Modules I

15-150

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Stephanie Balzer
Carnegie Mellon University

An important idea in computer science

Abstraction

What is abstraction? What does it entail?

Separation of **specification** from **implementation**.

Specification: externally visible promise to deliver

Implementation: internal choice of how to deliver promise

An important idea in computer science

Client 

Specification

What?

Abstraction

Implementation

How?

An important idea in computer science

Client 

Specification

What?

Abstraction

information hiding

representation independence

Implementation

How?

→ Implementations can be replaced as long as satisfy specification!

An important idea in computer science

Benefits of abstraction:

- Code evolution without disturbing client code.
- Reasoning: client only needs to consider specification.
- Separate development: specifications as blueprint.
- The only means by which programs become scalable.

Abstractions in SML

Abstraction at small: functions

- specification: function type with pre-/post-condition
- implementation: function body

Abstraction at large: modules

- Offer a way to combine what belongs together in one unit.
- Specification: **signature**.
- Implementation: **structure**.
- Composing structures using **functors**.

Abstractions in SML

Abstraction at large: modules

- Offer a way to combine what belongs together in one unit.
- Specification: **signature**.
- Implementation: **structure**.
- Composing structures using **functors**.

SML Basis Library makes use of modules. E.g.,

```
Int.toString
```

is a function inside a **structure** called `Int`. The structure `Int` has the **signature** called `INTEGER` ascribed.

- Let's unravel…

Modules at an example: queue

Queue, a **first-in first-out** (FIFO) data structure.

dequeue,
yields a queue with
this element removed

enqueue v_7,
yields a queue with
element added at end

- We can describe queue abstractly by specifying a new queue type, along with operations on that type.
  - That's a **signature**.
- Now we implement it in a **structure**.

Queue signature

```
signature QUEUE =
sig
  (* specification *)
end
```

name

specification

Queue signature

```
signature QUEUE =
sig

end
```

Queue signature

```
signature QUEUE =
sig

   type 'a queue                    (* abstract type *)

end
```

Postulate existence of an alpha queue

polymorphic

type name

No details on how represented.

Queue signature

```
signature QUEUE =
sig

  type 'a queue              (* abstract type *)

  (* Operations on an alpha queue *)

end
```

Postulate existence of an alpha queue

Queue signature

```
signature QUEUE =
sig

  type 'a queue          (* abstract type *)

end
```

Queue signature

```
signature QUEUE =
sig
  type 'a queue          (* abstract type *)
  val empty : 'a queue
```

end

Queue signature

```
signature QUEUE =
sig

  type 'a queue              (* abstract type *)

  val empty : 'a queue

  val enq : 'a queue * 'a -> 'a queue




end
```

Queue signature

```
signature QUEUE =
sig

  type 'a queue              (* abstract type *)

  val empty : 'a queue
  val enq : 'a queue * 'a -> 'a queue
  val null : 'a queue -> bool



end
```

Queue signature

```
signature QUEUE =
sig
  type 'a queue              (* abstract type *)
  val empty : 'a queue
  val enq : 'a queue * 'a -> 'a queue
  val null : 'a queue -> bool
  exception Empty
  (* deq (q) raises Empty if q is empty *)
  val deq : 'a queue -> 'a * 'a queue
end
```

Let's implement QUEUE in a structure

Implementation I: **represent** alpha queue as a single list.

List represents alpha queue elements in arrival order.

Single list representation of alpha queue

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Ascribe signature **QUEUE** to structure **Queue1**

```
structure Queue1 : QUEUE =
struct
```

name

```
end
```

Ascribe means:
The structure provides all the items specified in the signature. (It may contain additional items, e.g., helper functions, which will not be visible outside the structure, e.g., to a client.)

Single list representation of alpha queue

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

```
structure Queue1 : QUEUE =
struct
```


```
end
```

Single list representation of alpha queue

```
structure Queue1 : QUEUE =
struct
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

```
end
```

Single list representation of alpha queue

```
structure Queue1 : QUEUE =
struct
  type 'a queue = 'a list

end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

representation type

Single list representation of alpha queue

transparent ascription

```
structure Queue1 : QUEUE =
struct
  type 'a queue = 'a list

end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

representation type

Transparent ascription means:
The representation type of the abstract type alpha queue will be visible outside the structure, e.g., to the client.

Single list representation of alpha queue

```
structure Queue1 : QUEUE =
struct
  type 'a queue = 'a list



end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Single list representation of alpha queue

```
structure Queue1 : QUEUE =
struct
  type 'a queue = 'a list
  val empty = nil



end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Single list representation of alpha queue

```
structure Queue1 : QUEUE =
struct
  type 'a queue = 'a list
  val empty = nil
  fun enq (q,x) = q @ [x]



end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Single list representation of alpha queue

