Imperative Programming

15-150

Lecture 21: November 21, 2024

Stephanie Balzer
Carnegie Mellon University

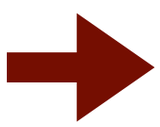
Functional programming

Functional programming

So far we have used the term "functional programming" as a synonym for pure programming.

Functional programming

So far we have used the term "functional programming" as a synonym for pure programming.



But what does **pure** really mean?

Functional programming

So far we have used the term "functional programming" as a synonym for pure programming.

- But what does **pure** really mean?
- Well, the prototypical answer is, **without** any **side-effects**.

Functional programming

So far we have used the term "functional programming" as a synonym for pure programming.

- But what does **pure** really mean?
- Well, the prototypical answer is, **without** any **side-effects**.
- But what does that really mean? 🤔

Functional programming

So far we have used the term "functional programming" as a synonym for pure programming.

- But what does **pure** really mean?
- Well, the prototypical answer is, **without** any **side-effects**.

Let's reconsider the correctness proofs that we carried out.

Functional programming

So far we have used the term "functional programming" as a synonym for pure programming.

- But what does **pure** really mean?
- Well, the prototypical answer is, **without** any **side-effects**.

Let's reconsider the correctness proofs that we carried out.

- Can you think of an implicit assumption that we made when proving a function correct, ensuring that our reasoning is valid?

Functional programming

So far we have used the term "functional programming" as a synonym for pure programming.

- But what does **pure** really mean?
- Well, the prototypical answer is, **without** any **side-effects**.

Let's reconsider the correctness proofs that we carried out.

- Can you think of an implicit assumption that we made when proving a function correct, ensuring that our reasoning is valid?
- We assumed that it suffices to *only* consider the function specification and implementation, *nothing else*.

Functional programming

So far we have used the term "functional programming" as a synonym for pure programming.

- But what does **pure** really mean?
- Well, the prototypical answer is, **without** any **side-effects**.

Let's reconsider the correctness proofs that we carried out.

- Can you think of an implicit assumption that we made when proving a function correct, ensuring that our reasoning is valid?
- We assumed that it suffices to *only* consider the function specification and implementation, *nothing else*.
- We carried out per-function (aka **local**) **reasoning**.

Functional programming

Let's reconsider the correctness proofs that we carried out.

- Can you think of an implicit assumption that we made when proving a function correct, ensuring that our reasoning is valid?
- We assumed that it suffices to *only* consider the function specification and implementation, *nothing else*.
- We carried out per-function (aka **local**) **reasoning**.

Functional programming

Let's reconsider the correctness proofs that we carried out.

- Can you think of an implicit assumption that we made when proving a function correct, ensuring that our reasoning is valid?
- We assumed that it suffices to *only* consider the function specification and implementation, *nothing else*.
- We carried out per-function (aka **local**) **reasoning**.

Functional programming validates local reasoning and guarantees that:

Functional programming

Let's reconsider the correctness proofs that we carried out.

- Can you think of an implicit assumption that we made when proving a function correct, ensuring that our reasoning is valid?
- We assumed that it suffices to *only* consider the function specification and implementation, *nothing else*.
- We carried out per-function (aka **local**) **reasoning**.

Functional programming validates local reasoning and guarantees that:

- Repeated evaluation of an expression yields the same result.

Functional programming

Let's reconsider the correctness proofs that we carried out.

- Can you think of an implicit assumption that we made when proving a function correct, ensuring that our reasoning is valid?
- We assumed that it suffices to *only* consider the function specification and implementation, *nothing else*.
- We carried out per-function (aka **local**) **reasoning**.

Functional programming validates local reasoning and guarantees that:

- Repeated evaluation of an expression yields the same result.
- Sequential and parallel evaluation of independent sub-expressions produces the same result.

Effects (impure or imperative programming)

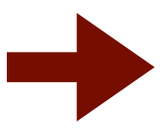
Effects (impure or imperative programming)

In the presence of effects, local reasoning* breaks down.

*(Local reasoning can be re-established by using program logics such as separation logic.)

Effects (impure or imperative programming)

In the presence of effects, local reasoning* breaks down.

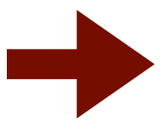


Effect, aka anything else that we can observe when evaluating an expression other than the returned value.

*(Local reasoning can be re-established by using program logics such as separation logic.)

Effects (impure or imperative programming)

In the presence of effects, local reasoning* breaks down.



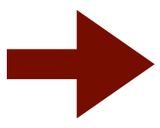
Effect, aka anything else that we can observe when evaluating an expression other than the returned value.

Examples of effects:

*(Local reasoning can be re-established by using program logics such as separation logic.)

Effects (impure or imperative programming)

In the presence of effects, local reasoning* breaks down.



Effect, aka anything else that we can observe when evaluating an expression other than the returned value.

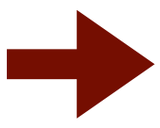
Examples of effects:

- When two functions share state, mutations by one affect the other.

*(Local reasoning can be re-established by using program logics such as separation logic.)

Effects (impure or imperative programming)

In the presence of effects, local reasoning* breaks down.



Effect, aka anything else that we can observe when evaluating an expression other than the returned value.

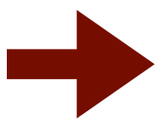
Examples of effects:

- When two functions share state, mutations by one affect the other.
- A non-terminating function will cause its caller to diverge too.

