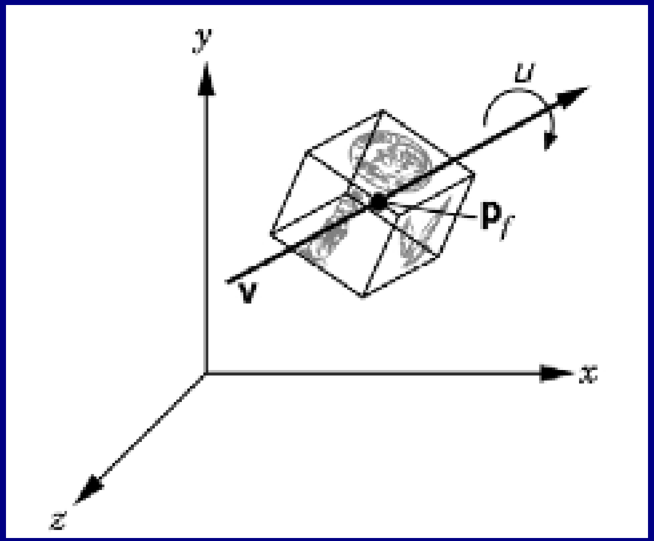
Now *any* sequence of translate/scale/rotate operations can be combined into a single homogeneous matrix by multiplication.