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

```
structure Queue1 : QUEUE =
struct
  type 'a queue = 'a list
  val empty = nil
  fun enq (q,x) = q @ [x]
  val null = List.null



end
```

Single list representation of alpha queue

```
structure Queue1 : QUEUE =
struct
  type 'a queue = 'a list
  val empty = nil
  fun enq (q,x) = q @ [x]
  val null = List.null
  exception Empty
 fun deq (x::q) = (x,q)
   | deq (nil) = raise Empty
end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Single list representation of alpha queue

```
structure Queue1 : QUEUE =
struct
  type 'a queue = 'a list
  val empty = nil
  fun enq (q,x) = q @ [x]
  val null = List.null
  exception Empty
 fun deq (x::q) = (x,q)
     | deq (nil) = raise Empty
end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Let's interact with `Queue1`:

```
val q = Queue1.enq(Queue1.enq(Queue1.empty, 1), 2)
```

What is the type of `q`?

```
int Queue1.queue
```

Single list representation of alpha queue

Let's interact with `Queue1`:

```
val q = Queue1.enq(Queue1.enq(Queue1.empty, 1), 2)
```

What is the type of `q`?

```
int Queue1.queue
```

Why? Because:

First, the signature specifies that queues have type `'a queue`, with `'a` representing the element type. That is `int` here.

Second, we have implemented queues using a structure called `Queue1`. The type is defined inside the structure, so the type has the qualified name `'a Queue1.queue`, here with `'a` instantiated with `int`.

Single list representation of alpha queue

Let's interact with `Queue1`:

```
val q = Queue1.enq(Queue1.enq(Queue1.empty, 1), 2)
```

What is the type of `q`?

```
int Queue1.queue
```

Also, `q` will be bound to:

```
[1, 2]
```

We can see the representation type list because of transparent ascription (and because list is defined in the Basis Library and thus in the top-level scope).

Single list representation of alpha queue

Let's interact with `Queue1`:

```
val q = Queue1.enq(Queue1.enq(Queue1.empty, 1), 2)
```

What is the type of `q`?

```
int Queue1.queue
```

Next, let's consider:

```
val (a, b) = Queue1.deq q
val (c, _) = Queue1.deq q
val (d, _) = Queue1.deq b
```

What are the bindings for `a`, `c`, and `d`?

```
[1/a, 1/c, 2/d]
```

Single list representation of alpha queue

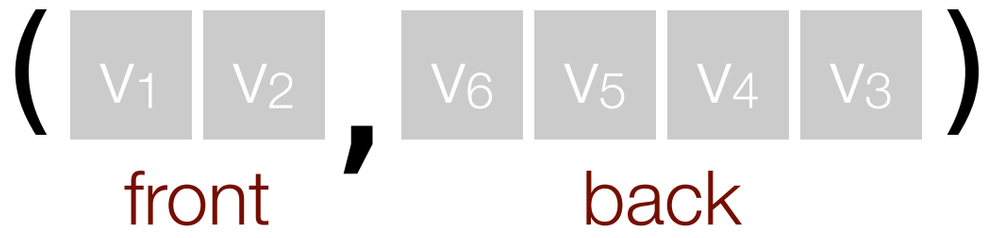
How long does enqueuing take?

```
fun enq (q,x) = q @ [x]
```

- **O**(n), with n the number of elements in **q**.
- Can we improve that?
  - Yes, let's choose a different representation type!

Pair of list representation of alpha queue

Implementation II: **represent** alpha queue as a pair of list.



Abstraction function:

```
front @ (rev back)
```

Represents alpha queue elements in arrival order.

Pair of list representation of alpha queue

opaque ascription

```
structure Queue2 :> QUEUE =
struct
  type 'a queue =
    'a list * 'a list
end
```

representation type

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Opaque ascription means that the representation type of the abstract type alpha queue will be hidden outside the structure, e.g., from the client. ML will only print a dash.

→ Ensures information hiding and integrity of data structure.

→ Not guaranteed with transparent ascription!

Pair of list representation of alpha queue

```
structure Queue2 :> QUEUE =
struct
  type 'a queue =
    'a list * 'a list
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

```
end
```

Pair of list representation of alpha queue

```
structure Queue2 :> QUEUE =
struct
  type 'a queue =
     'a list * 'a list

  val empty = (nil, nil)

  fun enq ((front, back), x) = (front, x::back)

end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Satisfies requirement that `f @ (rev(x::b))` constitutes queue elements in arrival order.

