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From Bits through Integers

18-213/18-613

Introduction to Computer Systems

2nd Lecture, January 18, 2023

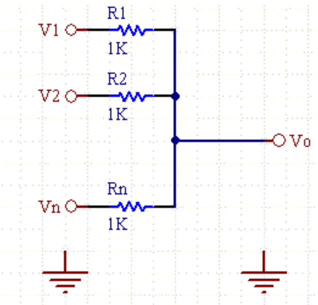
Bits, Bytes, and Integers

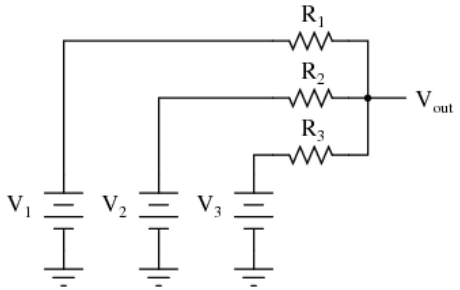
- Representing information as bits — CSAPP 2.1
- Bit-level manipulations
- Integers — CSAPP 2.2
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting — CSAPP 2.3
- Byte Ordering — CSAPP 2.1.3

Analog Computers

- Before digital computers there were analog computers.

- Consider a couple of simple analog computers:
  - A simple circuit can allow one to adjust voltages using variable resistors and measure the output using a volt meter:
  - A simple network of adjustable parallel resistors can allow one to find the average of the inputs.





https://www.daycounter.com/Calculators/Voltage-Summer/Voltage-Summer-Calculator.phtml

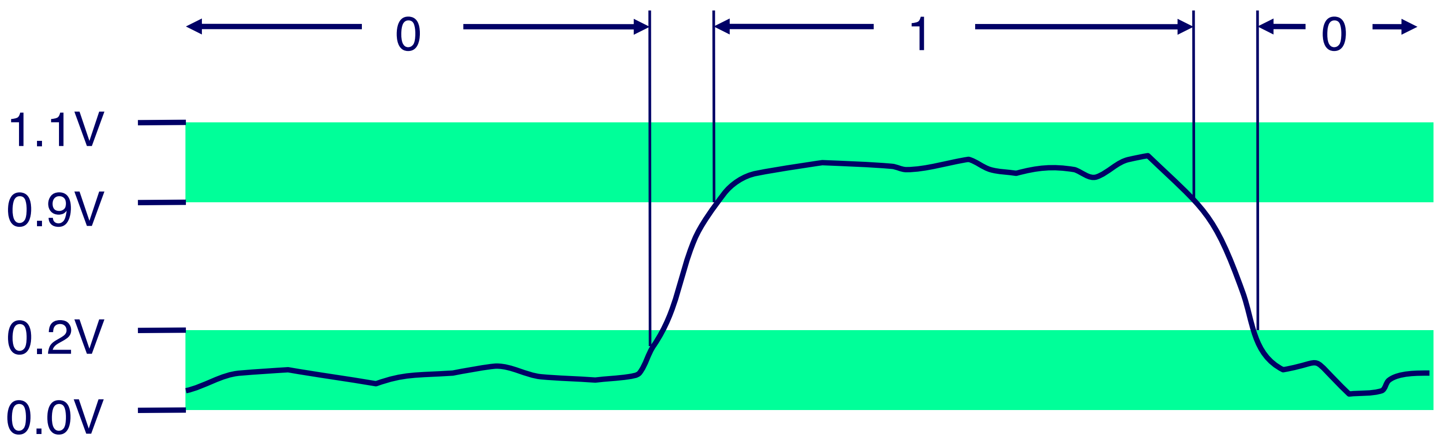
https://www.quora.com/What-is-the-most-basic-voltage-adder-circuit-without-a-transistor-op-amp-and-any-external-supply

Needing Less Accuracy, Precision is Better

- We don't try to measure exactly
  - We just ask, is it high enough to be "On", or
  - Is it low enough to be "Off".

- We have two states, so we have a binary, or 2-ary, system.
  - We represent these states as 0 and 1

- Now we can easily interpret, communicate, and duplicate signals well enough to know what they mean.



Binary Representation

- Binary representation leads to a simple binary, i.e. base-2, numbering system
  - 0 represents 0
  - 1 represents 1
  - Each “place” represents a power of two, exactly as each place in our usual “base 10”, 10-ary numbering system represents a power of 10
- By encoding/interpreting sets of bits in various ways, we can represent different things:
  - Operations to be executed by the processor, numbers, enumerable things, such as text characters
- As long as we can assign it to a discrete number, we can represent it in binary

Binary Representation: Simple Numbers

- For example, we can count in binary, a base-2 numbering system
  - 000, 001, 010, 011, 100, 101, 110, 111, …
    - 000 = $0*2^2 + 0*2^1 + 0*2^0 = 0$ (in decimal)
    - 001 = $0*2^2 + 0*2^1 + 1*2^0 = 1$ (in decimal)
    - 010 = $0*2^2 + 1*2^1 + 0*2^0 = 2$ (in decimal)
    - 011 = $0*2^2 + 1*2^1 + 1*2^0 = 3$ (in decimal)
    - Etc.
- For reference, consider some base-10 examples:
    - 000 = $0*10^2 + 0*10^1 + 0*10^0$
    - 001 = $0*10^2 + 0*10^1 + 1*10^0$
    - 357 = $3*10^2 + 5*10^1 + 7*10^0$

Hexadecimal and Octal

- Writing out numbers in binary takes too many digits
- We want a way to represent numbers more densely such that fewer digits are required
  - But also such that it is easy to get at the bits that we want
- Any power-of-two base provides this property
  - Octal, e.g. base-8, and hexadecimal, e.g. base-16 are the closest to our familiar base-10.
  - Each has been used by “computer people” over time
  - Hexadecimal is often preferred because it is denser.

Hexadecimal

- Hexadecimal 00_{16} to FF_{16}
  - Base 16 number representation
  - Use characters '0' to '9' and 'A' to 'F'
- Consider 1A2B in Hexadecimal:
  - $1*16^3 + A*16^2 + 2*16^1 + B*16^0$
  - $1*16^3 + 10*16^2 + 2*16^1 + 11*16^0$ = 6699 (decimal)
  - The C Language prefixes hexadecimal numbers with "0x" so they aren't confused with decimal numbers
  - Write $FA1D37B_{16}$ in C as
    - 0xFA1D37B
    - 0xfa1d37b

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

```
18213: 0100 0111 0010 0101
        4    7    2    5
```

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- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting
- Byte Ordering

Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic
    - Encode “True” as 1 and “False” as 0

And

- A&B = 1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

- A|B = 1 when either A=1 or B=1

|	0	1
0	0	1
1	1	1

Not

- ~A = 1 when A=0

~	
0	1
1	0

Exclusive-Or (Xor)

- A^B = 1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- Operate on Bit Vectors
  - Operations applied bitwise

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
  --------     --------     --------     --------
  01000001     01111101     00111100     10101010
```

- All of the Properties of Boolean Algebra Apply

Example: Representing & Manipulating Sets

- Representation
  - Width w bit vector represents subsets of {0, …, w–1}
  - $a_j = 1$ if j ∈ A
    - 01101001 { 0, 3, 5, 6 }
    - *76543210*
    - 01010101 { 0, 2, 4, 6 }
    - *76543210*
- Operations
  - & Intersection 01000001 { 0, 6 }
  - | Union 01111101 { 0, 2, 3, 4, 5, 6 }
  - ^ Symmetric difference 00111100 { 2, 3, 4, 5 }
  - ~ Complement 10101010 { 1, 3, 5, 7 }

Bit-Level Operations in C

- Operations &, |, ~, ^ Available in C
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- Examples (Char data type)
  - `~0x41` →
    -
  - `~0x00` →
    -
  - `0x69 & 0x55` →
    -
  - `0x69 | 0x55` →
    -

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Bit-Level Operations in C

- **Operations &, |, ~, ^ Available in C**
  - Apply to any "integral" data type
    - `long, int, short, char, unsigned`
  - View arguments as bit vectors
  - Arguments applied bit-wise
- **Examples (Char data type)**
  - ~0x41 → 1011 1110
  - ~0x00 → 1111 1111
  - 0x69 & 0x55: 0x69 | 0x55:

