Very-Large-Scale Generalized Combinatorial Multi-Attribute Auctions: Lessons from Conducting $60 Billion of Sourcing

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Introduction

Drawing from our experiences of designing and fielding over 800 sourcing auctions totaling over $60 billion, I will discuss issues that arise in very-large-scale generalized combinatorial auctions, as well as solutions that work (and ones that do not). These are by far the largest (in terms of the number of items as well as the number of side constraints) and most complex combinatorial markets ever developed.

I will discuss how combinatorial and multi-attribute auctions can be soundly hybridized. I will address preference and constraint expression languages for the bidders and the bid taker, as well as techniques for effectively using them. I will discuss scalable optimization techniques for the market clearing (a.k.a. winner determination) problem. I will also address a host of other issues that this work uncovered, and I will study the significant efficiency gains and other benefits that followed. While the experiences are mainly from sourcing, I believe that the lessons learned apply to many other combinatorial reverse auctions, combinatorial auctions, and combinatorial exchanges.

Historical Backdrop on Sourcing

Sourcing, the process by which companies acquire goods and services for their operations, entails a complex interaction of prices, preferences, and constraints. The buyer's problem is to

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Early versions of parts of this article appeared in my article in *AI Magazine*, 2007, and the section on automated supply network configuration is largely based on our paper in *Interfaces*, 2006.

decide how to allocate the business across the suppliers. Sourcing professionals buy several trillion dollars worth of goods and services yearly.

Traditionally, sourcing decisions have been made via manual in-person negotiations. The advantage is that there is a very expressive language for finding, and agreeing to, win-win solutions between the supplier and the buyer. The solutions are implementable because operational constraints can be expressed and taken into account. On the downside, the process is slow, unstructured, and nontransparent. Furthermore, sequentially negotiating with the suppliers is difficult and leads to suboptimal decisions. (This is because what the buyer should agree to with a supplier depends on what other suppliers would have been willing to agree to in later negotiations.) The 1-to-1 nature of the process also curtails competition.

These problems have been exacerbated by a dramatic shift from plant-based sourcing to global corporate-wide (category-based rather than plant-based) sourcing since the mid-1990s. This transition is motivated by the desire of corporations to leverage its spend across plants in order to get better pricing and better understanding and control of the supply network while at the same time improving supplier relationships (see, e.g., (Smock 2004)). This transition has yielded significantly larger sourcing events that are inherently more complex.

During this transition, there has also been a shift to electronic sourcing where prospective suppliers submit offers electronically to the buyer. The buyer then decides, using software, how to allocate its business across the prospective suppliers. Advantages of this approach include speed of the process, structure and transparency, global competition, and simultaneous negotiation with all suppliers (which removes the difficulties associated with the speculation about later stages of the negotiation process, discussed above).

The most famous class of electronic sourcing systems–which became popular in the mid-1990s through vendors such as FreeMarkets (now part of Ariba), Frictionless Commerce (now part of SAP), and Procuri (now part of Ariba)–is the *reverse auction*. The buyer groups the items into lots in advance, and conducts an electronic descending-price auction for each lot. The lowest bidder wins. (In some cases "lowness" is not measured in terms of price, but in terms of an *ad hoc* score which is a weighted function that takes into account the price and some non-price attributes such as delivery time and reputation.)

Reverse auctions are not economically efficient, that is, they do not generally yield good allocation decisions. This is because the optimal bundling of the items depends on the suppliers' preferences (which arise, among other considerations, from the set, type, and time-varying state of their production resources), which the buyer does not know at the time of lotting. Lotting by the buyer also hinders the ability of small suppliers to compete. Furthermore, reverse auctions do not support side constraints, yielding two drastic deficiencies: 1) the buyer cannot express her business rules; thus the allocation of the auction is unimplementable and the "screen savings" of the auction do not materialize in reality, and 2) the suppliers cannot express their production efficiencies (or differentiation), and are exposed to bidding risks. In short, reverse auctions assume away the complexity that is inherent in the problem, and dumb down the events rather than embracing the complexity and viewing it as a driver of opportunity. It is therefore not surprising that there are strong broad-based signs that reverse auctions have fallen in disfavor.

The New Paradigm: *Expressive Commerce*

In 1997 it dawned on me that it is possible to achieve the advantages of both manual negotiation and electronic auctions while avoiding the disadvantages. The idea is to allow supply and demand to be expressed in drastically more detail (as in manual negotiation) while conducting the events in a structured electronic marketplace where the supply and demand are algorithmically matched (as in reverse auctions). The new paradigm, which we called *expressive commerce* (or *expressive competition*), was so promising that I decided to found a company, CombineNet, Inc., to commercialize it.

I began technology development in 1997 and founded the company in 2000. I then served as its Chairman and Chief Technology Officer / Chief Scientist. I left the company after its acquisition in 2010. It continues to operate under the same name.

In expressive commerce, the finer-grained matching of supply and demand yields higher allocative efficiency (i.e., a win-win between the buyer and the suppliers in aggregate). However, matching the drastically more detailed supply and demand is an extremely complex combinatorial optimization problem. We developed the fastest algorithms for optimally solving this problem (discussed later). These algorithms are incorporated into the market-clearing engine at the core of our flagship product, the *Advanced Sourcing Application Platform*.

Expressive commerce has two sides: *expressive bidding* and *expressive allocation evaluation* (also called *expressive bid taking*) (Sandholm and Suri 2001), which I now describe.

Expressive Bidding

With *expressive bidding*, the suppliers can express their offers creatively, precisely, and conveniently using expressive and compact statements that naturally correspond to the suppliers' business rules, production constraints and efficiencies, etc. Our expressive bidding takes several forms. Our software supports the following forms of expressive bidding, among others, all in the same event.

- Bidding on an arbitrary number of self-constructed packages of items (rather than being restricted to bidding on predetermined lots as in basic reverse auctions). The packages can be expressed in more flexible and more usable forms than that supported in vanilla combinatorial auctions. For example, the bidder can specify different prices on the items if the items are accepted in given proportions, and the bidder can specify ranges for these proportions, thus allowing an exponential number of packages to be captured by one compact expression.
- Conditional discount offers. Both the trigger conditions and the effects can be specified in highly flexible ways. For example, the trigger conditions can specify whether they should be evaluated before or after the effects of the current discount and other discounts are taken into account.
- Rich forms of discount schedules. (Simpler forms of discount schedules have already been addressed in the literature (Sandholm and Suri 2001a, Sandholm and Suri 2002, Hohner et al. 2003).) Figure 1 shows a fairly basic example. Richer forms allow the bidder to submit

multiple discount offers and to control whether and how they can be combined. Also, discount triggers can be expressed as dollars or units, and as a percentage or an absolute.

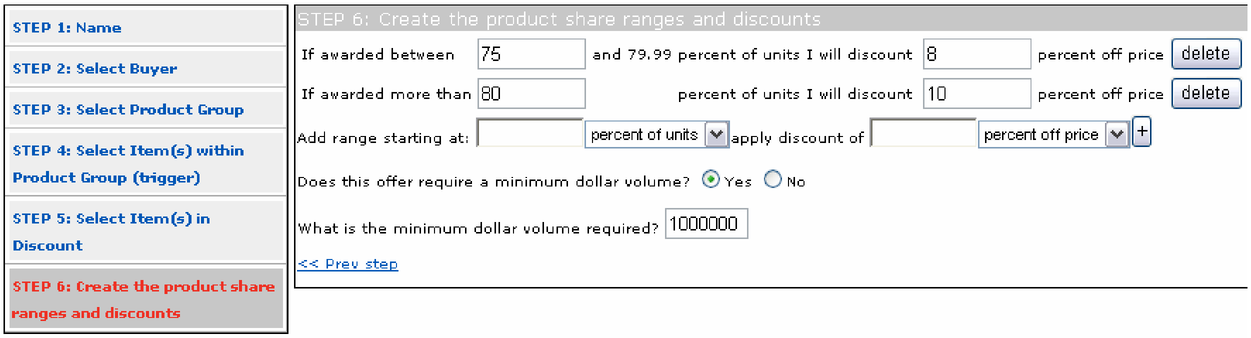
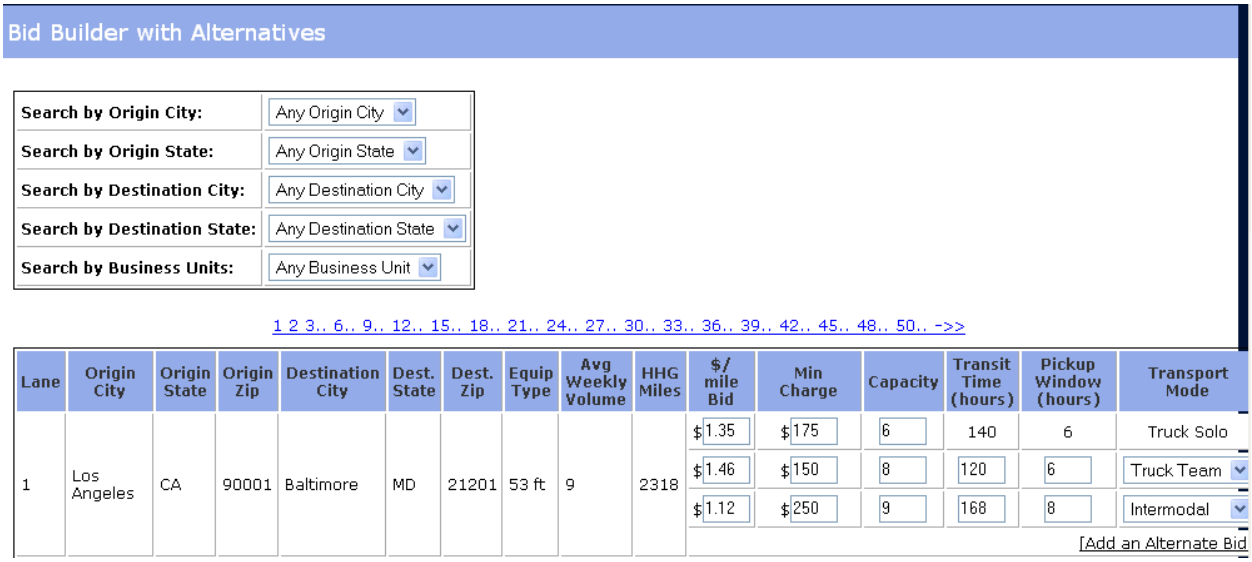


Figure 1. A relatively simple discount schedule. This screenshot is from an actual sourcing event. The scope of the trigger of the discount (STEP 4) can be different than the scope of the items to which the discount is to be applied (STEP 5).

- A broad variety of side constraints–such as capacity constraints (Sandholm and Suri 2001a).
- Multi-attribute bidding (Sandholm and Suri 2001a). This allows the buyer to leave the item specification partially open, so the suppliers can pick values for the item attributes–such as material, color, and delivery date–in a way that matches their production efficiencies. This is one way in which the suppliers can also express alternate items.
- Free-form expression of alternates. This fosters unconfined creativity by the suppliers.
- Expression of cost drivers. In many of our events, the buyer collects tens or hundreds of cost drivers (sometimes per item) from the suppliers. By expressing cost drivers, the bidder can concisely and implicitly price huge numbers of items and alternates. Figures 2 and 3 illustrate bidding with attributes and cost drivers.



