Lecture 21
Types in C

15-122: Principles of Imperative Computation (Spring 2019)
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Previous lectures have emphasized the things we *lost* by going to C:

- Many operations that would safely cause an error in C0, like dereferencing `NULL` or reading outside the bounds of an array, are *undefined* in C — we cannot predict or reason about what happens when we have undefined behaviors.
- It is not possible to capture or check the length of C arrays.
- In C, pointers and arrays are the same — and we declare them like pointers, writing `int *i`.
- The C0 types `string`, `char*` and `char[]` are all represented as pointers to `char` in C.
- C is not garbage collected, so we have to explicitly say when we expect memory to be freed, which can easily lead to memory leaks.

In this lecture, we will endeavor to look on the bright side and explore some of the new things that C gives us. But remember: with great power comes great responsibility. Today we will look at the different ways that C represents numbers and the general, though mostly implementation-defined, properties of these numbers that we frequently count on.

1 Numbers in C

In addition to the undefined behavior resulting from bad memory access (dereferencing a `NULL` pointer or reading outside of an array), there are other undefined behaviors in C. In particular:

- Division by zero is undefined. (In C0, this always causes an exception.)
- Shifting left or right by negative numbers or by too-large a number is undefined. (In C0, this always causes an exception.)
- Arithmetic overflow for *signed* types like **int** is undefined. (In C0, this is defined as modular arithmetic.)

This has some strange effects. If `x` and `y` are signed integers, then the expressions `x < x+1` and `x/y == x/y` are either `true` or undefined (due to signed arithmetic or overflow, respectively). So the compiler is allowed to pretend that these expressions are just `true` all the time. The compiler is also allowed to behave the same way C0 does, returning `false` in the first case when `x` is the maximum integer and raising an exception in the second case when `y` is 0. The compiler is also free to check for signed integer overflow and division by zero and start playing Rick Astley's "Never Gonna Give You Up" if either occurs, though this last option is unlikely in practice. Undefined behavior is unpredictable — it can and does change dramatically between different computers, different compilers, and even different versions of the same compiler.

The fact that signed integer overflow is undefined is particularly annoying. A check like `(x + 1 > x)`, which was a perfectly acceptable way to check that `x` was not `int_max()` in C0, is now a check that the compiler is allowed to optimize to just `true`, because the result of this expression, in C, is either `true` or undefined.

There are two ways of coping with signed integer overflow being undefined. One option is to use *unsigned* types, which are required to obey the laws of modular arithmetic: **unsigned int** instead of **int**. As an example, consider a simple function to compute Fibonacci numbers. There are even faster ways of doing this, but what we do here is to allocate an array on the stack, fill it with successive Fibonacci numbers, and finally return the desired value at the end.

```
unsigned int fib(unsigned int n) {
  unsigned int A[n+2];         /* stack-allocated array A */
  A[0] = 0;
  A[1] = 1;
  for (unsigned int i = 0; i <= n-2; i++)
    A[i+2] = A[i] + A[i+1];
  return A[n];  /* deallocates A just before actual return */
}
```

There's another solution, particular to the compiler, `gcc`, that we usually use to compile C programs. This compiler (as well as other modern C compilers like `clang`), has a flag `-fwrapv`. When we compile with `-fwrapv`, then the compiler promises it will treat overflow from addition and multiplication as signed two's complement modular arithmetic, exactly like C0 does.

2 Implementation-defined Behavior

In addition to **`int`**, which is a signed type, there are the signed types **`short`** and **`long`**, and **`unsigned`** versions of each of these types — **`short`** is smaller than **`int`** and **`long`** is bigger. The numeric type **`char`** is smaller than **`short`** and always takes up one byte. The maximum and minimum values of these numeric types can be found in the standard header file `<limits.h>`.

C, annoyingly, does not define whether **`char`** is signed or unsigned. A **`signed char`** is definitely signed, a **`unsigned char`** is unsigned. The type **`char`** can be either signed or unsigned — this is *implementation defined*.

It is often very difficult to say useful and precise things about the C programming language, because many of the features of C that we have to rely on in practice are not part of the C standard. Instead, they are things that the C standard leaves up to the implementation — implementation defined behaviors. Implementation-defined behaviors make it quite difficult to write code on one computer that will compile and run on another computer, because the other compiler may make completely different choices about implementation-defined behaviors.

The first example we have seen is that, while a **`char`** is always exactly one byte, we don't know whether it is signed or unsigned — whether it can represent integer values in the range $[-128, 128)$ or integer values in the range $[0, 256)$. And it is even worse, because a byte can be more than 8 bits! If you really want to mean "8 bits," you should say *octet*.

