AI: Representation and Problem Solving

Local Search

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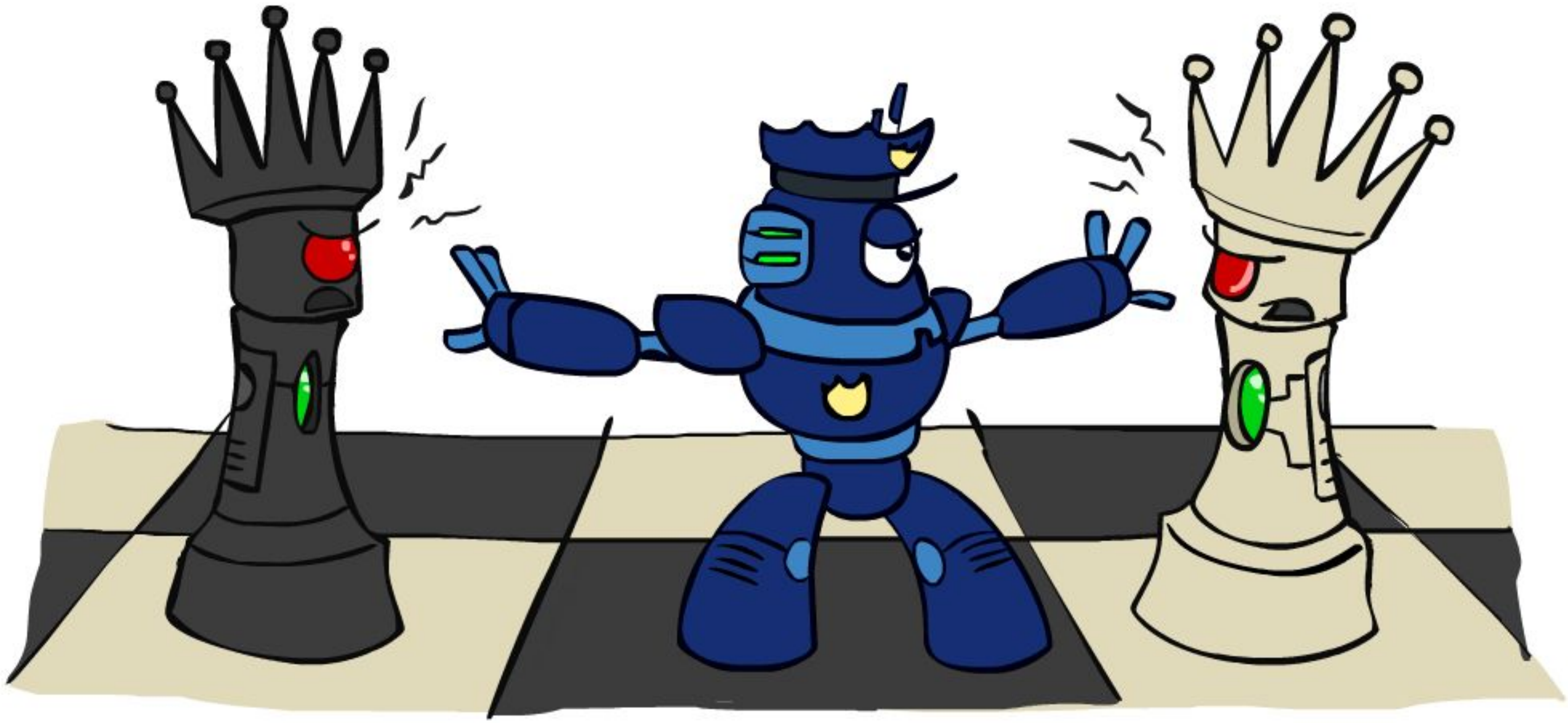
Slide credits: CMU AI, http://ai.berkeley.edu

Local Search

- Can be applied to identification problems (e.g., CSPs), as well as some planning and optimization problems

