Resolution Proof Example

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“Robot Doom” Domain

1. Jack owns a roomba
2. Every roomba owner is a robot enthusiast.
3. No robot enthusiast breaks a robot.
4. Either Jack or SENSOR MALFUNCTION broke my roomba.

Question: Did SENSOR MALFUNCTION break my roomba?

Write in First Order Logic

1. Jack owns a roomba

$$\exists x Roomba(x) \wedge Owns(Jack, x)$$

2. Every roomba owner is a robot enthusiast.

$$\forall x [\exists y Roomba(y) \wedge Owns(x,y)] \Rightarrow Robot\_enthusiast(x)$$

3. No robot enthusiast breaks a robot.

$$\forall x Robot\_enthusiast(x) \Rightarrow [\forall y Robot(y) \Rightarrow \neg Breaks(x,y)]$$

4. Either Jack or SENSOR MALFUNCTION broke my roomba.

$$Breaks(Jack, myRoomba) \vee Breaks(S.M., myRoomba)$$

Some Additional Facts:

$$Roomba(myRoomba)$$

$$\forall x Roomba(x) \rightarrow Robot(x)$$

Algorithm for Converting FOL to CNF

1. Eliminate Implications
2. Move negation inward
3. Standardize variables
4. Skolemization
5. Drop universal quantifiers
6. Apply distributivity

Convert to CNF
Step 1: Eliminate Implications

Replace p→q with not p or q

$$\exists x Roomba(x) \wedge Owns(Jack, x)$$

$$\forall x \neg (\exists y Roomba(y) \wedge Owns(x, y)) \vee Robot\_enthusiast(x)$$

$$\forall x \neg Robot\_enthusiast(x) \vee \forall y \neg Robot(y) \vee \neg Breaks(x, y)$$

$$Breaks(Jack, myRoomba) \vee Breaks(S.M., myRoomba)$$

$$Roomba(myRoomba)$$

$$\forall x \neg Roomba(x) \vee Robot(x)$$

Convert to CNF
Step 2: Move Negation Inward

Apply Demorgan's Laws, change quantifiers, etc.

$$\neg(A \lor B) \Leftrightarrow \neg A \land \neg B$$
$$\neg(A \land B) \Leftrightarrow \neg A \lor \neg B$$

$$\neg \forall x A(x) \Leftrightarrow \exists x \neg A(x)$$
$$\neg \exists x A(x) \Leftrightarrow \forall x \neg A(x)$$

$$\exists x Roomba(x) \land Owns(Jack, x)$$
$$\forall x(\forall y \neg Roomba(y) \lor \neg Owns(x, y)) \lor Robot\_enthusiast(x)$$
$$\forall x \neg Robot\_enthusiast(x) \lor \forall y \neg Robot(y) \lor \neg Breaks(x, y)$$
$$Breaks(Jack, myRoomba) \lor Breaks(S.M., myRoomba)$$
$$Roomba(myRoomba)$$
$$\forall x \neg Roomba(x) \lor Robot(x)$$

Convert to CNF
Step 3: Standardize Variables

If two variables have the same name, then change the name of one of the variables (no modification in this example)

$$\exists x Roomba(x) \wedge Owns(Jack, x)$$

$$\forall x (\forall y \neg Roomba(y) \vee \neg Owns(x, y)) \vee Robot\_enthusiast(x)$$

$$\forall x \neg Robot\_enthusiast(x) \vee \forall y \neg Robot(y) \vee \neg Breaks(x, y)$$

$$Breaks(Jack, myRoomba) \vee Breaks(S.M., myRoomba)$$

$$Roomba(myRoomba)$$

$$\forall x \neg Roomba(x) \vee Robot(x)$$

Convert to CNF
Step 4: Skolemization

"Skolemization" – the process of removing existential quantifiers by elimination.