```
    0110 1001            0110 1001
  & 0101 0101          | 0101 0101
-----------------    -----------------
    0100 0001            0111 1101
```

Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
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D	13	1101
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F	15	1111

Contrast: Logic Operations in C

- Contrast to Bit-Level Operators
  - **Logic Operations: &&, ||, !**
    - View 0 as "False"
    - Anything nonzero as "True"
    - Always return 0 or 1
    - Early termination
- Examples (char data type)
  - `!0x41` → `0x00`
  - `!0x00` → `0x01`
  - `!!0x41` → `0x01`

  - `0x69 && 0x55` → `0x01`
  - `0x69 || 0x55` → `0x01`
  - `p && *p` (avoids null pointer access)

Watch out for && vs. & (and || vs. |)…
Super common C programming pitfall!

Shift Operations

- Left Shift: `x << y`
  - Shift bit-vector **x** left **y** positions
    - Throw away extra bits on left
    - Fill with 0's on right
- Right Shift: `x >> y`
  - Shift bit-vector **x** right **y** positions
    - Throw away extra bits on right
  - Logical shift
    - Fill with 0's on left
  - Arithmetic shift
    - Replicate most significant bit on left
- Undefined Behavior
  - Shift amount < 0 or ≥ word size

Argument **x**	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Argument **x**	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11101000

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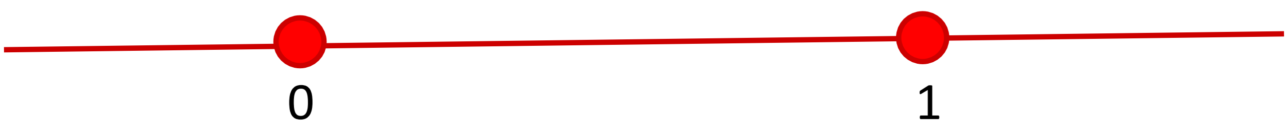
  - Addition, negation, multiplication, shifting

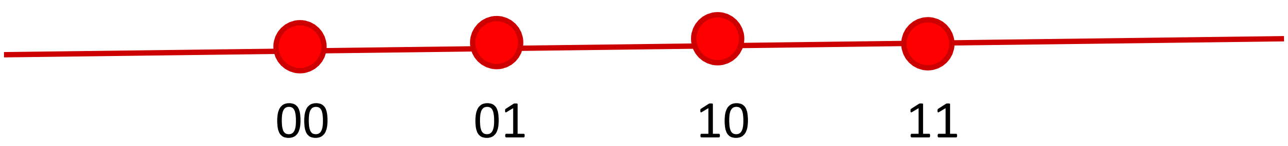
Binary Number Lines

- In binary, the number of bits in the data type size determines the number of points on the number line.
  - We can assign the points any meaning we'd like

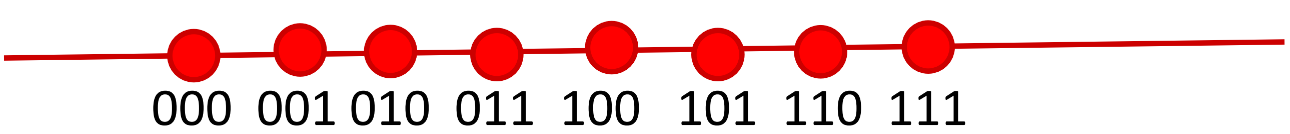
- Consider the following examples:
  - 1 bit number line



  - 2 bit number line

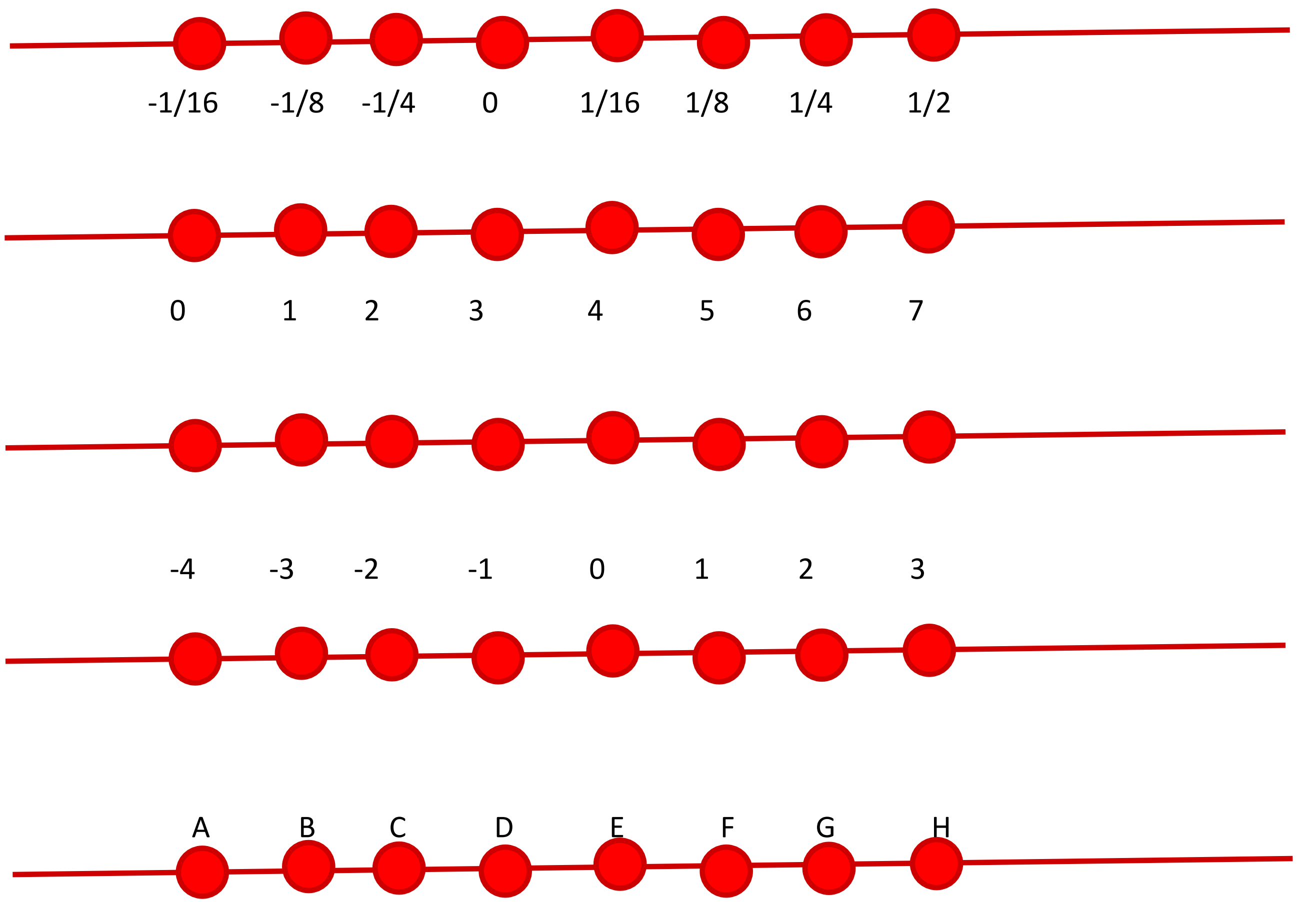


  - 3 bit number line



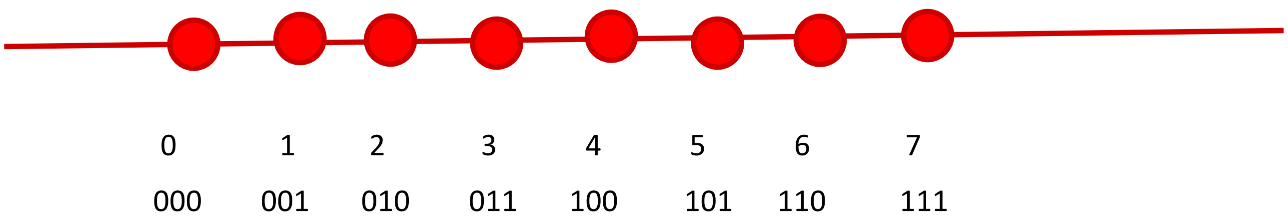
Some Purely Imaginary Examples

- 3 bit number line



Overflow

- Let’s consider a simple 3 digit number line:



- What happens if we add 1 to 7?
  - In other words, what happens if we add 1 to 111?

- 111+ 001 = 1 000
  - But, we only get 3 bits – so we lose the leading-1.
  - This is called overflow

- The result is 000

Modulus Arithmetic

- Let's explore this idea of overflow some more
  - 111 + 001 = 1 000 = 000
  - 111 + 010 = 1 001 = 001
  - 111 + 011 = 1 010 = 010
  - 111 + 100 = 1 011 = 011
  - …
  - 111 + 110 = 1 101 = 101
  - 111 + 111 = 1 110 = 110
- So, arithmetic "wraps around" when it gets "too positive"

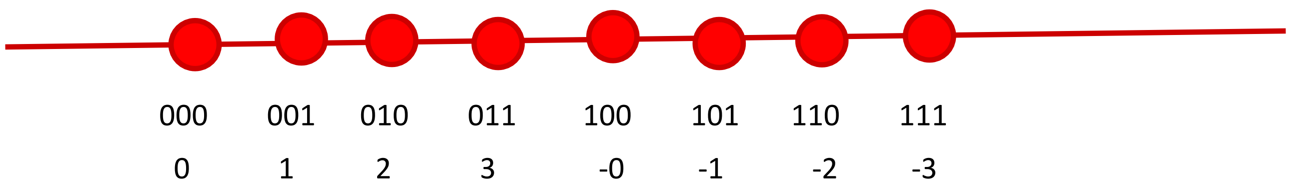
Unsigned and Non-Negative Integers

- We'll use the term "ints" to mean the finite set of integer numbers that we can represent on a number line enumerated by some fixed number of bits, i.e. *bit width*.

- We normally represent unsigned and non-negative int using simple binary as we have already discussed
  - An "unsigned" int is any int on a number line, e.g. of a data type, that doesn't contain any negative numbers
  - A non-negative number is a number greater than or equal to (>=) 0 on a number line, e.g. of a data type, that does contain negative numbers

How represent negative Numbers?

- We could use the leading bit as a *sign bit*:
  - 0 means non-negative
  - 1 means negative



- This has some benefits
  - It lets us represent negative and non-negative numbers
  - 0 represents 0

- It also has some drawbacks
  - There is a -0, which is the same as 0, except that it is different
  - How to add such numbers 1 + -1 should equal 0
    - But, by simple math, 001 + 101 = 110, which is -2?

A Magic Trick!

- Let's just start with three ideas:
  - 1 should be represented as 1
  - -1 + 1 = 0
  - We want addition to work in the familiar way, with simple rules.
- We want a situation where "-1" + 1 = 0
- Consider a 3 bit number:
  - 001 + "-1" = 0
  - 001 + 111 = 0
    - Remember 001 + 111 = 1 000, and the leading one is lost to overflow.
- "-1" = 111
  - Yep!

Negative Numbers

- Well, if 111 is -1, what is -2?
  - -1   - 1
  - 111 – 001 = 110

- Does that really work?
  - If it does -2 + 2 = 0
  - 110 + 010 = 1 000 = 000

- -2 + 5 should be 3, right?
  - 110 + 101 = 1 011 = 011

Finding –x the easy way

- Given a non-negative number in binary, e.g. 5, represented with a fixed bit width, e.g. 4
  - 0101
- We can find its negative by flipping each bit and adding 1
  - 0101 This is 5
  - 1010 This is the “ones complement of 5”, e.g. 5 with bits flipped
  - 1011 This is the “twos complement of 5”, e.g. 5 with the bits flipped and 1 added
  - 0101 + 1011 = 1 0000 = 0000
  - -x = ~x+1
- Because of the fixed width, the “two’s complement” of a number can be used as its negative.

Why Does This Work?

- Consider any number and its (ones) complement:
  - 0101
  - 1010

- They are called complements because complementary bits are set. As a result, if they are added, all bits are necessarily set:
  - 0101 + 1010 = 1111

- Adding 1 to the sum of a number and its complement necessarily results in a 0 due to overflow
  - (0101 + 1010) + 1 = 1111 + 1 = 1 0000 = 0000

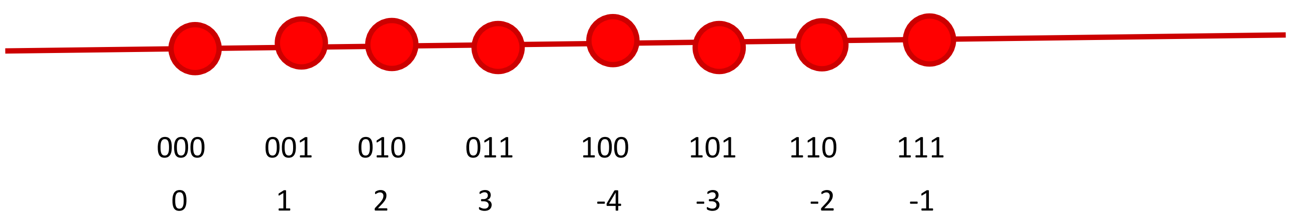
- And if x + y = 0, y must equal –x

Why Does This Work? *Cont.*

- If x + y = 0
  - y must equal –x
- So if x + (Complement(x) + 1) = 0
  - Complement(x) + 1 must equal –x
- Another way of looking at it:
  - if x + (Complement(x) + 1) = 0
  - x + Complement(x) = -1
  - x = -1 - Complement(x)
  - -x = 1 + Complement(x)

Visualizing Two's Complement

- Numbers "wrap around" with -1 at the very end



- A few things to note:
  - All negative numbers start with a "1"
    - E.g. 100 is "-4"
  - You can view the leading "1" as introducing a "-4"
    - E.g. 101 = 1*-4+0*2+1*1= -3
    - But 010 = 0*-4+1*2+0*1 = 2
  - -4 is missing a positive partner

Complement & Increment Examples

x = 0

	Decimal	Hex	Binary
0	0	00 00	00000000 00000000
~0	-1	FF FF	11111111 11111111
~0+1	0	00 00	00000000 00000000

x = Tmin (The most negative two's complement number)

	Decimal	Hex	Binary
x	-32768	80 00	10000000 00000000
~x	32767	7F FF	01111111 11111111
~x+1	-32768	80 00	10000000 00000000

Canonical counter example

Encoding Integers: Dense Form

Unsigned

$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