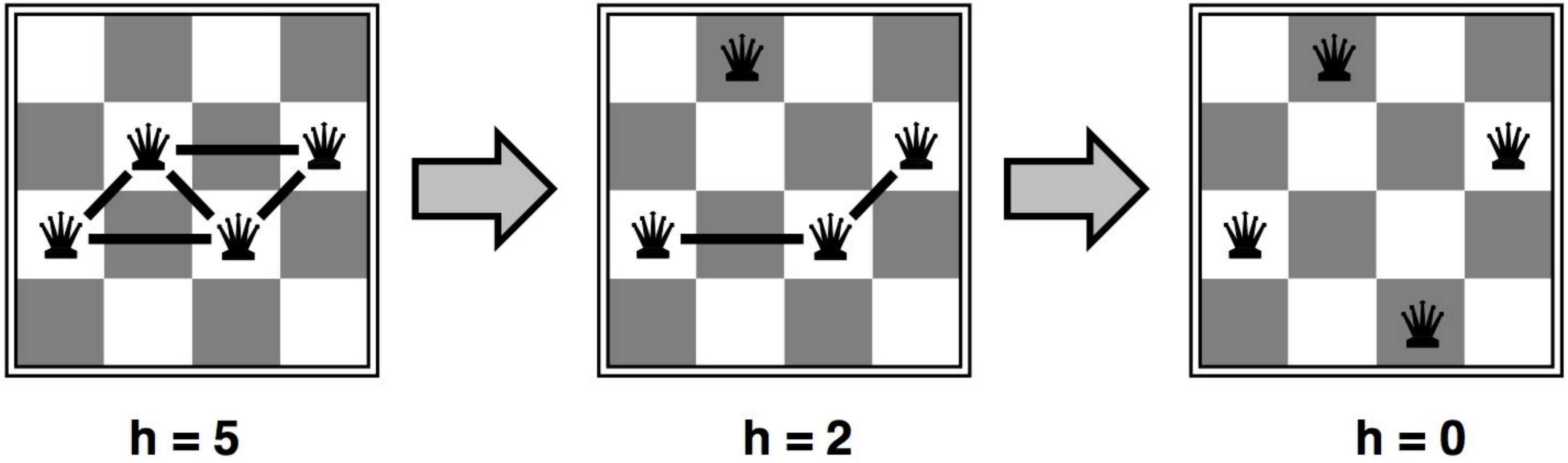
- For identification problems, we use a complete-state formulation, e.g., all variables assigned in a CSP (may not satisfy all the constraints)
- For planning problems, typically we make local decisions
  - E.g., not a plan all the way to the goal or not a deep search

Iterative Improvement for CSPs



Iterative Improvement for CSPs

- Start with an arbitrary assignment, iteratively *reassign* variable values
- While not solved,
  - Variable selection: randomly select a conflicted variable
  - Value selection with min-conflicts heuristic h: Choose a value that violates the fewest constraints (break tie randomly)
- For n-Queens: Variables $x_i \in \{1..n\}$; Constraints $x_i \neq x_j$, $|x_i - x_j| \neq |i - j|, \forall i \neq j$