General Rule: Arguments of the skolem function are all the universally quantified variables in whose scope the existential quantifier appears.

$$Roomba(R) \land Owns(Jack, R)$$

$$\forall x (\forall y \neg Roomba(y) \lor \neg Owns(x, y)) \lor Robot\_enthusiast(x)$$

$$\forall x \neg Robot\_enthusiast(x) \lor \forall y \neg Robot(y) \lor \neg Breaks(x, y)$$

$$Breaks(Jack, myRoomba) \lor Breaks(S.M., myRoomba)$$

$$Roomba(myRoomba)$$

$$\forall x \neg Roomba(x) \lor Robot(x)$$

Convert to CNF
Step 5: Drop Universal Quantifiers

Pretty Easy…

$$Roomba(R) \wedge Owns(Jack, R)$$

$$(\neg Roomba(y) \vee \neg Owns(x,y)) \vee Robot\_enthusiast(x)$$

$$\neg Robot\_enthusiast(x) \vee \neg Robot(y) \vee \neg Breaks(x,y)$$

$$Breaks(Jack, myRoomba) \vee Breaks(S.M., myRoomba)$$

$$Roomba(myRoomba)$$

$$\neg Roomba(x) \vee Robot(x)$$

Convert to CNF
Step 6: Apply Distributivity

Nothing much to do in this example…but you know how to do this well after problem 1.

$$Roomba(R)$$

$$Owns(Jack, R)$$

$$\neg Roomba(y) \lor \neg Owns(x, y) \lor Robot\_enthusiast(x)$$

$$\neg Robot\_enthusiast(x) \lor \neg Robot(y) \lor \neg Breaks(x, y)$$

$$Breaks(Jack, myRoomba) \lor Breaks(S.M., myRoomba)$$

$$Roomba(myRoomba)$$

$$\neg Roomba(x) \lor Robot(x)$$

Assert Negation of Conclusion

What we want to conclude: $Breaks(S.M.,myRoomba)$

Negation of conclusion: $\neg Breaks(S.M.,myRoomba)$

Resolution Rule
(simplified)

$$\frac{a \lor b, \neg a \lor c}{b \lor c}$$

Apply Resolution

$\neg Breaks(S.M, myRoomba)$

$Breaks(Jack, myRoomba) \vee Breaks(S.M., myRoomba)$

$Breaks(Jack, myRoomba)$

$\neg Robot\_enthusiast(x) \vee \neg Robot(y) \vee \neg Breaks(x, y)$

$\neg Robot\_enthusiast(Jack) \vee \neg Robot(myRoomba)$

$\neg Roomba(y) \vee \neg Owns(x, y) \vee Robot\_enthusiast(x)$

$\neg Robot(myRoomba) \vee \neg Roomba(y)$
$\vee \neg Owns(Jack, y)$

$Roomba(R)$
$Owns(Jack, R)$
$\neg Roomba(y) \vee \neg Owns(x, y) \vee Robot\_enthusiast(x)$
$\neg Robot\_enthusiast(x) \vee \neg Robot(y) \vee \neg Breaks(x, y)$
$Breaks(Jack, myRoomba) \vee Breaks(S.M., myRoomba)$
$Roomba(myRoomba)$
$\neg Roomba(x) \vee Robot(x)$

Apply Resolution (continued)

$\neg Robot(myRoomba) \vee \neg Roomba(y) \vee \neg Owns(Jack,y)$ $\quad$ $Roomba(R)$

$\neg Robot(myRoomba) \vee \neg Owns(Jack,R)$ $\quad$ $Owns(Jack,R)$

$\neg Robot(myRoomba)$ $\quad$ $\neg Roomba(x) \vee Robot(x)$

$\neg Roomba(myRoomba)$ $\quad$ $Roomba(myRoomba)$

FALSE

…So negated conclusion is wrong.
…So original statement must be right.

$Roomba(R)$

$Owns(Jack,R)$

$\neg Roomba(y) \vee \neg Owns(x,y) \vee Robot\_enthusiast(x)$

$\neg Robot\_enthusiast(x) \vee \neg Robot(y) \vee \neg Breaks(x,y)$

$Breaks(Jack,myRoomba) \vee Breaks(S.M.,myRoomba)$

$Roomba(myRoomba)$

$\neg Roomba(x) \vee Robot(x)$

Practical Aspects of Logic

- Jess Rule-Based System for Java

(http://herzberg.ca.sandia.gov)

- Wolfram-Alpha Logic Engine
- MATLAB toolbox