*(Local reasoning can be re-established by using program logics such as separation logic.)

Effects (impure or imperative programming)

In the presence of effects, local reasoning* breaks down.



Effect, aka anything else that we can observe when evaluating an expression other than the returned value.

Examples of effects:

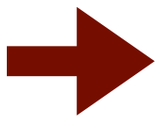
- When two functions share state, mutations by one affect the other.
- A non-terminating function will cause its caller to diverge too.

In the presence of effects, the **order of evaluation** matters.

*(Local reasoning can be re-established by using program logics such as separation logic.)

Effects (impure or imperative programming)

In the presence of effects, local reasoning* breaks down.

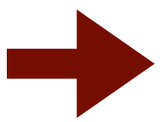


Effect, aka anything else that we can observe when evaluating an expression other than the returned value.

Examples of effects:

- When two functions share state, mutations by one affect the other.
- A non-terminating function will cause its caller to diverge too.

In the presence of effects, the **order of evaluation** matters.



Repeated evaluation of an expression may not yield the same result.

*(Local reasoning can be re-established by using program logics such as separation logic.)

Effects (impure or imperative programming)

In the presence of effects, local reasoning* breaks down.

→ Effect, aka anything else that we can observe when evaluating an expression other than the returned value.

Examples of effects:

- When two functions share state, mutations by one affect the other.
- A non-terminating function will cause its caller to diverge too.

In the presence of effects, the **order of evaluation** matters.

→ Repeated evaluation of an expression may not yield the same result.

→ Sequential and parallel evaluation of independent sub-expressions may not produce the same result.

*(Local reasoning can be re-established by using program logics such as separation logic.)

SML supports imperative programming

SML supports imperative programming

To reap all the benefits of functional programming, we have stayed entirely* in the pure fragment of SML until now.

*(Except for non-termination and exceptions.)

SML supports imperative programming

To reap all the benefits of functional programming, we have stayed entirely* in the pure fragment of SML until now.

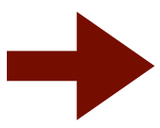
However, SML supports imperative features, such as reference cells, arrays, and commands for I/O.

*(Except for non-termination and exceptions.)

SML supports imperative programming

To reap all the benefits of functional programming, we have stayed entirely* in the pure fragment of SML until now.

However, SML supports imperative features, such as reference cells, arrays, and commands for I/O.



We may use effects locally to increase efficiency, for example.

*(Except for non-termination and exceptions.)

SML supports imperative programming

To reap all the benefits of functional programming, we have stayed entirely* in the pure fragment of SML until now.

However, SML supports imperative features, such as reference cells, arrays, and commands for I/O.

- We may use effects locally to increase efficiency, for example.
  - referred to as "benign effects"

*(Except for non-termination and exceptions.)

SML supports imperative programming

To reap all the benefits of functional programming, we have stayed entirely* in the pure fragment of SML until now.

However, SML supports imperative features, such as reference cells, arrays, and commands for I/O.

- We may use effects locally to increase efficiency, for example.
  - referred to as "benign effects"
- Expressions that engender effects typically are of `unit` type.

*(Except for non-termination and exceptions.)

Today's menu

Today's menu

- Shared state through mutable reference cells
  - reference type
  - typing and evaluation rules

Today's menu

- Shared state through mutable reference cells
  - reference type
  - typing and evaluation rules
- Aliasing

Today's menu

- Shared state through mutable reference cells
  - reference type
  - typing and evaluation rules
- Aliasing
- Race conditions

Today's menu

- Shared state through mutable reference cells
  - reference type
  - typing and evaluation rules
- Aliasing
- Race conditions
- Persistent versus ephemeral data

Today's menu

- Shared state through mutable reference cells
  - reference type
  - typing and evaluation rules
- Aliasing
- Race conditions
- Persistent versus ephemeral data
- Examples of benign effects

Mutable reference cells

Mutable reference cells

Reference type:

Mutable reference cells

Reference type: `t ref`

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

arbitrary SML type*

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

arbitrary SML type*

including t ref

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

Reference type values:

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

Reference type values:

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

Reference type values:

The type `t ref` represents mutable reference cells that store a value of type `t`.

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

Reference type values:

The type `t ref` represents mutable reference cells that store a value of type `t`.

Functions:

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

Reference type values:

The type `t ref` represents mutable reference cells that store a value of type `t`.

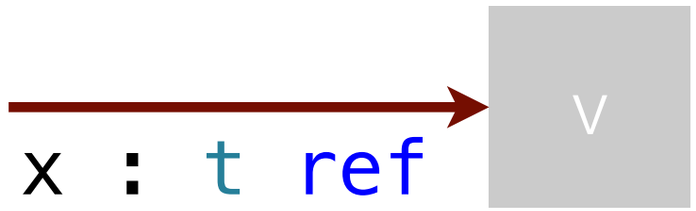
Functions: `ref : 'a -> 'a ref` allocation

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

Reference type values:



(where **v** : `t`)

The type `t ref` represents mutable reference cells that store a value of type `t`.

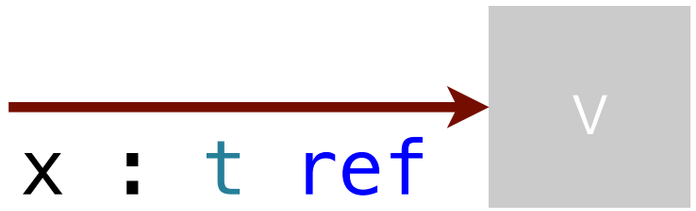
Functions:	`ref : 'a -> 'a ref`	allocation
	`! : 'a ref -> 'a`	read

*(Restriction: at top level, t must be monomorphic.)