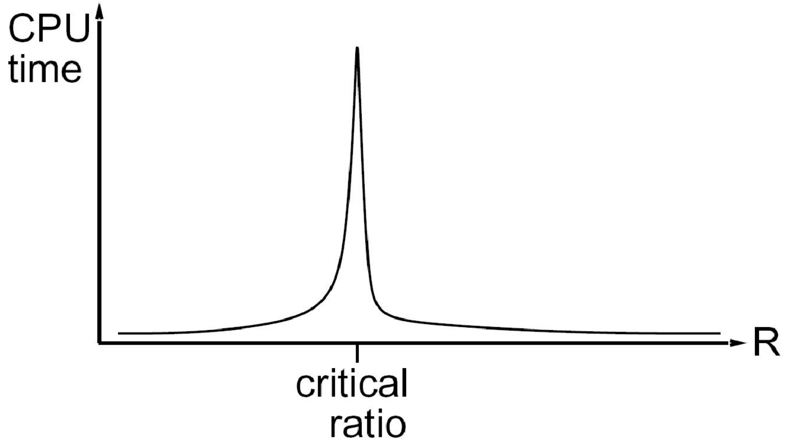


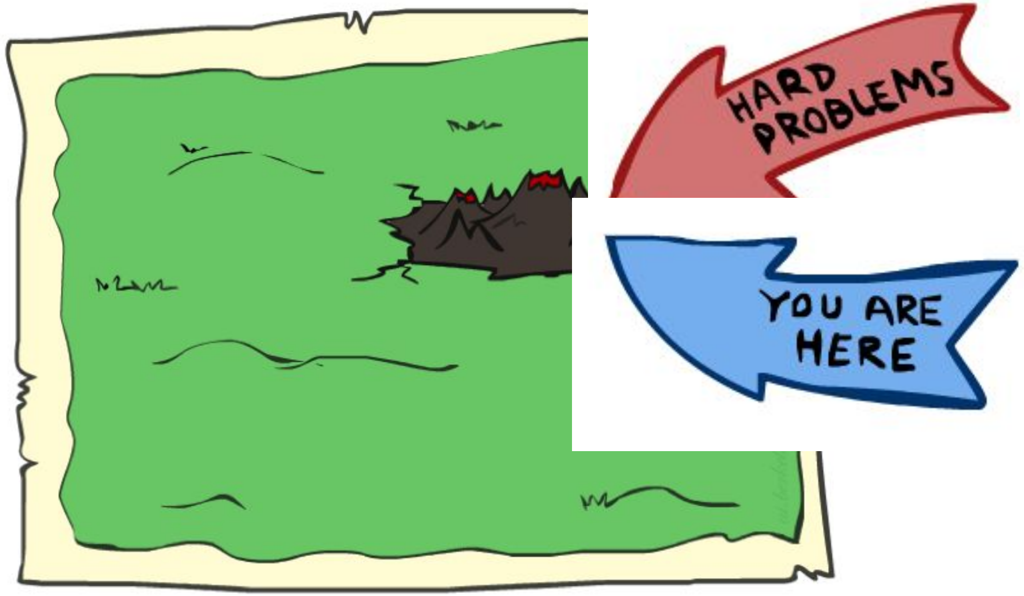
Iterative Improvement for CSPs

- Given random initial state, can solve n-queens in almost constant time for arbitrary n with high probability (e.g., n = 10,000,000)!

- Same for any randomly-generated CSP *except* in a narrow range of the ratio

$$R = \frac{\text{number of constraints}}{\text{number of variables}}$$





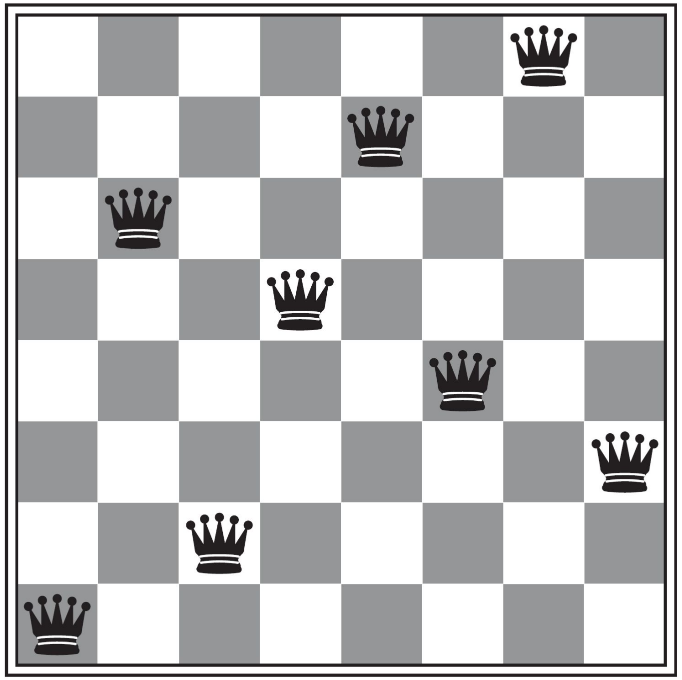
Local Search

- A local search algorithm is…
  - Optimal if it always finds a global minimum/maximum heuristic value

Will an iterative improvement algorithm for CSPs always find a solution?