In this class we are going to rely on a number of implementation-defined behaviors. For example, you can always assume that bytes are 8 bits on the computers we're using for this class in this decade. When it is important to *not* rely on integer sizes being implementation-defined, it is possible to use the types defined in `<stdint.h>`, which defines signed and unsigned types of specific sizes. In the systems that you are going to use for programming, you can reasonably expect a common set of implementation-defined behaviors: **`char`** will be an 8-bit integer (maybe signed, maybe unsigned) and so on.

This chart describes how the `<stdint.h>` types match up to the standard C types in most modern C compilers:

Signed		Unsigned	
C	`stdint.h`	`stdint.h`	C
`signed char`	`int8_t`	`uint8_t`	**`unsigned char`**
`short`	`int16_t`	`uint16_t`	**`unsigned short`**
`int`	`int32_t`	`uint32_t`	**`unsigned int`**
`long`	`int64_t`	`uint64_t`	**`unsigned long`**

However, please remember that we cannot count on this correspondence behavior in *all* C compilers!

There is another crucial numerical type: `size_t`, is the type used represent memory sizes and array indices. The **`sizeof`**`(ty)` operation in C actually returns just the size of a type in bytes, so `malloc` and `xmalloc` actually take one argument of type `size_t` and `calloc` and `xcalloc` take two arguments of type `size_t`. As we approach the third decade of the 21st century, we're increasingly using 64-bit systems and not dealing with 32-bit systems anymore. On 32-bit systems, `size_t` is usually 4-byte, 32-bit unsigned integer. We now usually expect `size_t` to be a 64-bit, 8-byte unsigned integer.

3 Casting Between Numeric Types

Now that we've introduced a bunch of different integer types, we need to see how to work with multiple integer types in the same program.

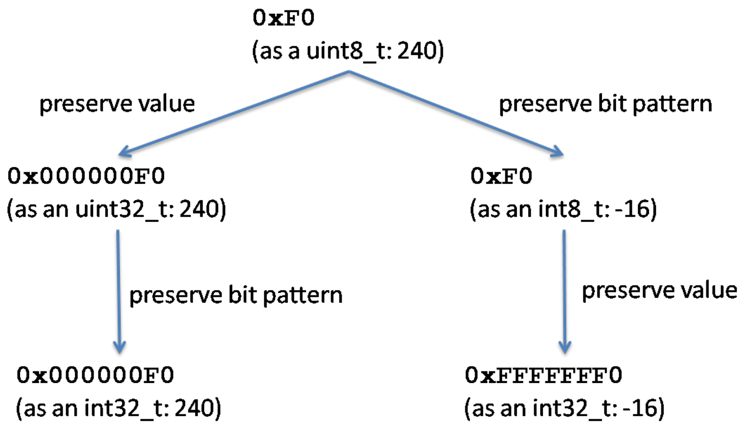
Imagine we have the hexadecimal value `0xF0` — represented as a sequence of bits as `11110000` — stored in an unsigned **`char`**, and we want to turn that value into an **`int`**. (This is a problem you will actually encounter later in this semester.) We can *cast* this character value to an integer value by writing `(`**`int`**`)e`.

```
unsigned char c = 0xF0;
int i = (int)c;
```

However, what will the value of this integer be? You can run this code and find out on your own, but the important thing to realize is that it's not clear, because there are two different stories we can tell.

In the first story, we start by transforming the unsigned **`char`** into an unsigned **`int`**. When we cast from a small unsigned quantity to a large unsigned quantity, we can be sure that the *value* will be preserved. Because the bits `11110000` are understood as the unsigned integer 240, the unsigned **`int`** will also be 240, written in hexadecimal as `0x000000F0`. Then, when we cast from an unsigned **`int`** to a signed **`int`**, we can expect the bits to remain the same (though this is really implementation defined), and because the interpretation of signed integers is two's-complement (also implementation defined) the final value will be 240.

In the second story, we first transform the unsigned **`char`** into a signed **`char`**. Again, the implementation-defined behavior we expect is that we will interpret the result as an 8-bit signed two's-complement quantity, meaning that `0xF0` is understood as -16. Then, when we cast from the small signed quantity (**`signed char`**) to a large signed quantity (**`int`**), we know the quantity -16 will be preserved, meaning that we will end up with a signed integer written in hexadecimal as `0xFFFFFFF0`. In order to preserve the value as we go from a small to a large signed quantity, all we have to do is use *sign extension* — copy the high-order bit into all the new spaces.



