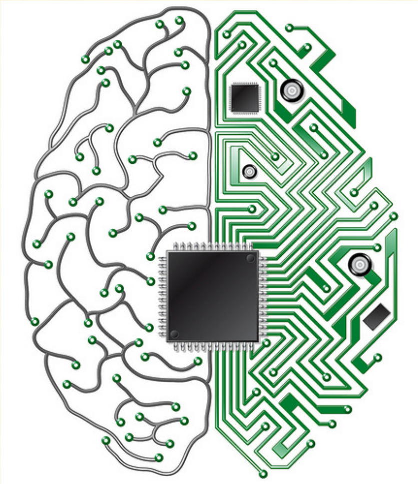
Algorithms in Nature

Path selection in networks: Steiner trees

Building efficient graphs

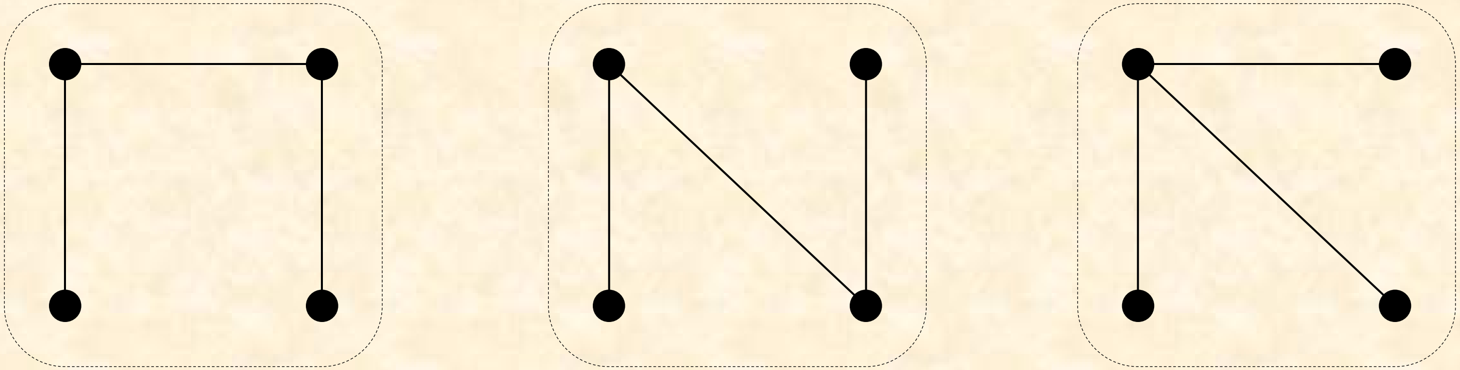
Building the shortest graphs

- Very often we have a set of points and want to find the shortest collection of edges that connect them up. For example,
  - Roads/railways connecting towns
  - Telephone/internet cables
  - Gas pipes
  - Connections in electronic circuits
  - Neurons connecting bits of your br



The shortest graph?

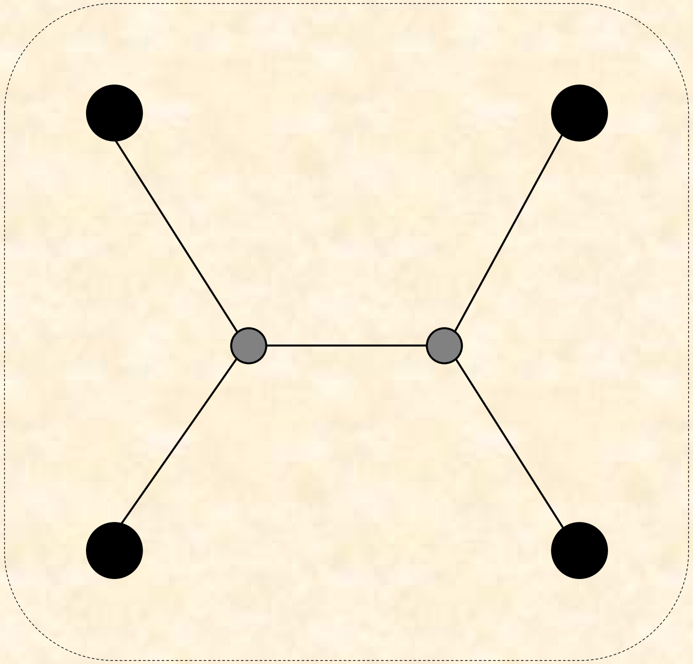
- Suppose we have 4 towns that we wish to connect up. Which of these do you think is shortest?