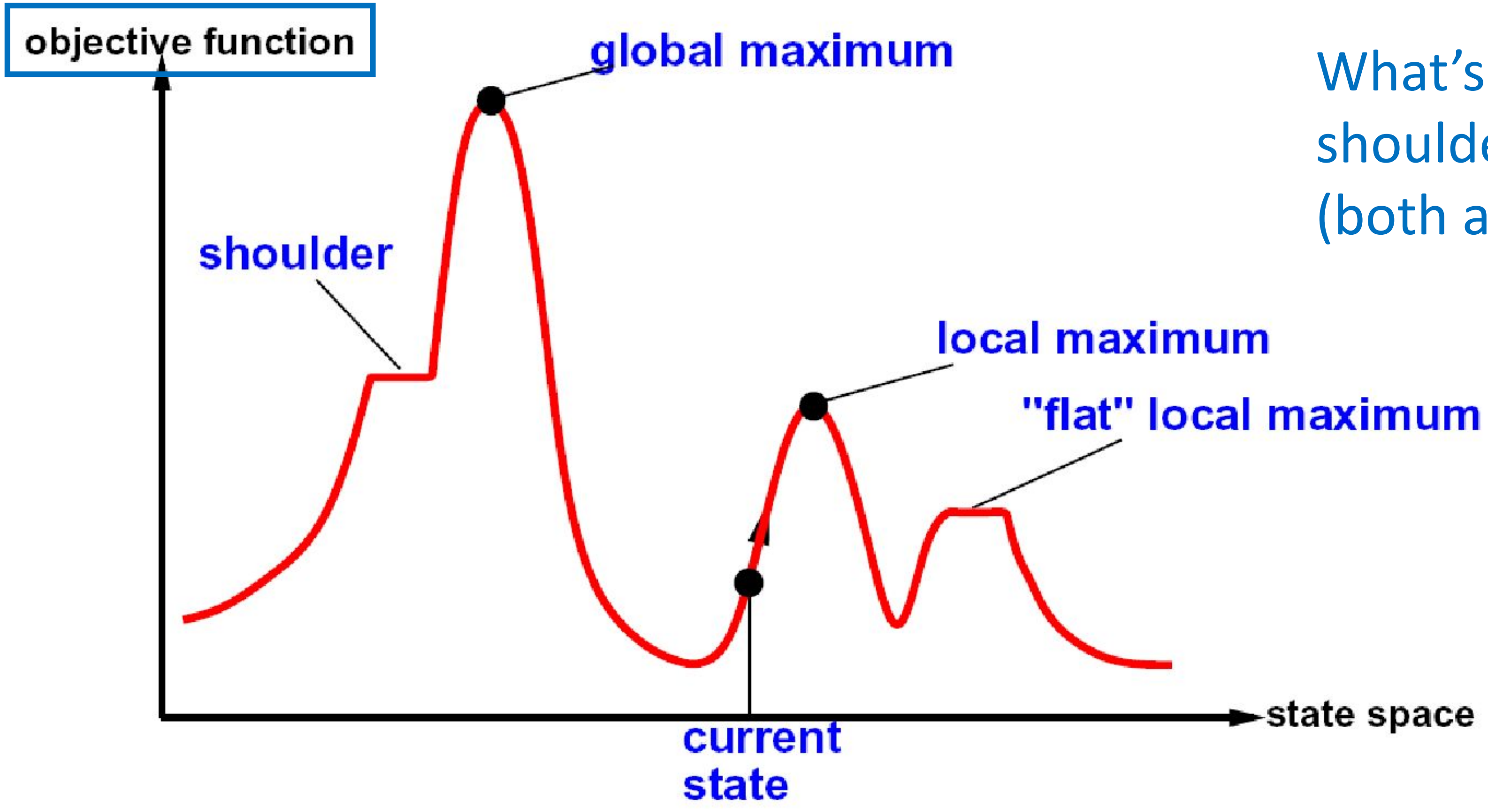
No! May get stuck in a local optimum

$h = 1$



State-Space Landscape

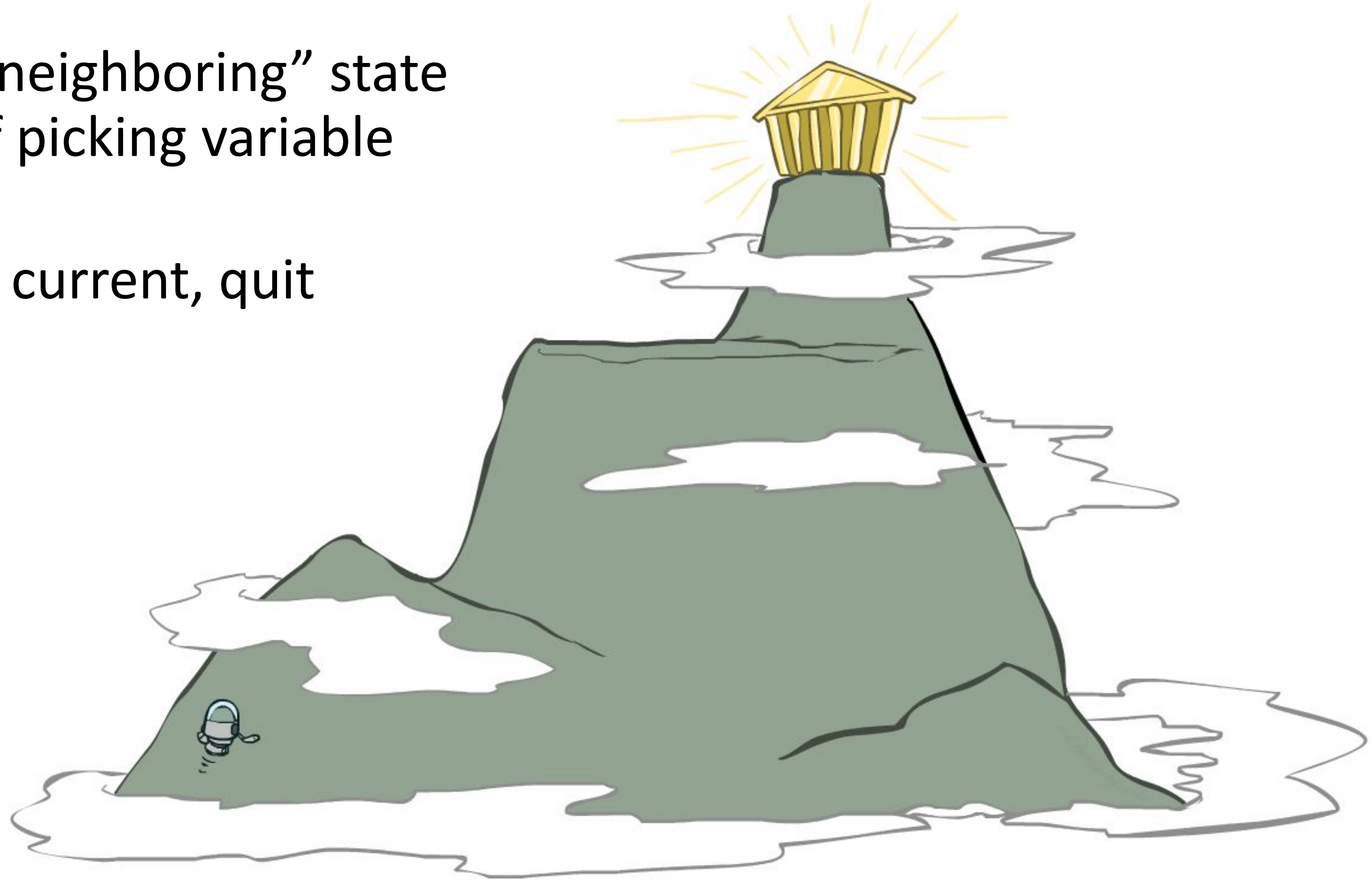
In identification problems, could be a function measuring how close you are to a valid solution, e.g., $-1 \times$ #conflicts in n-Queens/CSP



What's the difference between shoulder and flat local maximum (both are plateaux)?

Hill Climbing (Greedy Local Search)

- Simple, general idea:
  - Start wherever
  - Repeat: move to the best “neighboring” state (successor state) instead of picking variable randomly
  - If no neighbors better than current, quit



Hill Climbing (Greedy Local Search)

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

current ← MAKE-NODE(*problem*.INITIAL-STATE)
loop do
neighbor ← a highest-valued successor of *current*
if neighbor.VALUE ≤ current.VALUE **then** **return** *current*.STATE
current ← *neighbor*