Bid Builder with Alternatives

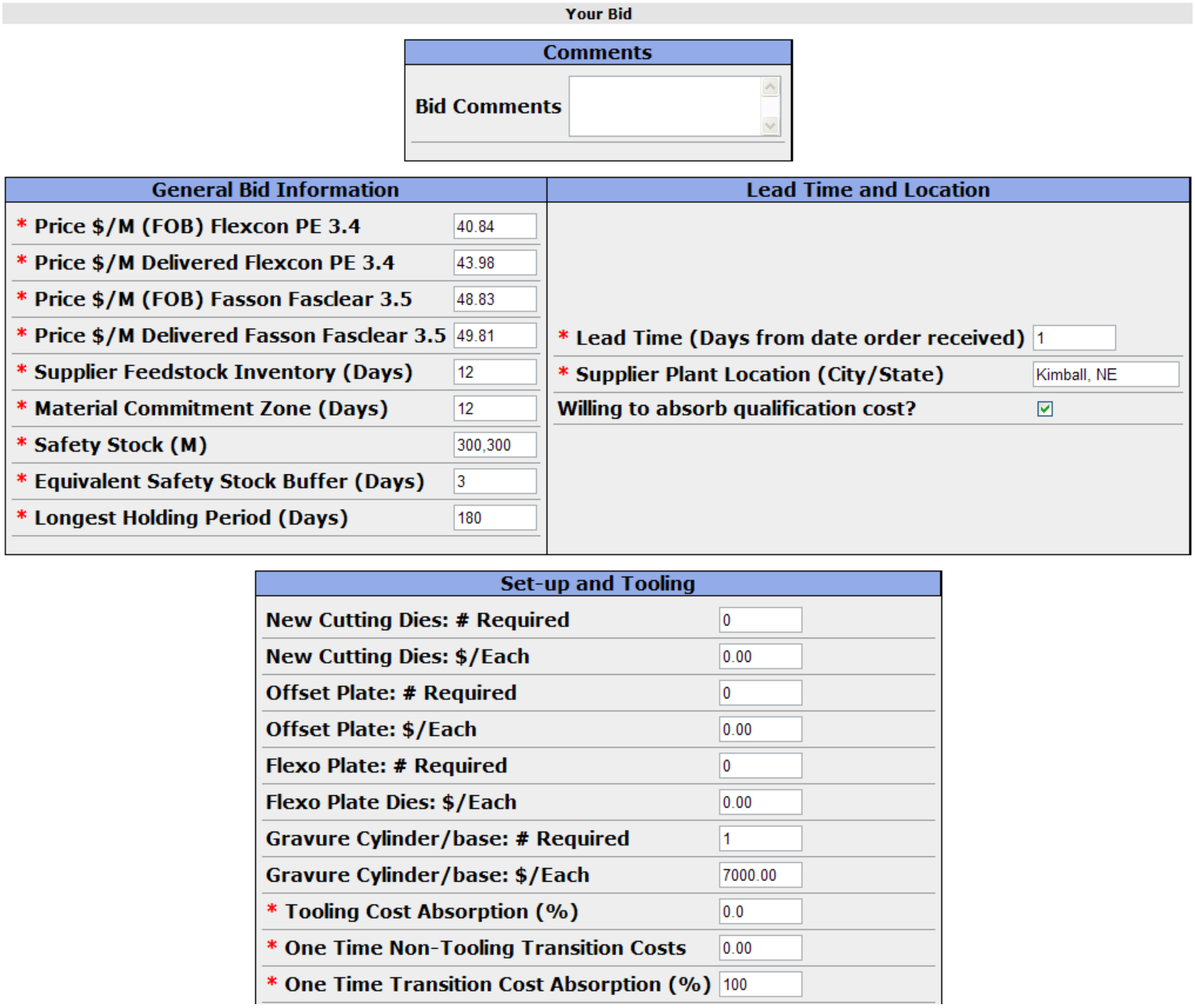
Search by Origin City:	Any Origin City
Search by Origin State:	Any Origin State
Search by Destination City:	Any Destination City
Search by Destination State:	Any Destination State
Search by Business Units:	Any Business Unit

1 2 3.. 6.. 9.. 12.. 15.. 18.. 21.. 24.. 27.. 30.. 33.. 36.. 39.. 42.. 45.. 48.. 50.. ->>

Lane	Origin City	Origin State	Origin Zip	Destination City	Dest. State	Dest. Zip	Equip Type	Avg Weekly Volume	HHG Miles	$/ mile Bid	Min Charge	Capacity	Transit Time (hours)	Pickup Window (hours)	Transport Mode
1	Los Angeles	CA	90001	Baltimore	MD	21201	53 ft	9	2318	$1.35	$175	6	140	6	Truck Solo
										$1.46	$150	8	120	6	Truck Team
										$1.12	$250	9	168	8	Intermodal

[Add an Alternate Bid

Figure 2. A simple example of bidding with alternates, cost drivers, attributes, and constraints. This piece of screen is from an actual event for sourcing truckload transportation. The figure shows part of the bid by one bidder on one item.



Your Bid

Comments	
Bid Comments	

General Bid Information	
* Price $/M (FOB) Flexcon PE 3.4	40.84
* Price $/M Delivered Flexcon PE 3.4	43.98
* Price $/M (FOB) Fasson Fasclear 3.5	48.83
* Price $/M Delivered Fasson Fasclear 3.5	49.81
* Supplier Feedstock Inventory (Days)	12
* Material Commitment Zone (Days)	12
* Safety Stock (M)	300,300
* Equivalent Safety Stock Buffer (Days)	3
* Longest Holding Period (Days)	180

Lead Time and Location	
* Lead Time (Days from date order received)	1
* Supplier Plant Location (City/State)	Kimball, NE
Willing to absorb qualification cost?	☑

Set-up and Tooling	
New Cutting Dies: # Required	0
New Cutting Dies: $/Each	0.00
Offset Plate: # Required	0
Offset Plate: $/Each	0.00
Flexo Plate: # Required	0
Flexo Plate Dies: $/Each	0.00
Gravure Cylinder/base: # Required	1
Gravure Cylinder/base: $/Each	7000.00
* Tooling Cost Absorption (%)	0.0
* One Time Non-Tooling Transition Costs	0.00
* One Time Transition Cost Absorption (%)	100

Figure 3. An example of bidding with cost structures and attributes. This is part of an actual event where we sourced printed labels.

All of these expressive bidding features of our software have been extensively used by our customers. The software supports bidding through both web-based interfaces and spreadsheets. In some cases, catalog prices from databases have also been used.

Our expressive bidding approach is flexible in the sense that different suppliers can bid in different ways, using different offer constructs. In fact, some suppliers may not be sophisticated enough to bid expressively at all, yet they can participate in the same sourcing events using traditional bidding constructs in the same system. This paves a smooth road for adoption, which does not assume sudden process changes at the participating organizations.

Benefits of Expressive Bidding

The main benefit of expressive bidding is that it leads to greater efficiency of the allocation. In business terms, it creates a win-win between the buyer and the suppliers in aggregate. There are several reasons for this.

First, because the suppliers and the buyer can express their preferences completely (and easily), the market mechanism can make better (economically more efficient and less wasteful) allocation decisions, which translates to higher societal welfare. In other words, the method yields better matching of supply and demand because they are expressed in more detail. The savings do not come from lowering supplier margins, but from reducing economic inefficiency. With expressive bidding, the suppliers can offer specifically what they are good at, and at lower prices because they end up supplying in a way that is economical for them. (They can consider factors such as production costs and capacities, raw material inventories, market conditions, competitive pressures, and strategic initiatives.) This creates a win-win solution between the suppliers and the buyer. For example, in the sourcing of transportation services, a substantial increase in economic efficiency comes from bundling multiple deliveries in one route (back-haul deliveries and multi-leg routes). This reduces empty driving, leading to lower transportation costs and yielding environmental benefits as well: lower fuel consumption, less driver time, less frequent need to replace equipment, and less pollution.

Second, suppliers avoid exposure risks. In traditional inexpressive markets, the suppliers face exposure problems when bidding. That makes bidding difficult. To illustrate this point, consider a simple auction of two trucking tasks: the first from Pittsburgh to Los Angeles, and the second from Los Angeles to Pittsburgh. If a carrier wins one of the tasks, he has to factor in the cost of driving the other direction empty. Say that his cost for the task then is $1.60 per mile. On the other hand, if he gets both tasks, he does not have to drive empty, and his cost is $1.20 per mile. When bidding for the first task in an inexpressive auction, it is impossible to say where in the $1.20 - $1.60 range he should bid, because his cost for the first task depends on whether he gets the second task, which in turns depends on how other carriers will bid. Any bid below $1.60 exposes the carrier to a loss in case he cannot profitably win the second task. Similarly, bidding above $1.20 may cause him to lose the deal on the first task although it would be profitable to take on that task in case he wins the second task. In an expressive auction, the buyer can price each of the tasks separately, and price the package of them together, so there is no exposure problem. (For example, he can bid $1.60 per mile for the first task, $1.60 per mile for the second task, and $1.20 per mile for the package of both tasks. Of course, he can also include a profit margin.) Therefore bidding is easier: the bidder does not have to speculate what other suppliers will bid in the later auctions. Also, the tasks get allocated optimally because no bidder gets stuck with an undesirable bundle, or misses the opportunity to win when he is the most efficient supplier. Furthermore, when there is an exposure problem, the suppliers hedge against it by higher prices. Removal of the bidders' exposure problems thus also lowers the buyer's sourcing cost.

Third, by expressive bidding with side constraints (such as capacity constraints), each supplier can bid on all bundles of interest without being exposed to winning so much that handling the business will be unprofitable or even infeasible. This again makes bidding easier because – unlike in inexpressive markets – the supplier does not have to guess which packages to commit his capacity to. (In an inexpressive market, making that guess requires counterspeculating what the other suppliers are going to bid, because that determines the prices at which this supplier can win different alternative packages.) This also leads to more efficient allocations compared to those in inexpressive markets because in those markets each bidder needs to make guesses as to what parts of the business he should bid on, and those might not be the parts for which he really is the most efficient supplier.

Fourth, expressive bidding allows more straightforward participation in markets because the strategic counterspeculation issues that are prevalent in non-combinatorial markets can be mitigated, as discussed above.[1] This leads to wider access of the benefits of ecommerce because less experienced market participants are raised to an equal playing field with experts. This yields an increase in the number of market participants, which itself leads to further economic efficiency and savings in sourcing costs. Broader access also stems from the buyer not lotting the items and thus facilitating competition from small suppliers as well.

Fifth, in basic reverse auctions, the buyer has to pre-bundle items into lots, but he cannot construct the optimal lotting because it depends on the suppliers' preferences. With expressive commerce, items do not have to be pre-bundled. Instead, the market determines the optimal lotting (specifically, the optimizer determines the optimal allocation based on the expressive bids and the expressions from the buyer). This way, the economically most efficient bundling is reached, weeks are not wasted on pre-bundling, and suppliers that are interested in different bundles compete. As a side effect, small suppliers' bids taken together end up competing with large suppliers.

Sixth, expressive bidding fosters creativity and innovation by the suppliers. This aspect is highly prized by both the suppliers and buyers. It can also yield drastic savings for the buyer due to creative construction of lower-cost alternates.

Overall, expressive bidding yields both lower prices and better supplier relationships. In addition to the buyers (our customers), also suppliers are providing very positive feedback on the approach. They especially like that they 1) also benefit from expressive bidding (unlike in traditional reverse auctions where their profit margins get squeezed), 2) can express their production efficiencies, and 3) can express differentiation and creative offers. In fact, suppliers like expressive commerce so much that they agree to participate in expressive commerce even in events that they boycotted when basic reverse auctions had been attempted. Furthermore, perhaps the best indication of supplier satisfaction is the fact the suppliers are recommending the use of our approach and software to buyers.