```
short int x =  15213;
short int y = -15213;
```

- C does not mandate using two's complement
  - But, most machines do, and we will assume so
- C `short` (2 bytes long)

	Decimal	Hex	Binary
x	15213	**3B 6D**	**00111011 01101101**
y	-15213	**C4 93**	**11000100 10010011**

- Sign Bit
  - For 2's complement, most significant bit indicates sign
    - 0 for nonnegative, 1 for negative

Numeric Ranges

- Unsigned Values
  - *UMin* = 0

    000…0
  - *UMax* = $2^w - 1$

    111…1

- Two's Complement Values
  - *TMin* = -2^{w-1}

    100…0
  - *TMax* = $2^{w-1} - 1$

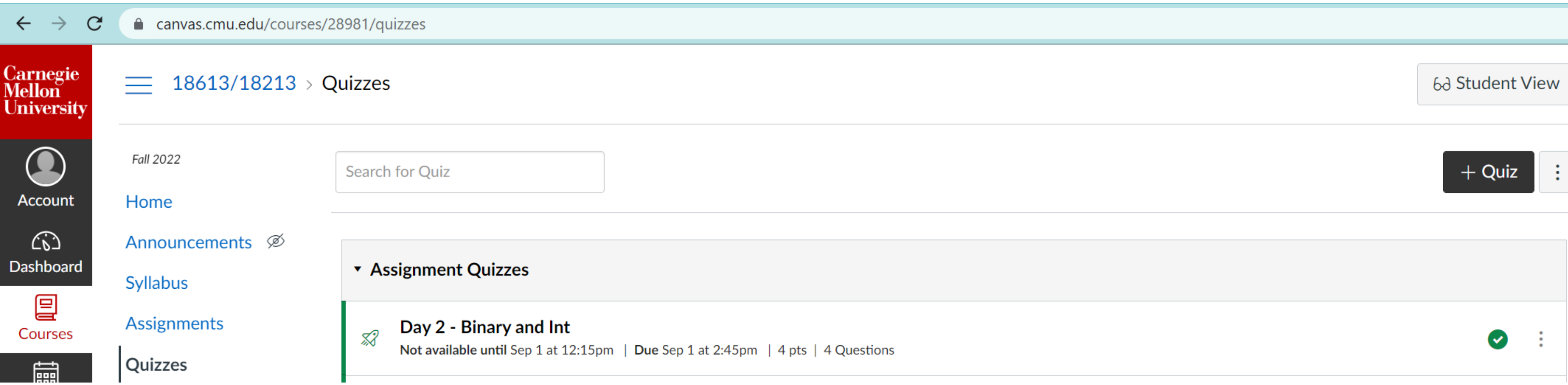
    011…1
  - Minus 1

    111…1

Values for *W* = 16

	Decimal	Hex	Binary
UMax	65535	FF FF	11111111 11111111
TMax	32767	7F FF	01111111 11111111
TMin	-32768	80 00	10000000 00000000
-1	-1	FF FF	11111111 11111111
0	0	00 00	00000000 00000000

Quiz Time!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Day 2 - Binary and Int
Not available until Sep 1 at 12:15pm | Due Sep 1 at 2:45pm | 4 pts | 4 Questions

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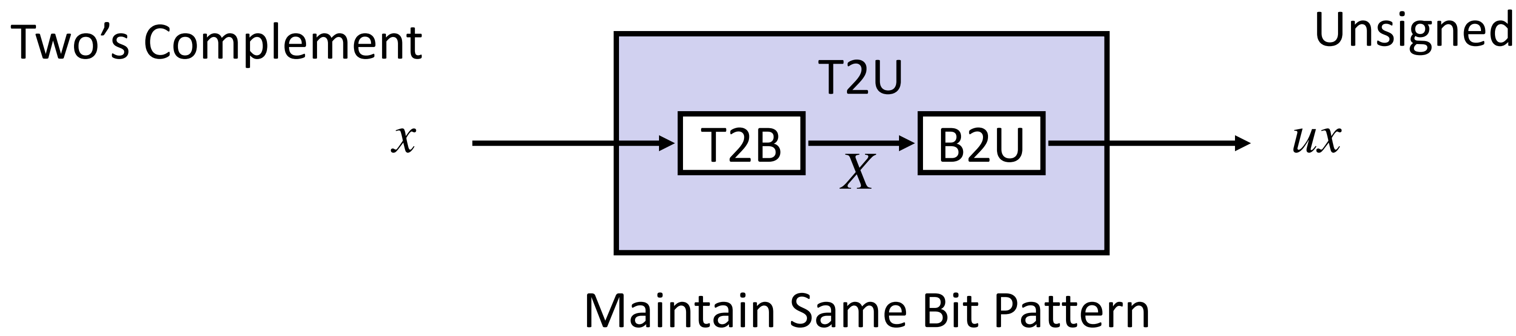
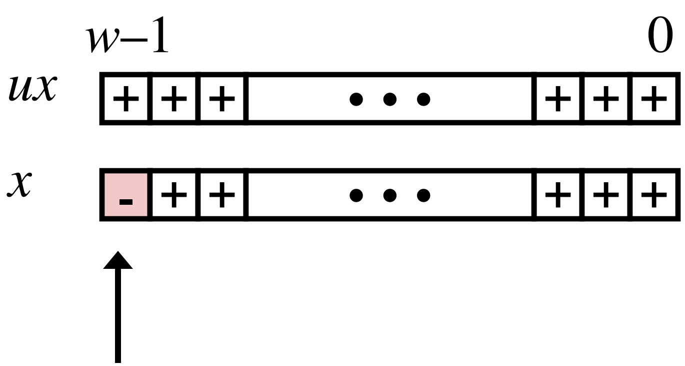
Mapping Signed ↔ Unsigned

Bits	Signed		Unsigned
0000	0		0
0001	1		1
0010	2		2
0011	3	=	3
0100	4	↔	4
0101	5		5
0110	6		6
0111	7		7
1000	-8		8
1001	-7		9
1010	-6		10
1011	-5	+/- 16	11
1100	-4	↔	12
1101	-3		13
1110	-2		14
1111	-1		15

Relation between Signed & Unsigned

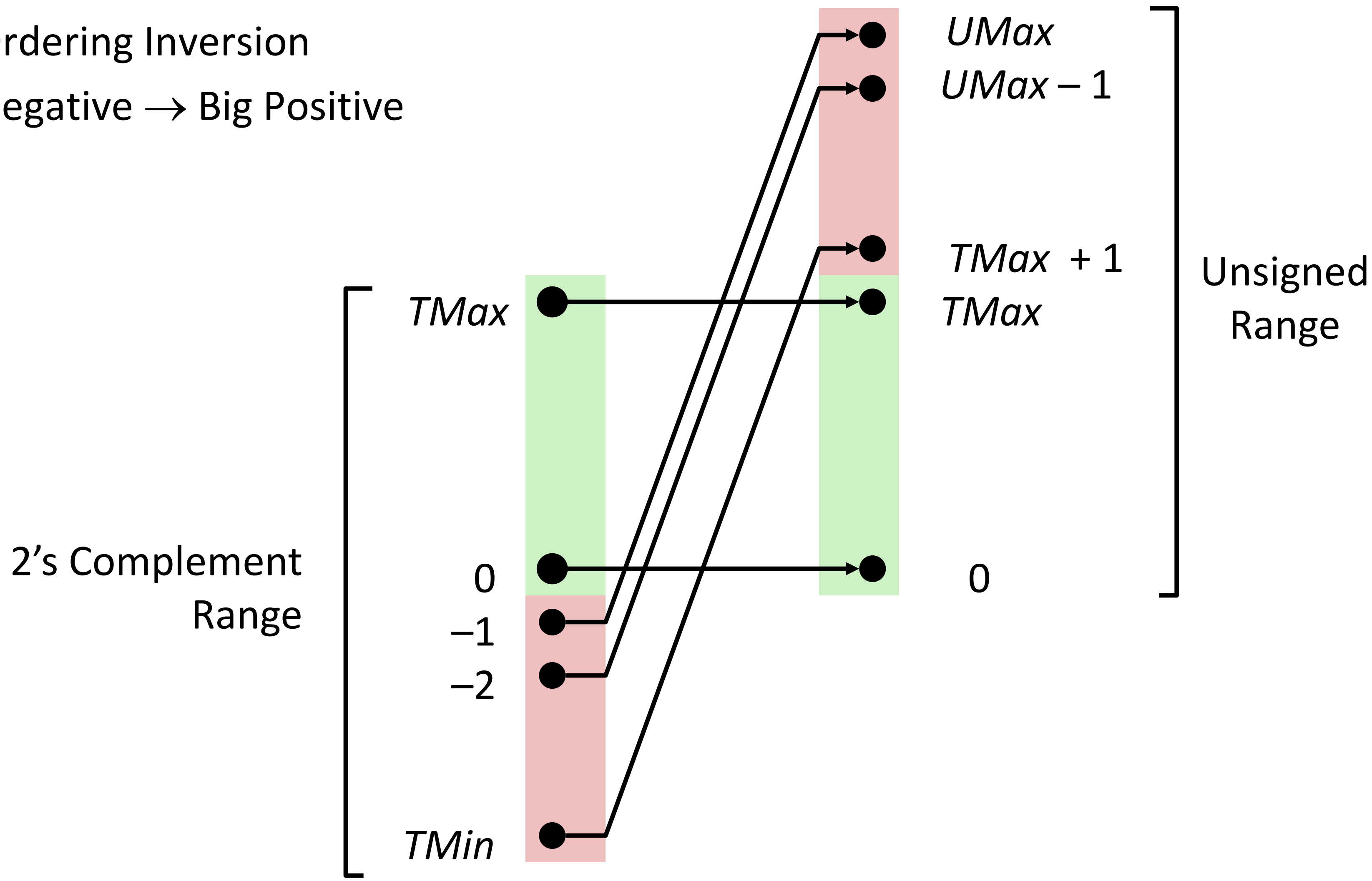




Large negative weight
becomes
Large positive weight

Conversion Visualized

- 2's Comp. → Unsigned
  - Ordering Inversion
  - Negative → Big Positive



Signed vs. Unsigned in C

- Constants
  - By default are considered to be signed integers
  - Unsigned if have "U" as suffix

