15-150 Fall 2024

Lecture 9

Types and Polymorphism

Announcement

Sections A-D go to **MM 103**
Sections E-L go to **PH 100**

Types in Programming

- Program organization and documentation
- Making sure bit sequences in memory are interpreted correctly
- Providing information to the compiler

Goals for today

- Apply type-checking rules for ML expressions
- State what it means for a function to be **polymorphic**
- Determine **the most general type** for a given expression
- Define **parameterized datatypes** and use them correctly

Type safety

A static check provides a runtime guarantee (modulo termination)

static guarantee	runtime guarantee
e has type t	if e ==> v then v : t

Type Analysis

- There are ***syntax-directed*** rules for figuring out when e has type t.

e is well-typed, with type t, if and only if this is ***provable*** from these rules.

We say “e has type t” or write “e : t”,
possibly with assumptions like “x : int and y : int”

Polymorphism

Monomorphic rev

```
fun rev ([ ]:int list) :int list = [ ]
  | rev (x::xs) = rev xs @ [x]
```

(x::xs): t list if x:t and xs: t list

datatype __ list = nil |:: **of** __* __ list (x::xs): t list if x:t and xs: t list

datatype 'a list = nil |:: **of** 'a * 'a list

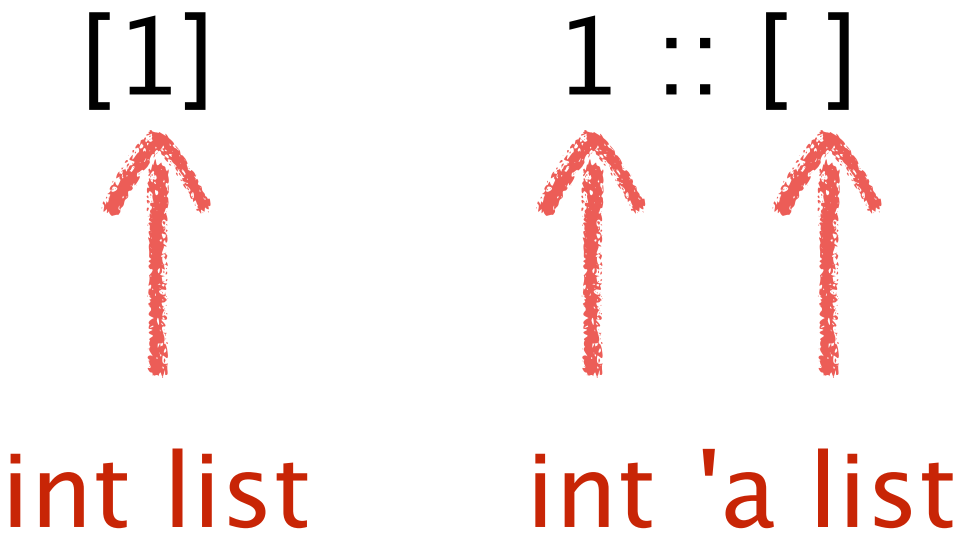
↑

“alpha”

infixr ::

[ ]:   'a list

[ ]: 'a list



[1]

