Notes 9, Computer Graphics 2, 21 Feb. 1995

Glossary of Signal Processing Terms

signal: a function of space or time, e.g. a picture is a 2-D signal.
digital signal: discrete in space and value.
analog signal: continuous in space and value.
DSP: digital signal processing

filter: operator that takes a signal as input and produces a signal as output.
linear filter: S is a linear filter if $S[\alpha f(x) + \beta g(x)] = \alpha S[f](x) + \beta S[g](x)$ for any α, β.
linear shift-invariant filter (LSI): S is linear, shift-invariant if it is linear and shifted input $f(x+t)$ yields output $S[f](x+t)$ for any shift t. Equivalently, if $S[f]$ can be written as the convolution of f with some impulse response h. In this case, we would write $S[f](x) = (f \circledast h)(x)$ or $S[f] = f \circledast h$.
convolution: integral (if signals are continuous) or sum (if discrete) of product of signal and shifted filter. See notes for precise formulas.
nonlinear filter: a filter that is non linear, e.g. computes products, squares, or square roots of sample values. Examples: Sobel edge filter, median filter. Less common than LSI filters.

low pass filter: filter that passes (allows through) low frequencies (those below the cutoff frequency) and stops (kills) high frequencies. Its frequency response will have a box-like shape. A sinc is the "ideal" low pass filter. Synonym: blur.
high pass filter: filter that passes high frequencies and stops low frequencies.
support: the domain over which a filter's impulse response is nonzero. Larger support typically makes a filter more expensive.
FIR filter: a Finite Impulse Response filter; one with finite support. Straightforward to compute. Examples: box, triangle.
IIR filter: an Infinite Impulse Response filter; one with infinite support. Generally harder to compute and less common than FIR filters. Computed using recursive filters, if they're computed at all. Examples: sinc, Gaussian.
recursive filter: a filter whose output is written in terms of previous samples of its output, e.g. $b[x] = \alpha b[x-1] + a[x]$.

box: a function that is 0 except for some interval around 0, within which it is constant. In the spatial domain, a box is often used as a cheap blur filter. A box in

the frequency domain makes an ideal low pass filter.
triangle: a function that is 0 except for some interval around 0, within which its shape is an isosceles triangle. Standard definition: triangle = box$\circledast$box.
Gaussian: a.k.a. the "normal probability density function", given by $Ae^{-x^2/(2\sigma^2)}$, where σ=standard deviation. To get unit integral, use $A = 1/(\sigma\sqrt{2\pi})$. Sometimes used as a low pass filter.
sinc: the "ideal" low pass filter. $\mathrm{sinc}(x) = \sin(\pi x)/(\pi x)$. Has value 0 for all integer x except $x = 0$, where it has value 1.

spatial domain: in image processing, the domain of pixel coordinates x and y. In audio processing, you speak of the "time domain" instead.
frequency domain: in image processing, the domain of horizontal and vertical frequencies, ω_x and ω_y.
angular frequency: frequency in radians per unit distance, typically denoted ω.
rotational frequency: frequency in cycles per unit distance, typically denoted f, where $\omega = 2\pi f$.

Fourier Transform: transformation of a signal into an integral of complex exponentials of all frequencies.
convolution theorem: states that convolution in the spatial domain is equivalent to multiplication in the frequency domain.
DFT: discrete Fourier transform. A variant of the Fourier transform for discrete, periodic signals. Involves finite sums, not infinite integrals.
FFT: fast Fourier transform. A fast algorithm for computing the DFT.
complex exponential: $e^{i\theta} = \cos\theta + i\sin\theta$ where $i = \sqrt{-1}$. A complex-valued sinusoid.
roots of unity: values of $z = e^{i\theta}$ for which $z^n = 1$ for some integer n. These complex constants are coefficients in the DFT.
butterfly: a computation step of the FFT involving one complex multiply and two complex add/subtracts.

impulse, delta function: a function $\delta(x)$ with value 0 everywhere, except at $x = 0$. It has unit integral. The continuous delta function has value infinity at $x = 0$, while the discrete delta function has value 1 at $x = 0$.
impulse response, point spread function: output of a filter when given an impulse as input. Inverse Fourier transform of a filter's frequency response.
gain: ratio of output amplitude to input amplitude.
frequency response: function which gives the gain of a filter as a function of frequency. (A complex function.) The Fourier transform of the filter's impulse response.
magnitude response: magnitude of the frequency response as a function of frequency.
phase response: phase of the frequency response as a function of frequency: atan(imaginary

part/real part).
dB (decibels): unitless measure of a number with a large dynamic range. $\mathrm{dB}(x) = -20\log_{10}(x)$. Popular with electrical engineers. Frequency responses and signal to noise ratios are often measured in decibels. A factor of 2 is about 6 dB.

sampling: conversion of a continuous signal to a discrete signal.
impulse train: a sequence of uniformly-spaced, equal-height impulses.
spectrum: Fourier transform of a signal.
bandwidth: highest frequency in a bandlimited signal.
bandlimited: a signal that has a highest frequency.
sampling frequency: frequency of samples, $\omega_s = 2\pi/T$ where T is the sample spacing.
Nyquist frequency: half the sampling frequency. Frequencies above this will alias if the signal is sampled.
aliasing: masquerading of high frequencies as low frequencies that occurs when a signal is improperly sampled.

reconstruction: conversion of a discrete signal into a continuous signal.
resampling: sampling a discrete signal again at a new set of sample points.
decimation: downsampling, scaling a signal down (samples farther apart than previously).
interpolation: upsampling, scaling a signal up (samples closer together than previously).
point sampling: sampling a signal at a single point, as opposed to filtering before sampling. Also known as "pixel replication", "sample and hold", or "box reconstruction".
rastering: an artifact of poor reconstruction in which the sampling grid is evident. It's bad when point sampling is used, better for bilinear interpolation.