15-745 Lecture 2

Dataflow Analysis
Basic Blocks
Related Optimizations
SSA

Dataflow Analysis

- Last time we looked at code transformations
  - Constant propagation
  - Copy propagation
  - Common sub-expression elimination
  - …
- Today, dataflow analysis:
  - How to determine if it is legal to perform such an optimization
  - (Not doing analysis to determine if it is beneficial)

A sample program

```
int fib10(void) {
   int n = 10;
   int older = 0;
   int old = 1;
   ir
   int i;

   if (n <= 1) return n;
   for (i = 2; i<n; i++) {
      result = old + older;
      older = old;
      old = result;
   }
   return result;
}
```

What are those numbers?

```
1:    n <- 10
2:    older <- 0
3:    old <- 1
4:    result <- 0
5:    if n <= 1 goto 14
6:    i <- 2
7:    if i > n goto 13

8:    result <- old + older
9:    older <- old
10:   old <- result
11:   i <- i + 1
12:   JUMP 7
13:   return result
14:   return n
```

Simple Constant Propagation

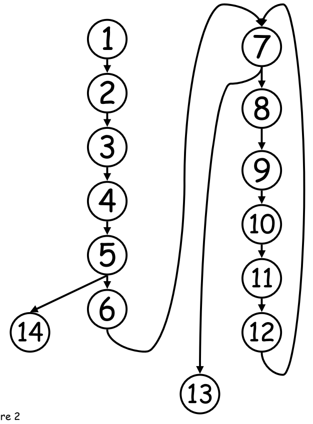
- Can we do SCP?
- How do we recognize it?
- What aren't we doing?
- Metanote:
  - keep opts simple!
  - Use combined power

```
1:    n <- 10
2:    older <- 0
3:    old <- 1
4:    result <- 0
5:    if n <= 1 goto 14
6:    i <- 2
7:    if i > n goto 13

8:    result <- old + older
9:    older <- old
10:   old <- result
11:   i <- i + 1
12:   JUMP 7
13:   return result
14:   return n
```

Reaching Definitions

- A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.



```
1:     n <- 10
2:     older <- 0
3:     old <- 1
4:     result <- 0
5:     if n <= 1 goto 14
6:     i <- 2
7:     if i > n goto 13
8:     result <- old + older
9:     older <- old
10:    old <- result
11:    i <- i + 1
12:    JUMP 7
13:    return result
14:    return n
```

Reaching Definitions (ex)

- 1 reaches 5, 7, and 14
- 2 reaches 8
- Older in 8 is reached by
  - 2
  - 9
- Tells us which definitions reach a particular use (ud-info)

```
1:     n <- 10
2:     older <- 0
3:     old <- 1
4:     result <- 0
5:     if n <= 1 goto 14
6:     i <- 2
7:     if i > n goto 13
8:     result <- old + older
9:     older <- old
10:    old <- result
11:    i <- i + 1
12:    JUMP 7
13:    return result
14:    return n
```

Reaching Definitions (ex)

- 1 reaches 5, 7, and 14

14, Really?

Meta-notes:

- (almost) always conservative
- only know what we know
- Keep it simple:
  - What opt(s), if run before this would help
  - What about:

```
1: x <- 0
2: if (false) x<-1
3: … x …
```

  - Does 1 reach 3?
  - What opt changes this?

```
1:     n <- 10
2:     older <- 0
3:     old <- 1
4:     result <- 0
5:     if n <= 1 goto 14
6:     i <- 2
7:     if i > n goto 13
8:     result <- old + older
9:     older <- old
10:    old <- result
11:    i <- i + 1
12:    JUMP 7
13:    return result
14:    return n
```

Calculating Reaching Definitions

- A definition of variable v at program point d reaches program point u if there exists a path of control flow edges from d to u that does not contain a definition of v.

- Build up RD stmt by stmt
- Stmt s, "d: v <- x op y", generates d
- Stmt s, "d: v <- x op y", kills all other defs(v)

Or,

- Gen[s] = { d }
- Kill[s] = defs(v) – { d }

Gen and kill for each stmt

```
                                 Gen   kill
1: n <- 10                       1
2: older <- 0                    2       9
3: old <- 1                      3       10
4: result <- 0                   4       8
5: if n <= 1 goto 14
6: i <- 2                        6       11
7: if i > n goto 13
8: result <- old + older         8       4
9: older <- old                  9       2
10:old <- result                 10      3
11:i <- i + 1                    11      6
12:JUMP 7
13:return result
14:return n
```

How can we determine the defs that reach a node?

We can use:

- control flow information
- gen and kill info

Computing in[n] and out[n]

- In[n]: the set of defs that reach the beginning of node n
- Out[n]: the set of defs that reach the end of node n

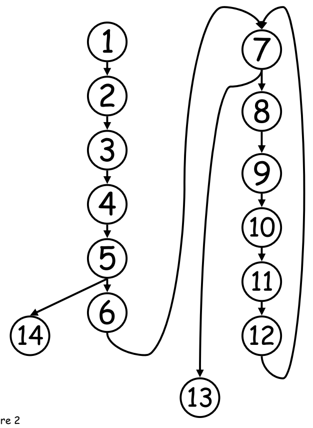
$$in[n] = \bigcup_{p \in pred[n]} out[p]$$

$$out[n] = gen[n] \cup (in[n] - kill[n])$$

- Initialize in[n]=out[n]={} for all n
- Solve iteratively

What is pred[n]?

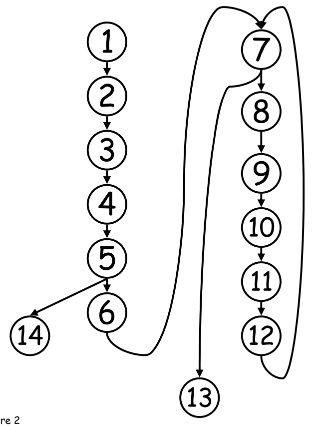
- Pred[n] are all nodes that can reach n in the control flow graph.
- E.g., pred[7] = { 6, 12 }



```
1:     n <- 10
2:     older <- 0
3:     old <- 1
4:     result <- 0
5:     if n <= 1 goto 14
6:     i <- 2
7:     if i > n goto 13
8:     result <- old + older
9:     older <- old
10:    old <- result
11:    i <- i + 1
12:    JUMP 7
13:    return result
14:    return n
```

What order to eval nodes?

- Does it matter?
- Lets do: 1,2,3,4,5,14,6,7,13,8,9,10,11,12



```
1:     n <- 10
2:     older <- 0
3:     old <- 1
4:     result <- 0
5:     if n <= 1 goto 14
6:     i <- 2
7:     if i > n goto 13
8:     result <- old + older
9:     older <- old
10:    old <- result
11:    i <- i + 1
12:    JUMP 7
13:    return result
14:    return n
```

Example:

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1-3
`4: result <- 0`	4	8		
`5: if n <= 1 goto 14`				
`6: i <- 2`	6	11		
`7: if i > n goto 13`				
`8: result <- old + older`	8	4		
`9: older <- old`	9	2		
`10: old <- result`	10	3		
`11: i <- i + 1`	11	6		
`12: JUMP 7`				
`13: return result`				
`14: return n`				

Example (pass 1)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1,2,3
`4: result <- 0`	4	8	1-3	1-4
`5: if n <= 1 goto 14`			1-4	1-4
`6: i <- 2`	6	11	1-4	1-4,6
`7: if i > n goto 13`			1-4,6	1-4,6
`8: result <- old + older`	8	4	1-4,6	1-3,6,8
`9: older <- old`	9	2	1-3,6,8	1,3,6,8,9
`10: old <- result`	10	3	1,3,6,8,9	1,6,8-10
`11: i <- i + 1`	11	6	1,6,8-10	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
`13: return result`			1-4,6	1-4,6
`14: return n`			1-4	1-4

Example (pass 2)

- Order: 1,2,3,4,5,14,6,7,13,8,9,10,11,12

$$in[n] = \bigcup_{p \in pred[n]} out[p] \qquad out[n] = gen[n] \cup (in[n] - kill[n])$$

	Gen	kill	in	out
`1: n <- 10`	1			1
`2: older <- 0`	2	9	1	1,2
`3: old <- 1`	3	10	1,2	1,2,3
`4: result <- 0`	4	8	1-3	1-4
`5: if n <= 1 goto 14`			1-4	1-4
`6: i <- 2`	6	11	1-4	1-4,6
`7: if i > n goto 13`			1-4,6,8-11	1-4,6,8-11
`8: result <- old + older`	8	4	1-4,6,8-11	1-3,6,8-11
`9: older <- old`	9	2	1-3,6,8-11	1,3,6,8-11
`10: old <- result`	10	3	1,3,6,8-11	1,6,8-11
`11: i <- i + 1`	11	6	1,6,8-11	1,8-11
`12: JUMP 7`			1,8-11	1,8-11
`13: return result`			1-4,6	1-4,6
`14: return n`			1-4	1-4

An Improvement: Basic Blocks

- No need to compute this one stmt at a time
- For straight line code:
  - In[s1; s2] = in[s1]
  - Out[s1; s2] = out[s2]
- Can we combine the gen and kill sets into one set per BB?