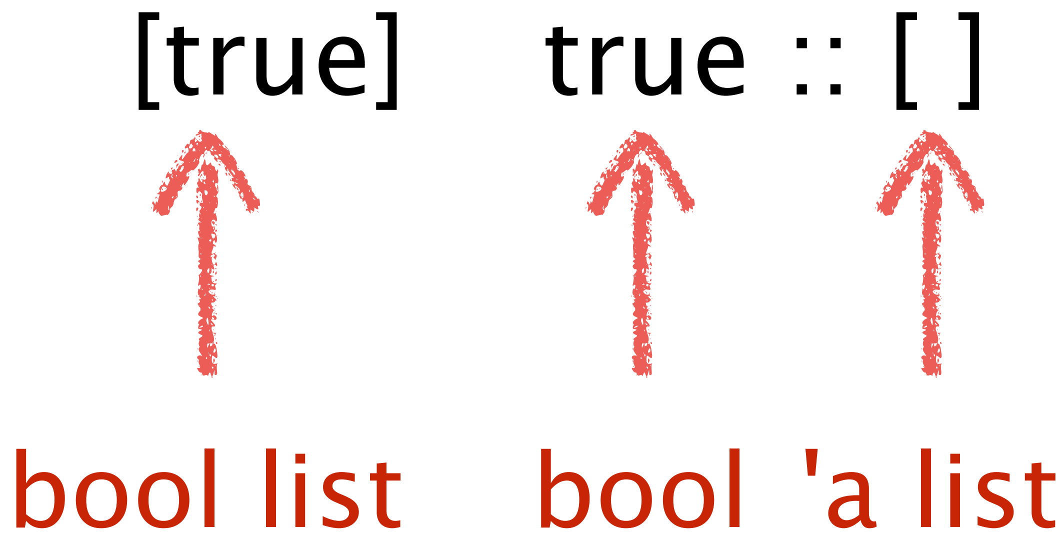
1 :: [ ]

int list

int 'a list

'a specialized to/instantiated as int, giving us int list

[ ]: 'a list

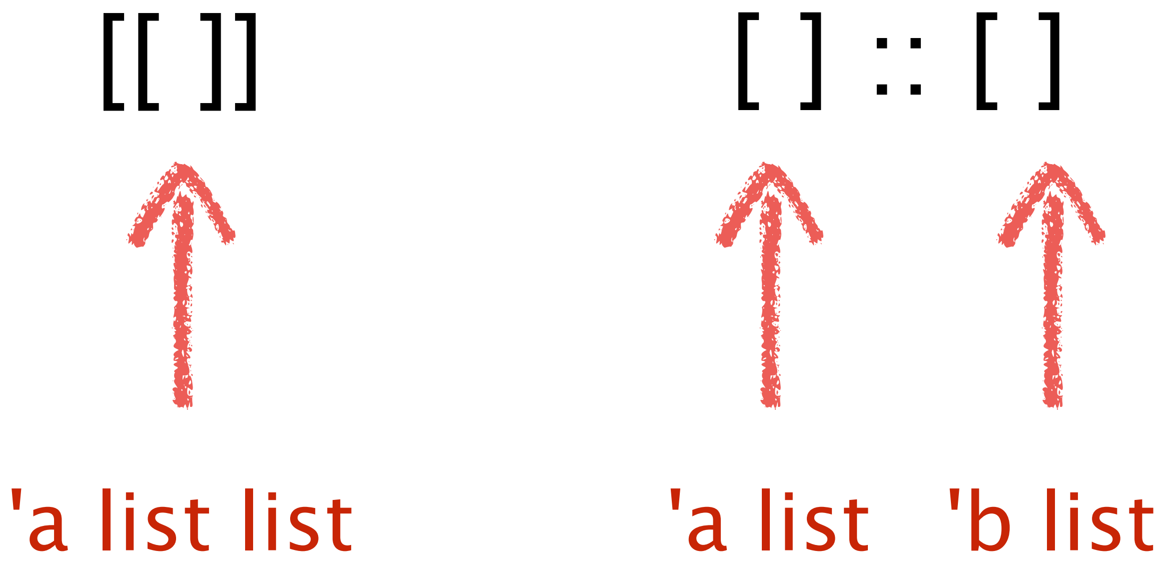


[true] true :: [ ]

bool list bool 'a list

'a specialized to/instantiated as bool, giving us bool list

'a list
is an instance of 'b



[[ ]]

[ ] :: [ ]

'a list list

'a list

'b list

'b specialized to/instantiated as 'a list, giving us list 'a list list

Polymorphic rev

```
fun rev ([ ]:'a list) : 'a list= [ ]
   | rev (x::xs) = rev xs @ [x]
```

In the scope of this declaration you can use rev with any list as an argument.