```
0U, 4294967259U
```

- Casting
  - Explicit casting between signed & unsigned same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

  - Implicit casting also occurs via assignments and procedure calls

```
tx = ux;
uy = ty;
```

```
int fun(unsigned u);
uy = fun(tx);
```

Casting Surprises

- Expression Evaluation
  - If there is a mix of unsigned and signed in single expression, ***signed values implicitly cast to unsigned***
  - Including comparison operations **<, >, ==, <=, >=**
  - Examples for *W* = 32: **TMIN = -2,147,483,648 ,** **TMAX = 2,147,483,647**

Constant$_1$	Constant$_2$	Relation	Evaluation
0	0U	==	unsigned
-1	0	<	signed
-1	0U	>	unsigned
2147483647	-2147483647-1	>	signed
2147483647U	-2147483647-1	<	unsigned
-1	-2	>	signed
(unsigned)-1	-2	>	unsigned
2147483647	2147483648U	<	unsigned
2147483647	(int) 2147483648U	>	signed

Summary
Casting Signed ↔ Unsigned: Basic Rules

- Bit pattern is maintained
- But reinterpreted
- Can have unexpected effects: adding or subtracting 2^w

- Expression containing signed and unsigned int
  - `int` is cast to `unsigned`!!

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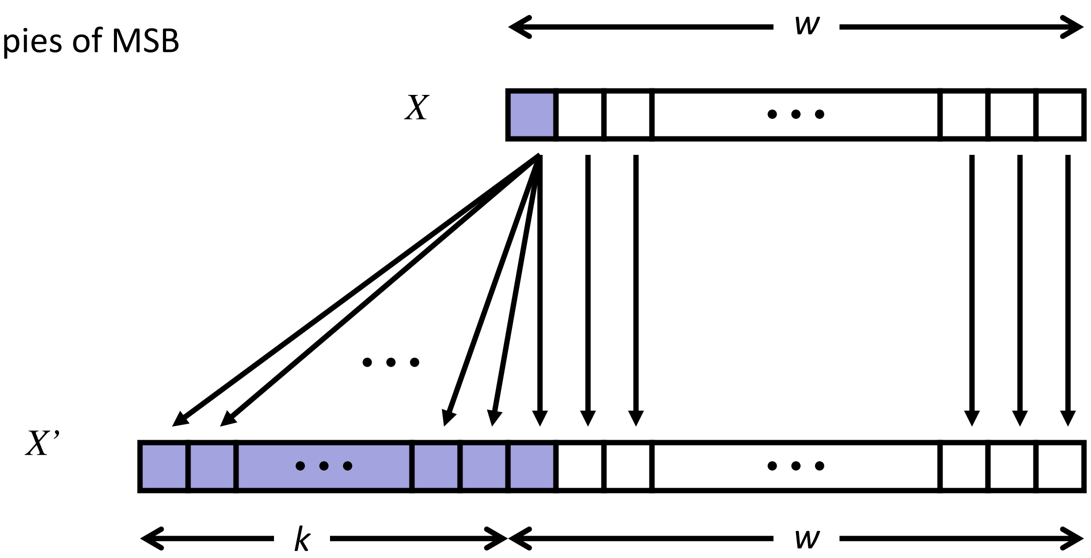
Sign Extension

- Task:
  - Given *w*-bit signed integer *x*
  - Convert it to *w*+*k*-bit integer with same value
- Rule:
  - Make *k* copies of sign bit:
  - $X' = x_{w-1}, \ldots, x_{w-1}, x_{w-1}, x_{w-2}, \ldots, x_0$

k copies of MSB



Sign Extension: Simple Example

Positive number

	-16	8	4	2	1
10 =	0	1	0	1	0

	-32	16	8	4	2	1
10 =	0	0	1	0	1	0

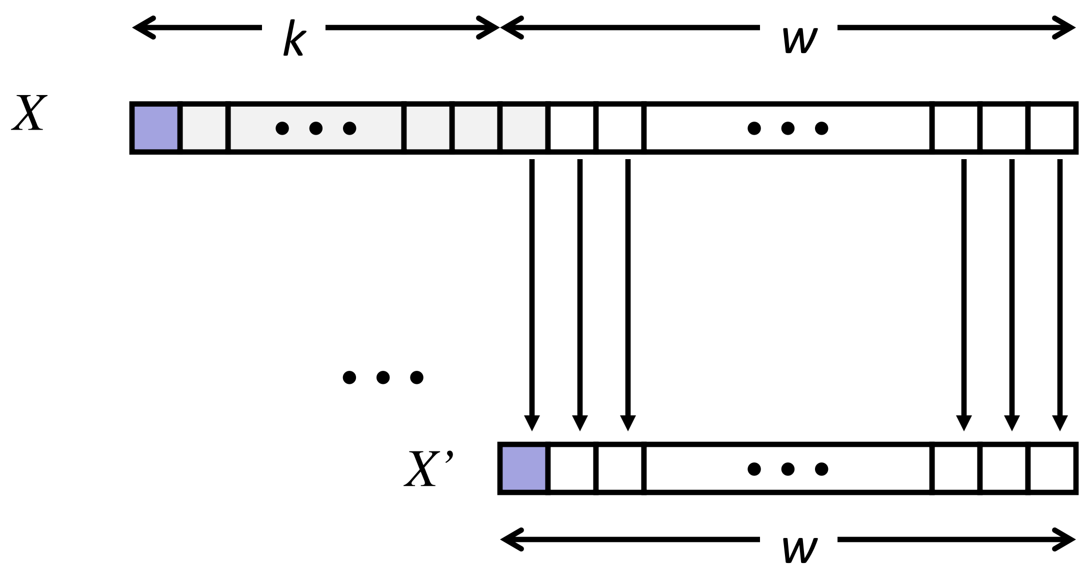
Negative number

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-32	16	8	4	2	1
-10 =	1	1	0	1	1	0

Truncation

- Task:
  - Given k+*w*-bit signed or unsigned integer *X*
  - Convert it to *w*-bit integer X' with same value for "small enough" X
- Rule:
  - Drop top *k* bits:
  - $X' = x_{w-1}, x_{w-2}, \ldots, x_0$



Truncation: Simple Example

No sign change

	-16	8	4	2	1
2 =	0	0	0	1	0

	-8	4	2	1
2 =	0	0	1	0

2 mod 16 = 2

	-16	8	4	2	1
-6 =	1	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

-6 mod 16 = 26U mod 16 = 10U = -6

Sign change

	-16	8	4	2	1
10 =	0	1	0	1	0

	-8	4	2	1
-6 =	1	0	1	0

10 mod 16 = 10U mod 16 = 10U = -6

	-16	8	4	2	1
-10 =	1	0	1	1	0

	-8	4	2	1
6 =	0	1	1	0

-10 mod 16 = 22U mod 16 = 6U = 6

Summary:
Expanding, Truncating: Basic Rules

- Expanding (e.g., short int to int)
  - Unsigned: zeros added
  - Signed: sign extension
  - Both yield expected result

- Truncating (e.g., unsigned to unsigned short)
  - Unsigned/signed: bits are truncated
  - Result reinterpreted
  - Unsigned: mod operation
  - Signed: similar to mod
  - For small (in magnitude) numbers yields expected behavior

Today: Bits, Bytes, and Integers

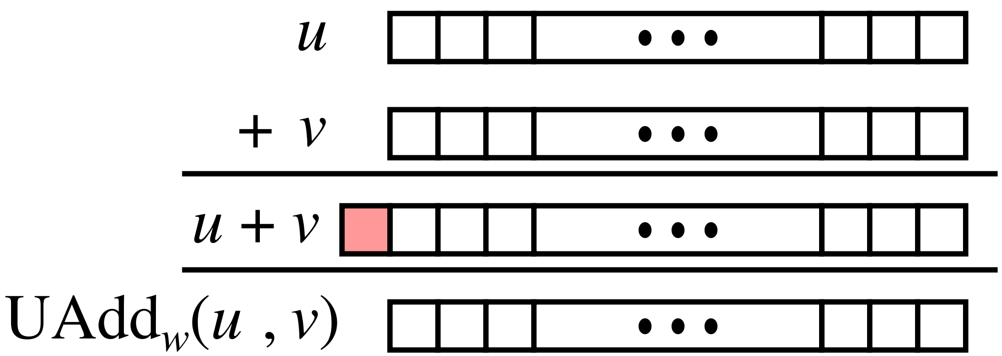
- Representing information as bits
- Bit-level manipulations
- Integers
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - **Addition, negation, multiplication, shifting**
- Byte Ordering

Unsigned Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- Standard Addition Function
  - Ignores carry output
- Implements Modular Arithmetic

$s \quad = \quad \text{UAdd}_w(u , v) \quad = \quad u + v \mod 2^w$

