INSTRUCTIONS

- Exam length: 80 minutes
- You are permitted to have one handwritten page of notes, double-sided
- No calculators or other electronic devices allowed

Name	
Andrew ID	

Andrew ID of Person On Your Left	Andrew ID of Person On Your Right

For staff use only

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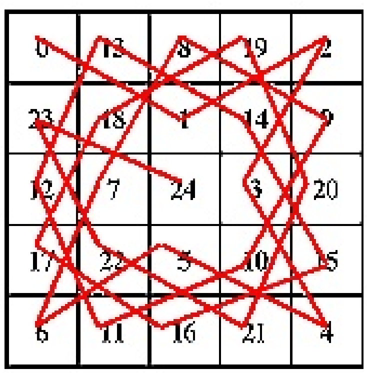
Q1. [37 pts] Knight's Tour (Search)

A knight in chess can move 2 squares forward, backward, left, or right, and then 1 square at a 90 degree angle. For example, it can move 2 squares forward and 1 square left, or 2 squares forward and 1 square right, or 2 squares left and 1 square forward, etc. **Please raise your hand and ask for clarification if you are unclear.**

(a) [5 pts] Put an X in every square that the knight K can move to in exactly one move.

			X		X		
		X				X	
				K			
		X				X	
			X		X		

Consider the problem of findings a Knight's Tour on an NxN grid. A knight starts on one square on the grid and **must move to legal knight positions without repeating any squares** until all squares on the entire grid have been visited. Below is a knight's tour on a 5x5 grid starting at 0 and ending at 24.



(b) [5 pts] We want to search for a knight's tour using our planning representation of states, actions, transitions, and goals. Let a state in this search problem be represented as the tuple of the knight's current location and the **set** of locations that the knight has visited already (current location included).

What is the smallest state space representation that we could use:

○ N^2 ○ $N^2 * N^2$ ● $N^2 * 2^{N^2}$ ○ $N^2 * (N^2!)$

(c) [5 pts] What is the maximum branching factor for the knights tour search problem on an arbitrarily large NxN grid?

Branching factor:
8

(d) [5 pts] Is a knights tour better solved using BFS or DFS? ○ BFS ● DFS

Why?
DFS, because all the solutions are at the same depth.

(e) [12 pts] Remember that the states in your search problem are represented as a tuple of the knight's current location and the set of all the places that the knight has visited already (current location included). Let $|set_s|$ be the size of the visited set in state s. For example, for a state s_0 where the knight is currently in (1,1) and has visited no other locations, $|set_{s_0}| = 1$. Assume that the actions (a single knight move) each cost 1.

On a 5x5 grid, you create the following heuristic so you can use A* search to find a knight's tour:

$$h(s) = (5 * 5) - |set_s|$$

.

Is this heuristic admissible? ● Yes ○ No

Is this heuristic consistent? ● Yes ○ No

Is this heuristic useful for A* to reduce the number of explored nodes in search compared to BFS or DFS?

○ Yes ● No

Why or Why not?

Why or Why not?
Because every state of the same depth has the same value, so it doesn't distinguish between them to find a more efficient path to the goal.

(f) [5 pts] A different heuristic for this problem has been proposed by Warnsdorff. It states: from the current location, select the action that leads to a location with the fewest possible next actions. What type of heuristic have we learned about that is most similar to this heuristic:

○ Informed search h(s) ● MRV ○ AC-3 Filtering ○ Hill Climbing

Why?
MRV because it tells you the ordering of which next state you should try - the one with the minimum remaining number of actions to take. It is not h(s) because the number of actions isn't a heuristic for the number of actions until the goal is met (also the number of actions changes as the different states are visited). It is not Filtering because you aren't using decisions at one level to remove possible values at the next level. It is not hill climbing because the decision isn't maximizing an objective function.

Q2. [16 pts] Knight's Tour Part II (LP/IP)

Consider again a knights tour covering an entire NxN grid. For example, we know it is possible to create a knights tour completely covering a 5x5 grid. We can also solve the knights tour using LP. Create binary variables $v_{x,y,k}$ such that the variable is 1 if the location (x, y) is visited kth on the knights tour and 0 otherwise $(1 \leq k \leq N^2)$. On an NxN board, we would have N^4 variables – there are N^2 variables for each of the N^2 locations representing if a location is visited k =first, second, third, ... N^2. The goal is for exactly N^2 of these N^4 variables to be 1, and the rest 0.

Using what you learned from HW4 about binary variables and summations for linear programs, create the following constraints and the objective **in inequality form** to define a knights tour using the variables $v_{x,y,k}$.