The order in which we do these two steps matters! Therefore, if we want to be clear about what result we want, we should cast in smaller steps to be explicit about how we want our casts to work:

```
unsigned char c = 0xF0;
int i1 = (int)(unsigned int) c;
int i2 = (int)(signed char) c;
assert(i1 == 240);
assert(i2 == -16);
```

The C standard does define which of these two things will happen when you cast directly from **`unsigned char`** to **`int`**. However, for the purposes of this class, where we're trying to teach you enough C to write clear and correct programs, it's worth obeying the following rules:

- Never cast between signed and unsigned types of *different sizes*. Only cast between signed and unsigned types with the same size (implementation defined to preserve bits) and between small and large types that are either both signed or both unsigned.
- When you cast from a large signed (or unsigned) type to a small signed (or unsigned) type, make sure that the type you're casting to can represent the number. (So, for instance, you can cast the **`int`** 17 to an signed **`char`**, but don't cast the **`int`** 1000 to a signed **`char`**, because a signed **`char`** can only represent numbers between -128 and 127, inclusive.)
- When you add, subtract, multiply, divide, compare, or do bitwise operations involving multiple variables, it's best to make sure that all the numbers you're working with have the same size and same signedness. One important "gotcha" here: if you just write the number `4`, it's treated as an **`int`** by default, so writing

  ```
  int64_t i = 1 << 40;
  ```

  will actually be undefined behavior, because `1` is (implementation-defined to be) a 32-bit quantity that can only be shifted by numbers between 0 and 31, inclusive. The fix, in this situation, is to write:

  ```
  int64_t i = 1;
  i = i << 40;
  ```

4 Other Types In C

C introduces a number of other types as well that we didn't have in C0. In particular, many C programs use **`enum`** types, **`union`** types, and the floating point types, **`float`** and **`double`**, which are used to represent fractional numbers like `0.25`. You'll learn much more about these other C types in later courses, like 15-213, but here are the basics.

4.1 Floating Point

The C type **`float`** allows writing numbers such as 0.1 and 3.14159265 as well as 2.2035×10^{-27} (entered as `2.2035E-27`) that have a fractional component. It also allows writing very large numbers such as 10^{20} that are not representable as **int**'s. **float** provides *floating point numbers* as a way to work with numbers other than integers, in particular (some) rational numbers. The size of a **float** is implementation-defined, although it is typically 32 bits in modern computers. Those 32 bits is all that is available to represent floating point numbers — the same as **int**'s — but the range these numbers are drawn from is much wider: 10^{20} and -10^{20} can both be entered as **float**'s although much larger than INT_MAX and much smaller than INT_MIN respectively, and so is 10^{-20}. What gives? *Precision*.

The following simple program divides 10^{20} by 10^{10} and multiplies the outcome by 10^{10}. We expect the final result to be 10^{20}.

```
void main() {
  float x = 10E20;
  float y = 10E10;
  printf("%f\n", (x/y)*y);
}
```

Instead, it outputs 9999999949672133165056.000000 — almost 10^{20} but not quite.

C offers another type for floating point numbers, **double**, which stands for *double precision* (its size is again implementation-defined and typically 64 bits in current hardware). The above example works as expected if we replace **float** with **double**, but a similar example which uses bigger numbers will suffer from the same problem: double precision is not infinite precision.

Precision loss during calculations, as just witnessed, makes it all but impossible to reason about programs that use floating point numbers. This is why **float** was left out of C0.

Here's another example:

```
  for (float res = 0.0; res != 5.0; res += 0.1) {
    printf("res = %f\n", res);
  }
  printf("Done!\n");
```

We would expect the loop to run some 50 times, then exit and print Done!. Instead, it keeps running, printing ever larger values of res:

```
# a.out
res = 0.000000
res = 0.100000
res = 0.200000
                ... [elided]
```

```
res = 2.600000
res = 2.700000
res = 2.799999
res = 2.899999
                ... [elided]
res = 4.999998
res = 5.099998
                ... [elided]
res = 8.799997
res = 8.899998
                ... [elided]
```

After a few iterations, the value of `res` starts deviating from what we expect — a single decimal digit followed by zeros — and eventually passes 5.0 because it is never equal to that value. How is this possible with simple numbers like `0.1` and `5.0`? These numbers, as we entered them in the program, are in decimal. The compiler automatically converts them into binary using a very similar procedure as what we saw for integers. Take `0.1`. We obtain the fractional part (or *mantissa*) by repeatedly multiplying this number by 2 and harvesting the digit to the left of the decimal point until we get 0.0:

$$\begin{array}{rcrclll}
0.1 & \times & 2 & = & \mathbf{0}.2 & \text{yields} & 0 \\
0.2 & \times & 2 & = & \mathbf{0}.4 & \text{yields} & 0 \\
0.4 & \times & 2 & = & \mathbf{0}.8 & \text{yields} & 0 \\
0.8 & \times & 2 & = & \mathbf{1}.6 & \text{yields} & 1 \quad \text{subtract 1} \\
0.6 & \times & 2 & = & \mathbf{1}.2 & \text{yields} & 1 \quad \text{subtract 1} \\
0.2 & \times & 2 & = & \mathbf{0}.4 & \text{yields} & 0 \\
 & & & \ldots & & &
\end{array}$$