An unexpected solution

- If we're restricted to roads between towns, then the first graph is the shortest.
- But there is a better solution, …

Steiner graph



Steiner tree

- ***Given***: connected undirected graph G=(V,E), weight for each edge $e \in$ E, subset of vertices *N* (known as terminals)
- ***Question***: find a subtree T of G, such that each vertex of *N* is on T and the total length of T is as small as possible
  - Steiner tree *spanning N*

Applications

- Wire routing of *very large scale integration* (VLSI)
- Other network design and facility location problems

Special cases

- $|N| = 1$: trivial
- $|N| = 2$: shortest path
- $N = V$: minimum spanning tree

The problem is NP hard

- Membership of ST in NP: trivial
- Hardness: Reduction from Exact Cover by 3-Sets (X3C)

X3C:

- 3q variables, $x_1 \ldots x_{3q}$
- n sets $C_1 \ldots C_n$ each containing 3 x variables ($|C| = 3$)
- For example, $C_1 = \{x_5, x_8, x_{12}\}$
- Goal – Are there *q* sets *C* that together cover all the variables *x*?

Reduction

- Construct a graph with a source node v, sink nodes x and intermediate nodes C.
- Set terminal nodes to be v and the x's.
- There is a Steiner tree with 4q edges iff there is a solution to X3C with q sets.

Approximation algorithms

- Several different algorithms that guarantee ratio 2 (or, more precise: $2 - 2/n$).

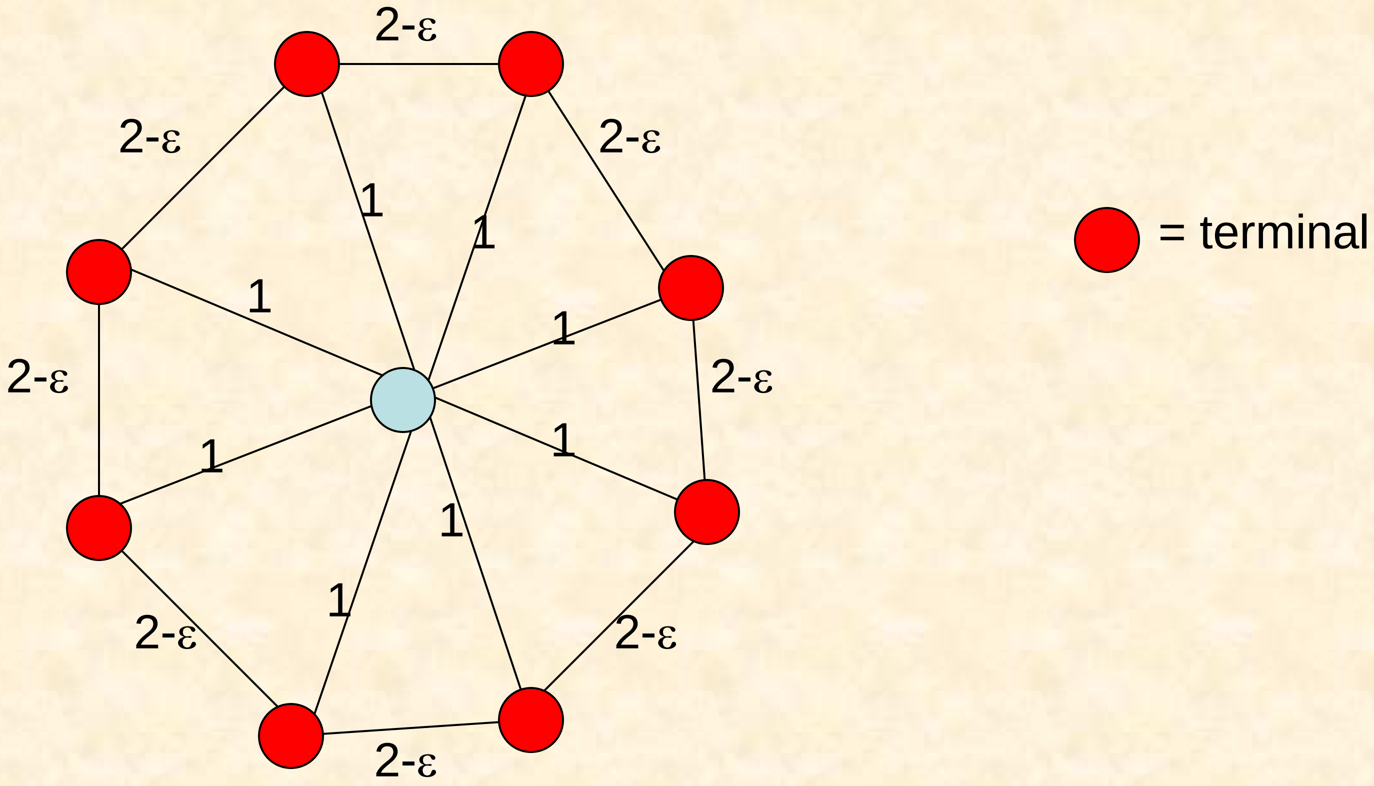
Shortest paths heuristic

- Start with a subtree T consisting of one terminal
- While T does not span all terminals
  - Select a terminal *x* not in T that is closest to a vertex in T.
  - Add to T the shortest path that connects *x* with T.