You may use $\sum$s over variables rather than writing out the addition, but you MUST specify what variable(s) is/are being summed over to receive credit.

(a) [5 pts] Consider the kth index in the knight's tour – $1 \leq k \leq N^2$. Write the constraint(s) that at most one location should be visited kth. (Yes, we know this is not sufficient but this is an exam and there isn't enough time!).

Constraint(s):
$\sum_{(x,y)} v_{x,y,k} \leq 1$

(b) [5 pts] Write the constraint(s) that each location (x, y) should be visited exactly one time:

Constraint(s):
$\sum_k v_{x,y,k} \leq 1$
$\sum_k -v_{x,y,k} \leq -1$

(c) [6 pts] Assume we have a bunch of additional linear constraints that relate the value of $v_{x,y,k}$ to variables reachable by one knight's move.

Which of the following objective functions for a linear program result in finding a knights tour of exactly length N^2? Select all that apply.

- ☒ min $(-\sum_k \sum_{(x,y)} v_{x,y,k})$
- ☒ min $(-\sum_{(x,y)} v_{x,y,N^2})$
- ☐ min $(-\sum_k (\prod_{(x,y)} v_{x,y,k}))$
- ☒ min $(c^T v)$ where c is a vector of all -1s
- ☐ min $(-\sum_k (\max_{(x,y)} v_{x,y,k}))$
- ☒ min $(-v_{1,1,1})$ where (1,1) is the top left corner

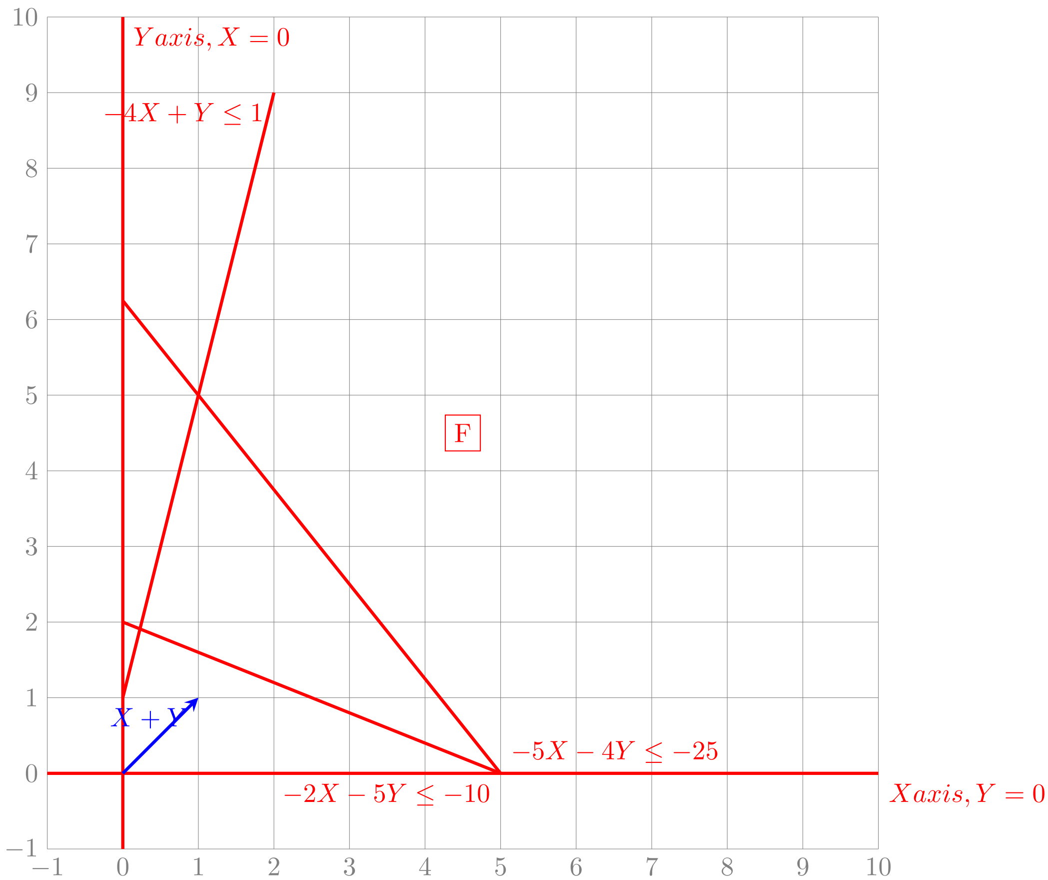
Q3. [19 pts] Linear and Integer Programming

You work for a bakery that makes cakes and cupcakes. The bakery is receiving a new shipment of supplies tomorrow and you must bake enough cakes and cupcakes to make room in storage for the shipment. Cakes require 5 eggs and 2 cups of flour and make a \$4 profit. Cupcakes require 4 eggs, 5 cups of flour, and the bakery loses \$1 for each cupcake sold. You are told you need to use at least 25 eggs and 10 cups of flour. Your boss wants you to not lose more than \$1 total when you sell your products (you can make a profit of course)! Baking takes a lot of effort, so you want to minimize the number of products you make.