```
unsigned char      1110 1001        E9         223
                 + 1101 0101      + D5       + 213
                 1 1011 1110       1BE         446
                   1011 1110        BE         190
```

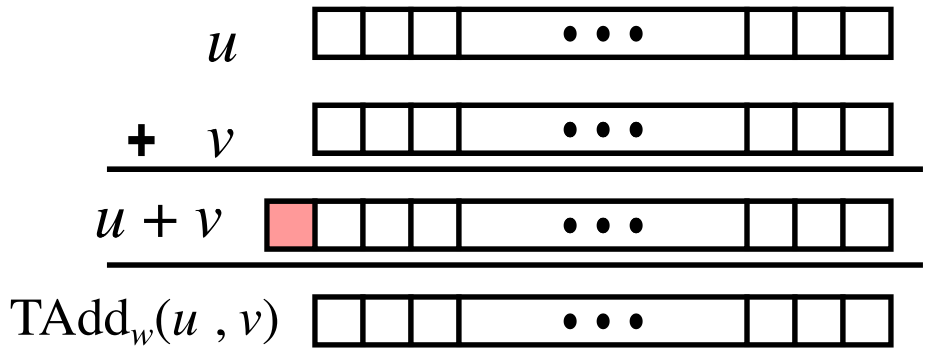
Hex	Decimal	Binary
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Two's Complement Addition

Operands: *w* bits

True Sum: *w*+1 bits

Discard Carry: *w* bits



- **TAdd and UAdd have Identical Bit-Level Behavior**
  - Signed vs. unsigned addition in C:

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

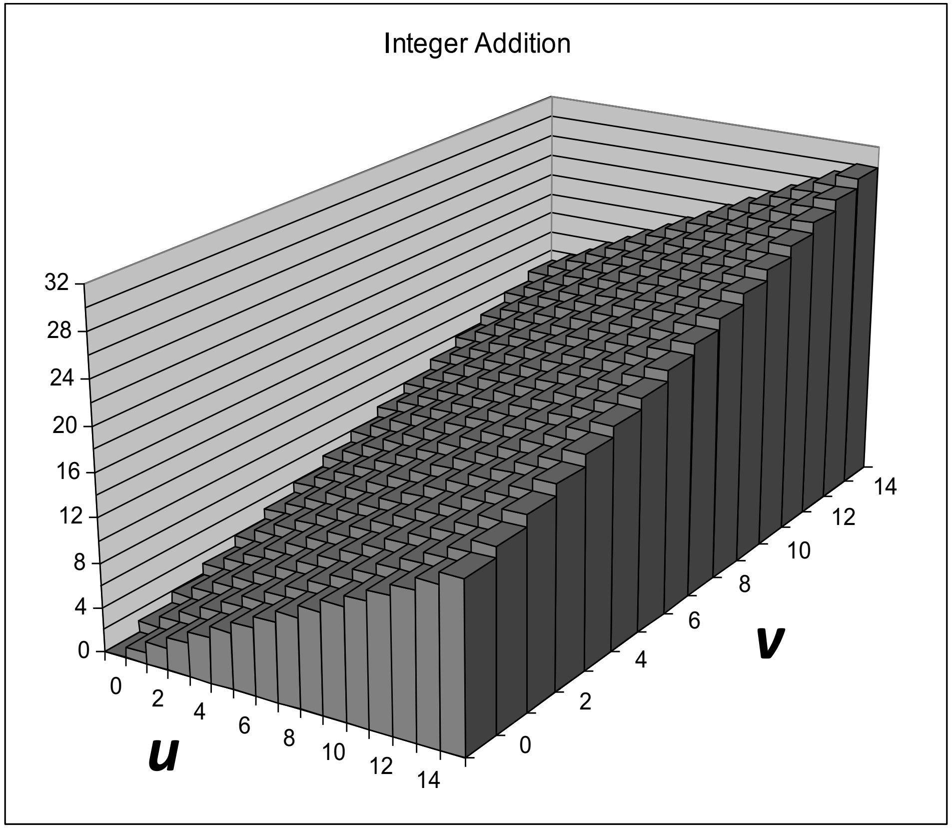
  - Will give `s == t`

1110 1001	E9	-23
+ 1101 0101	+ D5	+ -43
1 1011 1110	1BE	-66
1011 1110	BE	-66

Visualizing "True Sum" Integer Addition

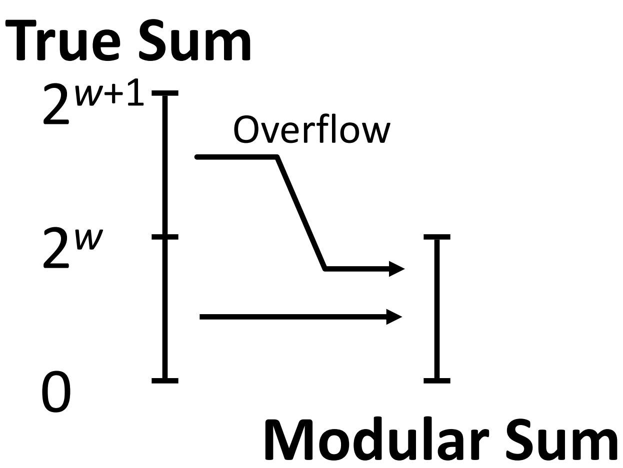
- Integer Addition
  - 4-bit integers *u*, *v*
  - Compute true sum $Add_4(u, v)$
  - Values increase linearly with *u* and *v*
  - Forms planar surface

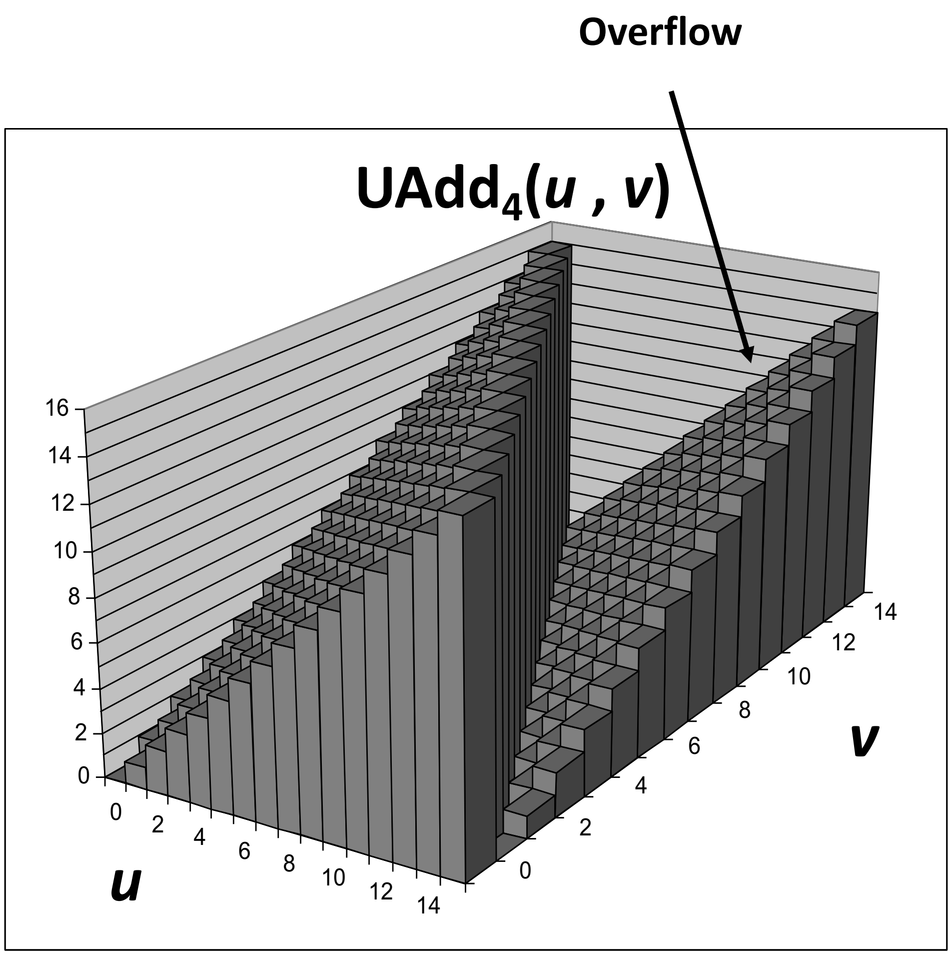
$Add_4(u, v)$



Visualizing Unsigned Addition

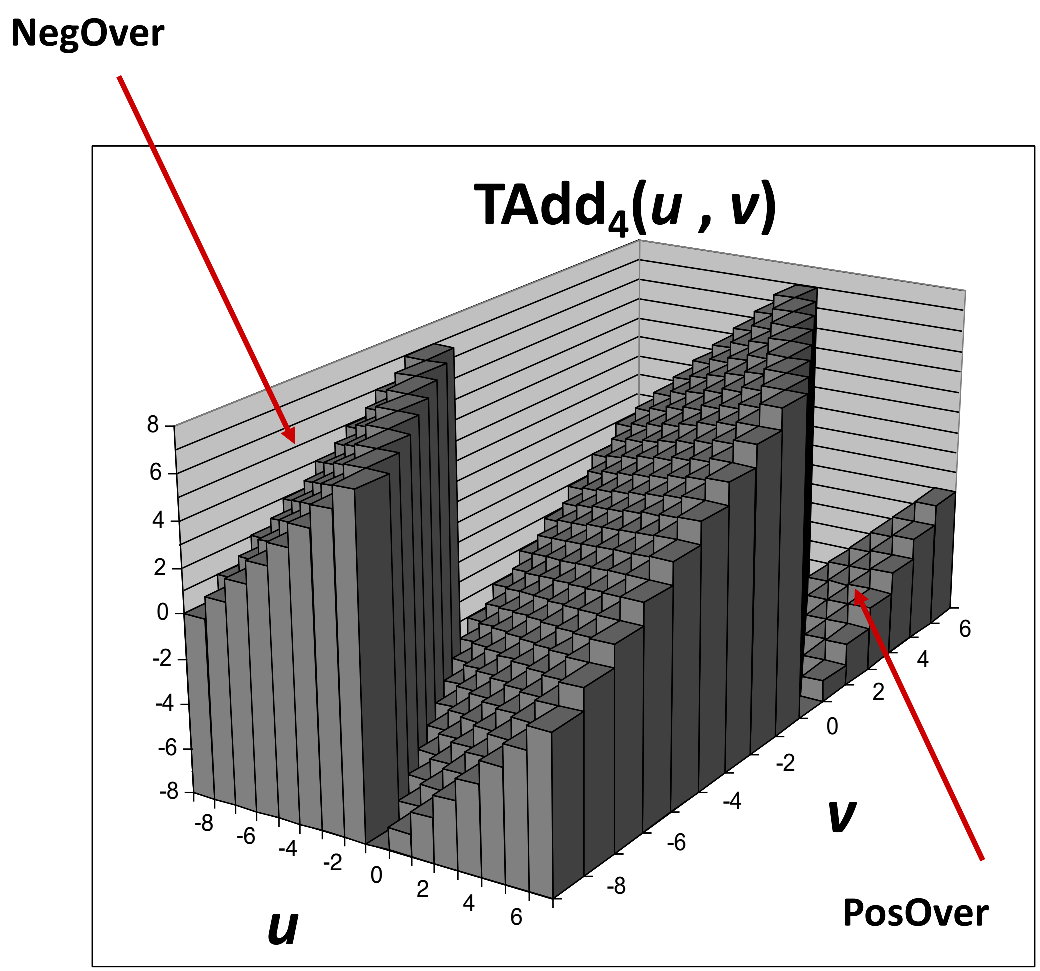
- Wraps Around
  - If true sum ≥ 2^w
  - At most once





Visualizing 2's Complement Addition

- Values
  - 4-bit two's comp.
  - Range from -8 to +7
- Wraps Around
  - If sum $\geq 2^{w-1}$
    - Becomes negative
    - At most once
  - If sum $< -2^{w-1}$
    - Becomes positive
    - At most once

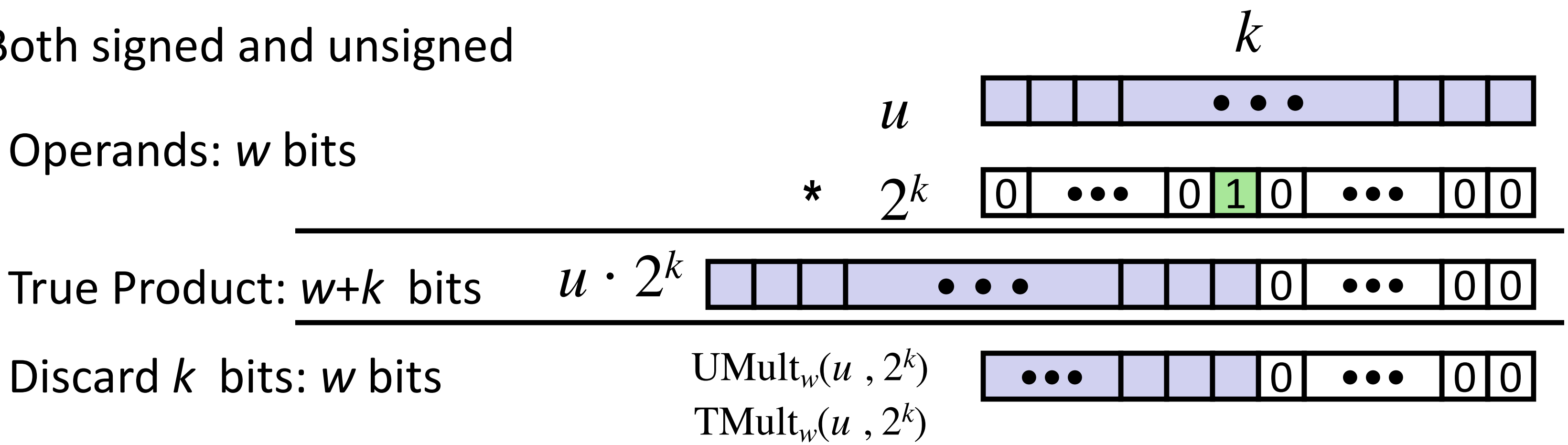


Multiplication

- Goal: Computing Product of *w*-bit numbers *x*, *y*
  - Either signed or unsigned