Mutable reference cells

Reference type: `t ref`

Reference type values:



(where **v** : `t`)

The type `t ref` represents mutable reference cells that store a value of type `t`.

Functions:

```
ref : 'a -> 'a ref          allocation
!   : 'a ref -> 'a          read
:=  : 'a ref * 'a -> unit   write
```

*(Restriction: at top level, t must be monomorphic.)

Allocation: `ref: 'a -> 'a ref`

Allocation: `ref: 'a -> 'a ref`

Evaluation rules:

`ref` e

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression `e`.

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Example:

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Example: `val r = ref (1 + 3)`

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Example: `val r = ref (1 + 3)`

evaluates to:

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Example: `val r = ref (1 + 3)`

evaluates to: `r` → 4

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Example: `val r = ref (1 + 3)`

evaluates to: `r` → 4

Here, `r : int ref` is bound to a reference to the reference cell containing the value `4 : int`.

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Typing rules: `ref e`

Allocation: `ref: 'a -> 'a ref`

Evaluation rules: `ref e`

1. Evaluate expression **e**.
2. If **e** reduces to a value **v**, create a new cell containing **v** and return the reference to it.

Typing rules: `ref e`

If `e : t`, then `ref e : t ref`.

Read: ! : 'a ref -> 'a

Read: ! : 'a ref -> 'a

Evaluation rules: !e

Read: ! : 'a ref -> 'a

Evaluation rules: !e

1. Evaluate expression e.

Read: ! : 'a ref -> 'a

Evaluation rules: !e

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

Read: ! : 'a ref -> 'a

Evaluation rules: !e

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

Example:

Read: ! : 'a ref -> 'a

Evaluation rules: !e

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

Example:
```
val r = ref (1 + 3)
val x = !r
```

Read: ! : 'a ref -> 'a

Evaluation rules: !e

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

Example:
```
val r = ref (1 + 3)
val x = !r
```

evaluates to:

Read: ! : 'a ref -> 'a

Evaluation rules: !e

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

Example:
```
val r = ref (1 + 3)
val x = !r
```

evaluates to: r → 4

Read: `!` : 'a ref -> 'a

Evaluation rules: `!e`

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

Example:
```
val r = ref (1 + 3)
val x = !r
```

evaluates to: `r` → 4 and `[4/x]`

Read: `!` : `'a ref -> 'a`

Evaluation rules: `!e`

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

Example:
```
val r = ref (1 + 3)
val x = !r
```

evaluates to: `r` → 4 and `[4/x]`

Here, `r : int ref` is bound to a reference to the cell containing the value `4 : int` and `x : int` is bound to `4`.

Read: ! : 'a ref -> 'a

Evaluation rules: !e

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

Read: ! : 'a ref -> 'a

Evaluation rules: !e

1. Evaluate expression **e**.
2. If **e** reduces to reference to a cell containing **v**, then return **v**.

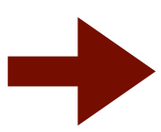
Typing rules: !e

Read: `!` : `'a ref -> 'a`

Evaluation rules: `!e`

1. Evaluate expression `e`.
2. If `e` reduces to reference to a cell containing `v`, then return `v`.

Typing rules: `!e`



If `e : t ref`, then `!e : t`.

Write: `:=` : `'a ref * 'a -> unit`

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Example:

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Example:

```
val r = ref (1 + 3)
r := (!r * 2)
```

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Example:
```
val r = ref (1 + 3)
r := (!r * 2)
```

evaluates to:

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Example:
```
val r = ref (1 + 3)
r := (!r * 2)
```

evaluates to: `r` → 8

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Example:
```
val r = ref (1 + 3)
r := (!r * 2)
```

evaluates to: `r` → 8 and `[()/it]`

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Example:
```
val r = ref (1 + 3)
r := (!r * 2)
```

evaluates to: `r` → `8` and `[()/it]`

Here, `r : int ref` is bound to a reference to the cell containing the value `8 : int` and `()` is returned.

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Typing rules: e_1 `:=` e_2

Write: `:=` : `'a ref * 'a -> unit`

Evaluation rules: e_1 `:=` e_2

1. Evaluate expression e_1.
2. If e_1 reduces to a reference `r`, then evaluate expression e_2.
3. If e_2 reduces to a value `v`, update contents of `r` to `v`, return `()`.

Typing rules: e_1 `:=` e_2

→ If e_1 `: t ref` and e_2 `: t`, then e_1 `:=` e_2 `: unit`.