([ ],[ ]):	'a list * 'b list
([1]::[[ ]]):	int list list
(1::[[ ]]):	not well–typed

Parameterized datatypes

```
datatype 'a tree = Empty
                 |Node of 'a tree * 'a * 'a tree
```

introduces a type constructor (tree)

and polymorphic value constructors Empty and Node:

```
Empty: 'a tree
Node: 'a tree * 'a * 'a tree -> 'a tree
```

Parameterized datatypes

```
datatype ('a,'b) mixed = A of 'a
                       | B of 'b
```

introduces a type constructor (mixed)

and polymorphic value constructors A and B:

```
A: 'a -> ('a, 'b) tree
B: 'b -> ('a, 'b) tree
```

Example

```
(* trav : 'a tree -> 'a list
   REQUIRES: true
   ENSURES:  trav(t) returns a list consisting of
             the elements in t, in the same order
             as seen during an in-order traversal of t.
*)

fun trav(Empty: 'a tree) : 'a list = [ ]
   | trav(Node(t1, x, t2)) = (trav t1) @ x :: (trav t2)
```

Example

```
(* trav : 'a tree -> 'a list
   REQUIRES: true
   ENSURES:  trav(t) returns a list consisting of
             the elements in t, in the same order
             as seen during an in-order traversal of t.
*)

fun trav(Empty: 'a tree) : 'a list = [ ]
    | trav(Node(t1, x, t2)) = (trav t1) @ x :: (trav t2)
```

trav (Node(Empty, 1, Empty)):   int list

zip

```
(* zip : 'a list * 'b list -> ('a * 'b) list
REQUIRES: true
ENSURES:  zip([a1,a2,...,an],[b1,b2,...,bm]) ≅
          [(a1,b1), (a2,b2), ..., (ak,bk)]
          with k = min(n,m) >= 0.
*)
```

```
(* zip : 'a list * 'b list -> ('a * 'b) list
REQUIRES: true
ENSURES:  zip([a1,a2,...,an],[b1,b2,...,bm]) ≅
          [(a1,b1), (a2,b2), ..., (ak,bk)]
          with k = min(n,m) >= 0.
*)

fun zip ([ ] : 'a list, B : 'b list) : ('a * 'b) list = [ ]
    | zip (A, [ ]) = [ ]
    | zip (a::A, b::B) =  (a,b)::zip(A,B)
```

```
(* zip : 'a list * 'b list -> ('a * 'b) list
   REQUIRES: true
   ENSURES:  zip([a1,a2,...,an],[b1,b2,...,bm]) ≅
             [(a1,b1), (a2,b2), ..., (ak,bk)]
             with k = min(n,m) >= 0.
*)

fun zip ([ ] : 'a list, B : 'b list) : ('a * 'b) list = [ ]
  | zip (A, [ ]) = [ ]
  | zip (a::A, b::B) =  (a,b)::zip(A,B)
```

```
  zip ([1,2,3,4,5],["a","b","c","d"]):          (int * string) list

evaluates to            [(1,"a"),(2,"b"),(3,"c"),(4,"d")]
```

