Algorithms Core 15-750 PROBLEM SET # 4 Spring 1999

DUE: 1:00PM Wednesday March 17 1999

1 The off-line k-server problem

In the k-server problem we have an edge-weighted graph G and k servers each server is located at a node of the graph, we may have more than one server at a node. At each time interval we get a request from a subset of $s \leq k$ nodes of G for a server. We must move s servers to each node that request a server. We may also move other servers to new nodes in anticipation of later requests. The cost to move a server from node v, were it is, to a new node w is equal to the min-cost path from v to w in G. In the off-line version of the problem we are given all the request ahead of time we must simply find the min-cost strategy to handle all the request at the requested times.

Problem 1.1 *Show how to reduce the off-line k-server problem to a min-cost max-flow problem of size at most $O(n \cdot t)$ nodes and $O(n^2 \cdot t)$ edges.*

2 Maximum Flow for Negative Capacities

In lecture we considered the case when all capacities are nonnegative. Suppose we would like to find a maximum flow where the flow on some edge (u, v) is in the interval $[2, 3]$, that is, the flow is at least 2 but no more than 3. We can model this by setting the capacity of (u, v) to 3 and the capacity of (v, u) to -2.

Problem 2.1 *Show how to reduce the problem of maximum flow with possibly negative capacities to two maximum flow problems both with nonnegative capacities.*

3 Maximum Planar Flow

In this problem we consider the Max-Flow problem where the flow graph is planar. We first consider a simpler problem, we say that G is a **circulation graph** if it has arc capacities c, in general c may be negative. A circulation graph has no source or sink. A circulation of G is a feasible flow on G.

A natural dual problem is finding potential functions. A **potential graph** is a graph $G = (V, E)$ plus assignment of a weight $w(e)$ to each arc of G, in general, the weights may be negative. A function p which assigns a real value to each node of G is a **potential function** for G if $w(u, v) \geq p(v) - p(u)$.

We first show there is a very nice relationship between flows in G and potential functions on G^* the geometric dual of G. Given an arc e in G the dual of e, denoted e^* is the arc in G^* from the face right of e to the face left of e.

Problem 3.1 *Show how that a planar circulation graph G has a circulation if and only if the dual G^* has a potential function, where $w(e^*) = c(e)$.*

Problem 3.2 *Show how to find a maximum flow in a planar flow graph in $O(n \log n)$ time when the capacities are nonnegative and the only source and only sink lie on the same face.*

Problem 3.3 *Show how to find a flow of some fixed size in a planar flow graph in $O(n^{1.5})$ time when there is only one source and only one sink.*

You may use the fact that single source shortest paths can be found in a planar graph in $O(n^{1.5})$ time.

4 Making a Palindrome

Suppose we have n words W where the longest word has length l. Give an $O(l^2 n^2)$ time algorithm to determine if we can form a palindrome using words in W. You are allowed to uses the same word more than once.

Hint: Try solving this problem on your own. But if after some time you have not found a solution come by and we can give you a hint.