- Result: Same as computing ideal, exact result *x*y* and keeping *w* lower bits.
- Ideal,exact results can be bigger than *w* bits
  - Worst case is up to 2w bits
    - Unsigned, because all bits are magnitude
    - Signed, but only for Tmin*Tmin, because anything added to Tmin reduces its magnitude and Tmax is less than Tmin.
- So, maintaining exact results…
  - would need to keep expanding word size with each product computed
  - Impossible in hardware (at least without limits), as all resources are finite
  - In practice, is done in software, if needed
    - e.g., by "arbitrary precision" arithmetic packages

Power-of-2 Multiply with Shift

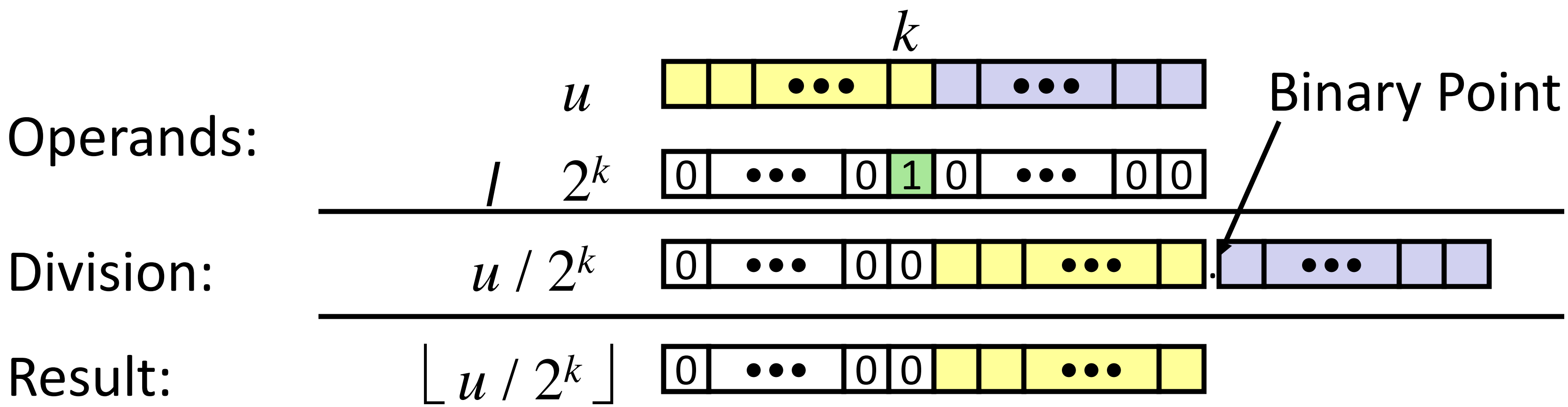
- Operation
  - `u << k` gives `u * 2`^{k}
  - Both signed and unsigned



- Examples
  - `u << 3 == u * 8`
  - `(u << 5) – (u << 3) == u * 24`
  - Most machines shift and add faster than multiply
    - Compiler generates this code automatically

Unsigned Power-of-2 Divide with Shift

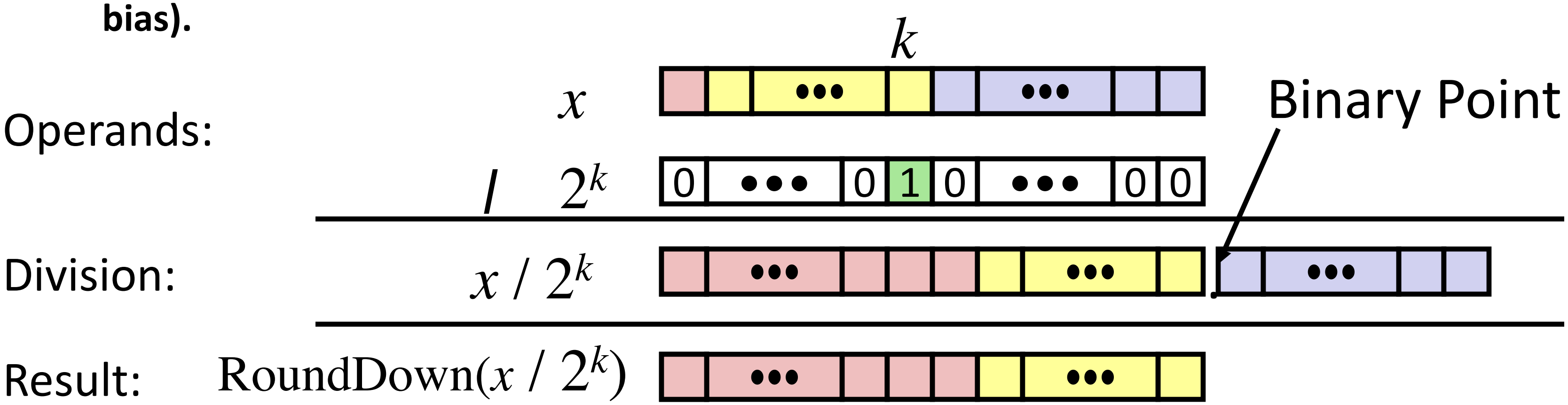
- Quotient of Unsigned by Power of 2
  - `u >> k` gives $\lfloor \texttt{u} / 2^k \rfloor$
    - **Uses logical shift**



	Division	Computed	Hex	Binary
`x`	15213	15213	`3B 6D`	`00111011 01101101`
`x >> 1`	7606.5	7606	`1D B6`	`00011101 10110110`
`x >> 4`	950.8125	950	`03 B6`	`00000011 10110110`
`x >> 8`	59.4257813	59	`00 3B`	`00000000 00111011`

Signed Power-of-2 Divide with Shift

- Quotient of Signed by Power of 2
  - `x >> k` gives $\lfloor x / 2^k \rfloor$
    - **Uses arithmetic shift**