Reference cells support pattern matching

Reference cells support pattern matching

We can pattern match on `ref`:

Reference cells support pattern matching

We can pattern match on `ref`:

```
(* containsZero : int ref -> bool *)

fun containsZero (ref 0) = true
  | containsZero _ = false
```

Reference cells support pattern matching

We can pattern match on `ref`:

```
(* containsZero : int ref -> bool *)

fun containsZero (ref 0) = true
  | containsZero _ = false
```

Reference cells support pattern matching

We can pattern match on `ref`:

```
(* containsZero : int ref -> bool *)

fun containsZero (ref 0) = true
  | containsZero _ = false
```

Reference cells support pattern matching

We can pattern match on `ref`:

```
(* containsZero : int ref -> bool *)

fun containsZero (ref 0) = true
  | containsZero _ = false
```

Reference cells support pattern matching

We can pattern match on `ref`:

```
(* containsZero : int ref -> bool *)

fun containsZero (ref 0) = true
  | containsZero _ = false

val d = ref 42
val false = containsZero d
```

Reference cells support pattern matching

We can pattern match on `ref`:

```
(* containsZero : int ref -> bool *)

fun containsZero (ref 0) = true
  | containsZero _ = false

val d = ref 42
val false = containsZero d
val false = containsZero (ref 7)
```

Reference cells support pattern matching

We can pattern match on `ref`:

```
(* containsZero : int ref -> bool *)

fun containsZero (ref 0) = true
  | containsZero _ = false

val d = ref 42
val false = containsZero d
val false = containsZero (ref 7)
val true = containsZeros (ref 0)
```

Sequential composition

Sequential composition

In the presence of effects, the **order of evaluation** matters.

Sequential composition

In the presence of effects, the **order of evaluation** matters.

For convenience, SML supports the semicolon expression:

$$(e_1;\ e_2)$$

Sequential composition

In the presence of effects, the **order of evaluation** matters.

For convenience, SML supports the semicolon expression:

$$(e_1; e_2)$$

Which is syntactic sugar for:

$$\texttt{let val \_ = } e_1 \texttt{ in } e_2 \texttt{ end}$$

Sequential composition

In the presence of effects, the **order of evaluation** matters.

For convenience, SML supports the semicolon expression:

$$(e_1;\ e_2)$$

Which is syntactic sugar for:

$$\texttt{let val \_ = } e_1 \texttt{ in } e_2 \texttt{ end}$$

1. Evaluate e_1, executing effects but ignoring any returned value.

Sequential composition

In the presence of effects, the **order of evaluation** matters.

For convenience, SML supports the semicolon expression:

$$(e_1;\ e_2)$$

Which is syntactic sugar for:

$$\texttt{let val \_ = } e_1 \texttt{ in } e_2 \texttt{ end}$$

1. Evaluate e_1, executing effects but ignoring any returned value.
2. Then, evaluate e_2, executing effects and return the value of e_2.

Sequential composition

In the presence of effects, the **order of evaluation** matters.

For convenience, SML supports the semicolon expression:

$$(e_1;\ e_2)$$

Which is syntactic sugar for:

$$\texttt{let val \_ = } e_1 \texttt{ in } e_2 \texttt{ end}$$

1. Evaluate e_1, executing effects but ignoring any returned value.
2. Then, evaluate e_2, executing effects and return the value of e_2.

Generalizes to:

$$(e_1;\ e_2;\ \ldots;\ e_n) : t_n$$

Sequential composition

Sequential composition

Example:

```
let
  val c = ref 10
in
  (print(Int.toString(!c));
   c)
end
```

Sequential composition

Example:

```
let
  val c = ref 10
in
  (print(Int.toString(!c));
   c)
end
```

What is the type of this `let` expression?

Sequential composition

Example:

```
let
  val c = ref 10
in
  (print(Int.toString(!c));
   c)
end
```

What is the type of this `let` expression? `int ref`

Sequential composition

Example:

```
let
  val c = ref 10
in
  (print(Int.toString(!c));
   c)
end
```

What is the type of this `let` expression? `int ref`

What is its value?

Sequential composition

Example:

```
let
  val c = ref 10
in
  (print(Int.toString(!c));
   c)
end
```

What is the type of this `let` expression? `int ref`

What is its value? `ref 10`

Sequential composition

Example:

```
let
  val c = ref 10
in
  (print(Int.toString(!c));
   c)
end
```

What is the type of this `let` expression? `int ref`

What is its value? `ref 10`

What its effect?

Sequential composition

Example:

```
let
  val c = ref 10
in
  (print(Int.toString(!c));
   c)
end
```

What is the type of this `let` expression? `int ref`

What is its value? `ref 10`

What its effect? prints 10

Sequential composition

Alternative implementation of previous example:

```
let
  val c = ref 10
  val _ = print(Int.toString(!c))
in
  c
end
```

Aliasing

Aliasing

Consider this code:

Aliasing

Consider this code:

```
val c = ref 10
val w = !c
val d = c
val () = d := 42
val v = !c
```

Aliasing

Consider this code:

```
val c = ref 10
val w = !c
val d = c
val () = d := 42
val v = !c
```

What values are **w** and **v** bound to?

Aliasing

Consider this code:

```
val c = ref 10
val w = !c
val d = c
val () = d := 42
val v = !c
```

What values are `w` and `v` bound to?

`w` is bound to `10`

Aliasing

Consider this code:

```
val c = ref 10
val w = !c
val d = c
val () = d := 42
val v = !c
```

What values are `w` and `v` bound to?

`w` is bound to `10`, `v` is bound to `42`.

Aliasing

Consider this code:

```
val c = ref 10
val w = !c
val d = c
val () = d := 42
val v = !c
```

What values are `w` and `v` bound to?

`w` is bound to `10`, `v` is bound to `42`.

Aliasing

Consider this code:

```
val c = ref 10
val w = !c
val d = c
val () = d := 42
val v = !c
```

d is now referring to the same cell as c

What values are `w` and `v` bound to?

