Written Assignment W1

Computer Graphics 2 (15-463)
13 Feb. 1995

Due noon Monday, 20 Feb. 1995 at my office (DH 4301A).

1. [20 pts.] Filter the discrete signal $a[t]$ with a box filter $h[t]$ of odd width N, yielding a signal $b[t]$. That is, $h[t] = 1/N$ for $|t| \leq M$, and $h[t] = 0$ for $|t| > M$, and $N = 2M + 1$.
a) For $M = 1$, write a general formula for $b[t]$ as a finite, weighted sum of terms of $a[t]$.
b) For large N, computing b gets expensive. Suppose you need to compute $b[t]$ for $t = 0 \ldots L$ for very large L. Find a fast way to compute it where the cost per output sample is constant, not $O(N)$. Hint: Write out the formula for $b[t+1] - b[t]$.
c) Describe, roughly, how this can be generalized to 2-D, to allow $N \times N$ box filter blur of an $L \times L$ image to be computed in constant time per output pixel. Don't worry too much about the perimeter of the image. How much extra memory will your fast algorithm require?

2. [10 pts.] Compute the 4-point DFT's of each of the following four sequences: $[1,0,0,0]$, $[1,1,0,0]$, $[1,0,1,0]$, $[1,1,1,1]$.

3. [20 pts.] Find the continuous Fourier transform of $f(x) = K \sin(\alpha x + \phi)$ and plot the magnitude of $F(\omega)$. Hint: first find the inverse Fourier transforms of $B\delta(\omega - \alpha)$ and $C\delta(\omega + \alpha)$, using the sifting property of delta functions given in Notes 7.

4. [30 pts.] As a notation for a finite sequence of nonzeros, we'll use $[p, q, r]$ as shorthand for the sequence $[\ldots, 0, 0, 0, p, q, r, 0, 0, 0, \ldots]$. With this notation, $[p, q] \circledast [r, s] = [pr, ps + qr, qs]$. (You might want to verify this for yourself with the general formula below.)

This problem explores the fast computation of linear convolution. Recall that linear convolution of f and g is:

$$a[x] = \sum_{t=-\infty}^{\infty} f[t]g[x - t] \text{ for any x}$$

and circular convolution is:

$$b[x] = \sum_{t=0}^{N-1} f[t]g[(x - t) \bmod N] \text{ for } x = 0 \ldots N - 1$$

Linear convolution differs from circular convolution because the latter assumes the signals are periodic, and its filtering wraps around.
a) Find the linear convolution and circular convolution of the signals f=[1, 1, 1, 1] and g=[1, 1, 1, 1] (in this case, $N = 4$).
b) For f and g of arbitrary length N, where values off either end are assumed to be zero, how long is their linear convolution (the nonzero portion of it)?
c) If we pad length-N signals f and g with enough zeros (lengthen the arrays, filling with zeros), then we can compute linear convolution using circular convolution. How many zeros are necessary?
d) In high level pseudocode, write out an algorithm for computing linear convolution of length-N signals f and g quickly using circular convolution. You can call FFT and InverseFFT as subroutines. It should take N, f, and g as inputs and produce a as output.
e) How fast is it (in big-Oh notation)?