options

```
datatype 'a option = NONE | SOME of 'a
```

lookup

```
(* lookup : ______________________________
   REQUIRES:
   ENSURES:



*)
```

```
(* lookup : ______________________________
  REQUIRES: true
  ENSURES:  lookup(eq, x, L) returns SOME(b) of the
            leftmost (a,b) in L for which eq(x,a) returns true, if
            there is  such an (a,b);
            returns NONE otherwise.
*)
```

```
(* lookup : _______________* 'a * ('a * 'b) list ->____________
   REQUIRES: true
   ENSURES:  lookup(eq, x, L) returns SOME(b) of the
             leftmost (a,b) in L for which eq(x,a) returns true, if
             there is  such an (a,b);
             returns NONE otherwise.
*)
```

```
(* lookup : ('a * 'a -> bool) * 'a * ('a * 'b) list -> __________
  REQUIRES: true
  ENSURES:  lookup(eq, x, L) returns SOME(b) of the
            leftmost (a,b) in L for which eq(x,a) returns true, if
            there is  such an (a,b);
            returns NONE otherwise.
*)
```

```
(* lookup : ('a * 'a -> bool) * 'a * ('a * 'b) list -> 'b option
   REQUIRES: true
   ENSURES:  lookup(eq, x, L) returns SOME(b) of the
             leftmost (a,b) in L for which eq(x,a) returns true, if
             there is  such an (a,b);
             returns NONE otherwise.
*)
```

```
  (* lookup : ('a * 'a -> bool) * 'a * ('a * 'b) list -> 'b option
    REQUIRES: true
    ENSURES:  lookup(eq, x, L) returns SOME(b) of the
              leftmost (a,b) in L for which eq(x,a) returns true, if
              there is  such an (a,b);
              returns NONE otherwise.
  *)

fun lookup(_: 'a * 'a -> bool, _ :'a, [ ]: ('a * 'b) list): 'b option = NONE

   |lookup(eq, x, (a,b) :: L) =   if eq(x,a) then SOME(b)
                                  else  lookup(eq, x, L)
```

```
fun lookup(_: 'a * 'a -> bool, _ :'a, [ ]: ('a * 'b) list): 'b option = NONE
    |lookup(eq, x, (a,b) :: L) =  if eq(x,a) then SOME(b)
                                  else lookup(eq, x, L)
```

val L = [(1,"a"),(2,"b"),(3,"c"),(4,"d")] : (int * string) list

lookup ((op =), 2, L): string option

evaluates to SOME "b"

lookup ((op =), __, L) evaluates to NONE

```
fun lookup(_: 'a * 'a -> bool, _ :'a, [ ]: ('a * 'b) list): 'b option = NONE
    |lookup(eq, x, (a,b) :: L) =   if eq(x,a) then SOME(b)
                                   else lookup(eq, x, L)
```

val L = [(1,"a"),(2,"b"),(3,"c"),(4,"d")] : (int * string) list

lookup ((op =), 2, L): string option

evaluates to SOME "b"

lookup ((op =), 5 , L) evaluates to NONE

Type Inference

Most General Types

Every well-typed expression has a
most general type

t is a most general type for e if and only if every instance of t is a type for e and every type for e is an instance of t

ML determines if your code is well-typed and
infers most general types, using a syntax-directed algorithm

Examples

1. **fun** square x = x * x * 1	square: int –> int
2. **fun** first(x,y) = x	first: 'a * 'b –> 'a
3. **fun** sqrf (f, x) = square (f(x))	('a –> int)*'a –>int
4. **fun** f x = f x	('a –> 'b)
5. **fun** h x = h (h x)	('a –> 'a)
6. **fun** id x = x	('a –> 'a)
7. id id 42	int

```
fun id x = x
```

: ('a -> 'a)

Function application is left-associative

f g x means (f g) x

id square 7 : int

square id 7 not well typed

square (id 7) : int

```
(* lookup : ('a * 'a -> bool) * 'a * ('a * 'b) list -> 'b option
   REQUIRES: true
   ENSURES:  lookup(eq, x, L) returns SOME(b) of the
             leftmost (a,b) in L for
             which eq(x,a) returns true, if there is such an (a,b);
             returns NONE otherwise.
*)

fun lookup(_: 'a * 'a -> bool, _ :'a, []: ('a * 'b) list): 'b option = NONE
   |lookup(eq, x, (a,b) :: L) =  if eq(x,a) then  SOME(b)
                                 else  lookup(eq, x, L)
```

In fact, if we omit the type annotations in our spec ML derives the following type

lookup : ('a * 'b -> bool) * 'a * ('b * 'c) list -> 'c option