The benefits of expressiveness can be further enhanced by multiple buyers conducting their sourcing in the same event. This provides an opportunity for the bidders to bundle across the demands of the buyers, and also mitigates the exposure risks inherent in participating in separate events. As an example, in Spring 2005 we conducted an event where Procter & Gamble and its two largest customers, Walmart and Target, jointly sourced their North America-wide truckload transportation services for the following year (Sandholm et al. 2006). This enabled the carriers to construct beneficial backhaul deliveries and multi-leg routes by packaging trucking lanes across the demand of the three buyers. This was a large event. Procter & Gamble's volume alone exceeded $885 million.

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[1] In fact, with full expressiveness, in principle, truthful bidding can be made a dominant strategy by using Vickrey-Clarke-Groves pricing. However, as I will discuss later, that is not practical here.

Expressive Allocation Evaluation by the Bid Taker

The second half of expressive commerce is *expressive allocation evaluation*, where the bid taker (i.e., buyer in the case of sourcing) expresses preferences over allocations using a rich, precise, and compact language that is also natural in the buyer's business. It can be used to express legal constraints, business rules, prior contractual obligations, and strategic considerations.

In our experience, different types of side constraints offer a powerful form of expressiveness for this purpose. For example, the buyer can state: "I don't want more than 200 winners (in order to avoid overhead costs)," "I don't want any one supplier to win more than 15% (in order to keep the supply network competitive for the long term)," "I want minority suppliers to win at least 10% (because that is the law)," "Carrier X has to win at least $3 million (because I have already agreed to that)," etc. Our system supports hundreds of types of side constraints.

Our system also has a rich language for the buyer to express how item attributes (such as delivery date or transshipment specifications) and supplier attributes (such as reputation) are to be taken into account when determining the allocation of business (Sandholm and Suri 2001b).

A professional buyer–with potentially no background in optimization–can set up a *scenario* in our system by adding constraints and preferences through an easy-to-use web-based interface. A simple example is shown in Figure 4. To set up each such expression, the buyer first chooses the template expression (e.g., "I don't want more than a certain number of winners," or "I want to favor incumbent suppliers by some amount") from a set of expressions that have been deemed potentially important for the sourcing event in question by the person who configured the event within our software tool. He then selects the *scope* to which that expression should apply: everywhere, or to a limited set of items, bid rounds, product groups, products, sites, and business groups. Finally, he selects the exact parameter(s) of the constraint, e.g., exactly how many winners are allowed. Constraints and preferences can also be uploaded from business rule databases. Once the buyer has defined the scenario consisting of side constraints and preferences, he calls the optimizer in our system to find the best allocation of business to suppliers under that scenario.

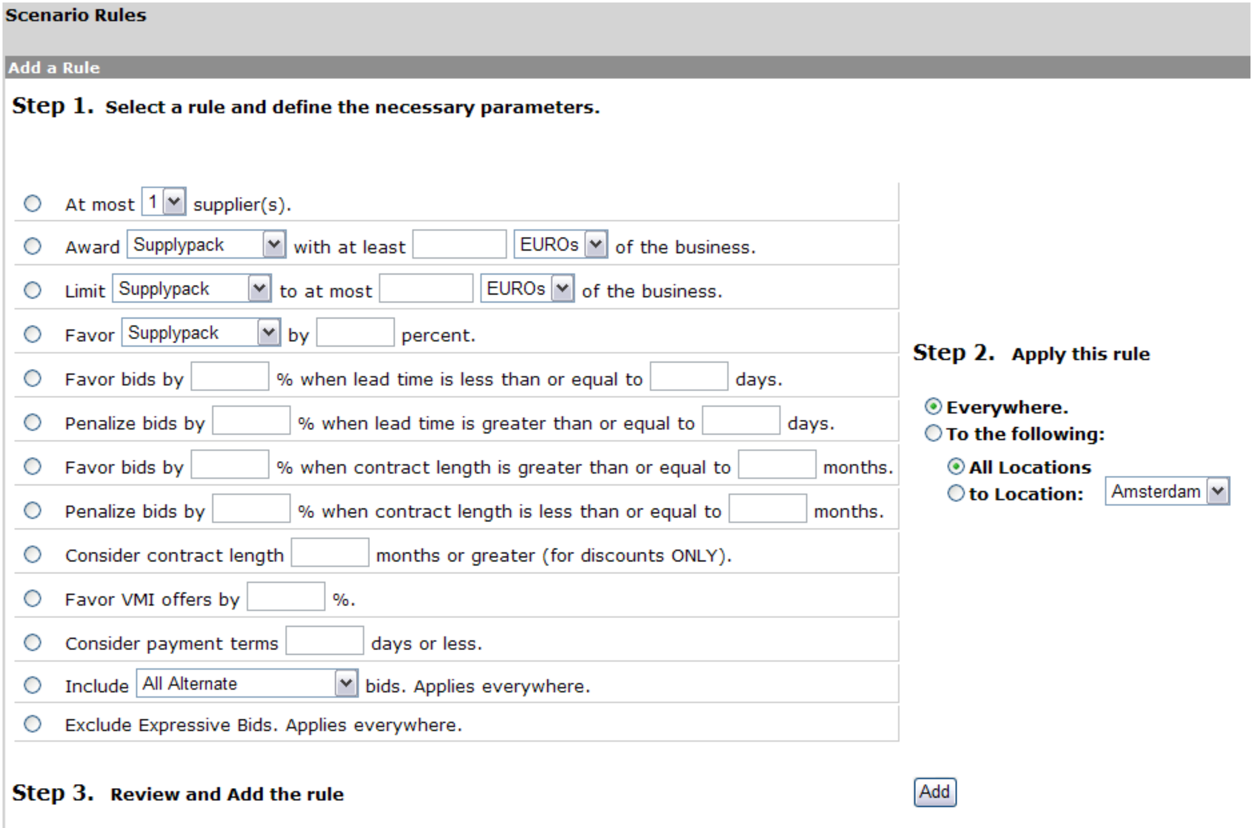

Figure 4. A user interface for expressive allocation evaluation by the bid taker. Every sourcing event has different expressiveness forms (selected from a library or preconfigured in a template). This particular sourcing event was one of the simpler ones in that relatively few expressions of constraints and preferences were included in the user interface. The buyer uses the interface as follows. First, on the left (Step 1), he selects which one of the expressiveness forms he wants to work on, and sets the parameter(s) for that form. Then, on the right (Step 2) he selects the scope to which this rule is to be applied. In Step 3, he presses the "Add" button to add the rule, and a restatement of the rule appears in natural language for verification (not shown). The buyer can then add more rules to the same scenario by repeating this process. Finally, the buyer presses the "Optimize" button (not shown) to find the optimal allocation for the scenario. This triggers the automated formulation of all these constraints and preferences – together with all the bid information that the bidders have submitted – into an optimization problem, and the solving of the problem via advanced tree search.

Our software takes these high-level supply and demand expressions, *automatically* converts them into an optimization model, and uses sophisticated tree search algorithms to solve the model. We have faced scenarios with over 2.6 million bids (on 160,000 items, multiple units of each) and over 300,000 side constraints, and solved them to optimality.

Scenario Navigation

Once (at least some of) the bids have been collected, the buyer can engage in *scenario navigation* with the system. At each step of that process, the buyer specifies a set of side constraints and preferences (these define the scenario), and runs the optimizer to find an optimal allocation for that scenario. This way the buyer obtains a quantitative understanding of how different side constraints and preferences affect the sourcing cost and all other aspects of the allocation. In our system, the buyer can change/add/delete any number of side constraints and preferences in between optimizations.

Studying our sourcing events around 2006, we found that a buying organization using our system will navigate an average of about 100 scenarios per sourcing event. The maximum we saw by then was 1107. To navigate such large numbers of scenarios, fast clearing is paramount.

Rapid clearing enables scenario navigation to be driven by the actual data (offers). In contrast, most prior approaches required the scenario (side constraints and preferences, if any) to be defined prior to analysis; there were insufficient time and expert modeling resources to try even a small number of alternative scenarios. Data-driven approaches are clearly superior because the actual offers provide accurate costs for the various alternative scenarios.

Benefits of Expressive Allocation Evaluation

Through side constraints and preference expressions, the buyer can include business rules, legal constraints, logistical constraints, and other operational considerations to be taken into account when determining the allocation. This makes the auction's allocation *implementable* in the real world: the plan and execution are aligned because the execution considerations are captured in the planning.

Second, the buyer can include prior (e.g., manually negotiated) contractual commitments in the optimization. This begets a sound hybrid between manual and electronic negotiation. For example, he may have the obligation that a certain supplier has to be allocated at least 80 truckloads. He can specify this as a side constraint in our system, and the system will decide which 80 truckloads (or more) are the best ones to allocate to that supplier in light of all other offers, side constraints, and preferences. This again makes the allocation implementable. (A poor man's way of accomplishing that would be to manually earmark some of the business to the prior contracts. Naturally, allowing the system to do that earmarking with all the pertinent information in hand yields better allocations.)

Third, the buyer obtains a quantitative understanding of the tradeoffs in his supply network by scenario navigation; that is, by changing side constraints and preferences and reoptimizing, the buyer can explore the tradeoffs in an objective manner. For example, he may add the side constraint that the supply base be rationalized from 200 to 190 suppliers. The resulting increase in procurement cost then gives the buyer an understanding of the tradeoff between cost and practical implementability. As another example, the buyer might ask: If I wanted my average supplier delivery-on-time rating to increase to 99%, how much would that cost? As a third example, the buyer might see what would happen if he allowed a supplier to win up to 20% of the business instead of only 15%. The system will tell the buyer how much the sourcing cost would decrease. The buyer can then decide whether the savings outweighs the added long-term strategic risks such as vulnerability to that supplier's default and the long-term financial downside of allowing one supplier to become dominant.

Fourth, quantitative understanding of the tradeoffs also fosters stakeholder alignment on the procurement team, because the team members with different preferences can discuss based on facts rather than opinions, philosophies, and guesswork. The buyer is typically not an individual but an organization of several individuals with different preferences over allocations. Finance people want low sourcing cost, plant managers want small numbers of suppliers, marketing people want a high average carrier-delivery-on-time rating, etc. Scenario navigation enables the organization to better understand the available tradeoffs.

Feedback to Bidders, and its Interaction with Scenario Navigation

Allowing the bid taker to conduct scenario navigation introduces interesting issues. If scenario navigation is allowed, the sourcing mechanism is not uniquely defined from the perspective of the bidders. This is because scenario navigation by the bid taker changes the rules of the game for the bidders. Therefore, one could argue that a sourcing mechanism with scenario navigation is not an auction at all. Beyond semantics, this also affects the bidders' incentives, and this is one reason why we (and to my knowledge all other expressive sourcing vendors nowadays) use the first-price (i.e., pay your winning bids) pricing rule instead of mechanisms that are incentive compatible without scenario navigation (notably the Vickrey-Clarke-Groves (VCG) mechanism). While our market clearing engine supports both first-price and VCG pricing, none of our customers have wanted to use the latter. Therefore, in the sourcing platform we offer the former.

Scenario navigation also has an interesting interaction with quotes[2]: if the bid taker changes the rules later on, then how should the price quote on a bundle (or any other kind of quote) be computed? The quote depends on the scenario, but when the scenario is not (yet) fixed, quotes are ill-defined.