`w` is bound to `10`, `v` is bound to `42`.

Aliasing

Consider this code:

```
val c = ref 10
val w = !c
val d = c
val () = d := 42
val v = !c
```

d is now referring to the same cell as c

assignment to d affects what can be read from c

What values are `w` and `v` bound to?

`w` is bound to `10`, `v` is bound to `42`.

Aliasing

Consider this code:

```
val c = ref 10
val w = !c
val d = c
val () = d := 42
val v = !c
```

d is now referring to the same cell as c

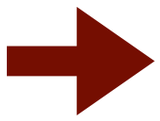
assignment to d affects what can be read from c

What values are `w` and `v` bound to?

`w` is bound to `10`, `v` is bound to `42`.

→ To account for aliasing, we must extend dynamics with a store.

Aliasing



To account for aliasing, we must extend dynamics with a store.

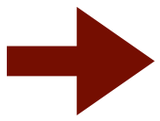
Aliasing

To account for aliasing, we must extend dynamics with a store.

For pure expressions:

$$e \implies e'$$

Aliasing



To account for aliasing, we must extend dynamics with a store.

For pure expressions:

```
e ==> e'
```

For impure expressions:

```
{s | e} ==> {s' | e'}
```

Aliasing

To account for aliasing, we must extend dynamics with a store.

For pure expressions:

```
e ==> e'
```

For impure expressions:

```
{s | e} ==> {s' | e'}
```

Aliasing

To account for aliasing, we must extend dynamics with a store.

For pure expressions:

```
e ==> e'
```

For impure expressions:

```
{s | e} ==> {s' | e'}
```

store,
i.e., all allocated
reference cells

Aliasing

To account for aliasing, we must extend dynamics with a store.

For pure expressions:

```
e ==> e'
```

For impure expressions:

```
{s | e} ==> {s' | e'}
```

store,
i.e., all allocated
reference cells

Aliasing

To account for aliasing, we must extend dynamics with a store.

For pure expressions:

```
e ==> e'
```

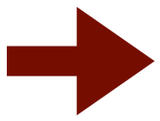
For impure expressions:

```
{s | e} ==> {s' | e'}
```

store,
i.e., all allocated
reference cells

evaluation may alter
the store!

Aliasing



To account for aliasing, we must extend dynamics with a store.

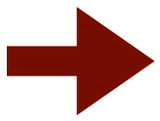
For pure expressions:

```
e ==> e'
```

For impure expressions:

```
{s | e} ==> {s' | e'}
```

Aliasing



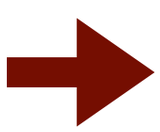
To account for aliasing, we must extend dynamics with a store.

For pure expressions:

```
e ==> e'
```

For impure expressions:

```
{s | e} ==> {s' | e'}
```



We won't go into any further details in 15-150.

Aliasing

To account for aliasing, we must extend dynamics with a store.

For pure expressions:

```
e ==> e'
```

For impure expressions:

```
{s | e} ==> {s' | e'}
```

We won't go into any further details in 15-150.

More on this in 15-312!

Aliasing

To account for aliasing, we must extend dynamics with a store.

For pure expressions:

```
e ==> e'
```

For impure expressions:

```
{s | e} ==> {s' | e'}
```

We won't go into any further details in 15-150.

More on this in 15-312!

Note: aliasing complicates reasoning about programs 😢

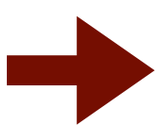
Extensional equivalence

Extensional equivalence

For pure programs:

Extensional equivalence

For pure programs:



extensional equivalence as defined until now

Extensional equivalence

For pure programs:

- extensional equivalence as defined until now
- allow equals to be replaced by equals ("referential transparency")

Extensional equivalence

For pure programs:

- extensional equivalence as defined until now
- allow equals to be replaced by equals ("referential transparency")

For imperative programs:

Extensional equivalence

For pure programs:

- extensional equivalence as defined until now
- allow equals to be replaced by equals ("referential transparency")

For imperative programs:

- extensional equivalence must additionally account for the store

Extensional equivalence

For pure programs:

- extensional equivalence as defined until now
- allow equals to be replaced by equals ("referential transparency")

For imperative programs:

- extensional equivalence must additionally account for the store
  - requires advanced program logics (even beyond 15-312)

Extensional equivalence

For pure programs:

- extensional equivalence as defined until now
- allow equals to be replaced by equals ("referential transparency")

For imperative programs:

- extensional equivalence must additionally account for the store
  - requires advanced program logics (even beyond 15-312)
- For pure expressions e and e', to show e ≅ e', we must show that e and e' are independent of any store.

Extensional equivalence

For imperative programs:

→ extensional equivalence must additionally account for the store

→ requires advanced program logics (even beyond 15-312)

→ For pure expressions e and e', to show e ≅ e', we must show that e and e' are independent of any store.

Extensional equivalence

For imperative programs:

- extensional equivalence must additionally account for the store
  - requires advanced program logics (even beyond 15-312)
- For pure expressions e and e', to show e ≅ e', we must show that e and e' are independent of any store.

Note:

Extensional equivalence

For imperative programs:

- extensional equivalence must additionally account for the store
  - requires advanced program logics (even beyond 15-312)
- For pure expressions e and e', to show e ≅ e', we must show that e and e' are independent of any store.