Improving the shortest paths heuristic

- Take the solution T from the heuristic
- Build the subgraph of G, induced by the vertices in T
- Compute a minimum spanning tree of this subgraph
- Repeat
  - Delete non-terminals of degree 1 from this spanning tree
  - Until there are no such non-terminals

Example where bound is met



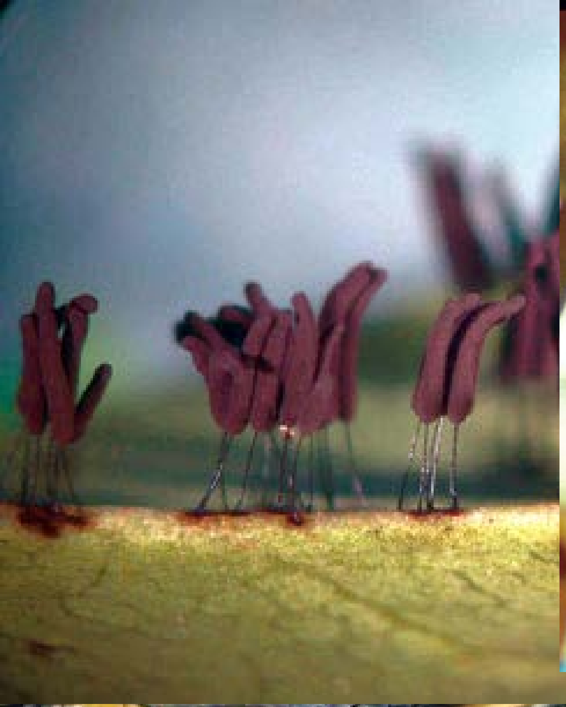
Variants

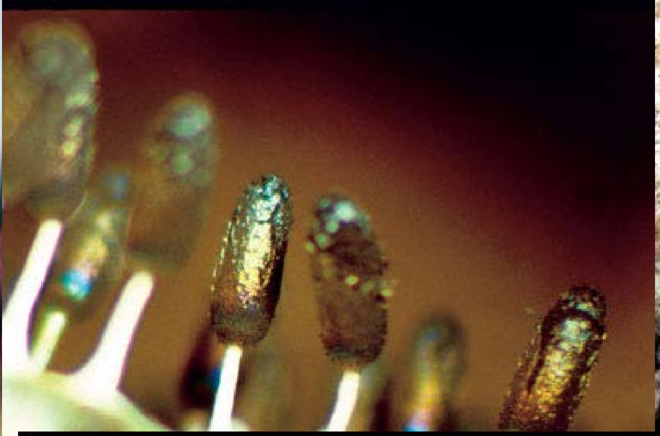
- Points in the plane
- Vertex weights
- Directed graphs

How can we find these solutions?