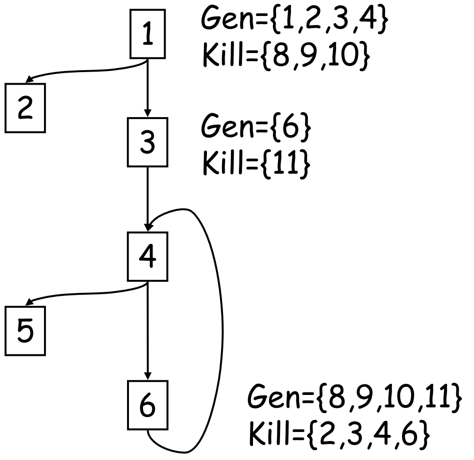
	Gen	kill
`1: i <- 1`	1	8,4
`2: j <- 2`	2	
`3: k <- 3 + i`	3	11
`4: i <- j`	4	1,8
`5: m <- i + k`	5	

- Gen[BB]={2,3,4,5}
- Kill[BB]={1,8,11}
- Gen[s1;s2]=
- Kill[s1;s2]=

BB sets

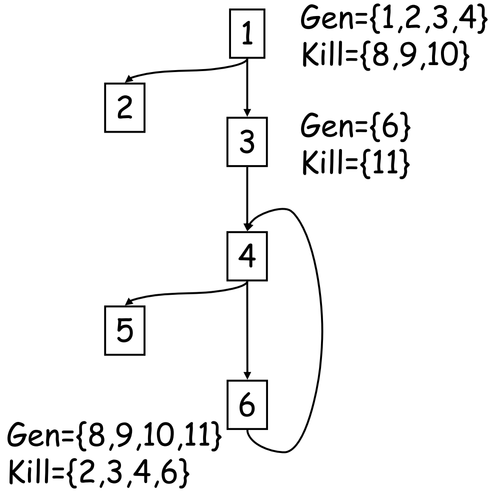
```
                                      Gen  kill
1    1:  n <- 10                      1
     2:  older <- 0                   2    9
     3:  old <- 1                     3    10
     4:  result <- 0                  4    8
     5:  if n <= 1 goto 14                       1,2,3,4   8,9,10
3    6:  i <- 2                       6    11    6         11
 4   7:  if i > n goto 13
     8:  result <- old + older        8    4
     9:  older <- old                 9    2
6    10: old <- result                10   3
     11: i <- i + 1                   11   6
     12: JUMP 7                                  8-11      2-4,6
 5   13: return result
2    14: return n
```

BB sets



In	out
	1,2,3,4

BB sets



In	out
	1,2,3,4
1,2,3,4	1,2,3,4,6
1-4,6,8-11	1-4,6,8-11
1-4,6,8-11	1,8-11

Forward Dataflow

- Reaching definitions is a forward dataflow problem:
  It propgates information from preds of a node to the node
- Defined by:
  - Basic attributes: (gen and kill)
  - Transfer function: $out[b]=F_{bb}(in[b])$
  - Meet operator: $in[b]=M(out[p])$ for all $p \in pred(b)$
  - Set of values (a lattice, in this case powerset of program points)
  - Initial values for each node b
- Solve for fixed point solution

How to implement?

- Values?
- Gen?
- Kill?
- F_{bb}?
- Order to visit nodes?
- When are we done?
  - In fact, do we know we terminate?

Implementing RD

- Values: bits in a bit vector
- Gen: 1 in each position generated, otherwise 0
- Kill: 0 in each position killed, otherwise 1
- F_{bb}: out[b] = (in[b] | gen[b]) & kill[b]
- Init in[b]=out[b]=0

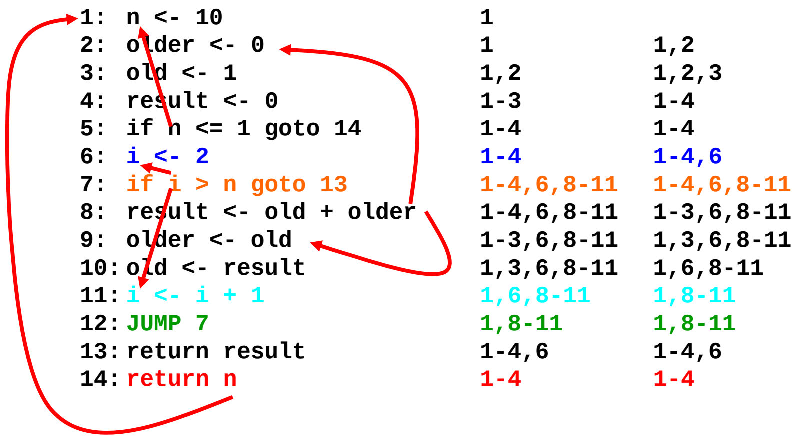
- When are we done?
- What order to visit nodes? Does it matter?

RD Worklist algorithm

```
Initialize: in[B] = out[b] = ∅
Initialize: in[entry] = ∅
Work queue, W = all Blocks in topological order
while (|W| != 0) {
    remove b from W
    old = out[b]
    in[b] = {over all pred(p) ∈ b} ∪ out[p]
    out[b] = gen[b] ∪ (in[b] - kill[b])
    if (old != out[b]) W = W ∪ succ(b)
}
```

Storing Rd information

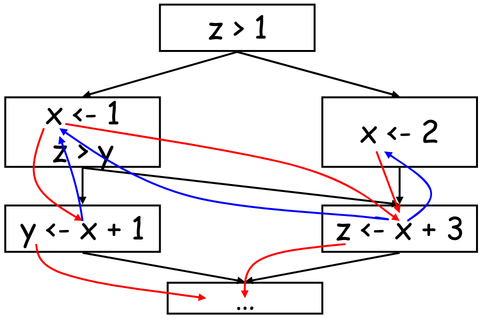
- Use-def chains: for each use of var x in s, a list of definitions of x that reach s



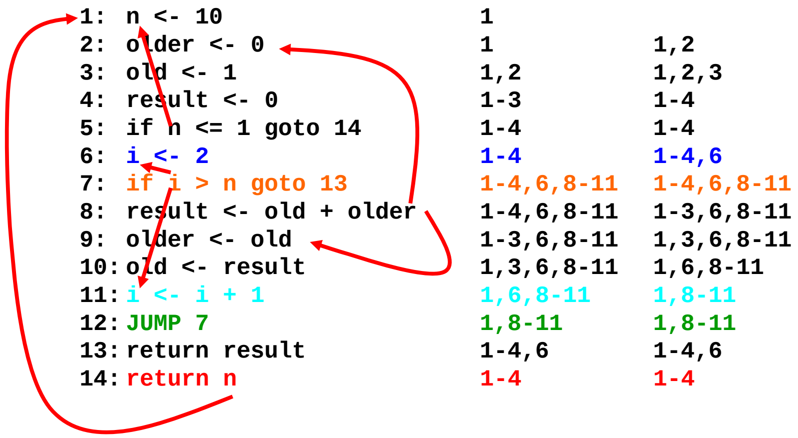
```
1:  n <- 10                 1
2:  older <- 0              1             1,2
3:  old <- 1                1,2           1,2,3
4:  result <- 0             1-3           1-4
5:  if n <= 1 goto 14       1-4           1-4
6:  i <- 2                  1-4           1-4,6
7:  if i > n goto 13        1-4,6,8-11    1-4,6,8-11
8:  result <- old + older   1-4,6,8-11    1-3,6,8-11
9:  older <- old            1-3,6,8-11    1,3,6,8-11
10: old <- result           1,3,6,8-11    1,6,8-11
11: i <- i + 1              1,6,8-11      1,8-11
12: JUMP 7                  1,8-11        1,8-11
13: return result           1-4,6         1-4,6
14: return n                1-4           1-4
```

Def-use chains are valuable too

- Def-use chain: for each definition of var x, a list of all uses of that definition
- Computed from liveness analysis, a backward dataflow problem
- Def-use and use-def are different



Using RD for Simple Const. Prop.



```
1:  n <- 10                     1
2:  older <- 0                  1             1,2
3:  old <- 1                    1,2           1,2,3
4:  result <- 0                 1-3           1-4
5:  if n <= 1 goto 14           1-4           1-4
6:  i <- 2                      1-4           1-4,6
7:  if i > n goto 13            1-4,6,8-11    1-4,6,8-11
8:  result <- old + older       1-4,6,8-11    1-3,6,8-11
9:  older <- old                1-3,6,8-11    1,3,6,8-11
10: old <- result               1,3,6,8-11    1,6,8-11
11: i <- i + 1                  1,6,8-11      1,8-11
12: JUMP 7                      1,8-11        1,8-11
13: return result               1-4,6         1-4,6
14: return n                    1-4           1-4
```

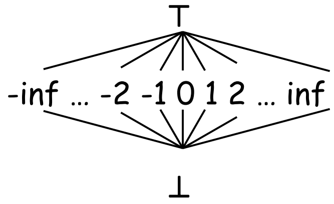
Better Constant Propagation

- What about:

x <- 1
if (y > z)
    x <- 1
a <- x

Better Constant Propagation

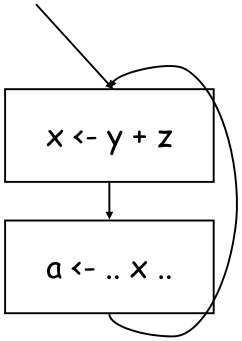
- What about: x <- 1
  if (y > z)
      x <- 1
  a <- x
- Lattice



- Meet:
  a <- a ∧ T
  ⊥ <- a ∧ ⊥
  c <- c ∧ c
  ⊥ <- c ∧ d (if c ≠ d)
- Init all vars to: bot or top?