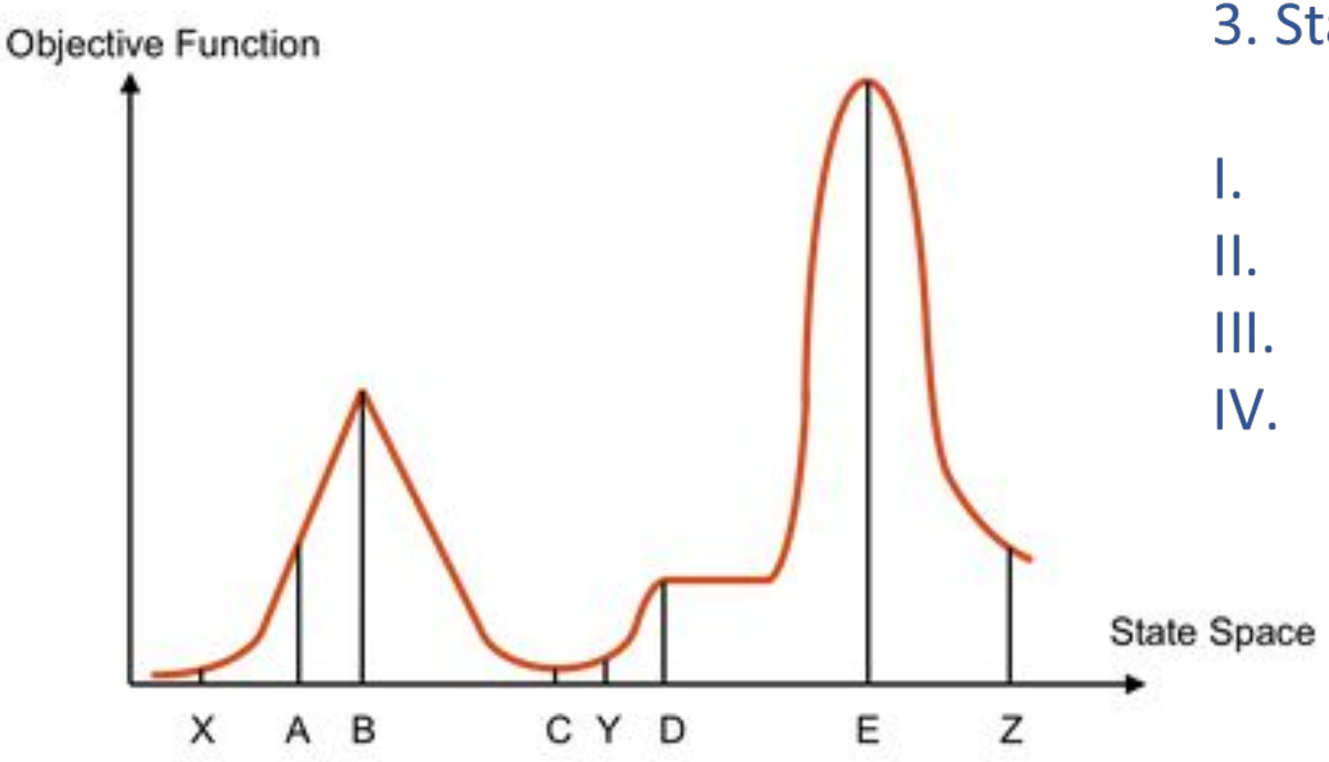
What if there is a tie?

Typically break ties randomly

What if we do not stop here? Make a sideway move if "="

- In 8-Queens, steepest-ascent hill climbing solves 14% of problem instances
  - Takes 4 steps on average when it succeeds, and 3 steps when it fails
- When allow for ≤100 consecutive sideway moves, solves 94% of problem instances
  - Takes 21 steps on average when it succeeds, and 64 steps when it fails

Poll 1 (on Piazza): Hill Climbing



1. Starting from X, where do you end up?
2. Starting from Y, where do you end up?
3. Starting from Z, where do you end up?