One practical solution that we used is to provide quotes (or other feedback) based on the current scenario with the understanding that the feedback may not be accurate in light of the final scenario. This is in line with the standard understanding that quotes are not accurate anyway because they may change as other bidders change their bids.

Another, very practical, but limited, approach that our software offers is to provide quotes of the kind that do not depend on the scenario, such as giving bidders feedback on their bids on individual items only, e.g., "you are currently the 7^{th}-lowest bidder on item x" or "your bid on item x is \$3.54 higher than the lowest bid". One can also offer feedback to bidders on what items are sparsely covered by bids; that helps bidders focus their bidding on "opportunities".

A third possibility in our system, which is sometimes used by our customers for fast auctions with only tens of items, is to force the bid taker to lock in the scenario before bidding starts. That enables accurate quotes but precludes the bid taker from conducting scenario navigation in a data-driven way based on the bids.

Another possible approach is to constrain how the bid taker can change the scenario. For instance, he may be allowed to tighten, but not loosen, constraints during the sourcing process. Depending on the constraints, nontrivial upper or lower bounds on quotes may then be well defined, and even bounds can give useful guidance to bidders on which bundles they should focus their valuation determination and bidding effort.

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[2] Sandholm (2002a) discusses how one can compute quotes for bundles in a combinatorial auction.

Our system supports sealed-bid events (winners are determined at the end), events that have multiple (usually 2-3) rounds (winners are determined and feedback provided at the end of each round), and "live" events (winners are determined and feedback provided every time any participant expresses anything new). All three formats have been used extensively by our customers. In the live and multi-round formats, we require that a bidder's change in his offer not make the allocation worse (for one, otherwise early bids would be meaningless because they could be pulled out). An idea for the future is to give detailed feedback only at the end of each bidding round but coarser feedback all the time. One could even use "mini-rounds" within each round for medium-granularity feedback, and so on.

Our customers have typically not been very concerned about collusion because the number of suppliers tends to be large, the bid taker often has the most negotiation power, and he knows what prices should be expected quite well. Live events run the risk of supporting collusion, while sealed-bid events tend to deter collusion because the parties of a potential coalition have an opportunity to "stab each other in the back" without the other parties observing their actions until the auction is over - when it is too late to respond. The multi-round format tries to get the best of both worlds: giving bidders feedback between rounds in order to help focus their valuation determination and bidding effort while at the same time having a back-stabbing opportunity at the end.

It is not known what kind of feedback is best in order to minimize sourcing cost. Under stylized assumptions in the single-item auction setting, the revenue equivalence theorem states that the open-cry format and sealed-bid format yield equal expected revenue. However, if bidders' valuation distributions are asymmetric, either can have higher expected revenue than the other (Maskin and Riley 2000). In one metal sourcing event we knew that one of the suppliers had a cost significantly below the rest. We therefore chose a sealed-bid format so that supplier could not know exactly how low he needed to bid to win; rather he would ensure winning by going lower than that. I will discuss research related to revenue-maximizing (cost-minimizing) combinatorial auctions in the last section of this paper.

Additional Tools for the Bid Taker

Sometimes the buying organization can get carried away with controlling the allocation with side constraints to the extent that no feasible allocation exists. To help the bid taker in that situation, we developed a *feasibility obtainer* as an extension of our market clearing (a.k.a. winner determination) technology where the optimizer finds a minimum-cost relaxation of the constraints if the scenario is over-constrained. Each constraint can have a different percentage relaxation cost. The feasibility obtainer can yield allocations that have high sourcing cost. To address this, we developed an *optimal constraint relaxer*. It finds a solution that minimizes the sum of sourcing cost and constraint relaxation cost.

Sometimes the bid taker can contact bidders to encourage certain bids, but this takes time and uses up favors with the bidders. We developed a methodology and algorithms for deciding what favors to ask using optimization. In addition to the usual inputs to the market clearing optimization, the bid taker can state how he expects he could improve the suppliers' offers by negotiating with them, and then asks the optimizer how he should negotiate in light of all of the inputs so as to minimize his total sourcing cost. For example, "If I can improve 5 discount schedules by 2% each, which 5 should I negotiate?"

Our system also provides coverage feedback to the bid taker so he can encourage bidders to submit bids on items that are thinly covered by bids.

The bid taker typically has the option to source items by means other than the current auction. He can buy from existing catalog prices, negotiate manually, or hold another auction. Which items should be bought in the current auction and which should be sourced in these other ways? Our system enables the bid taker to optimize this decision. This is accomplished by simply inserting *phantom bids* into our system to represent the cost at which items (or bundles) can be sourced by others means than the current auction. The clearing algorithm then optimizes as usual; whatever items are won by phantom bids are sourced by the other means. This approach is important because it allows some items not to be sourced in the auction if their bid prices are too high. This is a way of accomplishing automated demand reduction, and is used in most of our auctions. In some of our sourcing events–mainly single-item multi-unit ones–we also used another form of automated demand reduction where the bid taker specifies a price-quantity curve and if the current point is above the curve, demand is automatically reduced.

Time to Contract in Expressive Commerce

The time to contract is reduced from several months to weeks because no manual lotting is required, all suppliers can submit their offers in parallel, what-if scenarios can be rapidly generated and analyzed, and the allocation is implementable as is. This causes the cost savings to start to accrue earlier, and decreases the human hours invested.

Automated Item Configuration

An additional interesting aspect of bidding with cost drivers and alternates (e.g., using attributes) is that the market clearing (a.k.a. winner determination) algorithm not only decides who wins, but also ends up optimizing the configuration (setting of attributes) for each item. In deciding this, the optimizer, of course, also takes into account the buyer's constraints and preferences.

Automated Supply Network Configuration

In many of the sourcing events we conducted, we did not commit to a particular supply network up front, but rather collected offers for pieces of all possible supply networks. We then let winner determination–as a side effect–optimize the supply network multiple levels upstream from the buyer. This is in sharp contrast to the traditional approach where the supply network is designed first, and then one sources to the given network.

As an example of this new paradigm, in a sourcing event where Procter & Gamble (P&G) sourced in-store displays using our hosting service and technology, we sourced items from different levels of the supply network in one event: buying colorants and cardboard of different types, the service of printing, the transportation, the installation services, etc. (Sandholm et al. 2006). Some suppliers made offers for some of those individual items while others offered complete ready-made displays (which are, in effect, packages of the lower-level items), and some bid for partial combinations. The market clearing determined the lowest-cost solution (adjusted for P&G's constraints and preferences) and thus, in effect, configured the supply network multiple levels upstream.

I will now discuss this event in more detail. P&G uses pre-packed displays to help retailers merchandise its products. A display can contain different sizes of one product or contain multiple products, for example, Crest® toothpaste and Scope® mouthwash. Retail stores place displays in the aisles or in promotional areas when there is some special activity, such as a sale and/or a coupon for the brand. P&G spends $140 million annually in North America on these displays.

Based on individual product-promotion schedules and display requirements, managers typically used incumbent suppliers to design, produce, and assemble turnkey displays for easy setup in the stores. While these solutions were of high quality, there was little visibility into the costs and quality of alternate methods. P&G's corporate sourcing team thought that there could be a more efficient way to source displays, and wanted to understand the cost tradeoffs between buying the traditional turnkey displays and buying components, leveraging the size of P&G's entire operations.

Process

The P&G-CombineNet project team developed and executed a sourcing implementation designed to allocate P&G's annual spending on displays across a more efficiently utilized supplier base, while also improving the reliability and quality of display production and services. The plan contained three key elements:

- A bidding structure designed to capture component-specific information.
- A simple way for suppliers to understand and participate in the bidding process.
- Advantages for P&G's product managers that encouraged them to embrace the new process.

P&G's purchasing department invited all of the incumbents and some new suppliers to bid on the company's annual volume of displays. P&G's new capability to collect detailed cost information and solicit expressive or creative offers from suppliers allowed the purchasing organization to put up for bid each of the supply network cost drivers that contributed to the final cost of the display, such as display components as well as assembly and shipping costs that increase the base cost of the display materials. The purchasing department collected detailed information on the costs of materials, such as corrugated paper, film, and trays that hold the product, the costs of holding inventory, of freight, and of printing. It invited suppliers to bid on specification and then to make alternate off-specification bids that would allow suppliers to suggest ways to reduce the cost of the display. (For example, using three-color printing instead of four-color printing for the header card, which advertises the product, would reduce its cost.)

Of the 40 suppliers that participated in the sourcing event, some were manufacturers only, some were assemblers only, and some could manufacture and assemble. There were four display categories (pallets, special packs, Pigment/Dye/Quick trays, and wings and floor stands) covering 14 benchmark and unique displays. For roughly 500 display components, suppliers offered piece prices, substrate fluctuations, other fixed and variable costs, assembly rates, packaging, and freight. There were two online rounds of bidding followed by one round of off-line negotiation.

For suppliers, the flexibility of component-based bidding and the unique expressive bidding format allowed them to bid on their own terms, including volume discounts, bundled pricing, and alternate products or services. P&G encouraged the suppliers to submit two sets of bids, one identifying prices for full turnkey displays (including the aspects of production handled by others in their alliance networks) and a second bid for only those display components and services they could supply directly.

For P&G, the larger more complex set of data generated greater business insight when analyzed using our scenario navigation tool that enabled P&G to quickly and easily consider a large number of what-if scenarios by changing side constraints and preferences.

Results

The unconstrained savings were nearly 60% compared to previous year's prices. The implementable savings (that is, the savings P&G could achieve after applying its side constraints and preferences) were nearly 48% ($67 million annually). The collaborative planning produced insights into costs and strengthened P&G's relationships with its suppliers. P&G's annual procurement cycle dropped from 20 to 8 weeks, with the time for finding allocations to scenarios reduced from days to seconds.

P&G used our scenario navigation to assess the cost impact of constraints and preferences, such as favoring incumbent suppliers, and the cost of different mixes of display components. P&G gained the ability to separate the true cost of must-have components and services from nice-to-haves. This let P&G compare the cost of a supplier's turnkey display to the total cost of sourcing the display as its components and then managing the process. P&G realized it could allocate much of its spending more efficiently.

The bidding and award process also improved P&G's relationships with its suppliers by promoting collaboration and allowing suppliers to leverage their strengths. Our expressive bidding format gave suppliers an opportunity to bid on their own terms and did not commoditize their offerings. Both P&G and its suppliers benefited from a consolidated and easy-to-manage sourcing cycle.

Expressive Commerce as a Generalization of Combinatorial Auctions

A relatively simple early form of expressive commerce was a *combinatorial reverse auction* (Sandholm et al. 2002), where the only form of expressiveness that the suppliers have is package bidding, and the buyer has no expressiveness. A predecessor of that was a *combinatorial auction* where the bidders are the buyers (and there is only one unit of each item and no side constraints). Combinatorial auctions (Rassenti et al. 1982, Sandholm 1993, 2002b, Ledyard et al. 1997, Rothkopf et al. 1998, Kwasnica et al. 2005, Sandholm et al. 2005, Sandholm and Suri 2003, Hoos and Boutilier 2000, Boutilier 2002, and deVries 2003) enable bidders to express complementarity among items (the value of a package being more than the sum of its parts) via package bids. Substitutability (the value of a package being less than the sum of its parts) can also be expressed in some combinatorial auctions, usually using different languages for specifying mutual exclusivity between bids (Sandholm 2002a, Fujishima et al. 1999, Sandholm 2002b, Nisan 2000, and Hoos and Boutilier 2001).