Loop Invariant Code Motion

- When can expression be moved out of a loop?

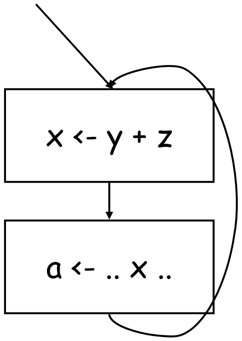


x <- y + z

a <- .. x ..

Loop Invariant Code Motion

- When can expression be moved out of a loop?
- When all reaching definitions of operands are outside of loop, expression is loop invariant
- Use ud-chains to detect
- Can du-chains be helpful?



x <- y + z

a <- .. x ..

Liveness (def-use chains)

- A variable x is live-out of a stmt s if x can be used along some path starting a s, otherwise x is dead.
- Why is this important?

- How can we frame this as a dataflow problem?

Liveness as a dataflow problem

- This is a backwards analysis
  - A variable is live out if used by a successor
  - Gen: For a use: indicate it is live coming into s
  - Kill: Defining a variable v in s makes it dead before s (unless s uses v to define v)
  - Lattice is just live (top) and dead (bottom)
- Values are variables
- In[n] = variables live before n
  = $out[n] - kill[n] \cup gen[n]$
- Out[n] = variables live after n
  = $\bigcup_{s \in succ(n)} In[s]$

Dead Code Elimination

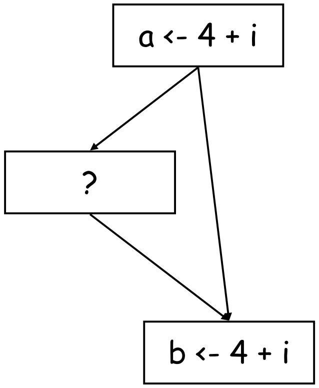
- Code is dead if it has no effect on the outcome of the program.
- When is code dead?

Dead Code Elimination

- Code is dead if it has no effect on the outcome of the program.
- When is code dead?
  - When the definition is dead, and
  - When the instruction has no side effects
- So:
  - run liveness
  - Construct def-use chains
  - Any instruction which has no users and has no side effects can be eliminated

---

When can we do CSE?



Available Expressions

- X+Y is "available" at statement S if
  - x+y is computed along every path from the start to S AND
  - neither x nor y is modified after the last evaluation of x+y

```
a <- b+c
b <- a-d
c <- b+c
d <- a-d
```

Computing Available Expressions

- Forward or backward?
- Values?
- Lattice?
- gen[b] =
- kill[b] =
- in[b] =
- out[b] =
- initialization?

Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- gen[b] = if b evals expr e and doesn't
  define variables used in e
- kill[b] = if b assigns to x,
  then all exprs using x are killed.
- out[b] = in[b] – kill[b] $\cup$ gen[b]
- in[b] = what to do at a join point?
- initialization?

Computing Available Expressions

- Forward
- Values: all expressions
- Lattice: available, not-avail
- gen[b] = if b evals expr e and doesn't
  define variables used in e
- kill[b] = if b assigns to x, exprs(x) are killed
  out[b] = in[b] – kill[b] $\cup$ gen[b]
- in[b] = An expr is avail only if avail on ALL edges, so: in[b] = $\cap$ over all p$\in$ pred(b), out[p]
- Initialization
  - All nodes, but entry are set to ALL avail
  - Entry is set to NONE avail

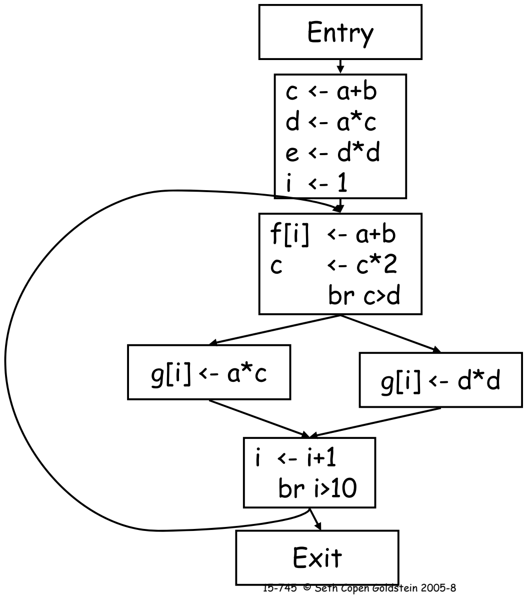
Constructing Gen & Kill

Stmt	Gen	Kill
t <- x op y	{x op y}-kill[s]	{exprs containing t}
t <- M[a]	{M[a]}-kill[s]	
M[a] <- b		
f(a, …)		{M[x] for all x}
t <- f(a,…)		

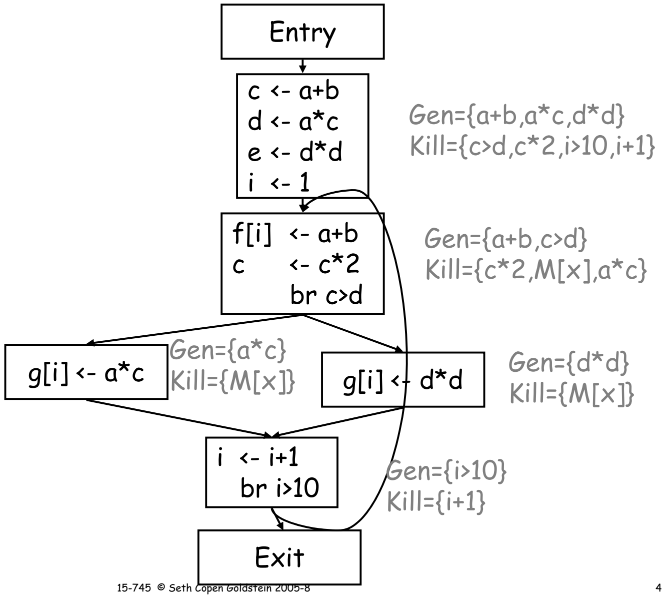
Constructing Gen & Kill

Stmt	Gen	Kill
t <- x op y	{x op y}-kill[s]	{exprs containing t}
t <- M[a]	{M[a]}-kill[s]	{exprs containing t}
M[a] <- b	{}	{for all x, M[x]}
f(a, …)	{}	{for all x, M[x]}
t <- f(a,…)	{}	{exprs containing t for all x, M[x]}

Example

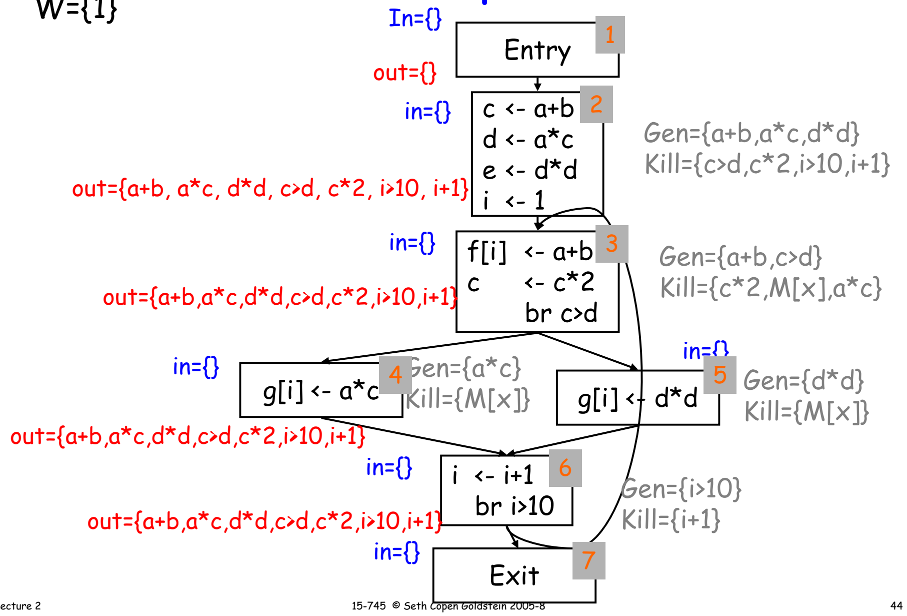


Example



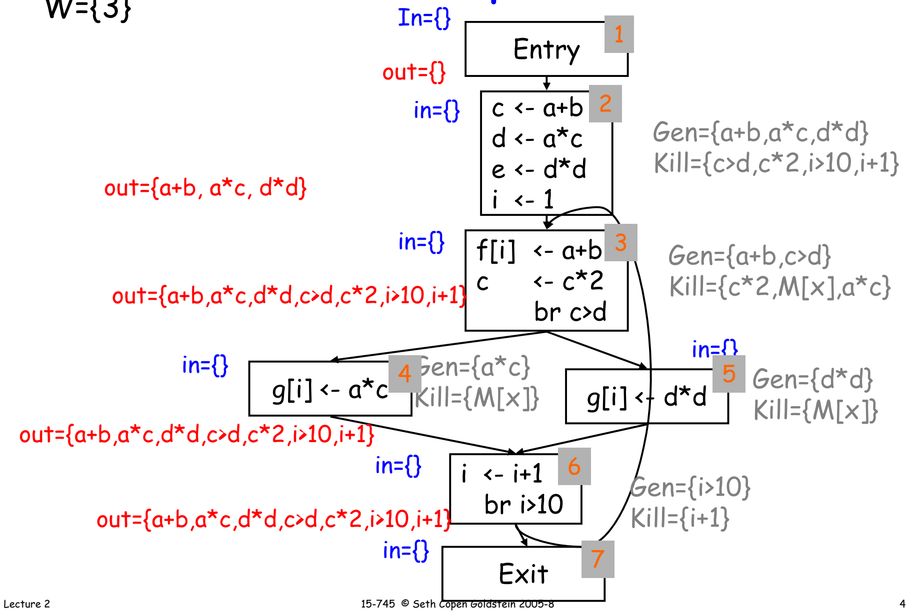
W={1}

Example



Example

W={3}



In={}
Entry 1
out={}

in={}
c <- a+b 2
d <- a*c
e <- d*d
i <- 1
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}

in={}
f[i] <- a+b 3
c <- c*2
br c>d
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}

in={}
g[i] <- a*c 4
Gen={a*c}
Kill={M[x]}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}

