Robotic Motion Planning:

curve tracing

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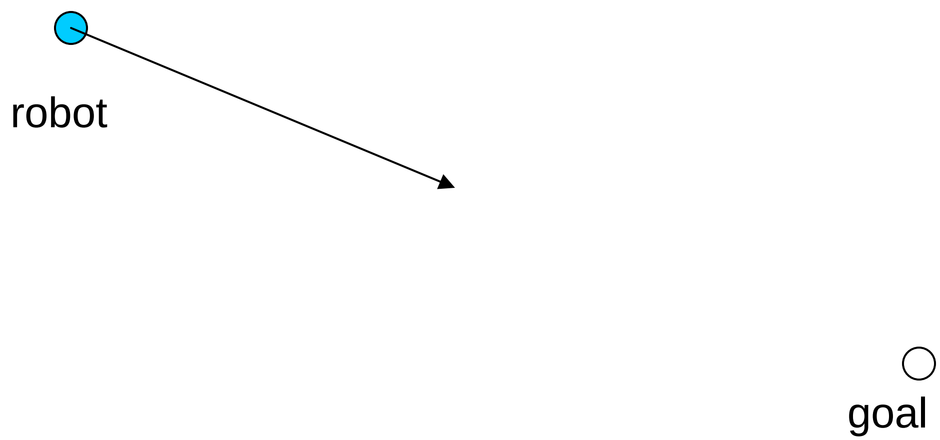
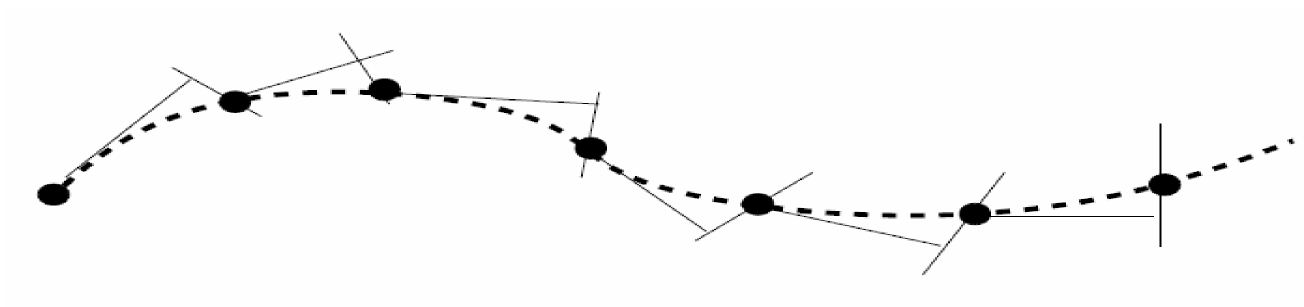
Move to Goal

- Distance $d(a,b) = ((a_x - b_x)^2 + (a_y - b_y)^2)^{½}$
- Gradient descent of d(a,b), i.e., decrease distance to the goal



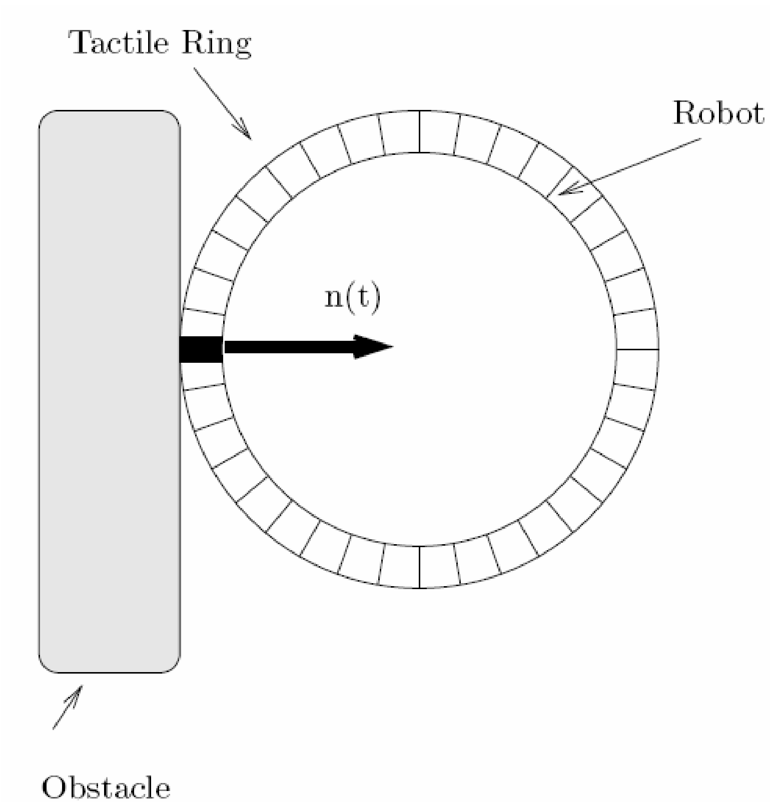
Circumnavigating Obstacles: Curve Tracing