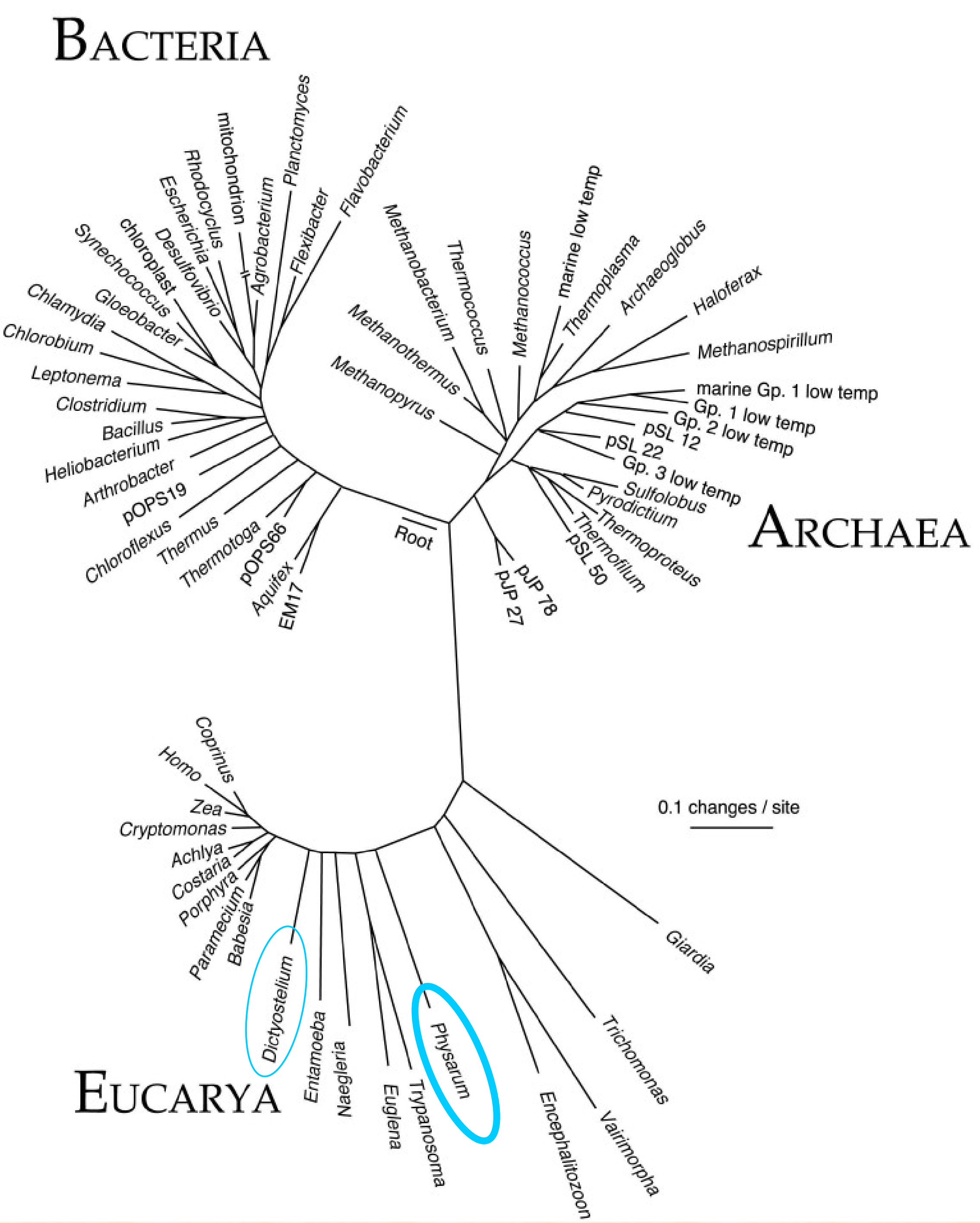
- We currently have no (fast) algorithm for finding the shortest Steiner graph between a given number of points.
- Nature, on the other hand, is quite good at it.

Slime molds



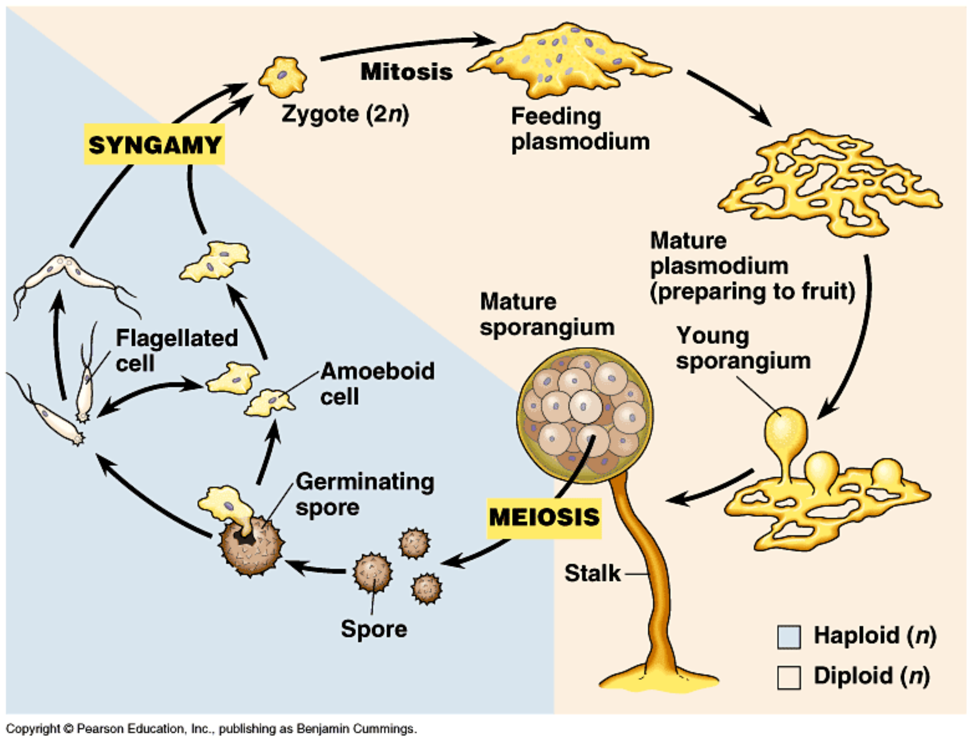


"Pretzel Slime Mold"
(*Hemitrichia serpula*)