Pair of list representation of alpha queue

```
structure Queue2 :> QUEUE =
struct
  type 'a queue =
     'a list * 'a list

  val empty = (nil, nil)

  fun enq ((front, back), x) = (front, x::back)

end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Pair of list representation of alpha queue

```
structure Queue2 :> QUEUE =
struct
  type 'a queue =
    'a list * 'a list

  val empty = (nil, nil)

  fun enq ((front, back), x) = (front, x::back)

  fun null (nil,nil) = true
    | null _ = false

end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Pair of list representation of alpha queue

```
structure Queue2 :> QUEUE =
struct
  type 'a queue =
    'a list * 'a list

  val empty = (nil, nil)

  fun enq ((front, back), x) = (front, x::back)

  fun null (nil,nil) = true
    | null _ = false

  exception Empty

  fun deq (x::front, back) = (x, (front, back))
    | deq (nil, nil) = raise Empty
    | deq (nil, back) = deq(rev back, nil)

end
```

```
signature QUEUE =
sig
 type 'a queue          (* abstract type *)
 val empty : 'a queue
 val enq : 'a queue * 'a -> 'a queue
 val null : 'a queue -> bool
 exception Empty
 val deq : 'a queue -> 'a * 'a queue
end
```

Pair of list representation of alpha queue

Let's interact with `Queue2`:

```
val q = Queue2.enq(Queue2.enq(Queue2.empty, 1), 2)
```

What is the type of `q`?

```
int Queue2.queue
```

Why? Because:

First, the signature specifies that queues have type `'a queue`, with `'a` representing the element type. That is `int` here.

Second, we have implemented queues using a structure called `Queue2`. The type is defined inside the structure, so the type has the qualified name `'a Queue2.queue`, here with `'a` instantiated with `int`.

Pair of list representation of alpha queue

Let's interact with `Queue2`:

```
val q = Queue2.enq(Queue2.enq(Queue2.empty, 1), 2)
```

What is the type of `q`?

```
int Queue2.queue
```

However, the binding for `q` will not be revealed because of opaque ascription:

```
val q = - : int Queue2.queue
```

Pair of list representation of alpha queue

Let's interact with `Queue2`:

```
val q = Queue2.enq(Queue2.enq(Queue2.empty, 1), 2)
```

What is the type of `q`?

```
int Queue2.queue
```

Next, let's consider:

```
val (a, b) = Queue2deq q
val (c, _) = Queue2.deq q
val (d, _) = Queue2.deq b
```

What are the bindings for `a`, `c`, and `d`?

```
[1/a, 1/c, 2/d]
```

Pair of list representation of alpha queue

How long does enqueuing take?

```
fun enq ((front, back), x) = (front, x::back)
```

- **O**(1)!
- Dequeuing can now take **O**(n) time.
- However, enqueuing and dequeuing n items will on take **O**(n) time total, so on average it is **O**(1).
  - The amortized cost is **O**(1).

Comparing the two implementations

Operation:	Queue1:	Queue2:

Comparing the two implementations

Operation:	Queue1:	Queue2:
`empty`	`[]`	`([], [])`

Comparing the two implementations

Operation:	Queue1:	Queue2:
`empty`	`[]`	`([], [])`
`enq 1`	`[1]`	`([], [1])`

Comparing the two implementations

Operation:	Queue1:	Queue2:
`empty`	`[]`	`([], [])`
`enq 1`	`[1]`	`([], [1])`
`enq 2`	`[1,2]`	`([], [2,1])`

Comparing the two implementations

Operation:	Queue1:	Queue2:
`empty`	`[]`	`([], [])`
`enq 1`	`[1]`	`([], [1])`
`enq 2`	`[1,2]`	`([], [2,1])`
`deq`	`[2]`	

Comparing the two implementations

Operation:	Queue1:	Queue2:
`empty`	`[]`	`([], [])`
`enq 1`	`[1]`	`([], [1])`
`enq 2`	`[1,2]`	`([], [2,1])`
		briefly this:
`deq`	`[2]`	`([1,2], [])`
		then this:
		`([2], [])`

Comparing the two implementations

Operation:	Queue1:	Queue2:
`empty`	`[]`	`([], [])`
`enq 1`	`[1]`	`([], [1])`
`enq 2`	`[1,2]`	`([], [2,1])`
		briefly this:
`deq`	`[2]`	`([1,2], [])`
		then this:
		`([2], [])`
`enq 3`	`[2,3]`	`([2], [3])`

Another example: dictionary

A dictionary is a collection of pairs of the form

```
(key, value)
```

where all keys must be unique within a dictionary.

```
signature DICT =
sig
  type key = string                    (* concrete type *)
  type 'a entry = key * 'a             (* concrete type *)
  type 'a dict                         (* abstract type *)


end
```

Let's use strings as keys for now

Permit value type to be polymorphic

Another example: dictionary

A dictionary is a collection of pairs of the form

```
(key, value)
```

where all keys must be unique within a dictionary.

```
signature DICT =
sig
  type key = string                    (* concrete type *)
  type 'a entry = key * 'a             (* concrete type *)
  type 'a dict                         (* abstract type *)



end
```

