# Social welfare maximization in an online fashion

Avrim Blum, Anupam Gupta, Yishay Mansour, Ankit Sharma

## PROBLEM STATEMENT

**How to allocate *limited resources* to an *online stream* of requests made by *self-interested agents*?**

## SCENARIO I : BUSES TO ROUTE ALLOCATION

Pittsburgh Port Authority has 'm' buses and it has to decide which routes to ply its buses on. *Different routes have different levels of public demand and need different number of buses.* How to decide which routes to pick to maximize the level of public satisfaction?

As if the difficulties of the Port Authority weren't enough, Pittsburgh mayor has gone on an expansion drive. Demand for new routes are being created as a result of this expansion.

What new routes will be created in future is unknown. And *once a bus is assigned to a route, it can't be retracted* (else people take to the streets….).

**Question: The chairman of the Port Authority has just got a request for a new route. Should the chairman assign a bus for this new route or should he decline it?**

*Remember his objective is to maximize public satisfaction.*

## SCENARIO II : CEO's DILEMMA

John has been assigned as the new CEO of MicroSmart. He is extremely enthusiastic and wants to initiate several new projects in the company.

But *new projects don't come for free*! Each project requires certain manpower, certain office space, and some number of servers.

And all these resources – manpower, office space, servers – come at a cost.

Different project managers have come to John with their ideas for new projects. John needs to decide which projects to assign the resources to, knowing that *once a project is assigned resources, they can't be reassigned*.

**Question: Which of the projects should John assign the limited resources to, knowing that potentially better project ideas may arise in future?**

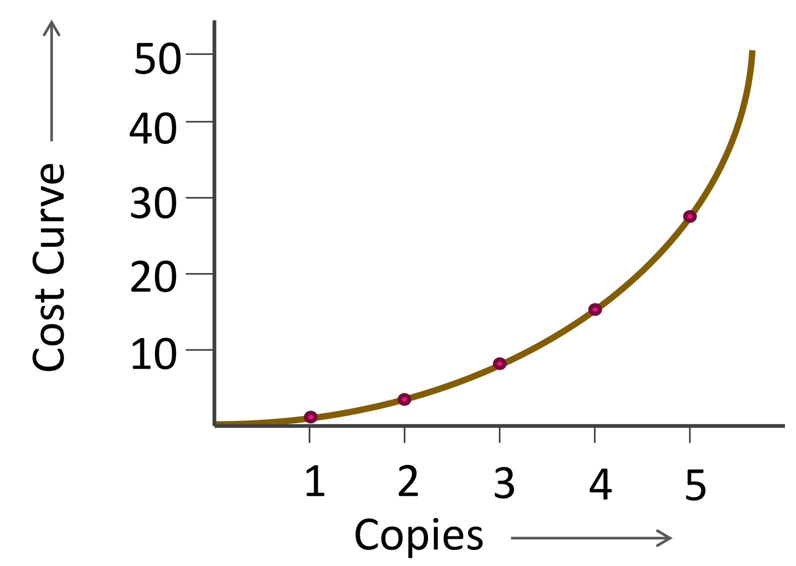
## EXTREME CASES OF LIMITATION

- **Extreme Limitation:** In case there are only a **fixed number of copies** of each resource, the problem is **NP hard** and has strong **inapproximability lower bounds.**

- **Zero Limitation:** In case we have unlimited number of copies of each resource, i.e., resources come at zero cost, optimal solution is assigning resources to all projects or routes.

## CASE OF MODERATE LIMITATION: COST CURVES

*What if additional copies of a resource can be had but at an increasing cost?*
Say our cost for the resource is given by ——— curve.
First copy of the resource costs '1', third copy costs '8', fifth copy costs '29'…




## OBJECTIVE

Given an online series of requests, choose which requests to satisfy with the objective of maximizing

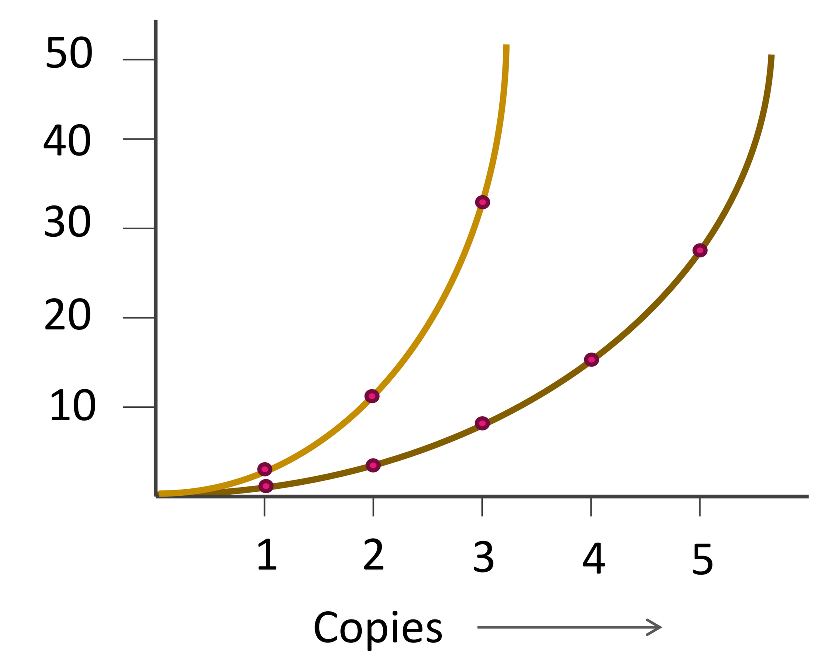
**Social welfare =**
**Value of accepted requests** minus
**Cost of resources to satisfy the requests**

