Recursion, Big-O, Tractability, & Search Algorithms

Brought to you by your TA's!

Recursion

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1. Make the problem smaller
2. Get the smaller problem's solution
    a. Pretend that recursion automatically solves the problem correctly!
3. Combine leftover solution with smaller problem's solution

Try it: Finding the sum of a list of ints

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1. Smaller problem: lst[1:]

8	2	6	9	16	35	21

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Try it: Finding the sum of a list of ints

1. Smaller problem: lst[1:]

8	2	6	9	16	35	21

2. 2 + 6 + 9 + 16 + 35 + 21 = 89
getSum(lst[1:]) # Smaller Result

Recursion

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   a. Pretend that recursion automatically solves the problem correctly!
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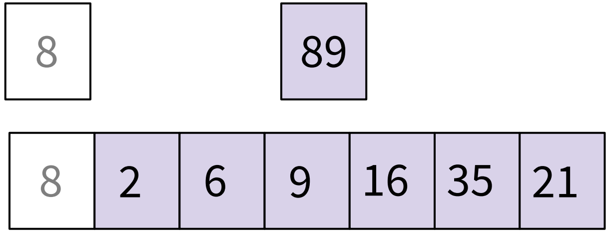
Try it: Finding the sum of a list of ints

1. Smaller problem: lst[1:]

2. 2 + 6 + 9 + 16 + 35 + 21 = 89
   getSum(lst[1:])

3. 8 + 89 = 97 **lst[0] + lst[1:]**



Recursion: Get Sum of List

- Base Case(s)
  - Length of list is 0 or 1
- Recursive Case
  - smallerProblem = lst[1:]
  - smallerResult = getSum(smallerProblem)
  - add it to the rest of the input
    - lst[0] + smallerResult

```
def getSum(lst):
    if len(lst) == 0:
        return 0
    elif len(lst) == 1:
        return lst[0]
    else:
        smallerProblem = lst[1:]
        smallerResult = getSum(lst[1:])
        return lst[0] + smallerResult
```

Multiple Recursive Calls: Fibonacci

- Fibonacci Numbers
  - 0, 1, 1, 2, 3, 5, 8, 13, 21, 34…
- Two Base Cases
  - 0th number: 0
  - 1st Number: 1
- Recursive Case
  - Adding the two numbers that came before to get the next number

```
def fib(n):
    if n == 0 or n == 1:
        return n
    else:
        return fib(n-1) + fib(n-2)
```

Recursion Reminders

- You have to call the function within its own body for it to be recursion
  - Make sure you're making the problem smaller when you're calling it
    - Otherwise it'll go on forever
- Your return types have to match!
  - Your base can't return an int while your recursive case returns a list

Big-O

Big-O

- Big-O: runtime it takes to execute a program based on its input
  - Simplest and tightest bound
    - O(n + 3) -> O(n)
    - O(n^2 + n) -> O(n^2)
- Common Big-O classes:
  - O(1)
  - O(logn)
  - O(n)
  - O(n^2)
  - O(2^n)

Determining Big-O

- For each line of a function, determine the runtime
  - Common O(1): print, return, >, <, initializing variables
  - Common O(n): in, .index()
  - When looking at loops:
    - Identify how many times the loop iterates
    - Identify the runtime of each line in the loop
    - Multiply the number of times the loop iterates by the longest runtime in the loop
- Runtime of the whole function is the runtime of the longest step

Loops Example

```
def f(number):
    for num in range(number):
        print(i)
def g(number):
    L = []
    for num in range(number):
        if num in L:
            L.append(num)
    return L
```

Loops Example

```
def f(number):
    for num in range(number):        O(n)
        print(i)
def g(number):
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    for num in range(number):
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O(n)

O(1)

O(n) * O(1) = O(n)
for the loop

Loops Example

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    for num in range(number):          O(n)      O(n) * O(1) = O(n)
                                                 for the loop
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    for num in range(number):       O(n)
        if num in L:
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Loops Example

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`    for num in range(number):`	O(n)	O(n) * O(1) = O(n) for the loop
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`    L = []`	O(1)	
`    for num in range(number):`	O(n)	
`        if num in L:`	O(n)	
`            L.append(num)`		
`    return L`		

Loops Example

def f(number):	
for num in range(number):	O(n)
print(i)	O(1)
def g(number):	
L = []	O(1)
for num in range(number):	O(n)
if num in L:	O(n)
L.append(num)	O(1)
return L	

O(n) * O(1) = O(n)
for the loop

Loops Example

```
def f(number):
    for num in range(number):        O(n)
        print(num)                   O(1)
def g(number):
    L = []                           O(1)
    for num in range(number):        O(n)
        if num not in L:             O(n)
            L.append(num)            O(1)
    return L                         O(1)
```

O(n) * O(1) = O(n)
for the loop

Loops Example

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def f(number):
    for num in range(number):        O(n)
        print(i)                     O(1)
def g(number):
    L = []                           O(1)
    for num in range(number):        O(n)
        if num in L:                 O(n)
            L.append(num)            O(1)
    return L                         O(1)
```