Expressiveness leads to more economical allocations of the items because bidders do not get stuck with partial bundles that are of low value to them. This has been demonstrated, for example, in auctions for bandwidth (McMillan 1994 and McAfee and McMillan 1996), transportation services (Sandholm 1993, 1996, 1991, and Caplice and Sheffi 2003), pollution rights, airport landing slots (Rassenti et al. 1982), and carrier-of-last-resort responsibilities for universal services (Kelly and Steinberg 2000).

However, package bids and exclusivity constraints are too impoverished a language for real-world sourcing. While any mapping from bundles to real numbers can be expressed in that language in principle, the real-world preferences in sourcing cannot be easily, naturally, and concisely expressed in it. Starting in 1997, we tackled this challenge and generalized the approach to expressive commerce, with the language constructs discussed above. Similar approaches have recently been adopted by others, but for drastically less complex (orders of magnitude smaller and less expressive) events (Hohner et al. 2003, Metty et al. 2005).

The use of our richer expressiveness forms (rather than mere canonical package bids with exclusivity constraints) is of key importance for several reasons:

- Bidders can express their preferences in the language that is natural in their domain.
- Bidders can express their preferences concisely. To illustrate this point, consider the following simple example. A bidder has no production efficiencies and thus has a fixed price for each item regardless of what other items he produces. However, he has a capacity constraint. In our bidding language, he can simply express a price for each item and a capacity constraint. In contrast, in the classical combinatorial auction bidding languages, the supplier would have to submit bids for an exponential number of packages.
- Due to this conciseness, the bids are easy to communicate to the bid taker.
- Our bidding constructs maintain the natural structure of the problem (rather than mapping this structure into a format that only allows package bids with exclusivity constraints). The clearing algorithms take advantage of that structure in many ways, for example, in generating cutting planes, deciding what variables to branch on, and so on.

Optimization Techniques to Enable Expressive Commerce

A significant challenge in making expressive commerce a reality is that the expressiveness makes the problem of allocating the business across the suppliers an extremely complex combinatorial optimization problem. Specifically, the *clearing problem* (a.k.a. *winner determination problem*) is that of deciding which bids to accept and reject (and to what extent in the case of partially acceptable bids) so as to minimize sourcing cost (adjusted for preferences) subject to satisfying all the demand and all side constraints. Even in the vanilla combinatorial reverse auction where the only form of bidding is package bidding, and no side constraints or preferences are allowed, the clearing problem is NP-complete and inapproximable in the worst case in polynomial time (Sandholm et al. 2002). Expressive commerce is a much richer problem; thus the NP-hardness and inapproximability carry over. (Müller et al. (2006) review the worst-case complexity of the clearing problem of different variants of combinatorial auctions.) Thus sophisticated techniques are required.

Prior to our system, no technology was capable of solving clearing problems of the scale and expressiveness that our customers required; for example, Hohner et al. (2003) found integer programming techniques to be effective for problems only as large as 500 items and 5,000 bids. In 2001, P&G gave us a trial instance of trucking services sourcing that took a competing optimization product 30 minutes to solve. Our system solved it optimally in 9 seconds. While that was already a decisive speed difference, since that time our technology development has yielded a further speed improvement of 2-3 orders of magnitude.

There is significant structure in the expressive commerce problem instances, and it is paramount that the optimizer be able to take advantage of the structure. Mixed integer programming (MIP) techniques, which use tree search, are quite good at this, and our software takes advantage of them. However, the techniques embodied in the leading general-purpose MIP solvers alone are not sufficient for the clearing problem.

Our system uses sophisticated tree search to find the optimal allocation. Given that the problem is NP-complete, in the worst-case the run-time is super-polynomial in the size of the input (unless P=NP). However, in real-world sourcing optimization the algorithms run extremely fast: the median run-time is less than a second and the average is 20 seconds, with some instances taking days. The algorithms are also anytime algorithms: they provide better and better solutions during the search process.

I began algorithm development in 1997, and over ten years CombineNet grew to have 16 people on my team working on the algorithms, half of them full time. The team has tested hundreds of techniques (some from the AI and operations research literature and some invented by us) to see which ones enhance speed on expressive commerce clearing problems. Some of the techniques are specific to market clearing, while others apply to combinatorial optimization more broadly. We published the first generations of our search algorithms (Sandholm 2002a, Sandholm and Suri 2003, and Sandholm et al. 2005). The new ideas in these algorithms included:

- different formulations of the basic combinatorial auction clearing problem (branching on items (Sandholm 2002a), branching on bids (Sandholm and Suri 2003 and Sandholm et al. 2005), and multi-variable branching (Gilpin and Sandholm 2011)),
- upper and lower bounding across components in dynamically detected decompositions (Sandholm and Suri 2003 and Sandholm et al. 2005),
- sophisticated strategies for branch question selection (Sandholm 2006, 2002a, Sandholm and Suri 2003, and Sandholm et al. 2005),
- dynamically selecting the branch selection strategy at each search node (Sandholm 2006 and Sandholm et al. 2005),
- the information-theoretic approach to branching in search (Gilpin and Sandholm 2011),
- sophisticated lookahead techniques (Sandholm 2006 and Gilpin and Sandholm 2011),
- solution seeding (Sandholm 2006),
- primal heuristics (Sandholm 2006 and Sandholm et al. 2005),

- identifying and solving tractable cases at nodes (Sandholm and Suri 2003, Sandholm et al. 2005, Sandholm 2006, and Conitzer et al. 2004),
- techniques for exploiting *part* of the remaining problem falling into a tractable class (Sandholm 2006 and Sandholm and Suri 2003),
- domain-specific preprocessing techniques (Sandholm 2002a),
- fast data structures (Sandholm 2002a, Sandholm and Suri 2003, and Sandholm et al. 2005),
- methods for handling reserve prices (Sandholm 2002a and Sandholm and Suri 2003), and
- incremental winner determination and quote computation techniques (Sandholm 2002a).

Sandholm (2006) provides an overview of the techniques.

We also invented a host of techniques for the search algorithms that we have decided to keep proprietary for now. They include different formulations of the clearing problem, new branching strategies, custom cutting plane families, cutting plane generation and selection techniques, etc.

Since around 2002, we have been using machine learning methods to predict how well different techniques will perform on specific instances at hand. (For this purpose, an instance is represented by about 50 hand-selected numeric features.) This information can be used to select the technique for the instance at hand, to give time estimates to the user, and so on. Our solver has several dozen important parameters and each of them can take on several values. Therefore, our machine learning approach of setting the parameters well on an instance by instance basis is significantly more challenging and more powerful than using machine learning to select among a handful of hardwired solvers, an approach that has been pursued in academia in parallel (e.g., (Leyton-Brown, Nudelman, and Shoham 2006)). More recently, machine learning-based parameter tuning for optimization algorithms has become a popular research topic in academia, see, for example, Xu et al. (2011).

While the literature on combinatorial auctions has mainly focused on a variant where the only form of expressiveness is package bidding (sometimes supplemented with mutual exclusion constraints between bids), in our experience with sourcing problems the complexity is dominated by rich side constraints. Thus we have invested significant effort into developing techniques that deal with side constraints efficiently. We have faced several hundred different types of real-world side constraints. Our system supports all of them. We abstracted them into eight classes from an optimization perspective so that the speed improvements we build into the solver for one type of side constraint get leveraged across all side constraint types within the class.

The resulting optimal search algorithms are often 10,000 times faster than others'. The main reason is that we specialize on a subclass of MIP problems and have over 100,000 real-world instances on which to improve its algorithms. This speed has allowed our customers to handle drastically larger and more expressive sourcing events. The events have sometimes had over 2.6 million bids (on 160,000 items, multiple units of each) and over 300,000 side constraints.

State-of-the-art general-purpose MIP solvers are inadequate also due to numeric instability. They err on feasibility, optimality, or both, on about 4% of the sourcing instances. We have invested significant effort on stability, yielding techniques that are significantly more robust.

Hosted Optimization for Sourcing Professionals

Our backend clearing engine, *ClearBox*, is industry-independent, and the interface to it is through our *Combinatorial Exchange Description Language (CEDL),* an XML-based language that allows ClearBox to be applied to a wide variety of applications by the company and its partners. See Figure 5.

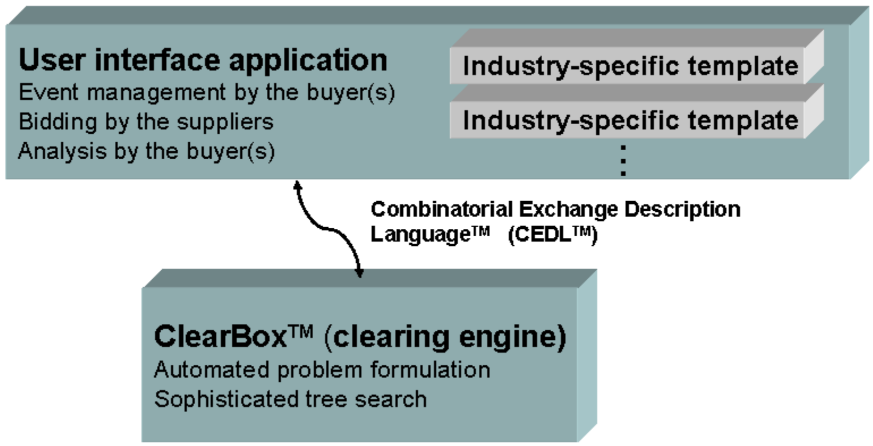

Figure 5. Advanced Sourcing Application Platform. The platform is hosted on a server farm with multiple instantiations of each component. The system also includes modules for clearing management, server farm management, secure databases, etc. (not shown).

Intuitive web-based interfaces designed for the buyer and for the suppliers bring the power of optimization to users with expertise in sourcing, not in optimization. The users express their preferences through interfaces that use sourcing terminology. The interfaces support simple click-through interaction rather than requiring the user to know special syntax. The approach allows sourcing professionals to do what they are best at (incorporating sourcing knowledge such as strategic and operational considerations) and the optimizer to do what it is best at (sifting through huge numbers of allocations to pick the best one).

For every event, separate front-ends are instantiated that support only those bidding and allocation evaluation features that are appropriate for that event. This makes the user interfaces easier and more natural to use by sourcing professionals. User training typically takes a few hours. New front ends typically take a few days or weeks to go from project specification to deployment. Instantiation of a front end can start from a clean slate with the entire configuration space of the system available. As an alternate, we also provide templates that are specific to industry and sourcing category, so as to accelerate the configuration process.

The user interfaces feed CEDL into ClearBox, and ClearBox then *automatically* formulates the optimization problem for our search algorithms. This contrasts with the traditional mode of using optimization, where a consultant with optimization expertise builds the model. The automated approach is drastically faster (seconds rather than months) and avoids errors.

Our web-based products and Software-as-a-Service (SaaS) business model make optimization available on demand. No client-side software installation is necessary. This also avoids hardware investments by customers. We buy the hardware and leverage it across customers, each with temporary load. The SaaS model allows us to quickly and transparently tune our algorithms, and to provide enhancements to all customers simultaneously. The solution is also offered through consulting firms.