Note:

- `ref` types are so called equality types

Extensional equivalence

For imperative programs:

- extensional equivalence must additionally account for the store
  - requires advanced program logics (even beyond 15-312)

- For pure expressions e and e', to show e ≅ e', we must show that e and e' are independent of any store.

Note:

- `ref` types are so called equality types

For `r : 'a ref` and `s : 'a ref`, `r = s` evaluates to `true`, if `r` and `s` are aliases, i.e., point to the same cell.

Race conditions

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n

val chk = ref 100

val _ = (deposit chk 50; withdraw chk 80)
```

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n

val chk = ref 100

val _ = (deposit chk 50; withdraw chk 80)
```

What is the value of `!chk`?

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n
```

```
val chk = ref 100

val _ = (deposit chk 50; withdraw chk 80)
```

What is the value of `!chk`?

```
70
```

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n

val chk = ref 100

val _ = (deposit chk 50; withdraw chk 80)
```

What is the value of !`chk`?

`70`

Now, if we parallelize, what is the value of !`chk`?

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n

val chk = ref 100

val _ = (deposit chk 50; withdraw chk 80)
```

What is the value of !`chk`?

`70`

Now, if we parallelize, what is the value of !`chk`?

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n

val chk = ref 100

val _ = (deposit chk 50, withdraw chk 80)
```

What is the value of !`chk`?

`70`

Now, if we parallelize, what is the value of !`chk`?

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n

val chk = ref 100

val _ = (deposit chk 50, withdraw chk 80)
```

What is the value of !`chk`?

`70`

Now, if we parallelize, what is the value of !`chk`?

We could end up with **`20`**, **`70`**, or **`150`**.

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n

val chk = ref 100

val _ = (deposit chk 50, withdraw chk 80)
```

Race conditions

In the presence of mutation, reasoning about parallel program becomes complicated.

```
fun deposit a n = a := !a + n

fun withdraw a n = a := !a - n

val chk = ref 100

val _ = (deposit chk 50, withdraw chk 80)
```

→ Mutation and parallelism leads to non-deterministic outcomes 😢

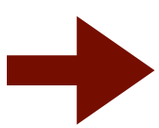
Persistent versus ephemeral data

Persistent versus ephemeral data

Pure programs:

Persistent versus ephemeral data

Pure programs:



yield persistent data structures

Persistent versus ephemeral data

Pure programs:

- yield persistent data structures
- facilitate reasoning and support deterministic parallelism

Persistent versus ephemeral data

Pure programs:

- yield persistent data structures
- facilitate reasoning and support deterministic parallelism

Imperative programs:

Persistent versus ephemeral data

Pure programs:

- yield persistent data structures
- facilitate reasoning and support deterministic parallelism

Imperative programs:

- yield ephemeral data structures

Persistent versus ephemeral data

Pure programs:

- yield persistent data structures
- facilitate reasoning and support deterministic parallelism

Imperative programs:

- yield ephemeral data structures
- complicate reasoning and demand concurrent scheduling

Persistent versus ephemeral data

Pure programs:

- yield persistent data structures
- facilitate reasoning and support deterministic parallelism

Imperative programs:

- yield ephemeral data structures
- complicate reasoning and demand concurrent scheduling

However, not all effects are evil.

Persistent versus ephemeral data

Pure programs:

- yield persistent data structures
- facilitate reasoning and support deterministic parallelism

Imperative programs:

- yield ephemeral data structures
- complicate reasoning and demand concurrent scheduling

However, not all effects are evil.

- When employed locally, effects can be **benign**.

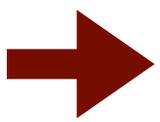
Benign effects

Benign effects

A **benign effect** is an effect (such as mutation) that is **localized** within some sufficiently small chunk of code (e.g., function or structure) so that external users can sue the code as **if it were purely functional**.

Benign effects

A **benign effect** is an effect (such as mutation) that is **localized** within some sufficiently small chunk of code (e.g., function or structure) so that external users can sue the code as **if it were purely functional**.



Benign effects can be useful, for instance, in improving efficiency.

Benign effects

A **benign effect** is an effect (such as mutation) that is **localized** within some sufficiently small chunk of code (e.g., function or structure) so that external users can sue the code as **if it were purely functional**.

→ Benign effects can be useful, for instance, in improving efficiency.

→ Because effect is local, local reasoning remains intact.

Benign effects

A **benign effect** is an effect (such as mutation) that is **localized** within some sufficiently small chunk of code (e.g., function or structure) so that external users can sue the code as **if it were purely functional**.

- Benign effects can be useful, for instance, in improving efficiency.
- Because effect is local, local reasoning remains intact.
- Let's look at some examples!

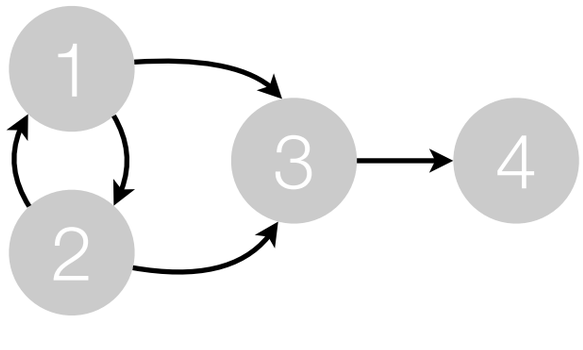
Example: graph reachability

Example: graph reachability

Consider this directed graph:

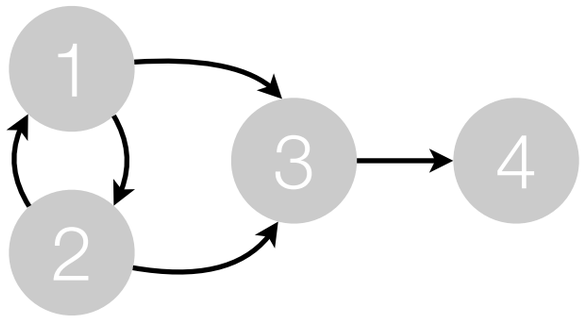
Example: graph reachability

Consider this directed graph:



Example: graph reachability

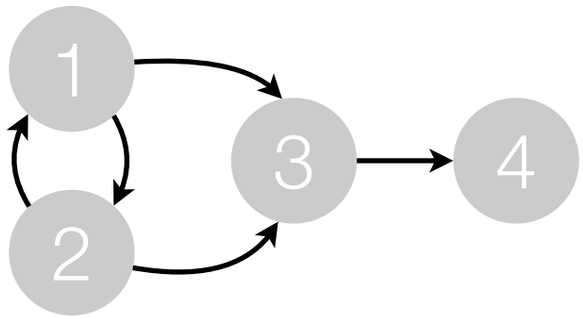
Consider this directed graph:



We can represent this graph as a function, giving for a node the nodes immediately reachable from it:

Example: graph reachability

Consider this directed graph:



We can represent this graph as a function, giving for a node the nodes immediately reachable from it:

```
type  graph = int -> int list

val G : graph = fn 1 => [2,3]
                 | 2 => [1,3]
                 | 3 => [4]
                 | _ => [ ]
```

Example: graph reachability

Example: graph reachability

Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

Example: graph reachability

Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

```
fun reach (g:graph) (x:int, y:int) : bool =
  let
    fun dfs n = (n=y) orelse (List.exists dfs (g n))
  in
    dfs x
  end
```

Example: graph reachability

Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

```
fun reach (g:graph) (x:int, y:int) : bool =
  let
    fun dfs n = (n=y) orelse (List.exists dfs (g n))
  in
    dfs x
  end
```

Example: graph reachability

Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

```
fun reach (g:graph) (x:int, y:int) : bool =
  let
    fun dfs n = (n=y) orelse (List.exists dfs (g n))
  in
    dfs x
  end
```

did we reach `y`?

Example: graph reachability

Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

```
fun reach (g:graph) (x:int, y:int) : bool =
  let
    fun dfs n = (n=y) orelse (List.exists dfs (g n))
  in
    dfs x
  end
```

did we reach `y`?

Example: graph reachability

Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

```
fun reach (g:graph) (x:int, y:int) : bool =
  let
    fun dfs n = (n=y) orelse (List.exists dfs (g n))
  in
    dfs x
  end
```

did we reach `y`?

neighbors of `n`

Example: graph reachability

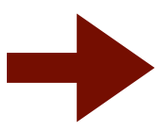
Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

```
fun reach (g:graph) (x:int, y:int) : bool =
  let
    fun dfs n = (n=y) orelse (List.exists dfs (g n))
  in
    dfs x
  end
```

Example: graph reachability

Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

```
fun reach (g:graph) (x:int, y:int) : bool =
  let
    fun dfs n = (n=y) orelse (List.exists dfs (g n))
  in
    dfs x
  end
```



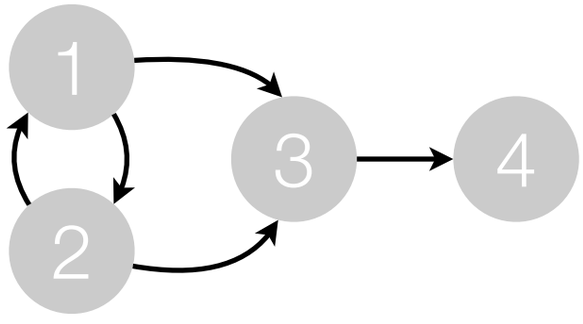
Problem: reach can loop on our example graph, which is cyclic!

Example: graph reachability

Now, let's define a function, `reach g (x,y)`, determining whether `y` is transitively reachable from `x` in graph `g`.

```
fun reach (g:graph) (x:int, y:int) : bool =
  let
    fun dfs n = (n=y) orelse (List.exists dfs (g n))
  in
    dfs x
  end
```

Problem: reach can loop on our example graph, which is cyclic!



Example: graph reachability

Example: graph reachability

We can fix this by recording who we have already visited.

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)
```

```
fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
   in
      dfs x
   end
```

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

`mem n L` checks whether `n` is in list `L`

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
   in
      dfs x
   end
```

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
   in
       dfs x
   end
```

reference that records visited nodes

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

only continue
if not has not yet been
visited

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

Example: graph reachability

We can fix this by recording who we have already visited.