3D transforms are modified similarly

Rigid Body Transformations







Rotation angle and line about which to rotate

Rigid Body Transformations

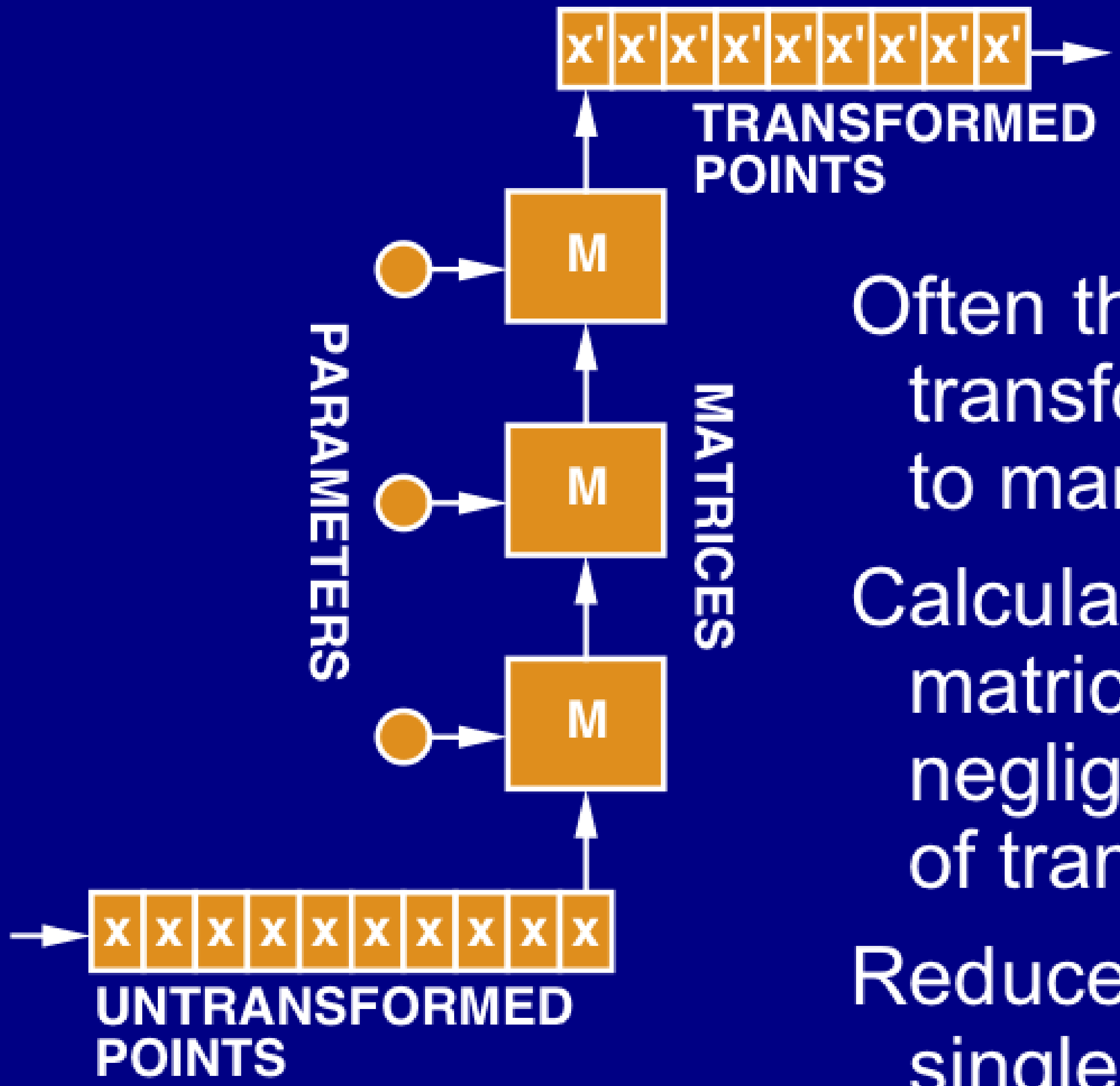
- A transformation matrix of the form

$$\begin{bmatrix} \mathbf{x_x} & \mathbf{x_y} & \mathbf{t_x} \\ \mathbf{y_x} & \mathbf{y_y} & \mathbf{t_y} \\ 0 & 0 & 1 \end{bmatrix}$$

where the upper 2x2 submatrix is a rotation matrix and column 3 is a translation vector, is a *rigid body transformation.*

- Any series of rotations and translations results in a rotation and translation of this form (and no change in the distance between vertices)

Sequences of Transformations



- Often the same transformations are applied to many points
- Calculation time for the matrices and combination is negligible compared to that of transforming the points
- Reduce the sequence to a single matrix, then transform

Collapsing a Chain of Matrices.

- Consider the composite function ABCD,   i.e. p' = ABCDp
- Matrix multiplication isn't commutative - the order is **important**
- But matrix multiplication is associative, so can calculate from right to left or left to right: ABCD = (((AB) C) D) = (A (B (CD))).
- Iteratively replace *either* the leading or the trailing pair by its product

Premultiply		Postmultiply
$M \leftarrow D$		$M \leftarrow A$
$M \leftarrow CM$	**or**	$M \leftarrow MB$
$M \leftarrow BM$		$M \leftarrow MC$
$M \leftarrow AM$		$M \leftarrow MD$

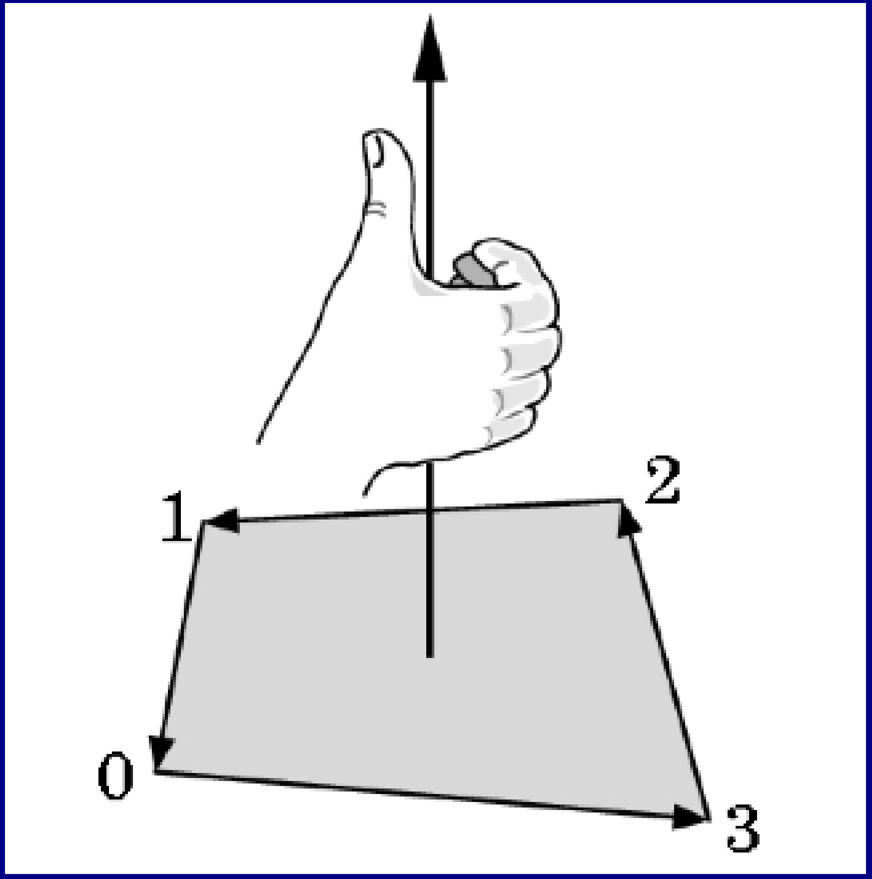
both give the same result.