Another example: dictionary

A dictionary is a collection of pairs of the form

```
(key, value)
```

where all keys must be unique within a dictionary.

```
signature DICT =
sig
  type key = string                    (* concrete type *)
  type 'a entry = key * 'a             (* concrete type *)
  type 'a dict
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Replace entry, if key already exists

Search tree representation of dictionary

Implementation: **represent** dictionary as a binary search tree, where

```
(key, value)
```

are stored in nodes.

- Representation **invariant**:
  - Tree must be sorted.
  - Keys must be unique.
- All functions declared by structure
  - may **assume** that received tree is sorted,
  - and must **assert** that returned tree is sorted.
  - (Similarly for key uniqueness.)

Search tree representation of dictionary

Explore :>

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree

end
```

```
signature DICT =
sig
  type key = string          (* concrete type *)
  type 'a entry = key * 'a   (* concrete type *)
  type 'a dict               (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

→ Transparent ascription can be useful for debugging purposes.

Search tree representation of dictionary

```
signature DICT =
sig
  type key = string          (* concrete type *)
  type 'a entry = key * 'a   (* concrete type *)
  type 'a dict               (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree

end
```

no other choice

Search tree representation of dictionary

```
signature DICT =
sig
  type key = string          (* concrete type *)
  type 'a entry = key * ‘a (* concrete type *)
  type 'a dict               (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree
```

→ Because datatype is not declared in signature, constructors (and thus pattern matching) are not available outside signature.

`end` → But bindings externally visible due to transparent ascription.

Search tree representation of dictionary

```
signature DICT =
sig
  type key = string          (* concrete type *)
  type 'a entry = key * ‘a (* concrete type *)
  type 'a dict               (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree



end
```

Search tree representation of dictionary

```
signature DICT =
sig
  type key = string          (* concrete type *)
  type 'a entry = key * 'a (* concrete type *)
  type 'a dict               (* abstract type *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

```
structure BST : DICT =
struct
  type key = string
  type 'a entry = key * 'a
  datatype 'a tree =
    Empty | Node of 'a tree * 'a entry * 'a tree
  type 'a dict = 'a tree
  val empty = Empty
  fun lookup ...
  fun insert ...
end
```

Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e' as (k',_), rt),
            e as (k,_)) =
```

Layered pattern matching

Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e' as (k',_), rt),
            e as (k,_)) =
```

Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e' as (k',_), rt),
            e as (k,_)) =
    (case String.compare(k,k') of
```

Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e' as (k',_), rt),
            e as (k,_)) =
    (case String.compare(k,k') of
       EQUAL => Node(lt, e, rt)
```

Replace existing entry with new one

Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e' as (k',_), rt),
                e as (k,_)) =
    (case String.compare(k,k') of
      EQUAL => Node(lt, e, rt)
```

Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e' as (k',_), rt),
                e as (k,_)) =
    (case String.compare(k,k') of
       EQUAL => Node(lt, e, rt)
     | LESS => Node(insert(lt,e), e', rt)
```

Search tree representation of dictionary

```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e' as (k',_), rt),
                e as (k,_)) =
    (case String.compare(k,k') of
       EQUAL => Node(lt, e, rt)
     | LESS => Node(insert(lt,e), e', rt)
     | GREATER => Node(lt, e', insert(rt,e)))
```

Search tree representation of dictionary

```
(* lookup : 'a dict -> key -> 'a option *)
fun lookup tree key =
 let
     fun lk (Empty) = NONE
     | lk (Node(left, (k,v), right)) =
       (case String.compare(key,k) of
         EQUAL => SOME(v)
         | LESS => lk left
         | GREATER => lk right)
in
   lk tree
end
```

Search tree representation of dictionary

Let's interact with **BST**:

```
val d = BST.insert(BST.insert(BST.insert(
          BST.empty,("a",1)),("b",2)),("c",3))
```

What is the type of **d**?

```
int BST.dict
```

The binding for **d** will be revealed because of opaque ascription. However, because the tree datatype is not declared in the signature, a client cannot pattern match on its constructors.

Now consider: `val look = BST.lookup d`

What is the type of **look**?

```
BST.key -> int option
```

Search tree representation of dictionary

Let's interact with **BST**:

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val d = BST.insert(BST.insert(BST.insert(
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int BST.dict
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Now consider: `val look = BST.lookup d`

What is the type of **look**?

```
BST.key -> int option
```

Now consider:
```
val x = look "e"
val y = look "a"
```
Bindings: `[NONE/x, (SOME 1)/y]`

That’s all for today.

Next time we discuss functors!