Modules II

15-150

Lecture 17: 🎃👻🎃, 2024

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Announcement: midterm II

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When and where:

- Thursday, **November 7**, **11:00am—12:20pm**.
- **MM 103** (Sections A—D), **PH 100** (Sections E—L).

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Be on time; next lecture starts at 12:30pm!

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Scope:

- Lectures: 1—15.
- Labs: 1—8 and midterm review section of Lab 10.
- Assignments: up to including Exceptions/Regex.

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What you may have on your desk:

- Writing utensils, we provide paper, something to drink/eat, tissues.
- 8.5” x 11” cheatsheet (back and front), handwritten or typeset.
- No cell phones, laptops, or any other smart devices.

Recap

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Abstraction through separating specification from implementation:

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- Specification: externally visible promise deliver.
- Implementation: internal choice of how to deliver promise.

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  - Any two implementations that satisfy specifications are indistinguishable to the client and thus equal.

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- Specification: externally visible promise deliver.
- Implementation: internal choice of how to deliver promise.
- Allows us to hide implementation details from the client.
    - Representation independence: the client becomes independent of the choice of internal representation.
    - Any two implementations that satisfy specifications are indistinguishable to the client and thus equal.
- Facilitates modular reasoning (component-wise reasoning).

Recap

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SML modules facilitate abstraction:

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- Specification: **signature**.
- Implementation: **structure**.

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- Structures can **hide** auxiliary, implementation-specific components, not specified by signature.

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- Specification: **signature**.
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SML modules allow us to control the “flow of information”:

- Structures can **hide** auxiliary, implementation-specific components, not specified by signature.
- **Transparent ascription**: for undefined type specified in signature, representation type chosen by structure is revealed.

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SML modules facilitate abstraction:

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SML modules allow us to control the “flow of information”:

- Structures can **hide** auxiliary, implementation-specific components, not specified by signature.
- **Transparent ascription**: for undefined type specified in signature, representation type chosen by structure is revealed.
- **Opaque ascription**: for undefined type specified in signature, representation type chosen by structure is hidden.

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specification	signature	type
implementation	structure	value
mapping	functor	function

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- Let's resume our dictionary example!

Example: dictionary

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(key, value)
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Let's use strings as keys for now

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Permit value type to be polymorphic

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Replace entry, if key already exists

Search tree representation of dictionary

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Implementation: **represent** dictionary as a binary search tree, where

`(key, value)`

are stored in nodes.

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Tree must be sorted.

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- Representation **invariant**:
  - Tree must be sorted.
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  - (Similarly for key uniqueness.)

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structure BST : DICT =
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➡ Transparent ascription can be useful for debugging purposes.

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no other choice

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Because datatype is not declared in signature, constructors (and thus pattern matching) are not available outside signature.

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But bindings externally visible due to transparent ascription.

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Search tree representation of dictionary

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```
(* ins : 'a dict * 'a entry -> 'a dict *)
fun insert (Empty, e) = Node(Empty, e, Empty)
  | insert (Node(lt, e’ as (k’,_), rt),
            e as (k,_)) =
```

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Layered pattern matching

Search tree representation of dictionary

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Replace existing entry with new one

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        | GREATER => Node(lt, e’, insert(rt,e)))
```

Search tree representation of dictionary

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```
(* lookup : 'a dict -> key -> 'a option *)
fun lookup tree key =
 let
    fun lk (Empty) = NONE
    | lk (Node(left, (k,v), right)) =
      (case String.compare(key,k) of
        EQUAL => SOME(v)
        | LESS => lk left
        | GREATER => lk right)
  in
    lk tree
  end
```

Search tree representation of dictionary

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```
val d = BST.insert(BST.insert(BST.insert(
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The binding for **d** will be revealed because of opaque ascription. However, because the tree datatype is not declared in the signature, a client cannot pattern match on its constructors.

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Now consider:

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Now consider: `val look = BST.lookup d`

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BST.key -> int option
```

Now consider:

Search tree representation of dictionary

Let's interact with **BST**:

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val d = BST.insert(BST.insert(BST.insert(
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int BST.dict
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Now consider: `val look = BST.lookup d`

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Let’s reconsider our DICT signature

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signature DICT =
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  type key = string                          (* concrete type *)
  type 'a entry = key * 'a                   (* concrete type *)
  type 'a dict                               (* abstract type *)
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What if we needed keys other than strings?

We could try to make key polymorphic too.

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How to implement now?

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Used `String.compare`

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How to implement now?

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- What if we needed keys other than strings?
- We could try to make key polymorphic too.
- Keys should become comparable!