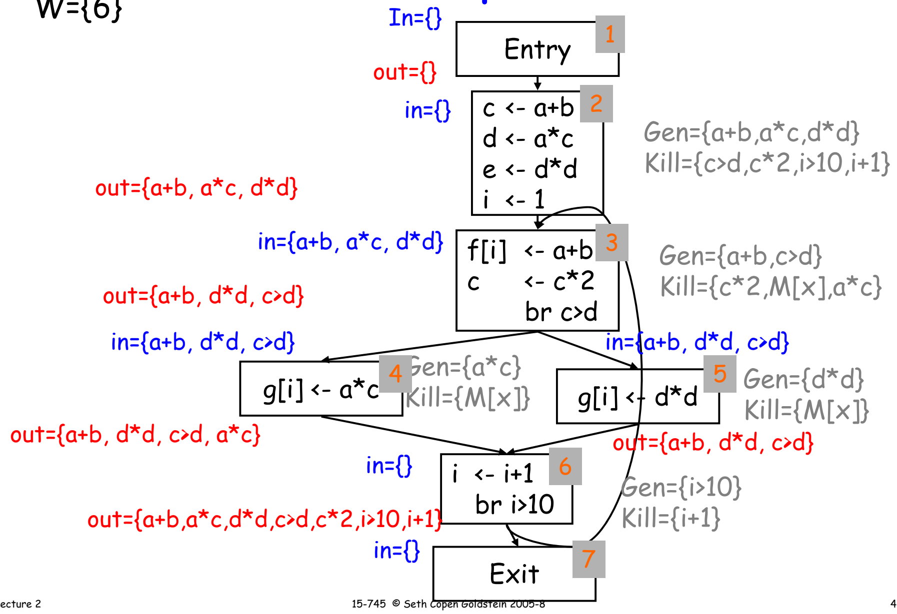
in={}
g[i] <- d*d 5
Gen={d*d}
Kill={M[x]}

in={}
i <- i+1 6
br i>10
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}

in={}
Exit 7

Example

W={6}



In={}
Entry 1
out={}

in={}
c <- a+b 2
d <- a*c
e <- d*d
i <- 1
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}

in={a+b, a*c, d*d}
f[i] <- a+b 3
c <- c*2
br c>d
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}

in={a+b, d*d, c>d}
g[i] <- a*c 4
Gen={a*c}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}

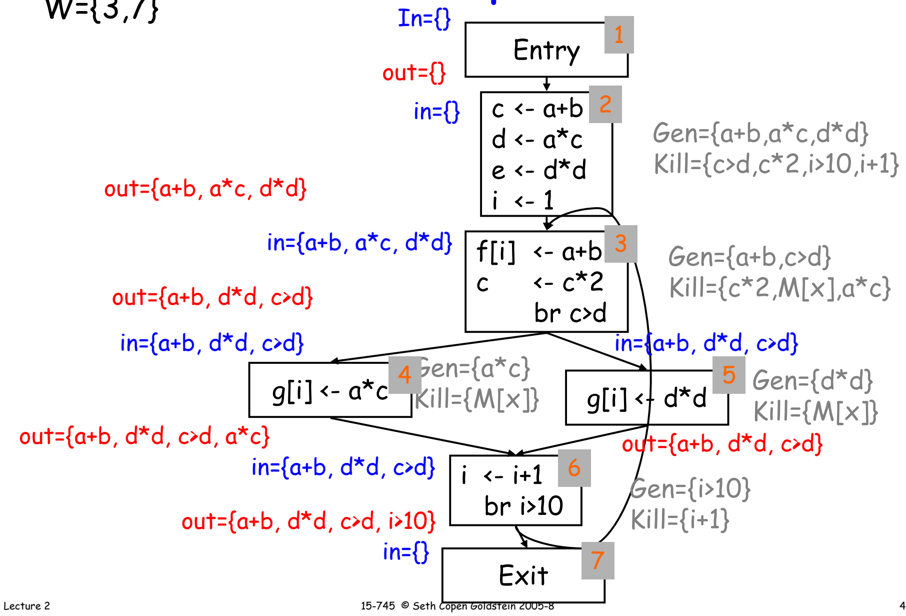
in={a+b, d*d, c>d}
g[i] <- d*d 5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d}

in={}
i <- i+1 6
br i>10
Gen={i>10}
Kill={i+1}
out={a+b,a*c,d*d,c>d,c*2,i>10,i+1}

in={}
Exit 7

Example

W={3,7}



In={}
Entry 1
out={}

in={}
c <- a+b 2
d <- a*c
e <- d*d
i <- 1
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}

in={a+b, a*c, d*d}
f[i] <- a+b 3
c <- c*2
br c>d
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}

in={a+b, d*d, c>d}
g[i] <- a*c 4
Gen={a*c}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}

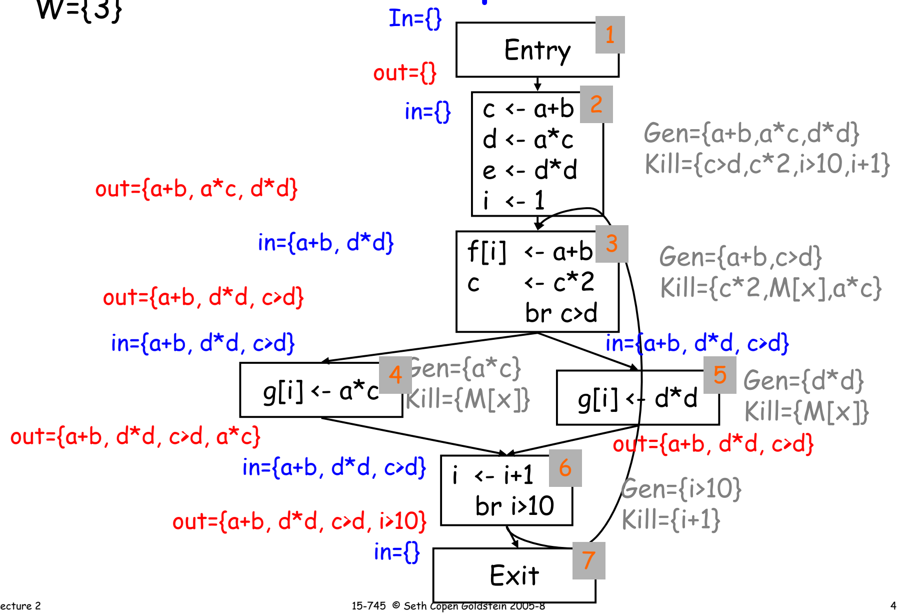
in={a+b, d*d, c>d}
g[i] <- d*d 5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d}

in={a+b, d*d, c>d}
i <- i+1 6
br i>10
Gen={i>10}
Kill={i+1}
out={a+b, d*d, c>d, i>10}

in={}
Exit 7

Example

W={3}



In={}
Entry 1
out={}

in={}
c <- a+b 2
d <- a*c
e <- d*d
i <- 1
Gen={a+b,a*c,d*d}
Kill={c>d,c*2,i>10,i+1}
out={a+b, a*c, d*d}

in={a+b, d*d}
f[i] <- a+b 3
c <- c*2
br c>d
Gen={a+b,c>d}
Kill={c*2,M[x],a*c}
out={a+b, d*d, c>d}

in={a+b, d*d, c>d}
g[i] <- a*c 4
Gen={a*c}
Kill={M[x]}
out={a+b, d*d, c>d, a*c}

in={a+b, d*d, c>d}
g[i] <- d*d 5
Gen={d*d}
Kill={M[x]}
out={a+b, d*d, c>d}

in={a+b, d*d, c>d}
i <- i+1 6
br i>10
Gen={i>10}
Kill={i+1}
out={a+b, d*d, c>d, i>10}

in={}
Exit 7

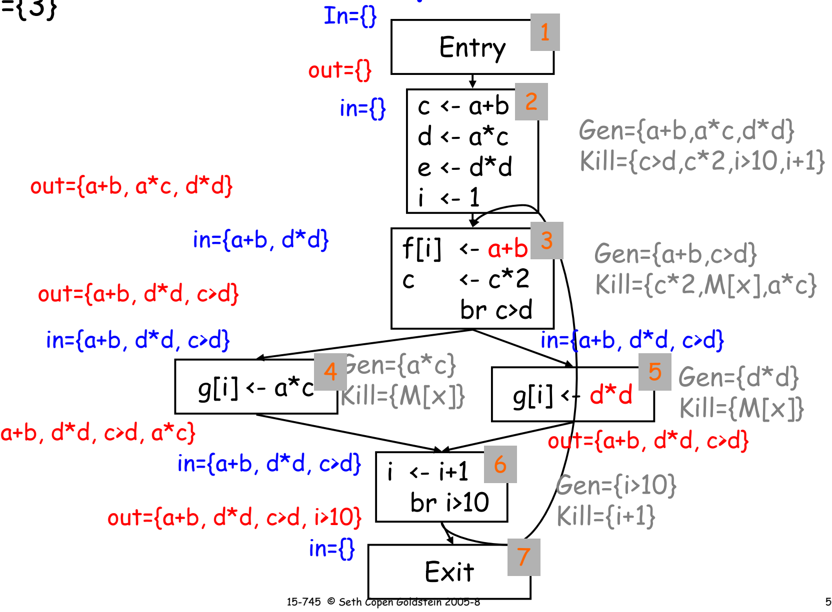
CSE

- Calculate Available expressions
- For every stmt in program

```
If expression, x op y, is available {
    Compute reaching expressions for x op y
        at this stmt
    foreach stmt in RE of the form t <- x op y
            rewrite at: t' <- x op y
                        t <- t'
    }
    replace x op y in stmt with t'
}
```