BACTERIA
ARCHAEA
EUCARYA
mitochondrion
Planctomyces
Flavobacterium
Rhodocyclus
Escherichia
Agrobacterium
Flexibacter
Desulfovibrio
chloroplast
Synechococcus
Gloeobacter
Chlamydia
Chlorobium
Leptonema
Clostridium
Bacillus
Heliobacterium
Arthrobacter
pOPS19
Chloroflexus
Thermus
Thermotoga
pOPS66
Aquifex
EM17
Root
Methanobacterium
Thermococcus
Methanococcus
marine low temp
Thermoplasma
Archaeoglobus
Haloferax
Methanospirillum
Methanothermus
Methanopyrus
marine Gp. 1 low temp
Gp. 1 low temp
Gp. 2 low temp
pSL 12
pSL 22
Gp. 3 low temp
Sulfolobus
Pyrodictium
Thermoproteus
Thermofilum
pSL 50
pJP 78
pJP 27
0.1 changes / site
Coprinus
Homo
Zea
Cryptomonas
Achlya
Costaria
Porphyra
Paramecium
Babesia
Dictyostelium
Entamoeba
Naegleria
Euglena
Trypanosoma
Physarum
Encephalitozoon
Vairimorpha
Trichomonas
Giardia

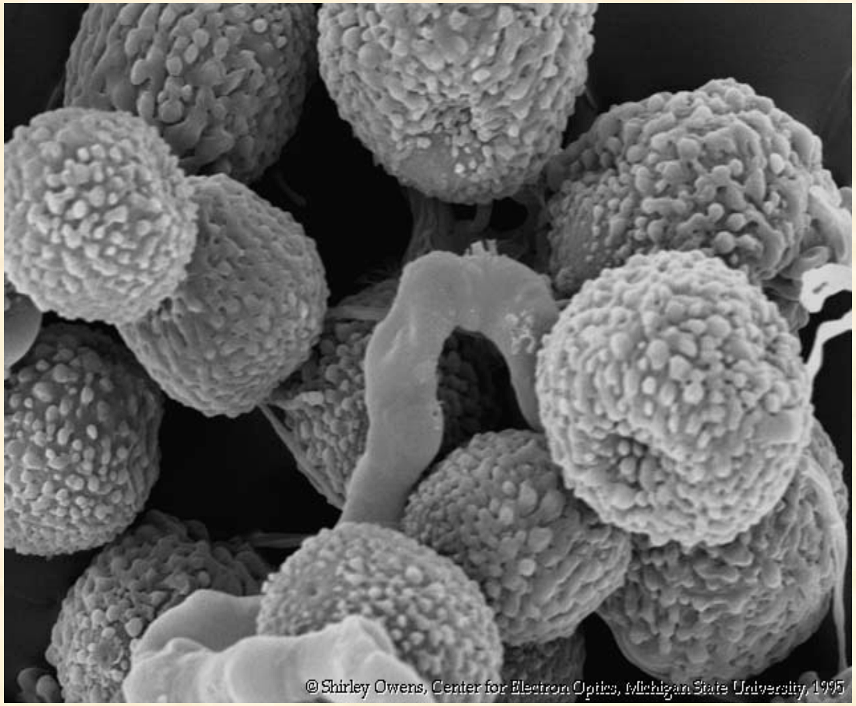
Eeeew! What is it?

- Kingdom Protista
  - True slime molds: Phylum Myxomycota
  - Cellular slime molds: Phylum Acrasiomycota
- True slime molds: nucleus replicates without dividing to form multinucleated feeding mass



Stages in Life Cycle

- **Spores (2n→n)**
  - 4-20 µm, pigmented ornamented; meiosis in spore = 4 nuclei; 3 degenerate



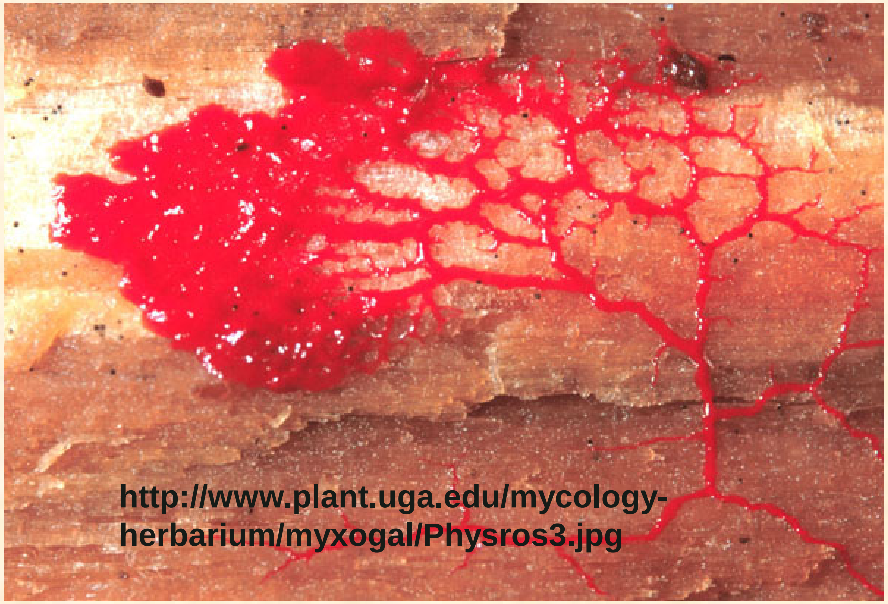
Stages in Life Cycle

- **Myxamoebae (n)**
  - feed, divide, convert to swarm cells, function as gametes; form microcysts under adverse conditions