What is a Normal?– refresher

Indication of outward facing direction for lighting and shading

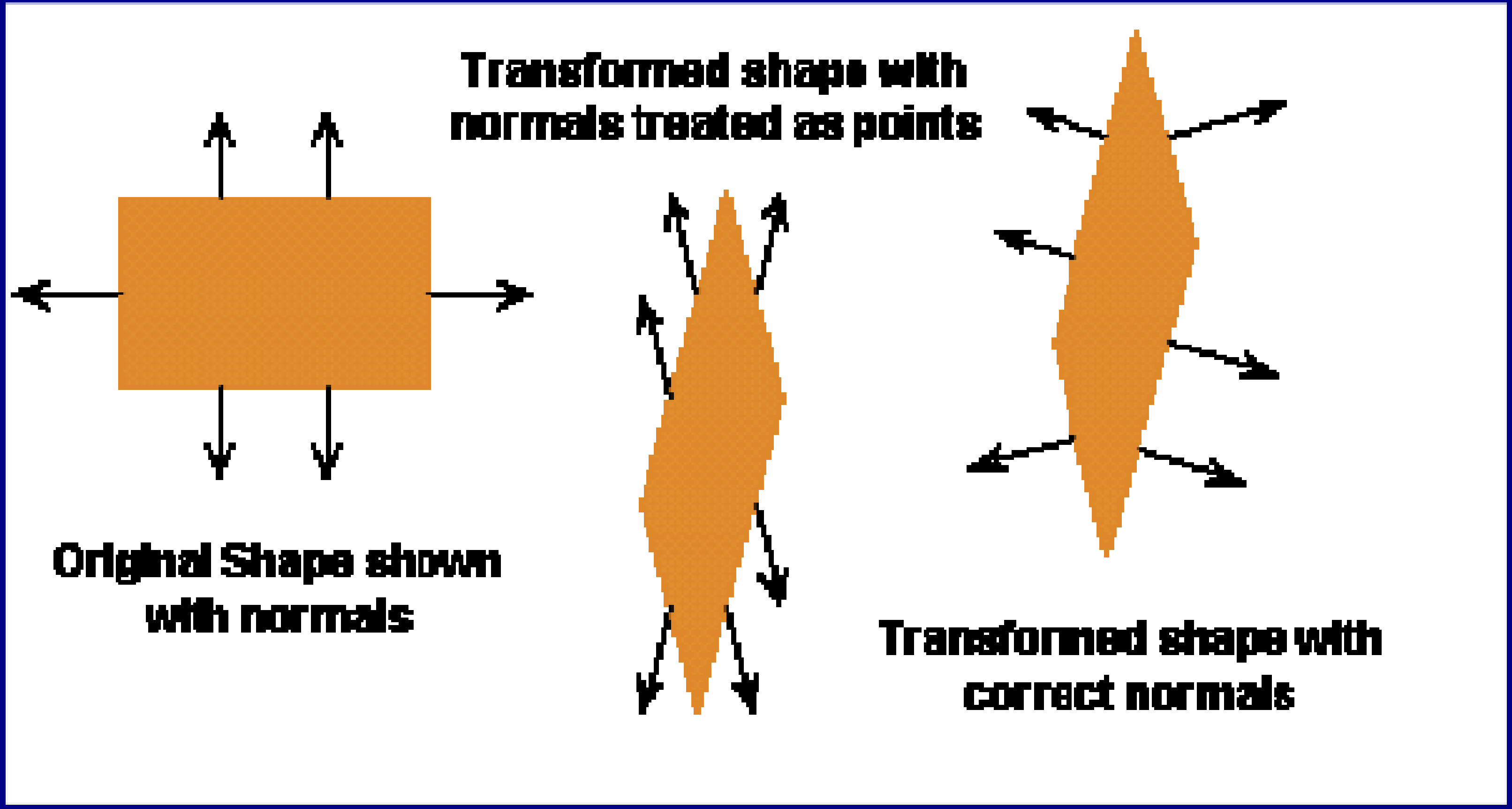
Order of definition of vertices in OpenGL