Example

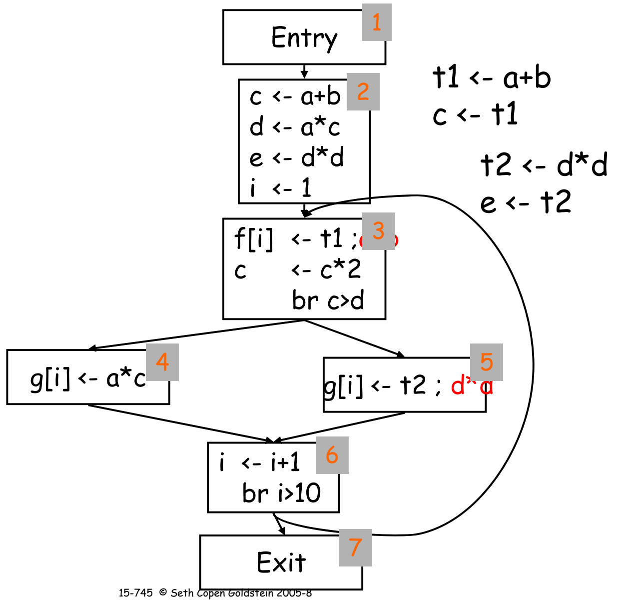
W={3}



Calculating RE

- Could be dataflow problem, but not needed enough, so …
- To find RE for x op y at stmt S
  - traverse cfg backward from S until
    - reach t <- x + y (& put into RE)
    - reach definition of x or y

Example



Dataflow Summary

	Union	intersection
Forward	Reading defs	Available exprs
Backward	Live variables	

Later in course we look at bidirectional dataflow

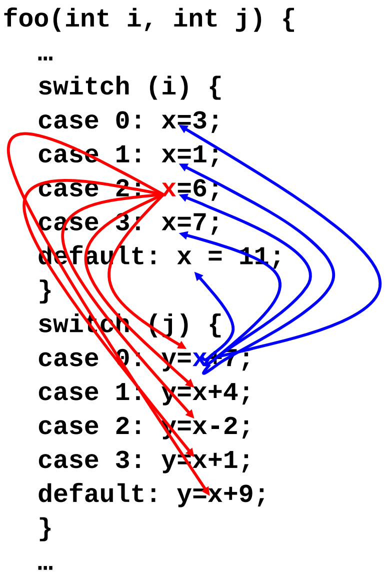
Dataflow Framework

- Lattice
- Universe of values
- Meet operator
- Basic attributes (e.g., gen, kill)
- Traversal order
- Transfer function

Def-Use chains are expensive

```
foo(int i, int j) {
  …
  switch (i) {
  case 0: x=3;break;
  case 1: x=1; break;
  case 2: x=6; break;
  case 3: x=7; break;
  default: x = 11;
  }
  switch (j) {
  case 0: y=x+7; break;
  case 1: y=x+4; break;
  case 2: y=x-2; break;
  case 3: y=x+1; break;
  default: y=x+9;
  }
  …
```

Def-Use chains are expensive



```
foo(int i, int j) {
  …
  switch (i) {
  case 0: x=3;
  case 1: x=1;
  case 2: x=6;
  case 3: x=7;
  default: x = 11;
  }
  switch (j) {
  case 0: y=x+7;
  case 1: y=x+4;
  case 2: y=x-2;
  case 3: y=x+1;
  default: y=x+9;
  }
  …
```

In general,
N defs
M uses
⇒ O(NM) space and time

A solution is to limit each var to ONE def site

Def-Use chains are expensive

```
foo(int i, int j) {
  …
  switch (i) {
  case 0: x=3; break;
  case 1: x=1; break;
  case 2: x=6;
  case 3: x=7;
  default: x = 11;
  }
  x1 is one of the above x's
  switch (j) {
  case 0: y=x1+7;
  case 1: y=x1+4;
  case 2: y=x1-2;
  case 3: y=x1+1;
  default: y=x1+9;
  }
  …
```

A solution is to limit each var to ONE def site

Advantages of SSA

- Makes du-chains explicit
- Makes dataflow analysis easier
- Improves register allocation
  - Automatically builds Webs
  - Makes building interference graphs easier
- For most programs reduces space/time requirements

SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
  - assign to a fresh variable at each stmt.
  - Each use uses the most recently defined var.
  - (Similar to Value Numbering)

Straight-line SSA

```
a ← x + y
b ← a + x
a ← b + 2
c ← y + 1
a ← c + a
```

Straight-line SSA

$a \leftarrow x + y$
$b \leftarrow a + x$
$a \leftarrow b + 2$
$c \leftarrow y + 1$
$a \leftarrow c + a$

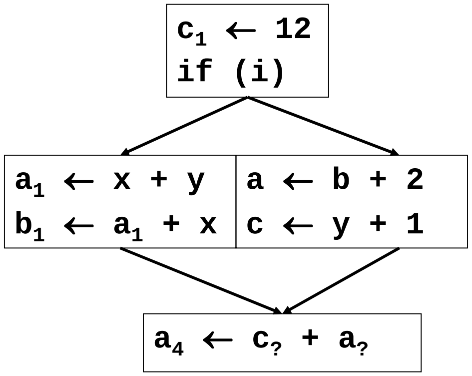
$a_1 \leftarrow x + y$
$b_1 \leftarrow a_1 + x$
$a_2 \leftarrow b_1 + 2$
$c_1 \leftarrow y + 1$
$a_3 \leftarrow c_1 + a_2$

SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
  - assign to a fresh variable at each stmt.
  - Each use uses the most recently defined var.
  - (Similar to Value Numbering)
- What about at joins in the CFG?

Merging at Joins

$c \leftarrow 12$
if (i) {
  $a \leftarrow x + y$
  $b \leftarrow a + x$
} else {
  $a \leftarrow b + 2$
  $c \leftarrow y + 1$
}
$a \leftarrow c + a$



$c_1 \leftarrow 12$
if (i)

$a_1 \leftarrow x + y$
$b_1 \leftarrow a_1 + x$

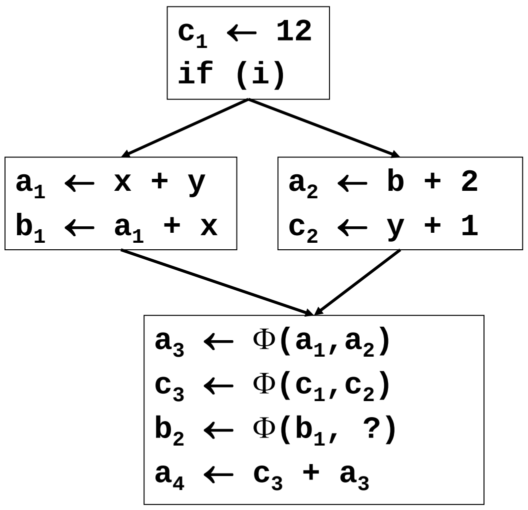
$a \leftarrow b + 2$
$c \leftarrow y + 1$

$a_4 \leftarrow c_? + a_?$

SSA

- Static single assignment is an IR where every variable is assigned a value at most once in the program text
- Easy for a basic block:
  - assign to a fresh variable at each stmt.
  - Each use uses the most recently defined var.
  - (Similar to Value Numbering)
- What about at joins in the CFG?
  - Use a notional fiction: A Φ function

Merging at Joins



$c_1 \leftarrow 12$
if (i)

$a_1 \leftarrow x + y$
$b_1 \leftarrow a_1 + x$

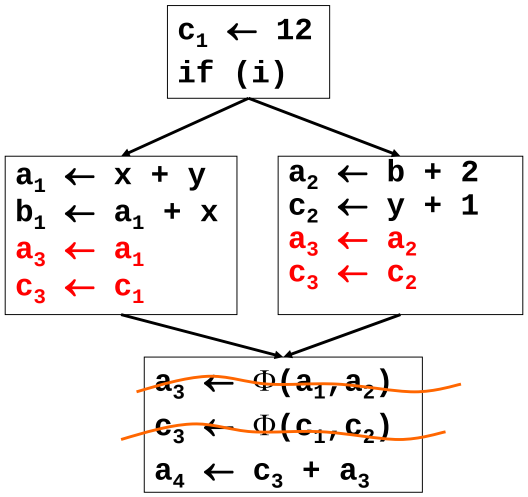
$a_2 \leftarrow b + 2$
$c_2 \leftarrow y + 1$