www.uoguelph.ca/~gbarron/MISCE2002/myxamo2.jpg

Stages in Life Cycle

- **Zygote (2n)**
  - Formed by fusion of myxamoebae or swarm cells; enlarges through synchronous nuclear division
- **Plasmodium (2n)**
  - Multinucleate, wall-less protoplasm



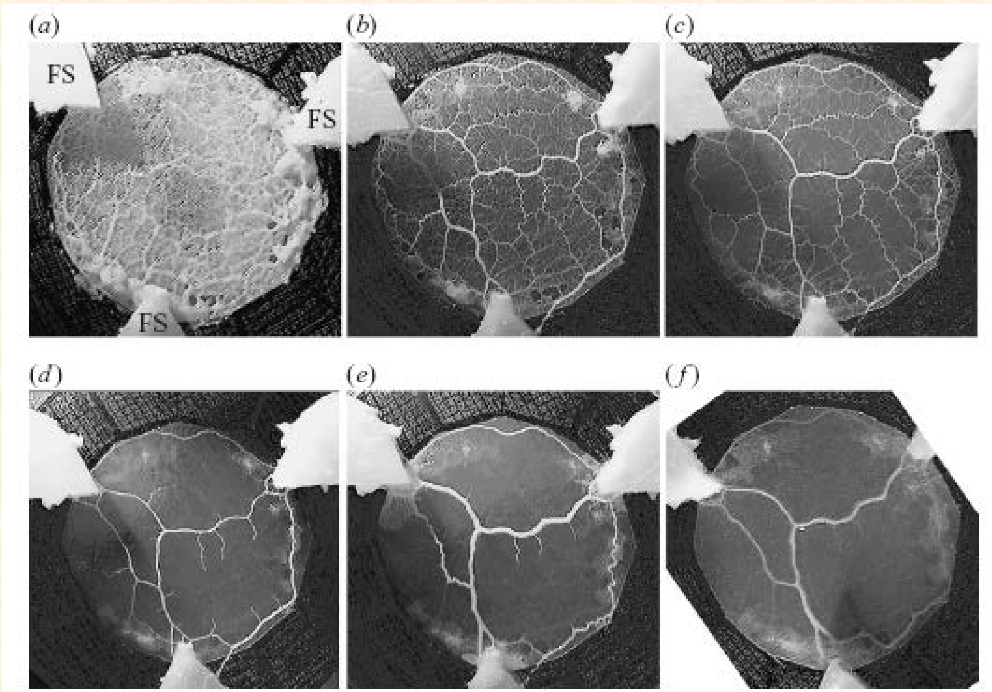
Why study them?

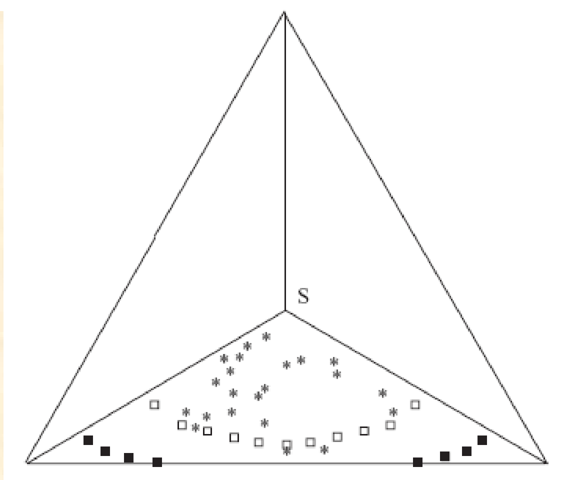
- Single, giant, multinucleated cell
  - Up to 20 meters in diameter!
- Biological information processing
  - Cell integrates sensory information and develops response
  - Solve maze
  - Minimal risk path
  - Robot control
- Phototactic and chemotactic
- Easily motivated by oats ☺

Information Processing

- “Intelligence” without a brain: Distributed computing
- Constraints:
  - Absorb nutrients
  - Maintain intracellular communication (remain connected)
  - Limit body mass

Efficient Pathfinding?