    - **Rounds to the left, not towards zero (Unlikely to be what is expected, introduces a bias).**

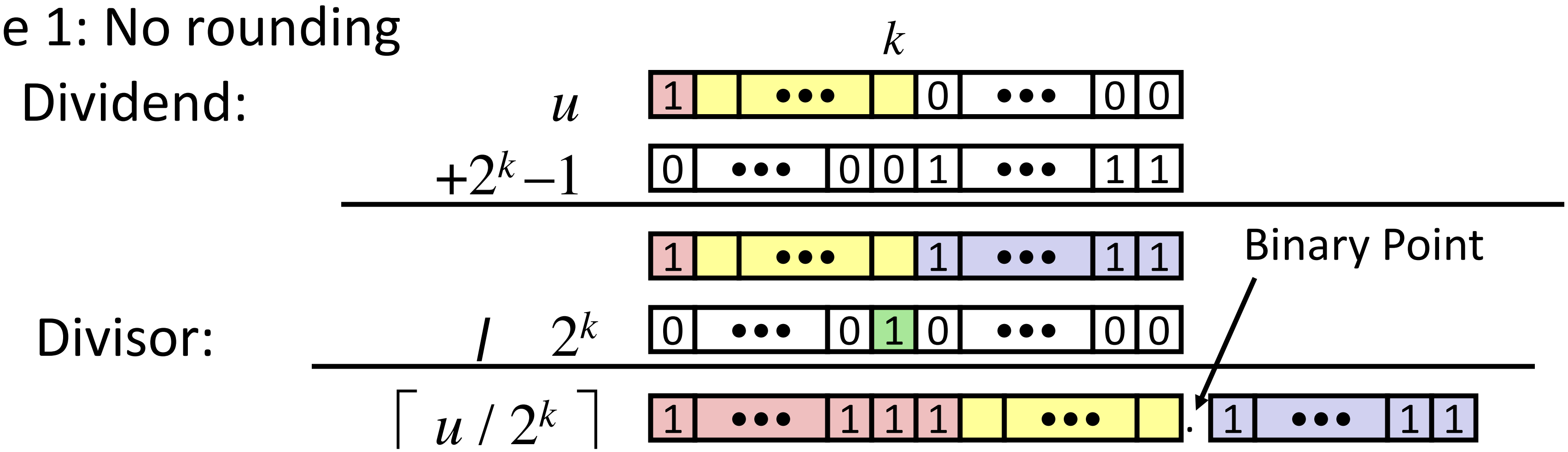


	Division	Computed	Hex	Binary
`x`	-15213	-15213	`C4 93`	`11000100 10010011`
`x >> 1`	-7606.5	-7607	`E2 49`	`11100010 01001001`
`x >> 4`	-950.8125	-951	`FC 49`	`11111100 01001001`
`x >> 8`	-59.4257813	-60	`FF C4`	`11111111 11000100`

Round-toward-0 Divide

- Quotient of Negative Number by Power of 2
  - Want $\lceil$ `x / 2`^{k} $\rceil$ (Round Toward 0)
  - Compute as $\lfloor$ `(x+(2`^{k}`-1))/ 2`^{k} $\rfloor$
    - In C: `(x + (1<<k)-1) >> k`
    - Biases dividend toward 0

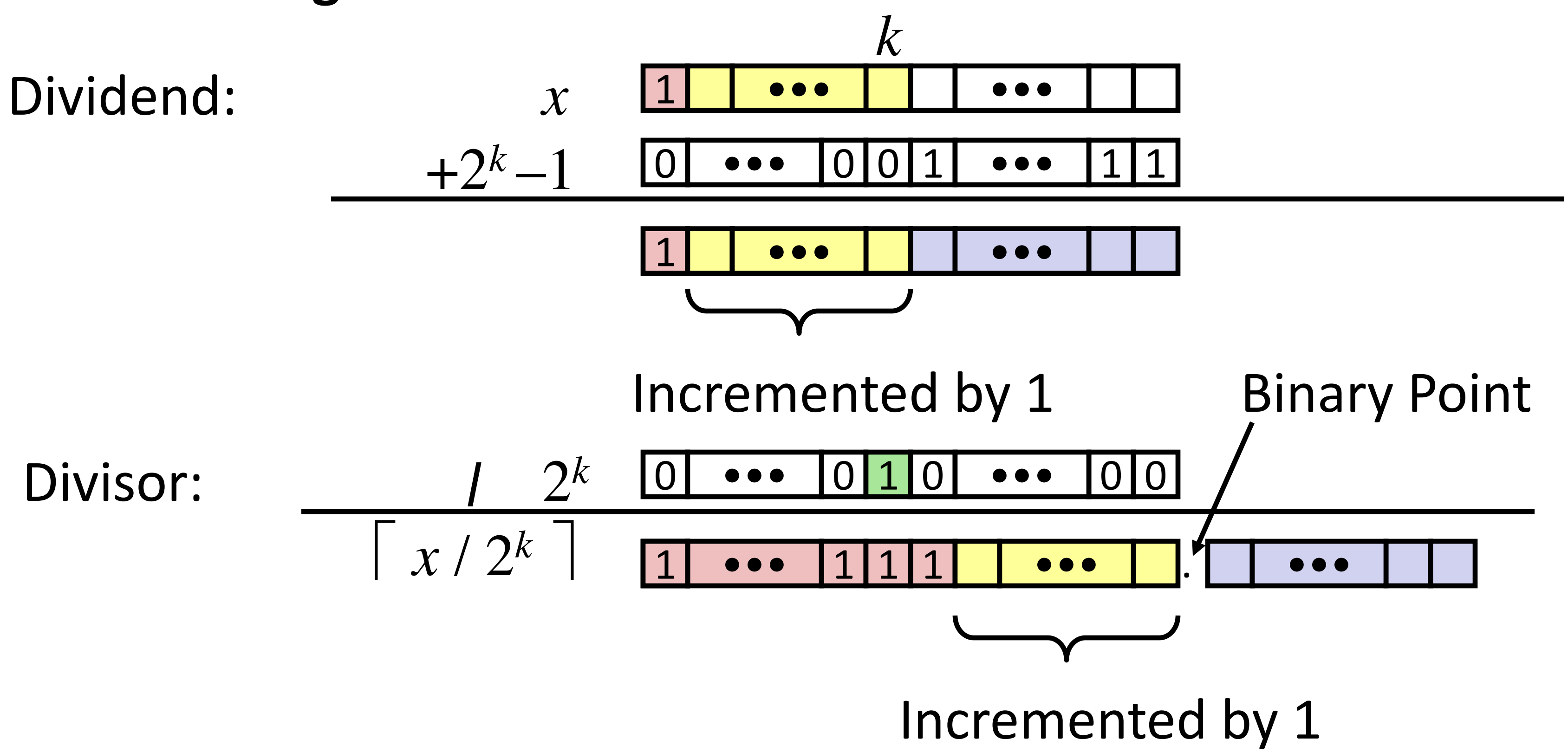
Case 1: No rounding



Biasing has no effect

Correct Power-of-2 Divide (Cont.)

Case 2: Rounding



Biasing adds 1 to final result

Today: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
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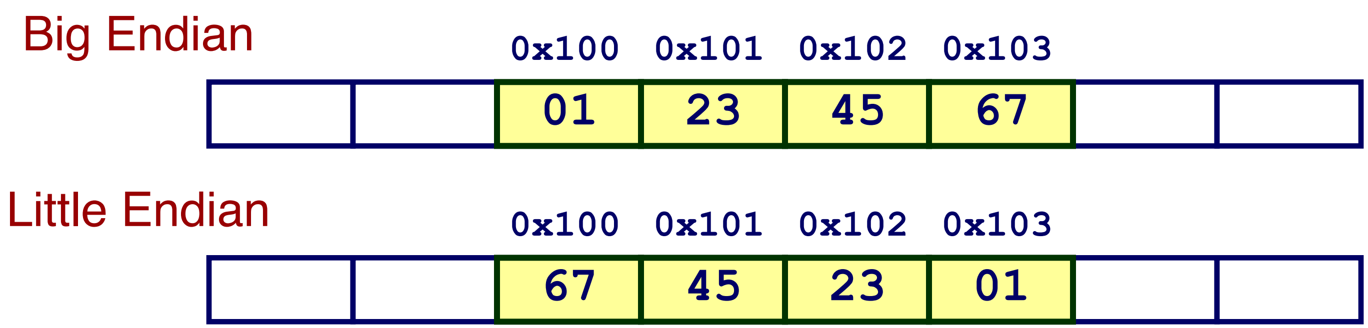
- Byte Ordering

Byte Ordering

- So, how are the bytes within a multi-byte word ordered in memory?
- Conventions
  - Big Endian: Sun (Oracle SPARC), PPC Mac, *Internet*
    - Least significant byte has highest address
  - Little Endian: *x86*, ARM processors running Android, iOS, and Linux
    - Least significant byte has lowest address
- Becomes a concern when data is communicated
  - Over a network, via files, etc.
- Important notes
  - Bits are not reversed, as the low order bit is the reference point.
  - Doesn't affect chars, or strings (arrays of chars), as chars are only one byte