$a_3 \leftarrow \Phi(a_1, a_2)$
$c_3 \leftarrow \Phi(c_1, c_2)$
$b_2 \leftarrow \Phi(b_1, ?)$
$a_4 \leftarrow c_3 + a_3$

The Φ function

- Φ merges multiple definitions along multiple control paths into a single definition.
- At a BB with p predecessors, there are p arguments to the Φ function.
  $x_{new} \leftarrow \Phi(x_1, x_1, x_1, \ldots, x_p)$
- How do we choose which x_i to use?
  - We don't really care!
  - If we care, use moves on each incoming edge

"Implementing" Φ



$c_1 \leftarrow 12$
if (i)

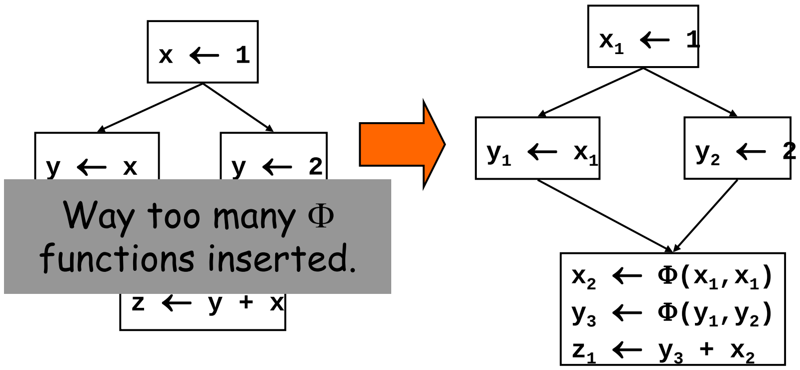
$a_1 \leftarrow x + y$
$b_1 \leftarrow a_1 + x$
$a_3 \leftarrow a_1$
$c_3 \leftarrow c_1$

$a_2 \leftarrow b + 2$
$c_2 \leftarrow y + 1$
$a_3 \leftarrow a_2$
$c_3 \leftarrow c_2$

~~$a_3 \leftarrow \Phi(a_1, a_2)$~~
~~$c_3 \leftarrow \Phi(c_1, c_2)$~~
$a_4 \leftarrow c_3 + a_3$

Trivial SSA

- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all live variables.



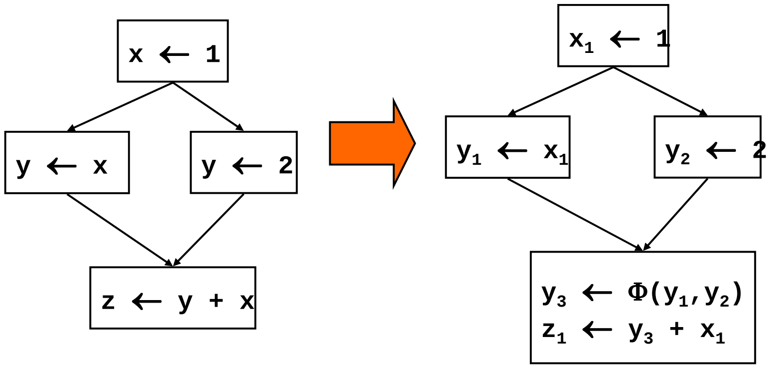
$x \leftarrow 1$
$y \leftarrow x$
$y \leftarrow 2$
$z \leftarrow y + x$

Way too many Φ functions inserted.

$x_1 \leftarrow 1$
$y_1 \leftarrow x_1$
$y_2 \leftarrow 2$
$x_2 \leftarrow \Phi(x_1, x_1)$
$y_3 \leftarrow \Phi(y_1, y_2)$
$z_1 \leftarrow y_3 + x_2$

Minimal SSA

- Each assignment generates a fresh variable.
- At each join point insert Φ functions for all variables with multiple outstanding defs.



$x \leftarrow 1$

$y \leftarrow x$

$y \leftarrow 2$

$z \leftarrow y + x$

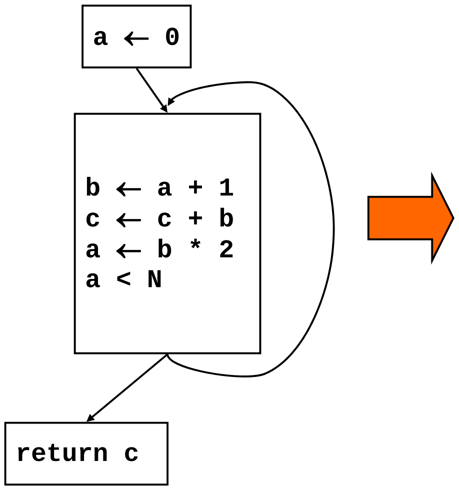
$x_1 \leftarrow 1$

$y_1 \leftarrow x_1$

$y_2 \leftarrow 2$

$y_3 \leftarrow \Phi(y_1,y_2)$
$z_1 \leftarrow y_3 + x_1$

Another Example

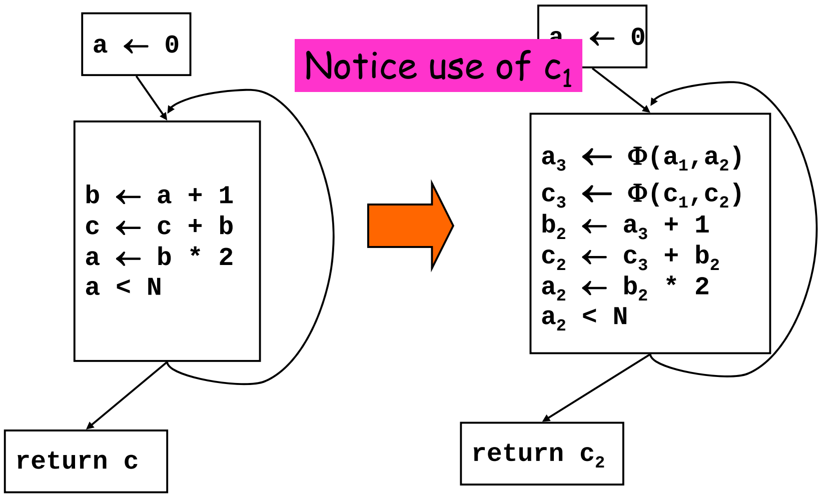


$a \leftarrow 0$

$b \leftarrow a + 1$
$c \leftarrow c + b$
$a \leftarrow b * 2$
$a < N$

return c

Another Example



$a \leftarrow 0$

$b \leftarrow a + 1$
$c \leftarrow c + b$
$a \leftarrow b * 2$
$a < N$

return c

Notice use of c_1

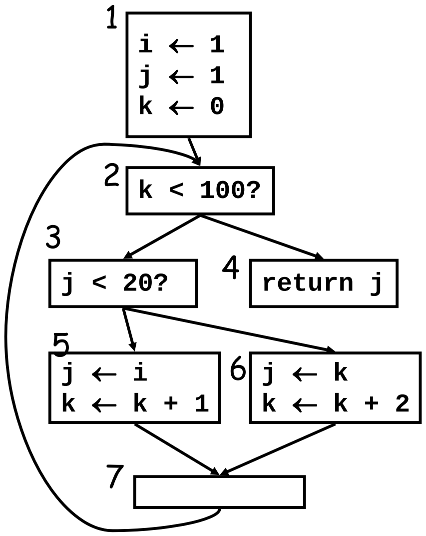
$a_1 \leftarrow 0$

$a_3 \leftarrow \Phi(a_1,a_2)$
$c_3 \leftarrow \Phi(c_1,c_2)$
$b_2 \leftarrow a_3 + 1$
$c_2 \leftarrow c_3 + b_2$
$a_2 \leftarrow b_2 * 2$
$a_2 < N$

return c_2

Lets optimize the following:

```
i=1;
j=1;
k=0;
while (k<100) {
  if (j<20) {
     j=i;
     k++;
  } else {
     j=k;
     k+=2;
  }
}
return j;
```