Three major constraints in optimization:

1. **Computational Constraint:** The problem is NP hard.
2. **Online Constraint:** The input to the problem is revealed slowly.
3. **Game Theoretic Constraint:** Requests are potentially made by *self-interested agents* who can misrepresent the value of the request in order to get it accepted.

## ALGORITHM: CREATE 'FAKE' PRICES

➢ For each resource, create 'fake price' curve (———):




Fake prices capture **how limited the resource** is – they increase faster, the costlier the resource is.

➢ **Assign resources to a project if the value of the project is at least the sum of fake prices of the resources it requires**.

## SUMMARY OF RESULTS

**Theorem I: Allocating through fake prices gives a *constant approximation* to social welfare when resources have 'nice' cost curves (polynomial and logarithmic cost curves).**

**Theorem II: Allocating through fake prices gives a *logarithmic approximation* when resources have arbitrary cost curves.**

*If the objective is to **maximize the profit** and not social welfare,*

**Theorem III: Allocating through fake prices plus random offsets, gives in expectation, a *logarithmic approximation* when resources have arbitrary cost curves.**

***The above results holds even when agents hold arbitrary combinatorial valuations.***

## HOW TO DETERMINE THE 'FAKE' PRICES?

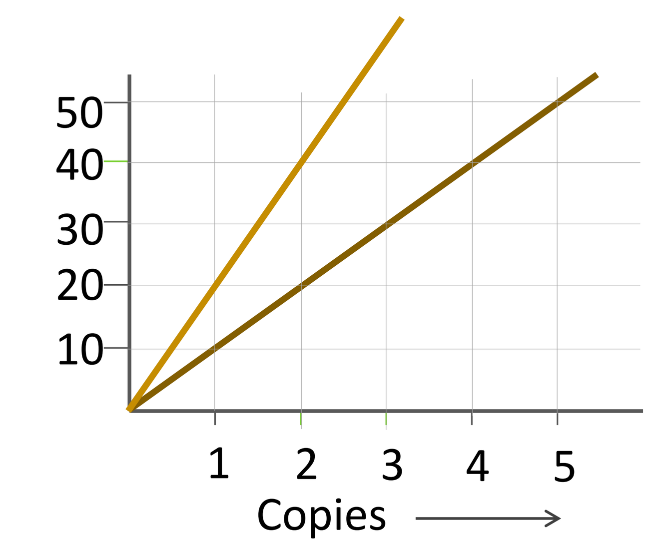
## CASE OF 'NICE' COST CURVES

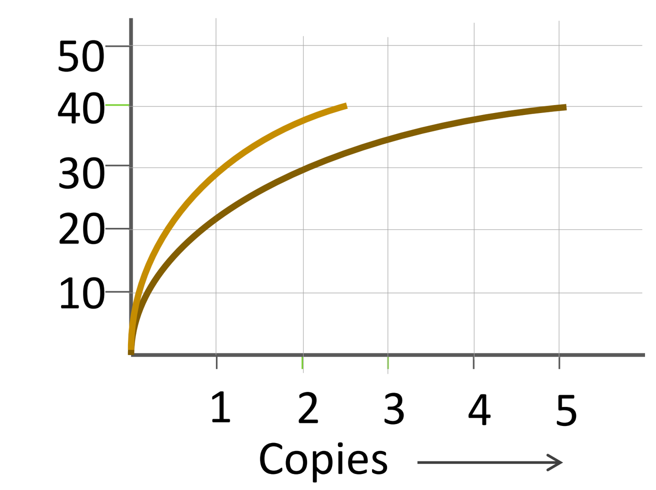
In case the cost curve is a

- **polynomial**: cost of $x^{th}$ copy is $\mathbf{x^d}$ (for some constant d)
- **logarithmic**: cost of $x^{th}$ copy is **ln(1+x)**

set the **price of $x^{th}$ copy = the cost of $(2*x)^{th}$ copy**.

**We call this 'Twice the Index Pricing Scheme'.**




## CASE OF ARBITRARY COST CURVES

Construction is more involved and builds on a paper by *Bartal, Gonen and Nisan [2005]*.