Byte Ordering Example

- Example
  - Variable x has 4-byte value of 0x01234567
  - Address given by &x is 0x100



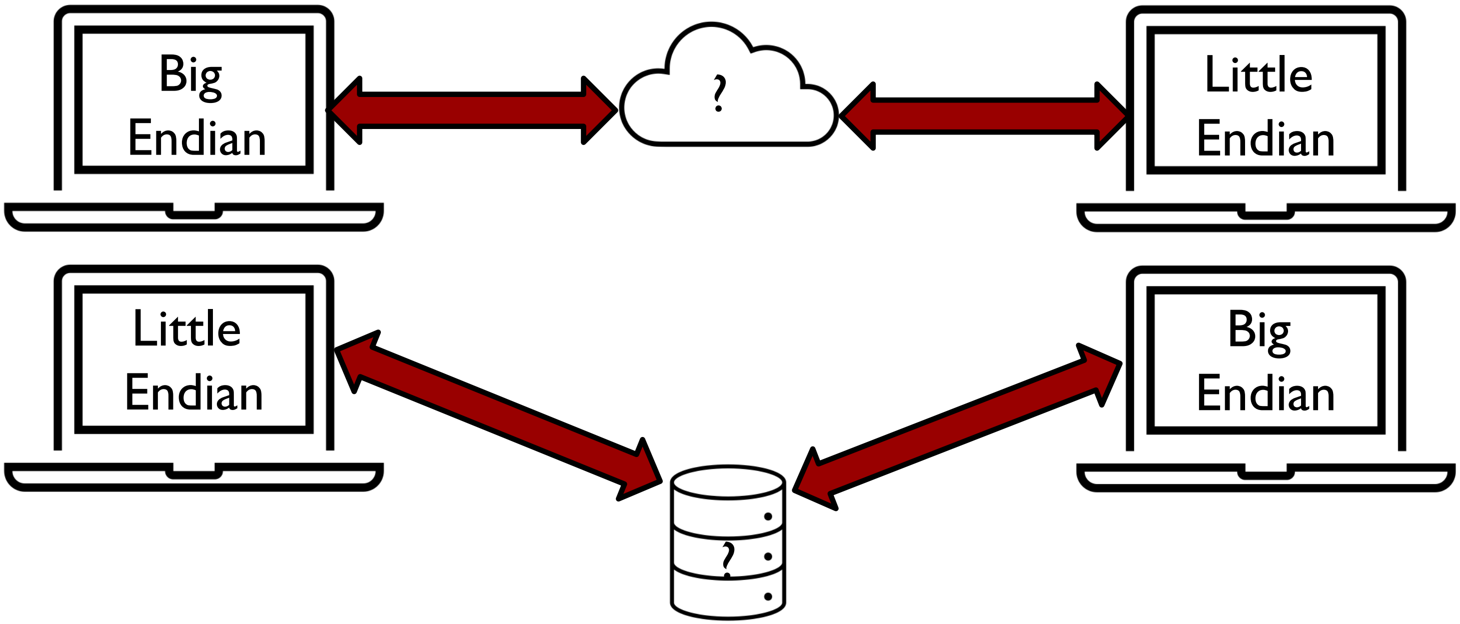
Reading Byte-Reversed Listings

- Disassembly
  - Text representation of binary machine code
  - Generated by program that reads the machine code
- Example Fragment

Address	Instruction Code	Assembly Rendition
8048365:	5b	pop %ebx
8048366:	81 c3 ab 12 00 00	add $0x12ab,%ebx
804836c:	83 bb 28 00 00 00 00	cmpl $0x0,0x28(%ebx)

- Deciphering Numbers
  - Value: `0x12ab`
  - Pad to 32 bits: `0x000012ab`
  - Split into bytes: `00 00 12 ab`
  - Reverse: `ab 12 00 00`

Byte Ordering: Data in Motion



- What happens when one host needs to send data to another host with a differing byte ordering convention?
- What about when values are stored in files by one host, but intended to be read by any host, e.g. the number of rows or columns or the pallet size of an image?
- There needs to be a convention for communication or storage, e.g. network byte ordering, image file format specification, etc.
  - One host, or the other, or both, or neither, may need to convert upon writing/transmitting and/or reading/receiving.

Today: Bits, Bytes, and Integers

- Representing information as bits — CSAPP 2.1
- Bit-level manipulations
- Integers — CSAPP 2.2
  - Representation: unsigned and signed
  - Conversion, casting
  - Expanding, truncating
  - Addition, negation, multiplication, shifting — CSAPP 2.3
- Byte Ordering — CSAPP 2.1.3

Questions?