Right hand rule



Transforming Normals

- It's tempting to think of normal vectors as being like porcupine quills, so they would transform like points
- Alas, it's not so.
- We need a different rule to transform normals.



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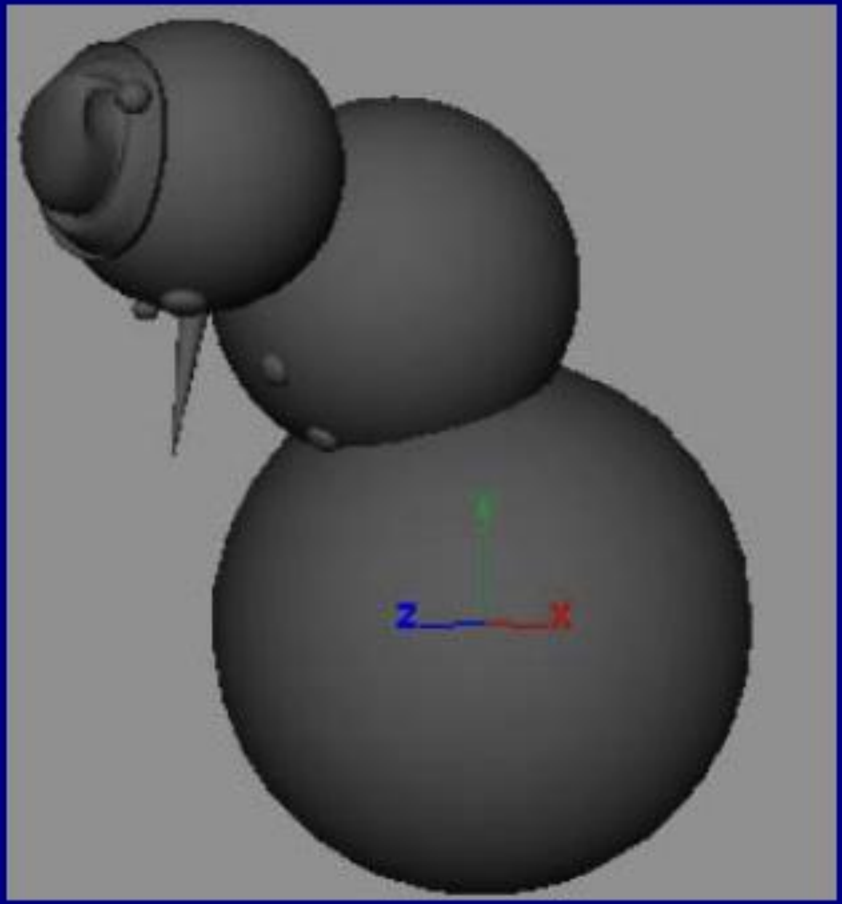
Examples

- Modeling with primitive shapes



Practice Problems

The slide on page 4 of this slide deck / pdf file shows three rigid body transformations. Refer to this slide for the problems below.

Give a transformation matrix for rigid body translation by vector **d** as shown in the first figure on page 4.

Give a transformation matrix (or sequence of matrices) for accomplishing the transformation shown in the second figure on page 4**.** This transformation results in a rotation of 45 degrees about the object center point $\mathbf{p_f}$.

Practice Problems

Give a set of OpenGL calls or pseudocode that may result in the transformation shown in the third figure on page 4. This is a rotation of angle u about the line in direction v passing through point $\mathbf{p}_f$. You may assume you have a function call for rotation of an angle about a given axis. If you do this problem as multiple operations, be clear and careful of the ordering of these operations.

Practice constructing shear matrices to achieve desired effects.