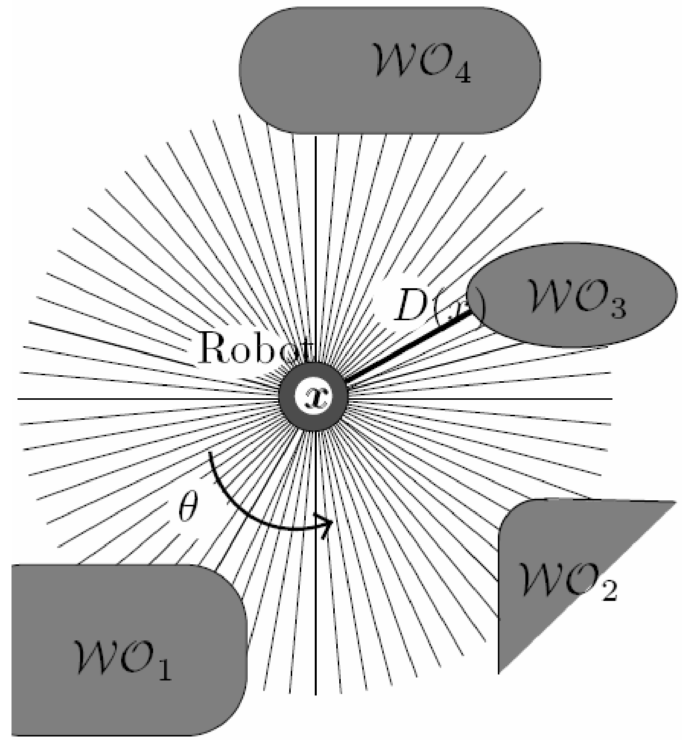


Predict: Tangent

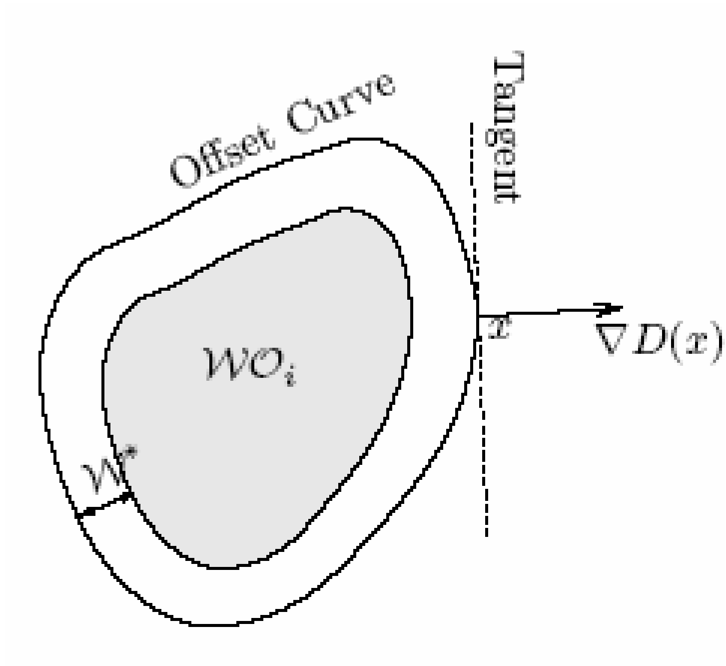
Correct: Something else

Normal (and hence Tangent) to Obstacle





Circumnavigate Obstacles: Boundary Following



Safety distance W*

D(x) = min d(x,c)

Normal is parallel to ∇D(x)