Impact

The new sourcing paradigm and technology has already had significant impact. Between December 2001 and June 2010[3], we used our system to host over 800 highly combinatorial sourcing events, totaling a spend of about $60 billion. The individual events ranged from $2 million to $7 billion, representing the most complex combinatorial auctions ever conducted. They spanned a broad range of categories such as:

- *transportation:* truckload, less-than-truckload, ocean freight, dray, bulk, intermodal, small parcel, air freight, train, fleet, freight forwarding, and other,
- *direct materials:* sugars/sweeteners, meat, vegetables, honey, starches, colorants, fibers/non-wovens, steel, fasteners, solvents, chemicals, casings, resins, and polymers,
- *packaging:* cans/ends, corrugates, corrugated displays, flexible film, folding cartons, labels, foam trays/pads, caps/closures, shrink and stretch films, bags, pulp, pallets, and printed instructions,
- *indirect materials:* Management, Repair, and Operations, a.k.a. MRO (electrical supplies, filters, pipes/valves/fittings, power transmissions, pumps, safety supplies, office supplies, lab supplies, file folders, solvents, and furnishings), chemicals (cylinder gasses, fuels, and other), technology (laptops/desktops and cameras), leased equipment, fleet vehicles, and promotional items, and
- *services:* security, janitorial, legal, patent/trademark, consulting, equipment maintenance, temp labor, marketing, customization, insurance, shuttling/towing, warehousing, pre-press, and advertising, and
- *healthcare:* pharmaceuticals as well as medical/surgical equipment and supplies, and
- *telecommunication:* sourcing wireless plans for employees of companies.

From this we conclude that the market design that we created for combinatorial multi-attribute markets indeed has an appropriate level of generality. We believe that it can serve as a model for the structuring of such markets in most applications.

At the point where we had cleared $35 billion of sourcing spend through our system, we conducted a study of those sourcing events. Over 60 buyer companies used our system and they

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[3] CombineNet was acquired in 2010. The company continues to operate, but my most recent data is as of the acquisition time.

were mostly among the Global 1000. A total of over 12,000 supplier companies bid in our system. We had delivered hard-dollar savings of $4.4 billion (12.6% of spend) to our customers (i.e., the buyers) in lowered sourcing costs. The savings were measured compared to the prices that the buyer paid for the same items the previous time the buyer sourced them (usually 12 months earlier). The $4.4 billion is the *implementable savings* that the system yielded after the buyer had applied side constraints and preferences; the *unconstrained savings* (which can be viewed as the savings arising from expressive bidding before the buyer expresses constraints and preferences) was $5.4 billion (15.4%). The savings figure is noteworthy especially taking into account that during the same time period, the prices in the largest segment, transportation, increased by 6-9% in the market overall.

The savings number does not include the savings obtained by suppliers, which are harder to measure because the suppliers' true cost structures are proprietary. However, there is strong evidence that the suppliers also benefited, so a win-win was indeed achieved: 1) suppliers that have participated in expressive commerce events are recommending the use of that approach to other *buyers*, 2) on numerous occasions, suppliers that boycotted reverse auctions (offered by other sourcing system vendors) came back to the "negotiation table" once we introduced expressive commerce for the sourcing event, and 3) suppliers are giving very positive feedback about their ability to express differentiation and provide creative alternatives.

The savings number also does not include savings that stem from reduced effort and compression of the event timeline from months to weeks or even days.

The cost savings were achieved while at the same time attaining the other advantages of expressive commerce discussed above, such as better supplier relationships (and better participation in the events), redesign of the supply network, implementable solutions that satisfy operational considerations, and solutions that strike tradeoffs in a data-driven way and align the stakeholders in the buying organization. See also Sandholm et al. (2006) and case studies at www.CombineNet.com.

CombineNet grew to 130 full-time employees (about half of them in engineering) and a dozen academics as advisors. The company has operations on four continents, with headquarters in Pittsburgh, Pennsylvania.

Summary of Key Lessons Learned

In this section I will distill some of the key lessons learned from this experience.

- (Generalization of) combinatorial auctions can be practical even in settings with tens of thousands of items, large numbers of units each item, hundreds of thousands of side constraints, and millions of bids.
- Combinatorial and multi-attribute auctions can be soundly hybridized into a general domain-independent market design. It can be embodied in a easy-to-use, application-independent, hosted software.
- Practicality in the large requires a bidding language that is compact and natural. The canonical bidding language of combinatorial auctions–namely package bidding (with

forms of exclusivity constraints between packages to express substitutability)–does not scale (if used alone).

- Bidders, as well as the bid taker, almost always have preferences that are so rich that they cannot be modeled in a linear program. Rather, both continuous and discrete variables are necessary for their modeling.
- For scalability of communicating the optimization problem to the market clearing engine, and for scalability of that optimization, one should retain the structure inherent in the bidding language within the optimization model (rather than expanding that input into a canonical language of package bids and exclusivity constraints).
- By retaining this structure, the clearing problem can typically be solved to provable optimality quickly, so the fact that the problem is NP-complete and worst-case inapproximable in polynomial time is not a prohibitive obstacle. Even on instances where the optimization does not solve to optimality quickly, typically solutions that are provably near optimal are found quickly – ones with much better quality guarantees than those provided by approximation algorithms that run in worst-case polynomial time.
- Typically the bid taker has additional preferences and constraints beyond cost minimization, but these tend to be hard to articulate up front. Rather, bid takers like to conduct scenario navigation with (at least some of) the bids in hand.
- Incentive compatible mechanisms seem to be undesirable, at least in sourcing. One reason is that such mechanisms are not actually incentive compatible in reality, since related auctions are conducted repeatedly over years with roughly the same bidders and items. Another reason is that scenario navigation compromises incentive compatibility even within a single sourcing event. Additional undesirable aspects of incentive compatible auctions are discussed in Rothkopf (1990), Sandholm (1996), Conitzer and Sandholm (2006), Ausubel and Milgrom (2006), and Rothkopf (2007). First-price (pay-your-winning-bids) mechanisms are natural and seem to work well.

Challenges and Future Work

Our experiences in developing and fielding large-scale combinatorial markets have uncovered many new issues that require further attention. In this section I will discuss some of the main ones.

Is More Expressiveness Always Better?

The experiences covered in the article show that increasing the expressiveness offered to the market participants tends to increase the efficiency of the allocation. But is that always so?

We recently developed a theory that ties the expressiveness of mechanisms to their efficiency in a domain-independent manner (Benisch and Sandholm 2010). We introduced two expressiveness measures, *maximum impact dimension*, which captures the number of ways that an agent can impact the outcome, and *shatterable outcome dimension*, which is based on the concept of *shattering* from computational learning theory. We derived an upper bound on the expected efficiency of any mechanism under its most efficient Nash equilibrium. Remarkably, it depends only on the mechanism's expressiveness. We proved that the bound increases strictly as we allow more expressiveness. We also showed that in some cases a small increase in expressiveness yields an arbitrarily large increase in the bound.

We then showed that in any private-values setting, the bound can always be reached in pure strategy Bayes-Nash equilibrium (while achieving budget balance in expectation). In contrast, without full expressiveness, dominant-strategy implementation is not always possible.

Finally, we studied *channel-based* mechanisms. They restrict the expressions of value through channels from agents to outcomes, and select the outcome with the largest sum. (Channel-based mechanisms subsume most combinatorial and multi-attribute auctions, the VCG mechanism, etc.) In this class, a natural measure of expressiveness is the number of channels allowed (this generalizes the *k*-wise dependence measure of expressiveness used earlier in the combinatorial auction literature (Conitzer et al. 2005)). We showed that our domain-independent measures of expressiveness increase strictly with the number of channels allowed. Using this bridge, our general results yield interesting implications, and a better understanding of problems such as the exposure problem. For example, even a slight lack of expressiveness can result in arbitrarily large inefficiency—unless agents have no private information.

The results mentioned in this section assume that the mechanism designer can create the mechanism *de novo* for the allowed expressiveness level. Future research should study the relationship between expressiveness and efficiency (and other objectives such as sourcing cost) when the mechanism designer is forced to stay within some standard allocation and pricing rules.

Revenue-Maximizing (Cost-Minimizing) Mechanism Design

As an example of how more expressiveness is not always better when operating within a given mechanism family, if one is using VCG, the bid taker can increase expected revenue (analogously, reduce sourcing cost) by reducing expressiveness via careful bundling. To see this, consider a simple auction for an apple and an orange with two bidders. One bidder wants the apple and the other wants the orange. Fully expressive VCG would allocate each item to the bidder who wants it, but there would be no competition and zero revenue. In contrast, if the bid taker bundles the items so they must sell together, VCG revenue will equal the lower of the two bidders' valuations for the bundle.[4]

More generally, it is open, even for just two items, how to design a revenue-maximizing combinatorial auction. The design problem itself is NP-complete (Conitzer and Sandholm 2004), so it is unlikely that a short characterization can exist. Instead, *automated mechanism design* holds promise for this (Sandholm et al. forthcoming). Designing the mechanism to make use of the prior of the bid taker seems to make sense in these settings where roughly the same event is conducted year after year and the bid taker has the opportunity to learn. Of course, the

[4] Vendors of inexpressive reverse auctions often justify bundling as a way to avoid cherry picking, that is, suppliers bidding on just the "desirable" items, which would lead to poor – or no – bid coverage of the "undesirable" items. This argument is incorrect, however, because items' desirability is determined by prices, which are set endogenously by the auction.

bidders knowing that the bid taker is learning for the purpose of cost minimization in the future may affect their bidding.

Automated Scenario Navigation

In basic scenario navigation, discussed earlier in this paper, the bid taker changes her preferences (hard and soft) and reoptimizes. She repeats this over and over (often hundreds of times) until she finds a solution that is satisfactory to her in terms of the tradeoff between her non-monetary preferences and economic value (sourcing cost in the case of sourcing). However, the space of scenarios to navigate is infinite, so there is no guarantee that even a reasonably good solution has been found. Furthermore, a good solution could have been found with much less scenario navigation than what the user actually went through.

To address these issues, we pioneered *automated scenario navigation* (Boutilier et al. 2004), and built a prototype of it on top of our sourcing system. Compared to basic scenario navigation, it enables a more systematic and less wasteful navigation of the scenario space. The system queries the sourcing team about their preferences, using, for example, tradeoff queries ("how much hassle would an extra supplier be in dollars? – give me an upper or lower bound in terms of dollars") and comparison queries ("which of these two allocations do you prefer?"). These two kinds of query are shown in Figures 6 and 7, respectively. The system decides the queries to pose in a *data-directed way* so as to only ask the team to refine its preferences *on an as-needed basis*. (This is desirable because internal negotiation in the team is costly in terms of time and goodwill.) Specifically, based on all the offers that the suppliers have submitted, and all answers to previous queries, the system strives to minimize maximum regret. At each iteration of automated scenario navigation, the system finds a *robust solution* that minimizes maximum regret (the regret is due to the fact that the sourcing team has not fully specified its preferences, so for some preferences that are still consistent with the answers so far, the system's recommended allocation is not optimal). As the other step of each iteration, the system poses a query to refine the team's preferences in order to be able to reduce the maximum regret further.

The maximum regret also provides a quantitative measure of when further negotiation within the team is no longer worth it, and the team should implement the current robust allocation.