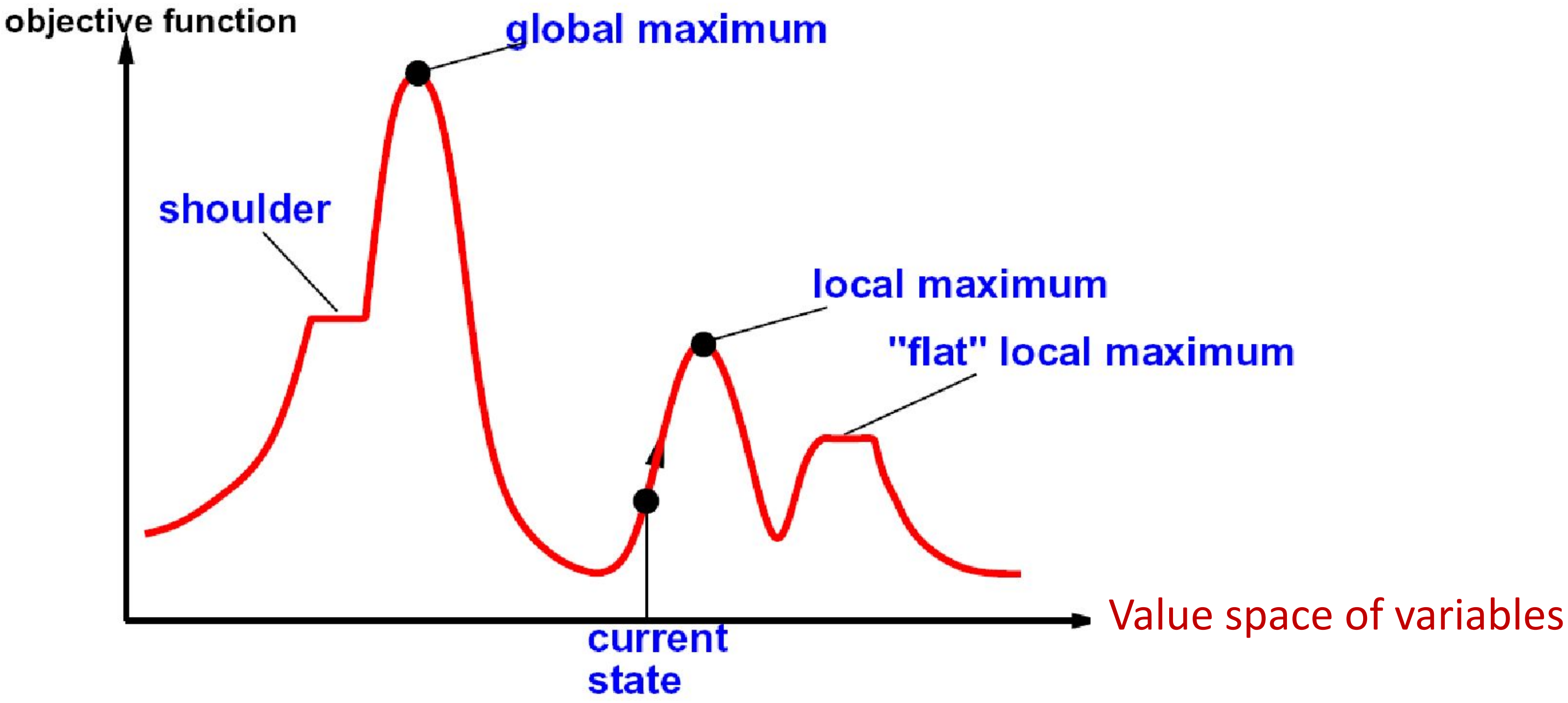
I. $X \rightarrow A, Y \rightarrow D, Z \rightarrow E$
II. $X \rightarrow B, Y \rightarrow D, Z \rightarrow E$
III. $X \rightarrow B, Y \rightarrow E, Z \rightarrow E$
IV. I don't know

Variants of Hill Climbing

- Random-restart hill climbing
  - “If at first you don’t succeed, try again.”
  - In what kind of landscape will random-restarts hill climbing work the best?
- Stochastic hill climbing
  - Choose randomly from the uphill moves, with probability dependent on the “steepness” (i.e., amount of improvement)
  - Converges slower than steepest ascent, but may find better solutions
- First-choice hill climbing
  - Generate successors randomly (one by one) until a better one is found
  - Suitable when there are too many successors to enumerate

Variants of Hill Climbing

- What if variables are continuous, e.g., find $x \in [0,1]$ that maximizes $f(x)$?
  - Gradient ascent
    - Use gradient to find best direction
    - Use the magnitude of the gradient to determine how big a step you move



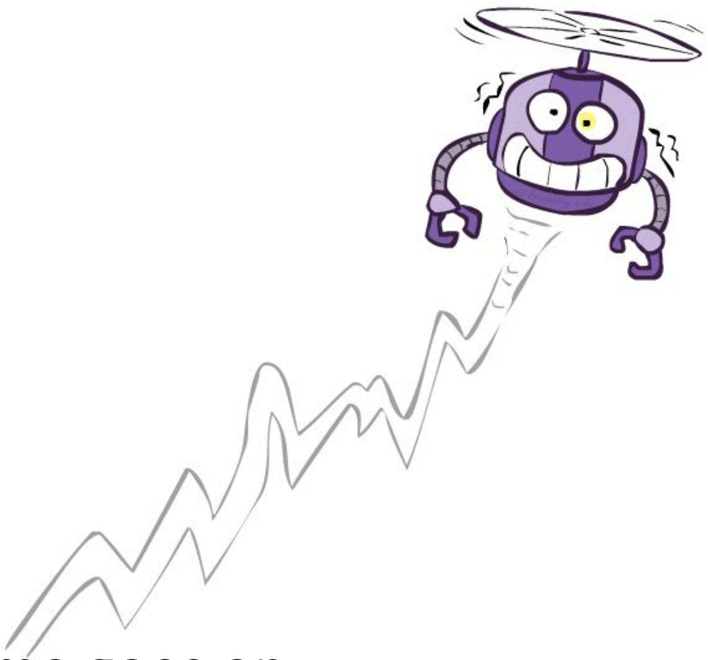
Random Walk

- Uniformly randomly choose a neighbor to move to
- Save the best you’ve seen so far
- Stop after K moves

- What happens to the solution as K increases?

