Algorithms, Winter 2020 at CIS

Homework 3

1. Let $A \in \mathbb{R}^{m \times n}$ be a matrix and $b \in \mathbb{R}^{m \times 1}$ be a vector. Prove that the set $\{x : Ax = b\}$ is convex.

2. Is $\max \left(\log \left(\frac{1}{(a^T x + b)^7} \right), \|Ax + b\|_2^5 \right)$ convex on its domain? Here the entries of a are non-negative, b is positive, and the entries of x are non-negative. Remember that $\|y\|_2 = \left(\sum_{i=1}^{n} y_i^2 \right)^{1/2}$ for a vector y.

3. Suppose B is the unit box in $\mathbb{R}^2$, so it consists of (x, y) for which $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Suppose C is the unit circle in $\mathbb{R}^2$ so it consists of (x, y) for which $x^2 + y^2 \leq 1$. Say which of $B \cap C$, and $B \setminus C$ are convex (there may be none or more than one).