1 $i \leftarrow 1$
$j \leftarrow 1$
$k \leftarrow 0$

2 $k < 100?$

3 $j < 20?$

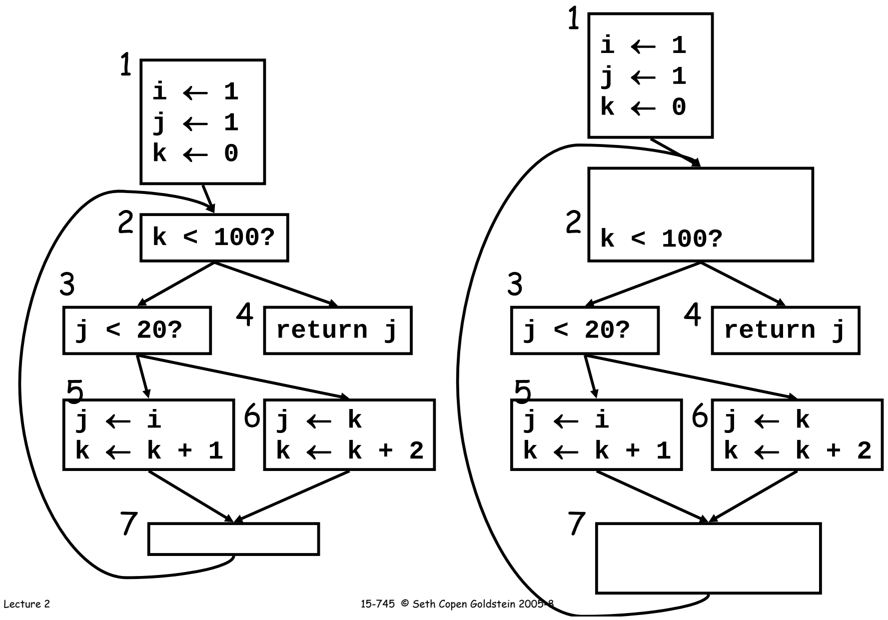
4 return j

5 $j \leftarrow i$
$k \leftarrow k + 1$

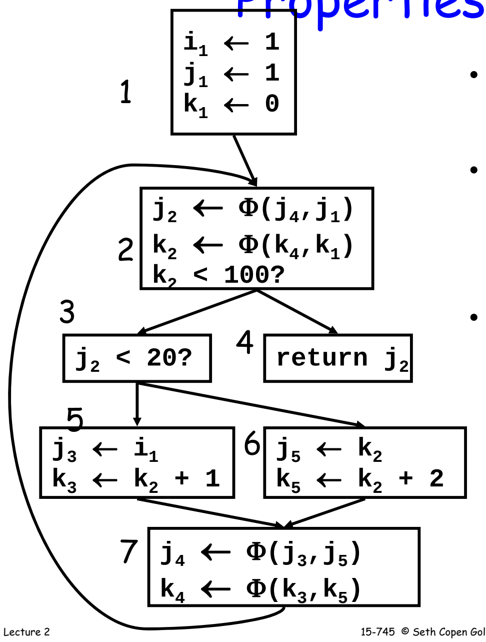
6 $j \leftarrow k$
$k \leftarrow k + 2$

7

First, turn into SSA



Properties of SSA



- Only 1 assignment per variable
- definitions dominate uses
- Can we use this to help with constant propagation?

Constant Propagation

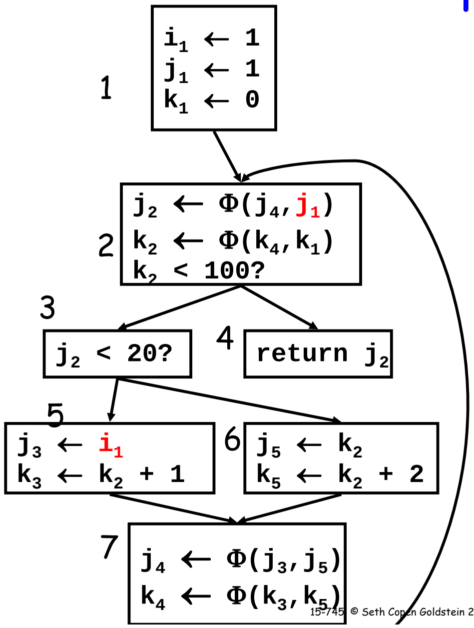
- If "v ← c", replace all uses of v with c
- If "v ← **Φ(c,c,c)**" replace all uses of v with c

```
W <- list of all defs
while !W.isEmpty {
        Stmt S <- W.removeOne
        if S has form "v <- Φ(c,…,c)"
          replace S with V <- c
        if S has form "v <- c" then
          delete S
          foreach stmt U that uses v,
            replace v with c in U
            W.add(U)
}
```

Other stuff we can do?

- Copy propagation
  delete "x ← Φ(y)" and replace all x with y
  delete "x ← y" and replace all x with y
- Constant Folding
  (Also, constant conditions too!)
- Unreachable Code
  Remember to delete all edges from unreachable block

Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, j_1)$
$k_2 \leftarrow \Phi(k_4, k_1)$
$k_2 < 100?$

3
$j_2 < 20?$

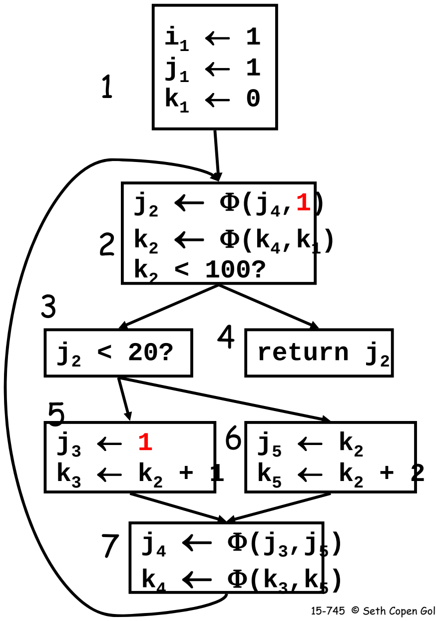
4
return j_2

5
$j_3 \leftarrow i_1$
$k_3 \leftarrow k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(j_3, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow \Phi(k_4, k_1)$
$k_2 < 100?$

3
$j_2 < 20?$

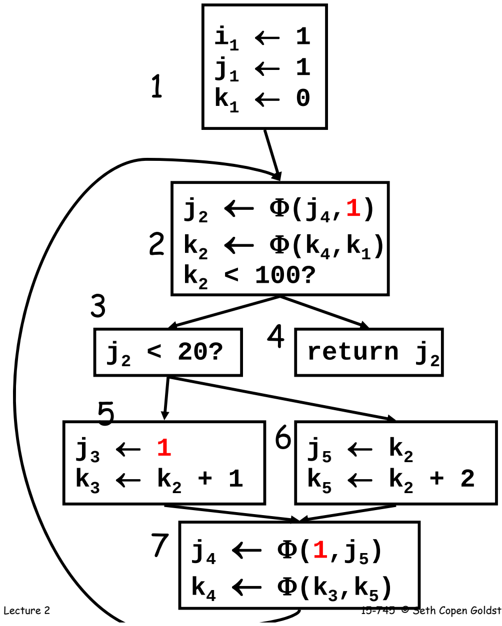
4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(j_3, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow \Phi(k_4, k_1)$
$k_2 < 100?$

3
$j_2 < 20?$

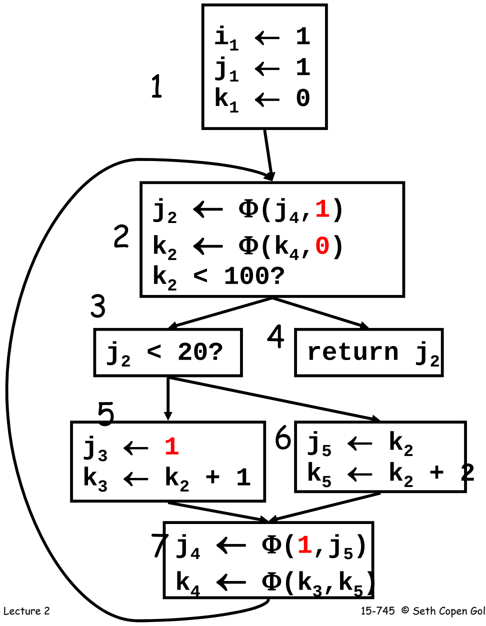
4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow \Phi(k_4, 0)$
$k_2 < 100?$

3
$j_2 < 20?$

4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

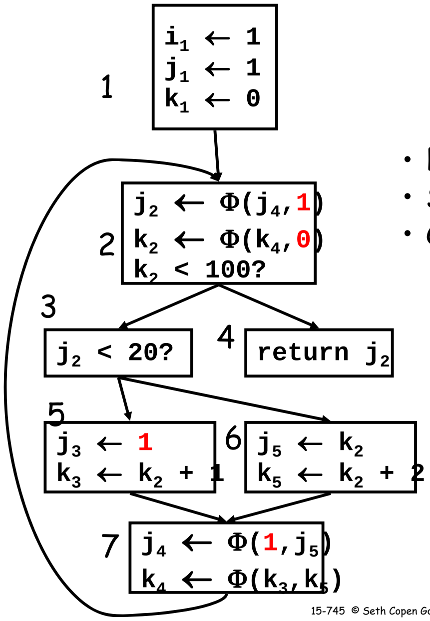
But, so what?