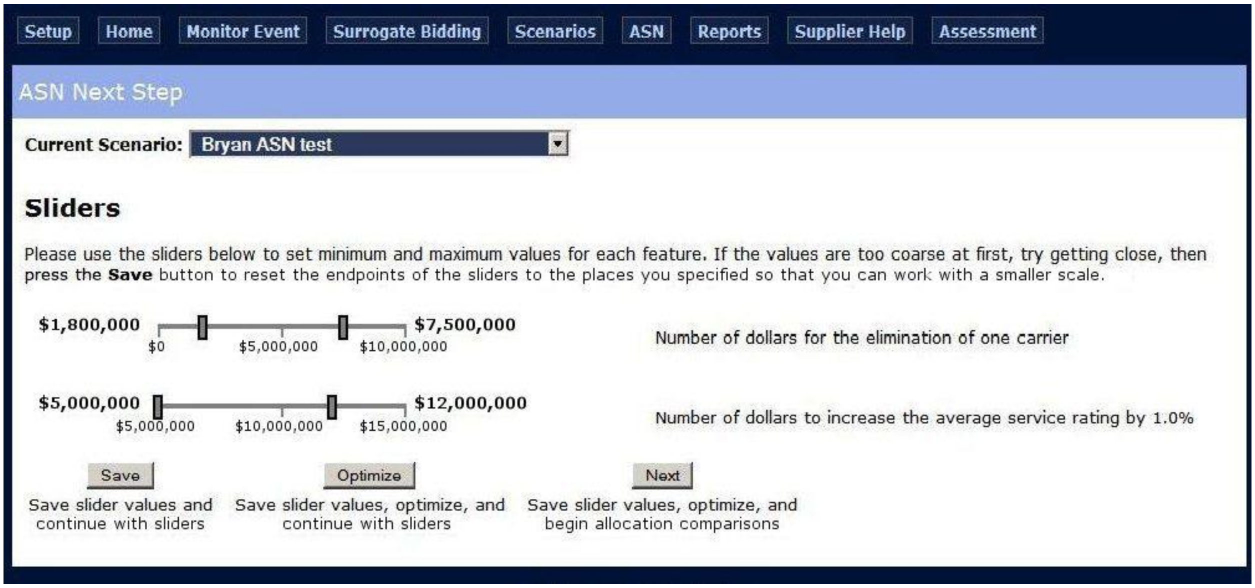

Figure 6. A tradeoff query in our system. The user gets to adjust the sliders to represent upper and lower bounds (in dollars) on the value of different aspects of allocations. That gives information to the system about her preferences.

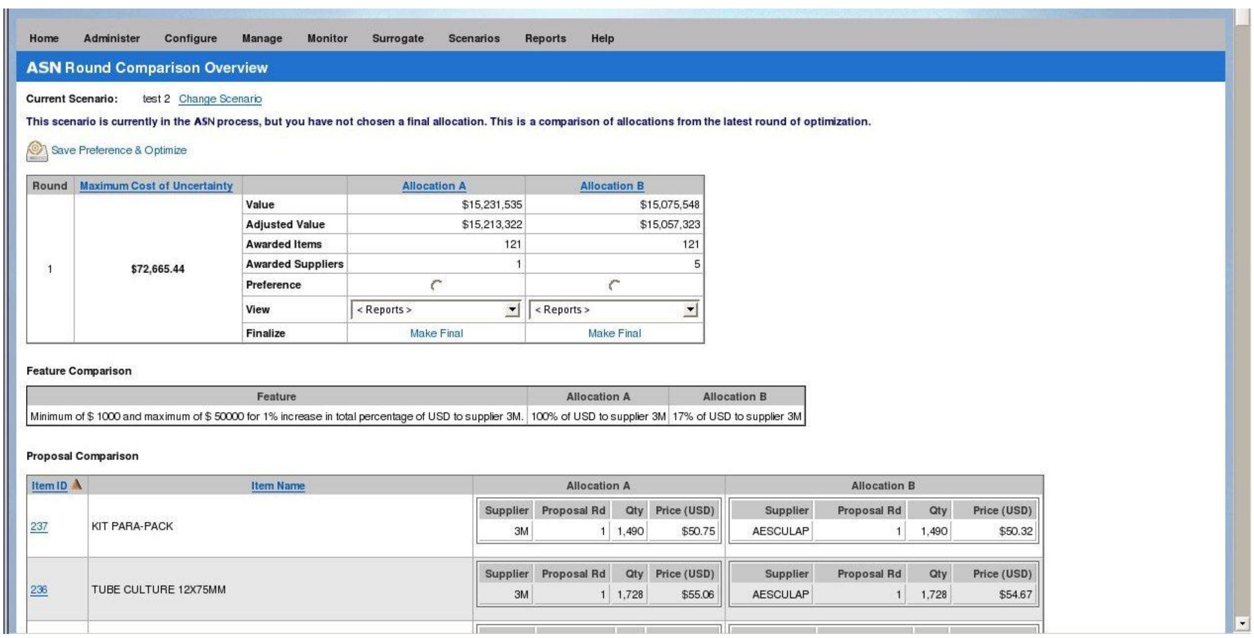

Figure 7. A comparison query in our system in a medical sourcing event. In the top matrix, the user gets to click on which allocation she prefers. That gives information to the system about her preferences.

With automated scenario navigation, provably good (i.e., ones with low maximum regret) solutions tend to be found while asking the user relatively little of her preferences. On the downside, the optimization problem of finding a regret-minimizing allocation is harder than the clearing problem discussed in most of this paper. We discuss automated scenario navigation in detail, including its different design dimensions and algorithms, in Boutilier et al. (2004).

Eliciting the Bidders' Preferences

The section above discussed how the bid taker's preference information can be frugally collected on an as-needed basis to make good allocation decisions. Perhaps even more important is how the bidders' preference information gets collected. Most, but not all, game-theoretic mechanisms are so called *direct-revelation* mechanisms, where each agent (bidder) reveals its type completely up front. The revelation principle states that in the absence of computation/communication limitations, this restriction comes at no cost. However, in practice such mechanisms are problematic because the agents may need to determine their own preferences via costly deliberation (e.g., computing to generate plans such as routings and schedules (Sandholm 1993, 1996)) or information gathering, and communicating complete preferences may be undesirable from the perspective of privacy or conserving bandwidth. It would be highly desirable to be able to reach the right allocation without requiring all the preference information from all the bidders.

The bidders have to decide which combinations they want to evaluate (and how precisely) and communicate bids on. If they choose bundles which they don't win, evaluation effort and communication is wasted, and unnecessary private information is revealed. Also, economic efficiency will suffer if the bidders do not evaluate bundles on which they would win. However, it is difficult to anticipate which combinations one will win before knowing the others' bids!

As a general way of tackling this, in our academic research we introduced the idea of explicit *preference elicitation in combinatorial auctions* (Conen and Sandholm 2001). The idea is to supplement the clearing algorithm with an *elicitor* (software) that incrementally queries the agents about their preferences, and builds a model of them. The elicitor decides the next query to ask (and who to ask it from) based on the answers it has received from the agents so far. Once enough information has been elicited to determine the right (in any given sense) outcome, no more information is elicited.

Preference elicitation in multiagent systems is fundamentally different from traditional preference elicitation where there is only one party whose preferences are to be elicited, because what information is needed from an agent depends on what information other agents reveal. We showed that allowing the elicitor to condition the queries it asks of an agent on the answers of another agent (rather than eliciting the agents' preferences separately) can yield exponential savings in communication and preference determination effort.

Furthermore, if the elicitor conditions the query it poses to an agent on the answers given by other agents, the elicitor leaks information about the others' answers to the agent. This introduces the potential for new forms of strategic manipulations not present in single-shot mechanisms. Conen and Sandholm (2001) proposed a method that nevertheless motivates each bidder to answer the elicitor's questions truthfully in *ex post* equilibrium (so the method does not require any knowledge of prior probability distributions). This is accomplished by eliciting enough information to determine the optimal allocation and the VCG payments. This maintains the truth-promoting property even while allowing bidders to skip questions and to answer questions that were never asked. This yields a *push-pull mechanism* where the revelation of information is guided by where the bidders think they are competitive, and by where the elicitor knows that it does (and does not) need further information.

Preference elicitation in combinatorial auctions has become an entire research area. Sandholm and Boutilier (2006) provide a review, discussing a range of results for various query types and query policies. They also discuss the surprising power of bundle pricing. Ascending combinatorial auctions are a special case of preference elicitation that preceded the general approach. Parkes (2006) provides a review of those mechanisms.

To my knowledge, the general preference elicitation approach is not yet used in real combinatorial auctions. Rather, in the industrial work we and others have focused on designing bidding languages via which bidders can express their preferences compactly and naturally in a push-only way. Future work should integrate explicit preference elicitation into combinatorial auctions in practice.

Bidding Support Tools versus Extreme Expressiveness

There has been quite a bit of discussion about tools for bidding in combinatorial auctions, especially in transportation domains. Bidders spend significant effort constructing and submitting bids, and there is demand for tools to make this easier. There has been some research on developing such tools for bidding in trucking (Sandholm 1993, Song and Regan 2005, Ergun et al.). A key challenge, though, is that what bids a bidder should submit depends not only on the bidders' local information but also on what others have bid. Therefore, any purely bidder-side bidding support tool is inherently limited, and tools should consider what information other bidders have submitted. This would bring bidding tools close to the preference elicitation approach discussed above.

Also, as discussed throughout this paper, my view is that the market design should allow highly expressive, compact, natural bids rather than mere package bids. Taken to its extreme, under that view a bidder should be able to submit all the information he would submit to the bidding support tool to the auction directly! The auction clearing algorithm would then take the role of the bidding support tool as well, but would be able to make better, optimized decisions because it has the information from all bidders, not just one.

That said, one argument in favor of the current level of expressiveness – which does not require all the details about each bidders' local optimization problem to be submitted to the auction – is that the bidding language serves as an abstraction layer between the bidders and the auction. This allows different ways of generating bids (automated or manual) to be used by different bidders. This open architecture may serve to foster innovation in tools by the bidders. The bidding language that I described in this paper seems to be an appropriate interface layer since it has been successfully used across a wide range of sourcing categories and settings.

Planning versus Execution

There are at least two key challenges in combinatorial sourcing auctions related to the discrepancy between planning (i.e., sourcing) and execution (i.e., procurement):

- In (e.g., year-ahead) combinatorial sourcing auctions, aspects of complementarity, substitutability, and feasibility are not known at bidding time. Rather they become apparent only during the execution of the long-term contracts that the auction is used to construct. For example, if a supplier (carrier) wins 10 truckloads per week on a lane from Pittsburgh to Chicago and 10 truckloads per week on a lane from Chicago to Pittsburgh, it is not clear that those can be executed as backhaul deliveries due to execution-time constraints–such as pickup and delivery time windows–that arise dynamically. As another example, once a carrier agrees to a long-term contract via a sourcing event, it does not mean that the carrier always has the capacity available at the moment to take on every load that the buyer calls on him to carry throughout the year based on the contract.

- Current downstream execution systems, *contract management systems (transportation management systems* in the case of logistics), do not understand the expressive contracts that a combinatorial auction generates. Rather, such systems assume that item prices are determined by sourcing and then during execution, procurement personnel can simply order through the contract management systems using those prices.[5]

  One cannot fix this by simply splitting accepted package bids into item prices because an item's price should depend on what other items it gets procured with. This actually raises a deep issue of what a package bid means: does it mean that the items in the package have to always be procured together, or that the agreed-upon quantities of those items are ordered over the entire procurement period (year), or something in between? Similarly, does a discount trigger based on what is awarded at sourcing time or what is actually procured (in the entire year, per week, or per month)? Both practices have been used, for example, in truckload transportation sourcing.

One could try to address the problem by allowing bids to have their pricing conditioned on execution-time variables, for example, whether lanes execute as a bundle.[6] However, in that setting the market clearing engine would have to take into account what values those variables might end up taking. This would involve risk management issues. (One cannot postpone the market clearing until execution time; that would undermine the predictability of long-term contracts, which also allow the parties to make situation-specific investments in capacity, etc.) Furthermore, current downstream systems do not support that.