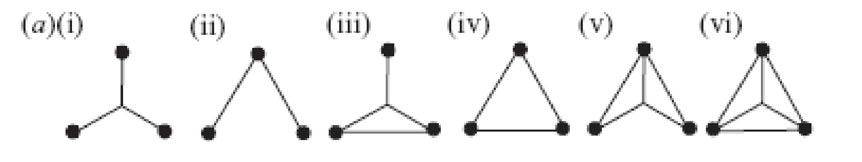




1. Grow *Physarum* on agar (forms plasmodium)
2. Add food sources (oats) at specific points
3. Wait & take pictures

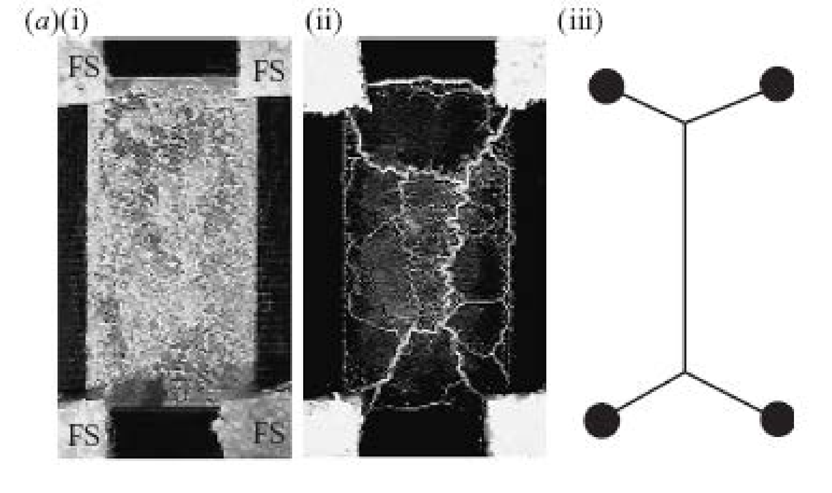
SMT and CYC

- SMT = Steiner's minimum tree:

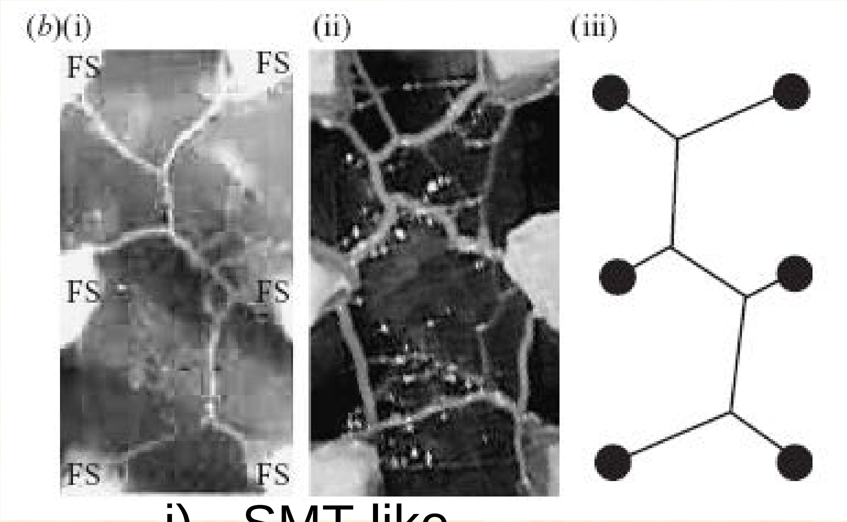


graph with least sum of edge lengths (NP-complete problem)