```
fun mem (n:int) = List.exists (fn x => n=x)

fun reachable (g:graph) (x:int, y:int) : bool =
  let
    val visited = ref []
    fun dfs n = (n=y) orelse
                (not (mem n (!visited)) andalso
                (visited := n::(!visited);
                 List.exists dfs (g n)))
  in
     dfs x
  end
```

update visited list

Example: random number generator

Example: random number generator

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```

bound

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

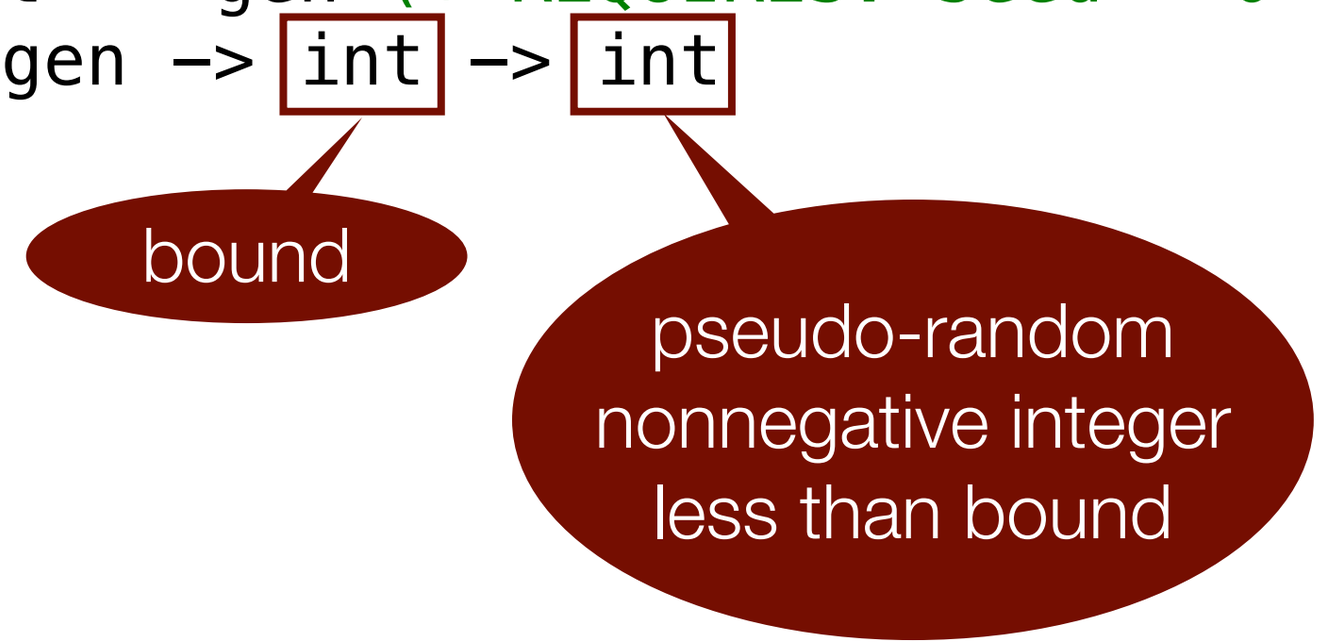
```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```

bound

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```



(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```

```
val G = R.init(12345)
val L = List.tabulate(42,fn _ => R.random G 1000)
```

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```

```
struct R :> RANDOM
  type gen = real ref
  val a = 16807.0
  val m = 2147483647.0
  fun next r = a * r - m*real(floor(a*r/m))
  val init = ref o real
  fun random g b = (g := next(!g);
                    floor( (!g/m)* (real b)))
end
```

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```

```
struct R :> RANDOM
  type gen = real ref
  val a = 16807.0
  val m = 2147483647.0
  fun next r = a * r - m*real(floor(a*r/m))
  val init = ref o real
  fun random g b = (g := next(!g);
                    floor( (!g/m)* (real b)))
end
```

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: random number generator

```
signature RANDOM =
sig
  type gen (*abstract *)
  val init: int -> gen (* REQUIRES: seed > 0 *)
  val random: gen -> int -> int
end
```

```
struct R :> RANDOM
  type gen = real ref
  val a = 16807.0
  val m = 2147483647.0
  fun next r = a * r - m*real(floor(a*r/m))
  val init = ref o real
  fun random g b = (g := next(!g);
                    floor( (!g/m)* (real b)))
end
```

(Reference: "ML for the Working Programmer" by Paulson. 1996.)

Example: stream memoization

Example: stream memoization

Previously, we had the following code inside our `Stream` structure:

Example: stream memoization

Previously, we had the following code inside our `Stream` structure:

```
(* delay : (unit -> 'front) -> 'a stream *)
fun delay (d) = Stream(d)

(* expose : 'a stream -> 'a front *)
fun expose (Stream(d)) = d ()
```

Example: stream memoization

Previously, we had the following code inside our `Stream` structure:

```
(* delay : (unit -> 'front) -> 'a stream *)
fun delay (d) = Stream(d)

(* expose : 'a stream -> 'a front *)
fun expose (Stream(d)) = d ()
```

Let's add a hidden reference cell that remembers the result of computing `d()`.

Example: stream memoization

Previously, we had the following code inside our `Stream` structure:

```
(* delay : (unit -> 'front) -> 'a stream *)
fun delay (d) = Stream(d)

(* expose : 'a stream -> 'a front *)
fun expose (Stream(d)) = d ()
```

- Let's add a hidden reference cell that remembers the result of computing `d()`.
- We will will leave `expose` as is, but change `delay`.

Example: stream memoization

Example: stream memoization

Updated function `delay`:

Example: stream memoization

Updated function `delay`:

```
fun delay (d) =
  let
    val cell = ref d
    fun memoFn () =
      let
        val r = d()
      in
        (cell := (fn () => r); r)
      end
    val _ = cell := memoFn
in
  Stream (fn () => !cell())
end
```

That's all for today.