The status quo is that the buyer might not procure the volumes that are sourced. This can occur due to his demand deviating from projections. It can also occur due to certain spot market opportunities. For example, in truckload transportation, certain spot bids are considered acceptable to take in order to reduce deadheading.[7] Similarly, on the supply side, bidders cannot always fulfill the procurement needs that arise related to a contract that they have won. In summary, the contracts that sourcing yields are soft on both sides.

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[5] Similar issues occur in upstream *spend analysis systems* because they do not support expressive contracts either.

[6] What values the execution-time variables take may be affected by other sourcing events (auctions or manual) that the bidders may be involved in. Thus the terms of a contract in one such auction could depend on the outcomes of other such auctions. However, this would be difficult to monitor and has the potential to lead to agency problems.

[7] The spot market opportunities should then be taken into account in the buyer's sourcing optimization. During sourcing time, they could be modeled stochastically. For example, in other work we have employed stochastic spot market modeling in the optimization used to decide which proposed Internet display advertising campaigns to accept versus reject (Boutilier et al. 2008).

In the future, it would be important to formalize execution issues into expressive contracts themselves, and to monitor them, in order to make the system function more efficiently. That, in turn, begets the need to optimize one's procurement against one's contracts. This can even involve reoptimization in light of the execution state and new projections.[8]

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References

Ausubel, L. and Milgrom, P. 2006. The lovely but lonely Vickrey auction. Chapter 1 of *Combinatorial Auctions*, Cramton, Shoham, and Steinberg, eds., MIT Press.

Benisch, M. and Sandholm, T. 2010. A Theory of Expressiveness in Mechanisms. Mimeo. Significantly extends over a AAAI-08 paper.

Boutilier, C. 2002. Solving concisely expressed combinatorial auction problems. In Proceedings of the National Conference on Artificial Intelligence (AAAI).

Boutilier, C., Parkes, D., Sandholm, T., and Walsh, W. 2008. Expressive Banner Ad Auctions and Model-Based Online Optimization for Clearing. In Proceedings of the National Conference on Artificial Intelligence (AAAI).

Boutilier, C., Sandholm, T., and Shields, R. 2004. Eliciting bid taker non-price preferences in (combinatorial) auctions. In Proceedings of the National Conference on Artificial Intelligence (AAAI).

Caplice, C. and Sheffi, Y. 2003. Optimization-based procurement for transportation services. *Journal of Business Logistics* **24**(2): 109-128.

Conen, W. and Sandholm, T. 2001. Preference Elicitation in Combinatorial Auctions: Extended Abstract. In Proceedings of the *ACM Conference on Electronic Commerce (EC)*.

Conitzer, V., Derryberry, J., and Sandholm, T. 2004. Combinatorial auctions with structured item graphs. In Proceedings of the National Conference on Artificial Intelligence (AAAI).

[8] If there are multiple expressive contracts covering some of the same items with the same supplier, a procurement sequence can be interpreted in multiple ways: different (combinations of) contracts could have been used to accomplish that procurement sequence. Some of those alternatives can be better interpretations than others, for example, in terms of which discounts get triggered. Curiously, this begets the opportunity to optimize the past!

Conitzer, V. and Sandholm, T. 2004. Self-Interested Automated Mechanism Design and Implications for Optimal Combinatorial Auctions. In Proceedings of the *ACM Conference on Electronic Commerce (EC)*.

Conitzer, V. and Sandholm, T. 2006. Failures of the VCG mechanism in combinatorial auctions and exchanges. In Proceedings of the *International Conference on Autonomous Agents and Multi-Agent Systems (AAMAS)*. Early version in the Agent-Mediated Electronic Commerce workshop, 2004.

Conitzer, V. and Sandholm, T., and Santi, P. 2005. Combinatorial Auctions with *k*-wise Dependent Valuations. In Proceedings of the *National Conference on Artificial Intelligence (AAAI)*.

Ergun, Ö, Kuyzu, G, and Savelsbergh, M. Bid Price Optimization for Simultaneous Truckload Transportation Procurement Auctions. Mimeo.

Kwasnica, A., Ledyard, J., Porter, D., and DeMartini, C. 2005. A new and improved design for multiobject iterative auctions. *Management Science* **51**(3): 419-434.

de Vries, S. and Vohra, R. 2003. Combinatorial auctions: A survey. *INFORMS Journal on Computing* **15**(3): 284-309.

Fujishima, Y., Leyton-Brown, K., and Shoham, Y. 1999. Taming the computational complexity of combinatorial auctions: Optimal and approximate approaches. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI).

Gilpin, A. and Sandholm, T. 2011. Information-theoretic approaches to branching in search. *Discrete Optimization*, 8, 147-159.

Hohner, G., Rich, J., Ng, E., Reid, G., Davenport, A., Kalagnanam, J., Lee, H., and An, C. 2003. Combinatorial and quantity-discount procurement auctions benefit Mars, Incorporated and its suppliers. *Interfaces* **33**: 23-25.

Hoos, H. and Boutilier, C. 2000. Solving combinatorial auctions using stochastic local search. In Proceedings of the National Conference on Artificial Intelligence (AAAI).

Hoos, H. and Boutilier, C. 2001. Bidding languages for combinatorial auctions. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI).

Kelly, F. and Steinberg, R. 2000. A combinatorial auction with multiple winners for universal services. *Management Science* **46**(4): 586-596.

Kothari, A., Parkes, D., and Suri, S. 2003. Approximately strategyproof and tractable multi-unit auctions. ACM Conference on Electronic Commerce, 166-175.

Ledyard, J., Porter, D., and Rangel, A. 1997. Experiments testing multiobject allocation mechanisms. *Journal of Economics and Management Strategy* **6**(3): 639-675.

Leyton-Brown, K., Nudelman, E., and Shoham Y. 2006. Empirical Hardness Models for Combinatorial Auctions. Chapter 19 of *Combinatorial Auctions*, Cramton, Shoham, and Steinberg, eds., MIT Press.

Maskin, E. and Riley, J. 2000. Asymmetric Auctions. *Review of Economic Studies* **67**, 413-438.

Metty, T., Harlan, R., Samelson, Q., Moore, T., Morris, T., Sorensen, R., Schneur, A., Raskina, O., Schneur, R., Kanner, J., Potts, K., and Robbins, J. 2005. Reinventing the supplier negotiation process at Motorola. *Interfaces* **35**(1): 7-23.

McMillan, J. 1994. Selling spectrum rights. *Journal of Economic Perspectives* **8**(3): 145-162.

Müller, R., Lehmann, D., and Sandholm, T. 2006. The winner determination problem. Chapter 12 of *Combinatorial Auctions*, Cramton, Shoham, and Steinberg, eds., MIT Press.

McAfee, P. and McMillan, J. 1996. Analyzing the airwaves auction. *Journal of Economic Perspectives* **10**(1): 159-175.

Nisan, N. 2000. Bidding and allocation in combinatorial auctions. ACM Conference on Electronic Commerce, 1-12.

Parkes, D. 2006. Iterative Combinatorial Auctions. Chapter 2 of the book *Combinatorial Auctions*, Cramton, Shoham, and Steinberg, eds., MIT Press.

Rassenti, S., Smith, V., and Bulfin, R. 1982. A combinatorial auction mechanism for airport time slot allocation. *Bell Journal of Economics* **13**(2): 402-417.

Rothkopf, M. 2007. Thirteen reasons why the Vickrey-Clarke-Groves process is not practical. *Operations Research* 55(2), 191-197.

Rothkopf, M., Pekec, A., and Harstad, R. 1998. Computationally manageable combinatorial auctions. *Management Science* **44**(8): 131-1147.

Rothkopf, M., Teisberg, T., and Kahn, E. 1990. Why are Vickrey auctions rare? *Journal of Political Economy* 98(1), 94-109.

Sandholm, T. 1991. A strategy for decreasing the total transportation costs among area-distributed transportation centers. In the *Nordic Operations Analysis in Cooperation: OR in Business* conference, Turku School of Economics, Finland.

Sandholm, T. 1993. An implementation of the contract net protocol based on marginal cost calculations. In Proceedings of the National Conference on Artificial Intelligence (AAAI).

Sandholm, T. 1996. Limitations of the Vickrey auction in computational multiagent systems. In Proceedings of the International Conference on Multiagent Systems. Extended version "Issues in Computational Vickrey Auctions" in *International Journal of Electronic Commerce* **4**: 107-129, 2000.

Sandholm, T. 2002a. Algorithm for optimal winner determination in combinatorial auctions. *Artificial Intelligence*, **135**:1-54. Early versions: ICE-98, IJCAI-99.

Sandholm, T. 2002b. eMediator: A next generation electronic commerce server. *Computational Intelligence*, **18**(4): 656-676. Early version: AAMAS-00.

Sandholm, T. 2006. Optimal winner determination algorithms. Chapter 14 of *Combinatorial Auctions*, Cramton, Shoham, and Steinberg, eds., MIT Press.

Sandholm, T. and Boutilier, C. 2006. Preference Elicitation in Combinatorial Auctions. Chapter 10 of the book *Combinatorial Auctions*, Cramton, Shoham, and Steinberg, eds., MIT Press.

Sandholm, T., Levine, D., Concordia, M., Martyn, P., Hughes, R., Jacobs, J., and Begg, D. 2006. Changing the game in strategic sourcing at Procter & Gamble: Expressive competition enabled by optimization. *Interfaces* **36**(1), 55-68.

Sandholm, T., Likhodedov, A., and Gilpin, A. Automated Design of Revenue-Maximizing Combinatorial Auctions. *Operations Research*, Special Issue on Computational Economics, forthcoming. Subsumes and extends over a AAAI-04 and AAAI-05 paper.

Sandholm, T. and Suri, S. 2001a. Market clearability. In Proceedings of the International Joint Conference on Artificial Intelligence (IJCAI).

Sandholm, T. and Suri, S. 2001b. Side constraints and nonprice attributes in markets. In Proceedings of the *IJCAI Workshop on Distributed Constraint Reasoning*, Seattle, WA. Journal version: *Games and Economic Behavior*, 55: 321-330, 2006.

Sandholm, T. and Suri, S. 2002. Optimal clearing of supply/demand curves. In Proceedings of the Annual International Symposium on Algorithms and Computation (ISAAC).

Sandholm, T. and Suri, S. 2003. BOB: Improved winner determination in combinatorial auctions and generalizations. *Artificial Intelligence* **145**: 3-58. Early version: AAAI-00.

Sandholm, T., Suri, S., Gilpin, A., and Levine, D. 2002. Winner determination in combinatorial auction generalizations. In Proceedings of the International Conference on Autonomous Agents and Multiagent Systems (AAMAS).

Sandholm, T., Suri, S., Gilpin, A., and Levine, D. 2005. CABOB: A fast optimal algorithm for winner determination in combinatorial auctions. *Management Science* **51**(3): 374-390. Early version: IJCAI-01.

Smock, D. 2004. How P&G buys plastics. *Modern Plastics.*

Song, J. and Regan, A. 2005. Approximation algorithms for the bid construction problem in combinatorial auctions for the procurement of freight transportation contracts. Transportation Research Part B: Methodological, 39(10), 914-933.

Xu, L., Hutter, F., Hoos, H., and Leyton-Brown, K. 2011. Hydra-MIP: Automated Algorithm Configuration and Selection for Mixed Integer Programming. RCRA workshop on Experimental Evaluation of Algorithms for Solving Problems with Combinatorial Explosion at the International Joint Conference on Artificial Intelligence (IJCAI).