Let’s reconsider our DICT signature

```
lookup:

insert:
```

➡ **Keys should become comparable!**

Let’s reconsider our DICT signature

```
lookup: ('a *'a -> order) -> ('a,'b) dict -> ‘a ->'b option

insert: ('a *'a -> order) -> (('a,'b) dict * ('a,'b) entry)
                          -> ('a,'b) dict
```

➡ **Keys should become comparable!**

Let’s reconsider our DICT signature

```
lookup: ('a *'a -> order) -> ('a,'b) dict -> ‘a ->'b option

insert: ('a *'a -> order) -> (('a,'b) dict * ('a,'b) entry)
                          -> ('a,'b) dict
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→ Keys should become comparable!

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- Keys should become comparable!
- Require a comparison function as an argument.

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```

- Keys should become comparable!
- Require a comparison function as an argument.
- Restricts polymorphism of keys!

Let’s update our BST structure accordingly

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```
structure BST : DICT =
struct
  type 'a key = ‘a

  type ('a,'b) entry = 'a key * 'b

  datatype ('a,'b) dict = Empty | Node of
             ('a,'b) dict * ('a,'b) entry * ('a,'b) dict

  val empty = Empty

  fun insert cmp d k =

  fun lookup cmp (d, k) =
end
```

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As specified by signature

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Again, binary search tree as representation type.

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Again, binary search tree as representation type.

This time, with polymorphic key.

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As before.

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Bodies of `insert` and `lookup` now use `cmp` instead of `String.compare`.

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Does this do the trick?

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Does this do the trick? Well, not quite.

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```
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- Does this do the trick? Well, not quite.
- What if a client provides different `cmp` functions to `insert` than to `lookup`, for example?

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For example, a client creates the following tree using `insert` and `Int.compare`:

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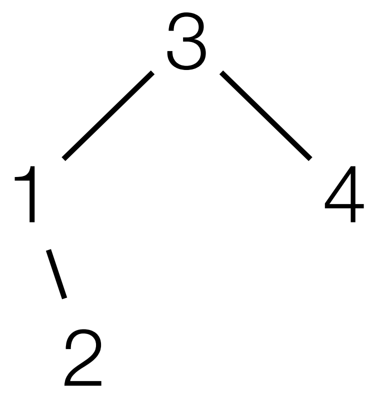

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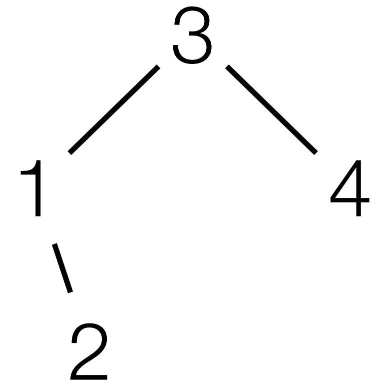

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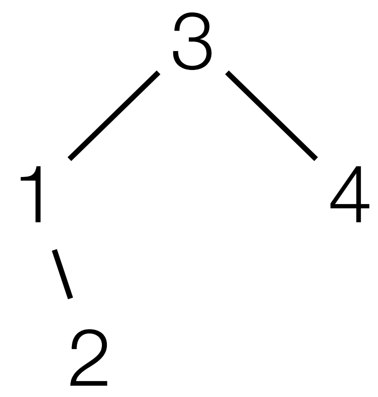

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- Does this do the trick? Well, not quite.
- What if a client provides different `cmp` functions to `insert` than to `lookup`, for example?
- Can we enforce the invariant, that all operations use the same comparison function by typing?
- Yes, but we need type classes for this!

Type classes

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A signature specifying a type and associated operations.

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  - No expectation that specification is exhaustive.

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Example:

```
signature ORDERED =
sig
  type t      (* parameter *)
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end
```

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Type class

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  - No expectation that specification is exhaustive.

Example:

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end
```

Signature `ORDERED` specifies an “ordered type class” to consist of a type `t` along with a comparison function `compare` for `t`.

Type classes

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Even though `t` is not concrete, it does not have to be abstract.

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We expect `t` to be some already existing type, hence use the comment **parameter**.

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- Signature `ORDERED` is said to be **descriptive**.

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- Signature `DICT` is in contrast **prescriptive**, defining an abstract type with all its operations, exhaustively.

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Type classes

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- We expect `t` to be some already existing type, hence use the comment **parameter**.
- Signature `ORDERED` is said to be **descriptive**.
- Signature `DICT` is in contrast **prescriptive**, defining an abstract type with all its operations, exhaustively.
- We tend to use transparent ascription for descriptive signatures (aka type classes), and opaque ascription for prescriptive signatures.

Perspective of types in signatures

Perspective of types in signatures

Concrete:

Signature dictates representation type, which is thus visible to client.

Abstract:

Signature hides representation type. Client code must work regardless of the representation type chosen by structure.

Parameter:

Client supplies the type, implementation must work with whatever the clients supplies.

Different ways of implementing ORDERED

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```
signature ORDERED =
sig
  type t     (* parameter *)
  val compare : t * t -> order
end
```