- CYC = plasmodium forms cyclical network
- Minimum tube length vs robustness

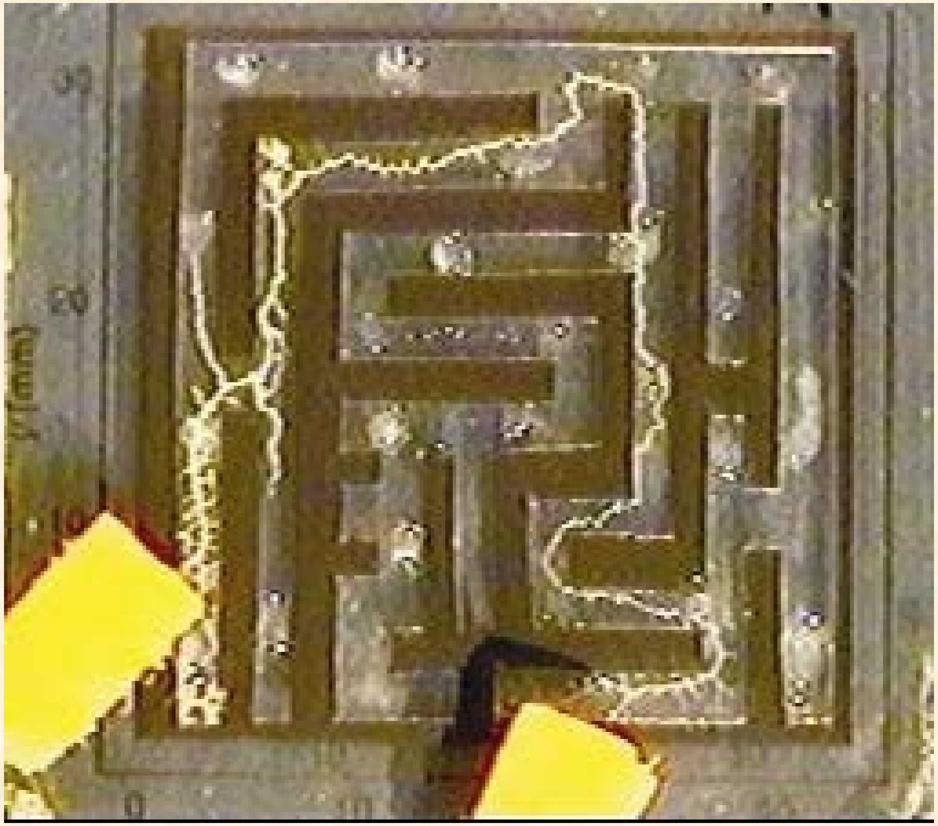


SMT-like



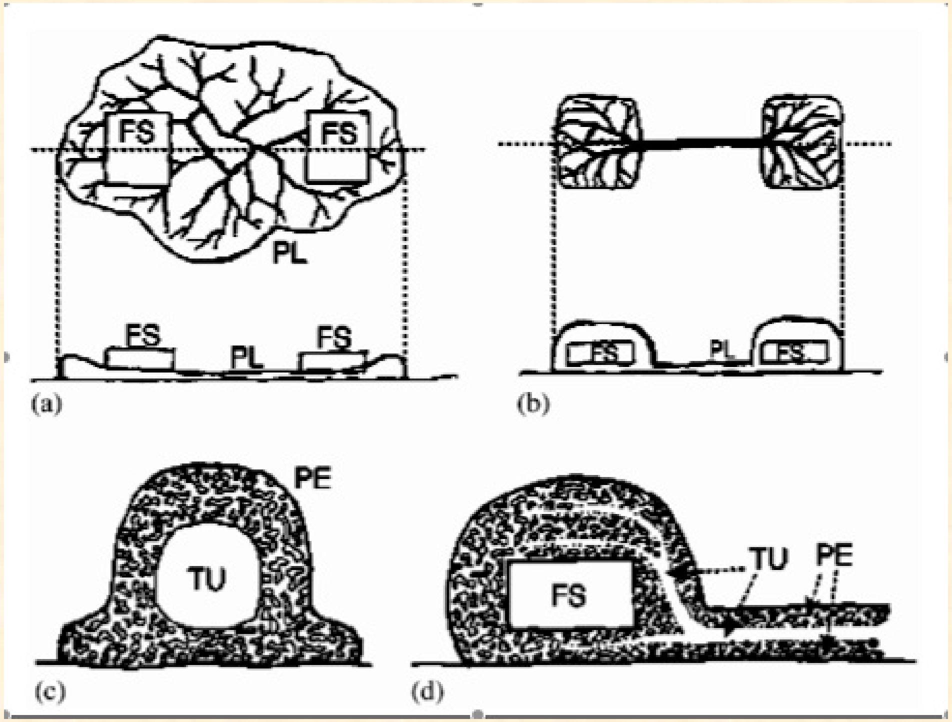
i) SMT-like
ii) combination

Maze Solving



Physical principles

- Mathematical model: feedback between thickness of tube and flux through it
  - More flux leads to wider tube
- Cytoplasmic streaming driven by rhythmic contractions of organism produces sheer stress to organize tubes



Mathematical model

- Cytosol is "shuttled" back and forth through the tubes--most of the slime mold's mass is at the food sources

Radius of tube

Pressure difference between ends of tube

Flux

$$Q_{ij} = \frac{\pi a_{ij}^4}{8\kappa} \frac{p_i - p_j}{L_{ij}},$$

Viscosity of sol

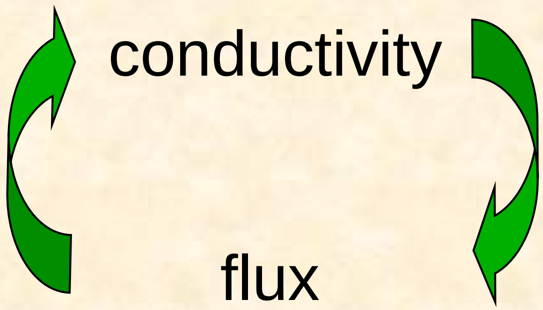
Length of tube

- Network of tubes "evolves" - conductivity D changes depending on flux through tube

$$\frac{\mathrm{d}}{\mathrm{d}t} D_{ij} = f(|Q_{ij}|) - D_{ij},$$

Evolution of network

- Positive feedback:



- Leads to:
  - Dead end cutting
  - Selection of solution path from other possibilities