Increase/Decrease/Same

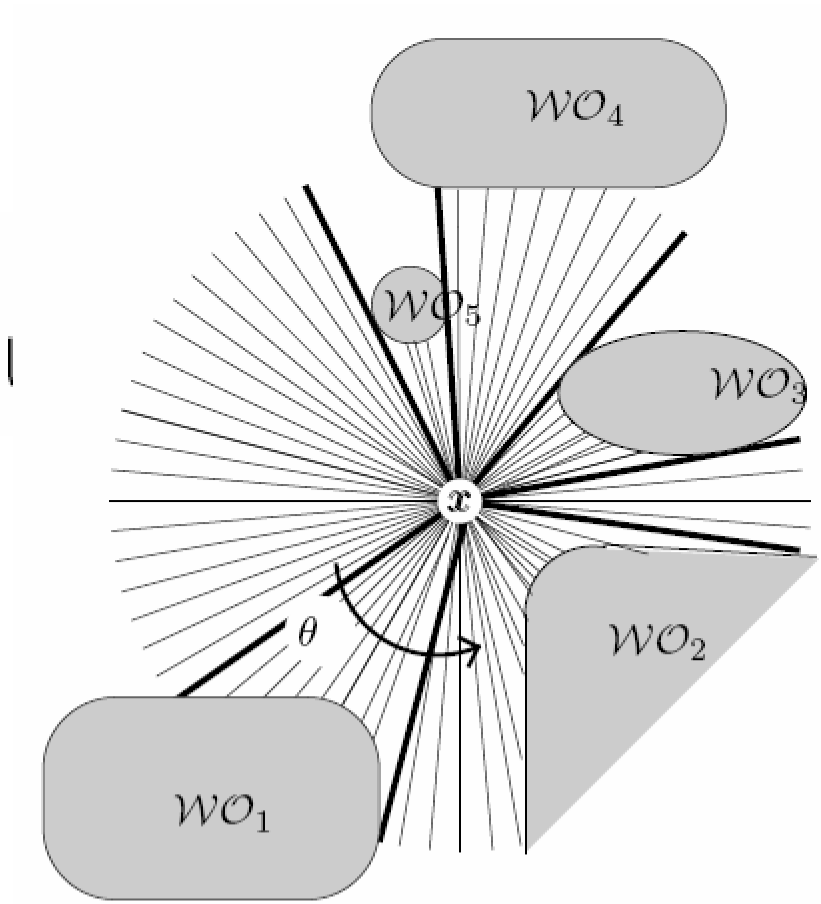
Tangent is orthogonal to both

$\dot{c}(t) = v \quad v$ is in $(n(c(t))^{\perp}$

Raw Distance Function

$$\rho(x,\theta) = \min_{\lambda \in [0,\infty]} d(x, x + \lambda[\cos\theta, \sin\theta]^T),$$

$$\text{such that} \quad x + \lambda[\cos\theta, \sin\theta]^T \in$$



Saturated raw distance function

$$\rho_R(x,\theta) = \begin{cases} \rho(x,\theta), & \text{if } \rho(x,\theta) < R \\ \infty, & \text{otherwise.} \end{cases}$$

Implicit Function Theorem

G(x) = D(x) – W*

Roots of G(x) trace the offset curve

DG(x) = DD(x), which is like a gradient in Euclidean spaces

Null of DG(x) is tangent, hence perp of DD(x) is too

THEOREM D.1.1 (Implicit Function Theorem) *Let* $f : \mathbb{R}^m \times \mathbb{R}^n \rightarrow \mathbb{R}^n$ *be a smooth vector-valued function,* $f(x, y)$*. Assume that* $D_y f(x_0, y_0)$ *is invertible for some* $x_0 \in \mathbb{R}^m$, $y_0 \in \mathbb{R}^n$*. Then there exist neighborhoods* X_0 *of* x_0 *and* Z_0 *of* $f(x_0, y_0)$ *and a unique, smooth map* $g : X_0 \times Z_0 \rightarrow \mathbb{R}^n$ *such that*

$$f(x, g(x, z)) = z$$

for all $x \in X_0$, $z \in Z_0$.

Correction

THEOREM D.2.1 (Newton-Raphson Convergence Theorem) *Let* $f : \mathbb{R}^n \to \mathbb{R}^n$ *and* $f(y^*) = 0$. *For some* $\rho > 0$, *let* f *satisfy*

- $Df(y^*)$ *is nonsingular with bounded inverse, i.e.,* $\|(Df(y^*))^{-1}\| \leq \beta$
- $\|Df(x) - Df(y)\| \leq \gamma \|x - y\|$ *for all* $x, y \in B_\rho(y^*)$, *where* $\gamma \leq \frac{2}{\rho\beta}$

Now consider the sequence $\{y^h\}$ *defined by*

$$y^{h+1} = y^h - (Df(y^h))^{-1} f(y^h),$$

for any $y^0 \in B_\rho(y^*)$. *Then* $y^h \in B_\rho(y^*)$ *for all* $h > 0$, *and the sequence* $\{y^h\}$ *quadratically converges onto* y^*, *i.e.,*

$$\|y^{h+1} - y^*\| \leq a\|y^h - y^*\|^2$$

where $a = \frac{\beta\gamma}{2(1-\rho\beta\gamma)} < \frac{1}{\rho}$.