Different ways of implementing ORDERED

```
structure IntLt : ORDERED =
struct
  type t = int
  val compare = Int.compare
end
```

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signature ORDERED =
sig
  type t     (* parameter *)
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end
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Different ways of implementing ORDERED

```
structure IntLt : ORDERED =
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  type t = int
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structure IntGt : ORDERED =
struct
  type t = int
  fun compare(x,y) = Int.compare(y,x)
end
```

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signature ORDERED =
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  type t      (* parameter *)
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Different ways of implementing ORDERED

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structure StringLt : ORDERED =
struct
  type t = string
  val compare = String.compare
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```

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Redefine DICT using type class ORDERED

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  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Redefine DICT using type class ORDERED

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```

Redefine DICT using type class ORDERED

Use type class as a parameter

```
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  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
end
```

Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key = ORDERED                   (* parameter *)
  type 'a entry = key * 'a                  (* concrete  *)
  type 'a dict                              (* abstract  *)
  val empty : 'a dict
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```

→ Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

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```

Use type class' type `t`

→ Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

Redefine DICT using type class ORDERED

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signature DICT =
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  structure Key = ORDERED              (* parameter *)
  type 'a entry = Key.t * 'a           (* concrete  *)
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➡ Any structure implementing `DICT` will comprise a sub-structure implementing `ORDERED`.

Redefine DICT using type class ORDERED

```
signature DICT =
sig
  structure Key = ORDERED              (* parameter *)
  type 'a entry = Key.t * 'a           (* concrete  *)
  type 'a dict                         (* abstract  *)
  val empty : 'a dict
  val lookup : 'a dict -> key -> 'a option
  val insert : 'a dict * 'a entry -> 'a dict
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Let’s define dictionaries with different keys!

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end
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structure IntLtDict : DICT =
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Only differ in **Key**!

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No, this is not possible! `IntGtDict.dict` and `IntLtDict.dict` are different types.

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ML type checker will thus prevent intermingling of dictionaries.

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No, this is not possible! `IntGtDict.dict` and `IntLtDict.dict` are different types.

ML type checker will thus prevent intermingling of dictionaries.

Remark: Had we implemented `dict` in terms of a representation type available in the client's scope, we should have used opaque ascription!

Is that it?

Have we solved the problem of inserting with one comparison function but looking up elements with a different one?

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YES!

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- A **functor** creates a structure, given a structure as an argument.

Let's write a functor that creates a structure ascribing to `DICT`, given a structure ascribing to `ORDERED` as an argument.

Avoid the bloat with a functor!

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```
functor TreeDict (K : ORDERED) : DICT =
struct




end
```

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Argument structure, of type `ORDERED`

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Argument structure, of type `ORDERED`

Note: ":" denotes typing, not ascription mode.

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Structured returned, transparently ascribing to `DICT`

Denotes ascription mode, as usual.

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```
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Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K




end
```

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a


end
```

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...

end
```

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) : DICT =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
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  (* code as before, but using Key.t and Key.compare *)
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```

Now, we can define our earlier dictionaries as:

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```

Now, we can define our earlier dictionaries as:

```
structure IntLtDict = TreeDict(IntLt)
structure IntGtDict = TreeDict(IntGt)
structure StringLtDict = TreeDict(StringLt)
```

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functor TreeDict (K : ORDERED) : DICT =
struct
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  structure Key = K
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end
```

Let's use opaque ascription instead!

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functor TreeDict (K : ORDERED) :> DICT =
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But now we hide the representation type for `Key.t`.

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- But now we hide the representation type for `Key.t`.
- But we want it to be known that `Key.t` is the same as the input key `K.t`!

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```
functor TreeDict (K : ORDERED) :> DICT =
struct
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- But now we hide the representation type for `Key.t`.
- But we want it to be known that `Key.t` is the same as the input key `K.t`!
- To rectify this, we need to add a `where` clause.

Avoid the bloat with a functor!

```
functor TreeDict (K : ORDERED) :> DICT
                           where type Key.t = K.t =
struct
  structure Key = K
  type 'a entry = Key.t * 'a
  datatype 'a dict = ...
  (* code as before, but using Key.t and Key.compare *)
end
```

- But now we hide the representation type for `Key.t`.
- But we want it to be known that `Key.t` is the same as the input key `K.t`!
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- Signatures can be descriptive (aka type classes), exposing a type parameter's operations, typically using opaque ascription.
- A functor creates a structure, given a structure as an argument.
  - Functor arguments are typically type classes to prevent code redundancy.

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→ **Functors only take a single structure as an argument.**

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- Functors only take a single structure as an argument.
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That's all for today. Happy 🎃👻🎃!