You will have to wait
til next time :)

Summary

- Dataflow framework
  - Lattice, meet, direction, transfer function, initial values
- Du-chains, ud-chains
- CSE
- SSA
  - One static definition per variable
  - Φ-functions

Conditional Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow \Phi(k_4, 0)$
$k_2 < 100?$

3
$j_2 < 20?$

4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

- Does block 6 ever execute?
- Simple CP can't tell
- CCP can tell:
  - Assumes blocks don't execute until proven otherwise
  - Assumes Values are constants until proven otherwise

Tracks:

- Blocks (assume unexecuted until proven otherwise)

- Variables (assume not executed, only with proof of assignments of a non-constant value do we assume not constant)

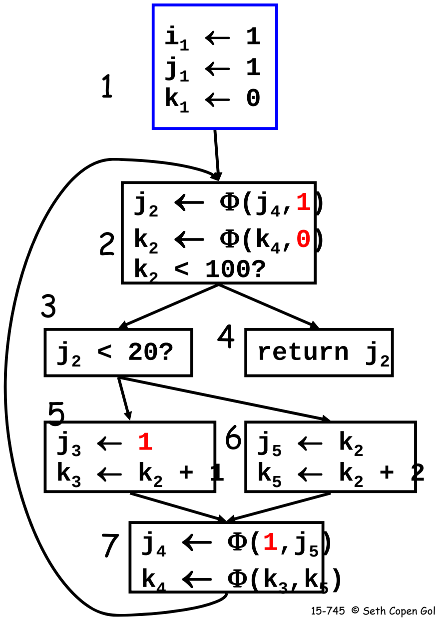
Use a lattice for variables:

TOP = we have evidence that variable can hold different values at different times

integers = we have seen evidence that the var has been assigned a constant with the value

BOT = not executed

Conditional Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow \Phi(k_4, 0)$
$k_2 < 100?$

3
$j_2 < 20?$

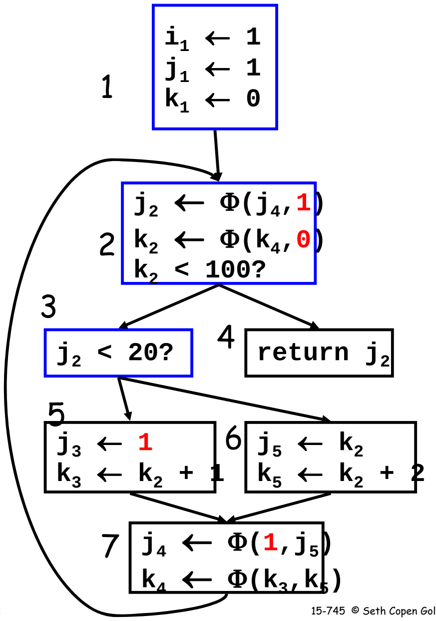
4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

Conditional Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow \Phi(k_4, 0)$
$k_2 < 100?$

3
$j_2 < 20?$

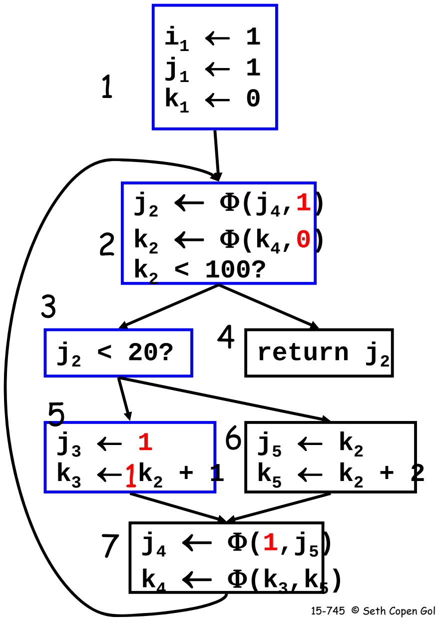
4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

Conditional Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow \Phi(k_4, 0)$
$k_2 < 100?$

3
$j_2 < 20?$

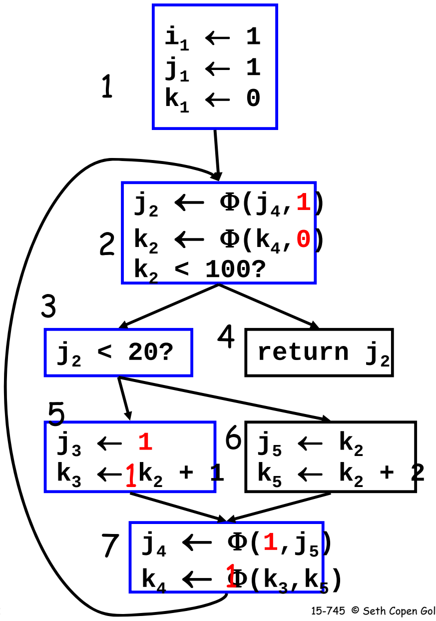
4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow$ 1 $k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1, j_5)$
$k_4 \leftarrow \Phi(k_3, k_5)$

Conditional Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow \Phi(k_4, 0)$
$k_2 < 100?$

3
$j_2 < 20?$

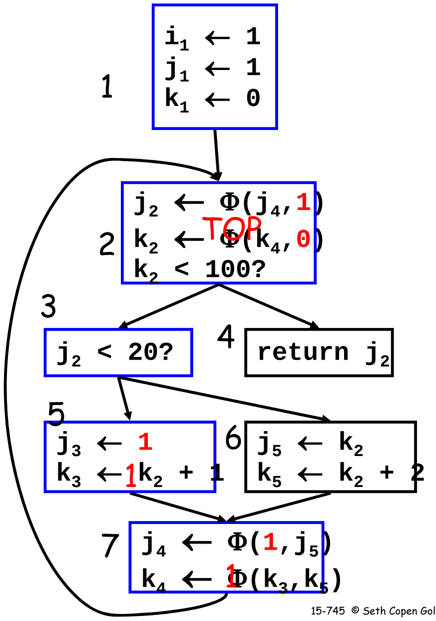
4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow$ 1 $k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1, j_5)$
$k_4 \leftarrow$ 1 $\Phi(k_3, k_5)$

Conditional Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4, 1)$
$k_2 \leftarrow$ TOP $\Phi(k_4, 0)$
$k_2 < 100?$

3
$j_2 < 20?$

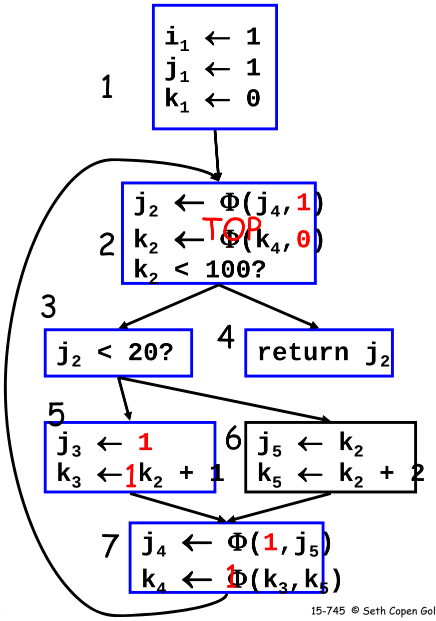
4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow$ 1 $k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1, j_5)$
$k_4 \leftarrow$ 1 $\Phi(k_3, k_5)$

Conditional Constant Propagation



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j_4,1)$
$k_2 \leftarrow \Phi(k_4,0)$
TOP
$k_2 < 100?$

3
$j_2 < 20?$

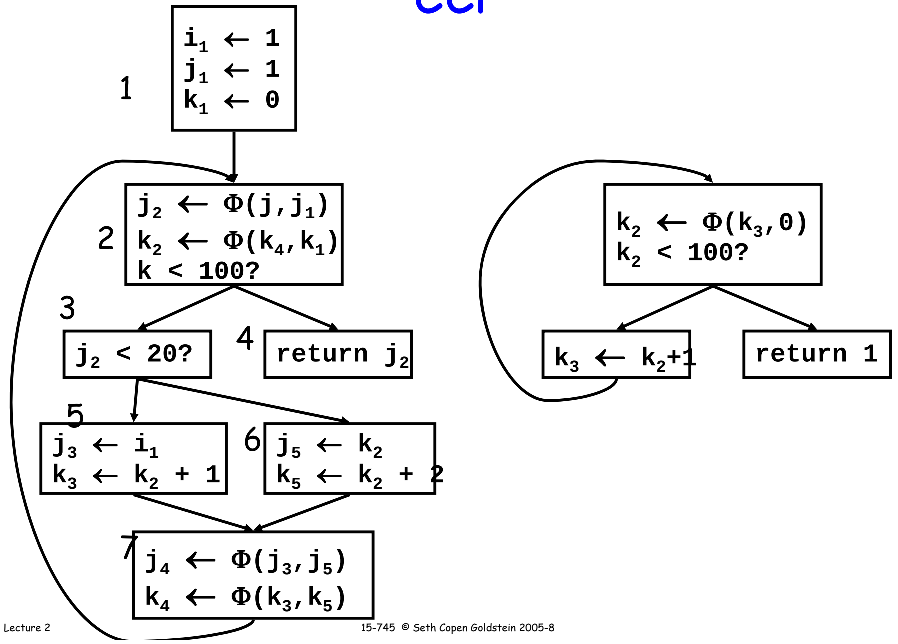
4
return j_2

5
$j_3 \leftarrow 1$
$k_3 \leftarrow 1 k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(1,j_5)$
$k_4 \leftarrow 1 \Phi(k_3,k_5)$

CCP



1
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 0$

2
$j_2 \leftarrow \Phi(j,j_1)$
$k_2 \leftarrow \Phi(k_4,k_1)$
$k < 100?$

3
$j_2 < 20?$

4
return j_2

5
$j_3 \leftarrow i_1$
$k_3 \leftarrow k_2 + 1$

6
$j_5 \leftarrow k_2$
$k_5 \leftarrow k_2 + 2$

7
$j_4 \leftarrow \Phi(j_3,j_5)$
$k_4 \leftarrow \Phi(k_3,k_5)$

$k_2 \leftarrow \Phi(k_3,0)$
$k_2 < 100?$

$k_3 \leftarrow k_2+1$

return 1