O(n) * O(1) = O(n) for the loop

O(n) * O(n) * O(1) = O(n^2) for the loop

Big-O Example

```
def addSomeNums(L):
    length = len(L)
    index = 1
    sum = 0
    while index < length:
        sum += L[index]
        index *= 2
    return sum
```

Big-O Example

```
def addSomeNums(L):
    length = len(L)              O(1)
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Big-O Example

```
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    length = len(L)          O(1)
    index = 1                O(1)
    sum = 0                  O(1)
    while index < length:    O(1) to check index < length, O(logn) iterations of the loop
        sum += L[index]
        index *= 2
    return sum
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*note that index is multiplied by 2 every time, so the loop can run a maximum of log n times. we simplify this in big-O terms to logn

Big-O Example

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        sum += L[index]      O(1)
        index *= 2           O(1)
    return sum               O(1)
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        sum += L[index]       O(1)
        index *= 2            O(1)
    return sum                O(1)
```

*note that index is multiplied by 2 every time, so the loop can run a maximum of log n times. we simplify this in big-O terms to logn

O(logn) overall

Big-O Example

```
def factorial(x):
    if x == 1:
        return 1
    else:
        return x * factorial(x - 1)
```

Big-O Example

```
def factorial(x):
    if x == 1:                         O(1)
        return 1
    else:
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```

Big-O Example

```
def factorial(x):
    if x == 1:                          O(1)
        return 1                        O(1)
    else:
        return x * factorial(x - 1)
```

Big-O Example

```
def factorial(n):
    if n == 1:                        O(1)
        return 1                      O(1)
    else:                             O(1)
        return n * factorial(n - 1)
```

Big-O Example

def factorial(n):		
if n == 1:	O(1)	
return 1	O(1)	
else:	O(1)	
return n * factorial(n - 1)	O(n)	after factorial is called for the first time, it will recursively be called n - 1 times, which is O(n)

Big-O Example

```
def factorial(n):
    if n == 1:                        O(1)
        return 1                      O(1)
    else:                             O(1)
        return n * factorial(n - 1)   O(n)
```

after factorial is called for the first time, it will recursively be called n - 1 times, which is O(n)

O(n) overall

Tractability

Tractability

- A problem is tractable if it has a reasonable efficient (polynomial) runtime. Otherwise, it is intractable.
- Polynomial runtimes:
  - O(1)
  - O(n)
  - O(logn)
  - O(nlogn)
  - O(n^2)
  - O(n^100)
- Non polynomial runtimes:
  - O(2^n)
  - O(n!)

P and NP

- P: the set of problems that can be solved in polynomial time
    - Examples
        - Linear search, binary search
        - Sorting a list
- NP: the set of problems that can be verified in polynomial time
    - Given the answer to the problem, we can verify in polynomial time that it is correct
    - Examples
        - Subset sum
        - Satisfying a circuit
        - All problems in P!

Search Algorithms

Linear Search

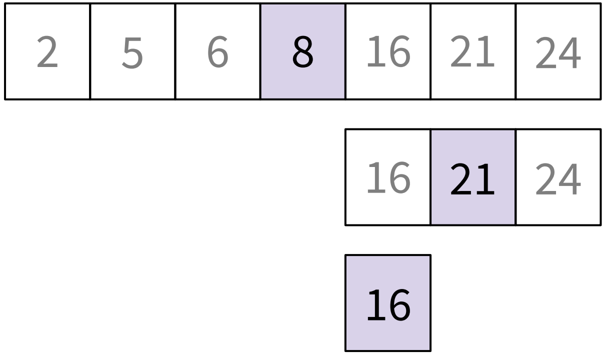
- Checks all values of input
- Best Case
  - Target is first element in list
- Worst Case
  - Target is last element of list or not in list
- What's the Big O?

```
def linearSearch(lst, target):
    for i in range(len(lst)):
        if lst[i] == target:
            return True
    return False


def recursiveLinearSearch(lst, target):
    if lst == [ ]:
        return False
    elif lst[0] == target:
        return True
    else:
        return recursiveLinearSearch(lst[1:],
    target)
```

Binary Search

- Input list has to be **sorted**
- We start by checking the **middle element**



- What's the Big O?

```
def binarySearch(lst, target):
    if lst == [ ]:
        return False
    else:
    midIndex = len(lst) // 2
    if lst[midIndex] == target:
        return True
    elif target < lst[midIndex]:
        return binarySearch(lst[:midIndex],
    target)
    else: # lst[midIndex] < target
        return binarySearch(lst[midIndex+1:],
    target)
```

Good luck on the final exam! :)