Notice that we have seen 0.2 before, and therefore the process repeats . . . infinitely. The number 0.0, at which point we would stop, never emerges. What is happening is that, while 0.1 has a finite mantissa in decimal (just one digit after the decimal point), it has an infinite mantissa in binary: 0.1_{10} is a periodic number in binary — $0.0\overline{0011}_2$. This means that a precise representation as a binary mantissa cannot be achieved with any fixed number of bits. Thus the value of `x` above is an approximation of 0.1 rather than exactly this number. It is printed out as `0.100000` thanks to rounding but, as we keep on adding it to itself with “`res += x`” errors accumulate since we are working with an approximation of 0.1, and these errors eventually manifest when printing `res`.

4.2 Union and enum types

As a way to introduce some additional features of C, consider a type of trees that carry their data in their leaves rather than in the inner nodes (we call them *leafy trees* and they are at the basis of data structures used in the implementation of database management systems and in other areas of computer science). A leafy tree can be a leaf carrying a value, it can be an inner node with no value but left and right children, it can also be empty which contains neither a value nor children.

Based on the fragment of C we know so far, we would use the following type to define the nodes of a leafy tree with integer values:

```
typedef struct ltree leafytree;
struct ltree {
  int nodetype;
  int data;
  leafytree *left;
  leafytree *right;
};
```

We use the field `nodetype` to distinguish the type of the node (a leaf, an inner node, or empty[1]). This representation wastes memory: inner nodes do not make use of the `data` field, while `left` and `right` are meaningless for a leaf, and furthermore all 3 for go unused for the empty tree. As we will see shortly, C provides a mechanism to mitigate this problem.

Before examining it, let's go back to `nodetype`: we need to pick three values to denote the three types of nodes, but we use the space for an entire **`int`** for them — more waste. C provides *enum types* as a way to shield the programmer both from picking constants whose values are to all effects irrelevant as long as they are distinct and in deciding exactly how much memory to allocate. In our example, this is done through the declaration

```
enum nodetype { INNER, LEAF, EMPTY };
```

From now on, we can use the mnemonic constants `INNER`, `LEAF` and `EMPTY` as type indicators for our various nodes.

Union types provide a way to view an area of memory as having more than one type. Here, the memory associated with a node needs to be viewed either as an **`int`** (for leaves) or as a pair of pointers (for inner nodes). We can do so using the following declarations:

[1] An alternative is to represent the empty leafy tree as `NULL`. We refrain from doing so to make this example more interesting.

```
typedef struct ltree leafytree;
struct innernode {        // type of an inner node
  leafytree *left;
  leafytree *right;
};
union nodecontent {       // contents of a non-empty node:
  int data;               // EITHER an int
  struct innernode node;  //     OR and inner node
};
struct ltree {
  enum nodetype type;
  union nodecontent content;
};
```

Here, **struct** innernode packages the two pointers needed for inner nodes. The type **union** nodecontent can contain either the integer data or the value node of type **struct** innernode — but not both. The compiler will decide how to organize the memory for this union type (on modern hardware, probably 16 byte viewed either as two 8-byte pointers or a 4-byte **int** and 12 unused bytes). Lastly, the definition of **struct** ltree contains the field type of enum type **enum** nodetype defined earlier, and the field content of union type **union** nodecontent.

We define an actual leafy tree as in the following example:

```
  leafytree *T = malloc(sizeof(leafytree));
  T->type = INNER;
  T->content.node.left = malloc(sizeof(leafytree));
  T->content.node.left->type = EMPTY;
  T->content.node.right = malloc(sizeof(leafytree));
  T->content.node.right->type = LEAF;
  T->content.node.right->content.data = 42;
```

Note that the type fields are assigned the symbolic constants in our enum type. Notice also that we access the alternatives of a union type using the dot notation, as in T->content.node.left. It is left to the discipline of the programmer to use these fields consistently, for instance not to access the data component of an INNER node.

Before we are done with this example, let's introduce a useful construct of C, especially in the presence of enum type definitions: the **switch** statement. The following recursive function adds all the values stored in (the leaves of) a leafy tree:

```
int add_tree(leafytree *T) {
  int n = 0;
  switch (T->type) {
  case INNER:
    n += add_tree(T->content.node.left);
    n += add_tree(T->content.node.right);
    break;

  case LEAF:
    n = T->content.data;
    break;

  default:
    n = 0;
  }

  return n;
}
```

The construct **switch** discriminates on the value of T->type, jumping to the appropriate **case** block. If a **case** block is not given for a value, the execution proceeds to the **default** block which is always the last (although a programmer may decide to omit it). Each block except the last typically needs to end with a **break** statement, otherwise the execution will proceed with the next block rather than exiting the **switch** statement.