Simulated Annealing

- Combines random walk and hill climbing
- Inspired by statistical physics
- Annealing – Metallurgy
    - Heating metal to high temperature then cooling
    - Reaching low energy state
- Simulated Annealing
    - Allow for downhill moves and make them rarer as time goes on
    - Escapes local maxima and reaches global maxima



Simulated Annealing

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
  inputs: problem, a problem
          schedule, a mapping from time to "temperature"

  current ← MAKE-NODE(problem.INITIAL-STATE)
  for t = 1 to ∞ do
     T ← schedule(t)
     if T = 0 then return current
     next ← a randomly selected successor of current
     ΔE ← next.VALUE – current.VALUE
     if ΔE > 0 then current ← next
     else current ← next only with probability e^(ΔE/T)
```

Control the change of temperature T (↓ over time)

Almost the same as hill climbing except for a *random* successor

Unlike hill climbing, move downhill with some probability

Poll 2:

Which of the following will make it more likely that we'll take a downward step?

A. Decrease T, decrease ΔE
B. Decrease T, increase ΔE
C. Increase T, decrease ΔE
D. Increase T, increase ΔE

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  inputs: $problem$, a problem
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  $current \leftarrow$ MAKE-NODE($problem$.INITIAL-STATE)
  for t = 1 **to** ∞ **do**
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Which of the following will make it more likely that we'll take a downward step?

A. Decrease T, decrease ΔE
B. Decrease T, increase ΔE
C. Increase T, decrease ΔE
D. $\boxed{\text{Increase } T\text{, increase } \Delta E}$

ΔE is negative but should be close to 0,
T should be big because of E's negative

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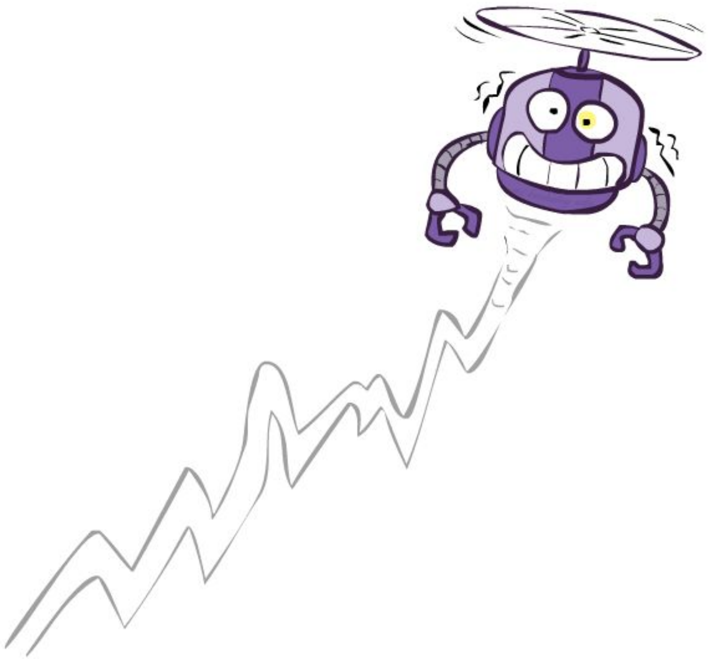
Simulated Annealing



- $P[\text{move downhill}] = e^{\Delta E/T}$
  - Bad moves are more likely to be allowed when T is high (at the beginning of the algorithm)
  - Really bad moves are less likely to be allowed

- Guarantee: If T decreased slowly enough, will converge to optimal state!

- But! In reality, the more downhill steps you need to escape a local optimum, the less likely you are to ever make them all in a row

Summary: Local Search

- Maintain a constant number of current nodes or states, and move to “neighbors” or generate “offspring” in each iteration
  - Do not maintain a search tree or multiple paths
  - Typically, do not retain the path to the node

- Advantages
  - Use little memory
  - The assignments of other variables give guidance on how the selected variable should be set
  - Can potentially solve large-scale problems or get a reasonable (suboptimal or almost feasible) solution