(a) [6 pts] Write the A, b, and c matrix that represent this problem in inequality form. Represent cakes as X and cupcakes as Y. Assume $x = [X, Y]^T$ in that order. Write all constraints including those that seem redundant.

A:	**b:**	**c:**
$[-1, 0]$ $[0, -1]$ $[-4, 1]$ $[-2, -5]$ $[-5, -4]$	$[0, 0, 1, -10, -25]^T$	$[1, 1]^T$

(b) [7 pts] Graph all the constraints and label them. Label the feasible region with the letter F. Draw and label the cost vector c starting at the origin (0,0). Additional graph paper is provided at the end of the test for your convenience but it will not be graded.



(c) [6 pts] What are the coordinates (X, Y) of the optimal objective? What is the objective value at that point? What profit do you make?

LP Coordinates:	**Objective Value:**	**Profit:**
(5,0)	5	\$20

Q4. [12 pts] Propositional Logic

The TAs are going on a trip. They decide that they will go swimming only if it is sunny. They will go canoeing only if they do not go swimming. If they go canoeing, they promise to be home by sunset. Today is cold and not sunny.

They translate these statements into the following logical sentence (knowledge base):

$$(\neg Sunny) \land (Cold) \land (Swim \rightarrow Sunny) \land (\neg Swim \rightarrow Canoe) \land (Canoe \rightarrow Sunset)$$

(a) [6 pts] Convert each of the implications into a disjunction.

$Swim \rightarrow Sunny$ $\neg Swim \lor Sunny$	$\neg Swim \rightarrow Canoe$ $Swim \lor Canoe$	$Canoe \rightarrow Sunset$ $\neg Canoe \lor Sunset$

(b) [2 pts] The TAs decide to use Resolution to show that they will be home at *Sunset*. In addition to the KB, what other sentence do they need to add to begin the proof?

Sentence
$(\neg Sunset)$

(c) [4 pts] In the space below, use your answers above and resolution to prove that *Sunset* is entailed by the knowledge base. You do NOT need to write every possible unit resolution to search for the goal. You SHOULD clearly put a box around each of the unit resolutions you do need to prove *Sunset*. Circle the conclusion.

Perform Resolution
Start with:

$$\neg Sunny \land Cold \land (\neg Swim \lor Sunny) \land (Swim \lor Canoe) \land (\neg Canoe \lor Sunset) \land (\neg Sunset)$$

Example:
Combine Sunny: $\neg Swim \land Cold \land (Swim \lor Canoe) \land (\neg Canoe \lor Sunset) \land (\neg Sunset)$
Combine Swim: $Canoe \land Cold \land (\neg Canoe \lor Sunset) \land (\neg Sunset)$
Combine Canoe: $Sunset \land Cold \land (\neg Sunset)$
End with contradiction like $Sunset \land \neg Sunset$. Therefore, it isn't possible to have $\neg Sunset$ in the KB, and *Sunset* is entailed.

Q5. [16 pts] Misc

Answer both of the following questions for 4 points. Answers should be 3-4 sentences.

(a) Describe the reason for the temperature function in simulated annealing. Be sure to comment on the way it is used in the algorithm and the effect it has on the search properties when you change it.

Describe:
The temperature function is used to control how quickly or slowly the algorithm changes from exploring to exploiting. If we make the temperature decrease quickly, we don't spent a lot of time exploring downward states. If we make the temperature decrease slowly, we allow more time to get out of local maxima and find the global maximum.

(b) Describe a way in which a consistent heuristic used in A* search for an in-car navigation system could be developed which adversely effects a population. Be sure to describe the features of the heuristic and also how the returned path would affect the population.

Describe:
Lots of examples. If you made a heuristic that penalizes in-city streets, you could wind up with paths that avoid these areas which can damage business that depends on the through-traffic.

EXTRA GRAPH PAPER FOR YOUR CONVENIENCE

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