AVL Trees

Rotations

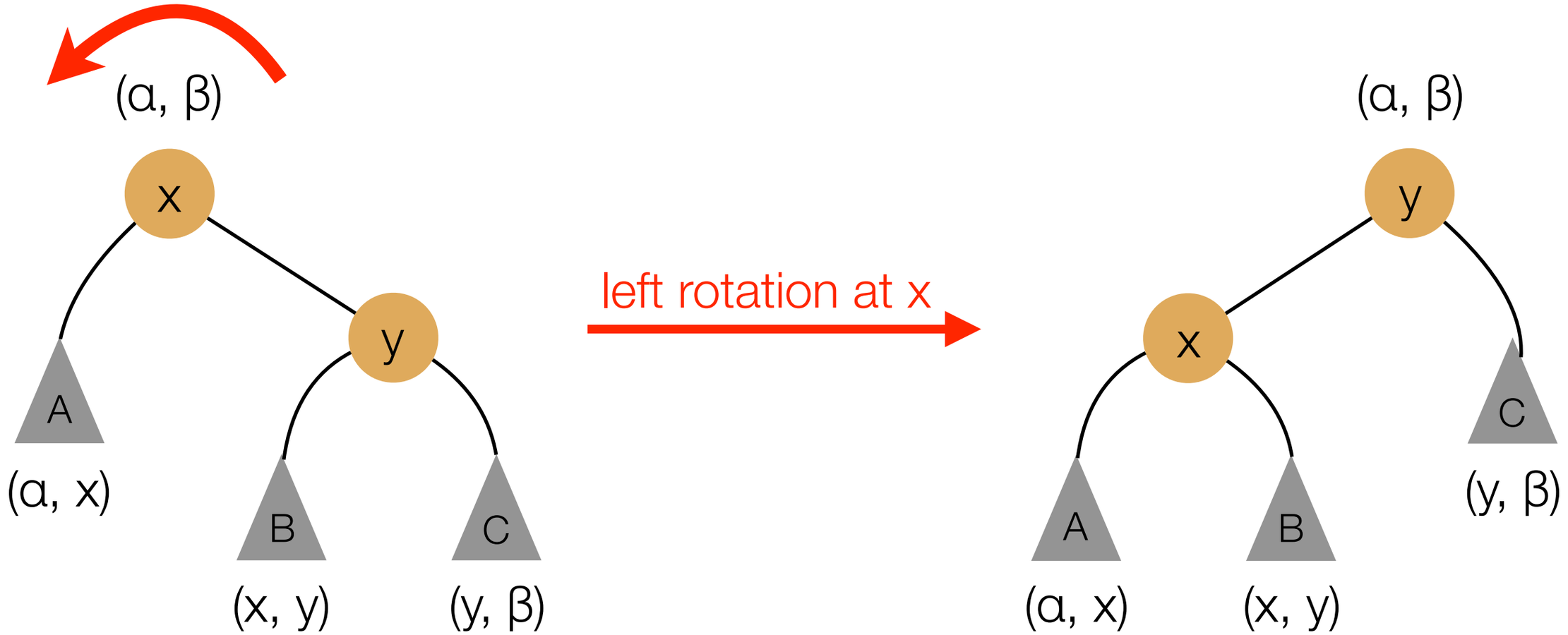
- Insertion into an AVL tree may compromise the height invariant.
- To restore the height invariant, the following basic rotations are performed:
  - **left rotation**
  - **right rotation**

→ Rotations are performed at **lowest** node violating height invariant.

→ Rotations **restore** the height of the tree to its previous height.

→ At **most two** rotations are needed to restore height invariant.

Left rotation

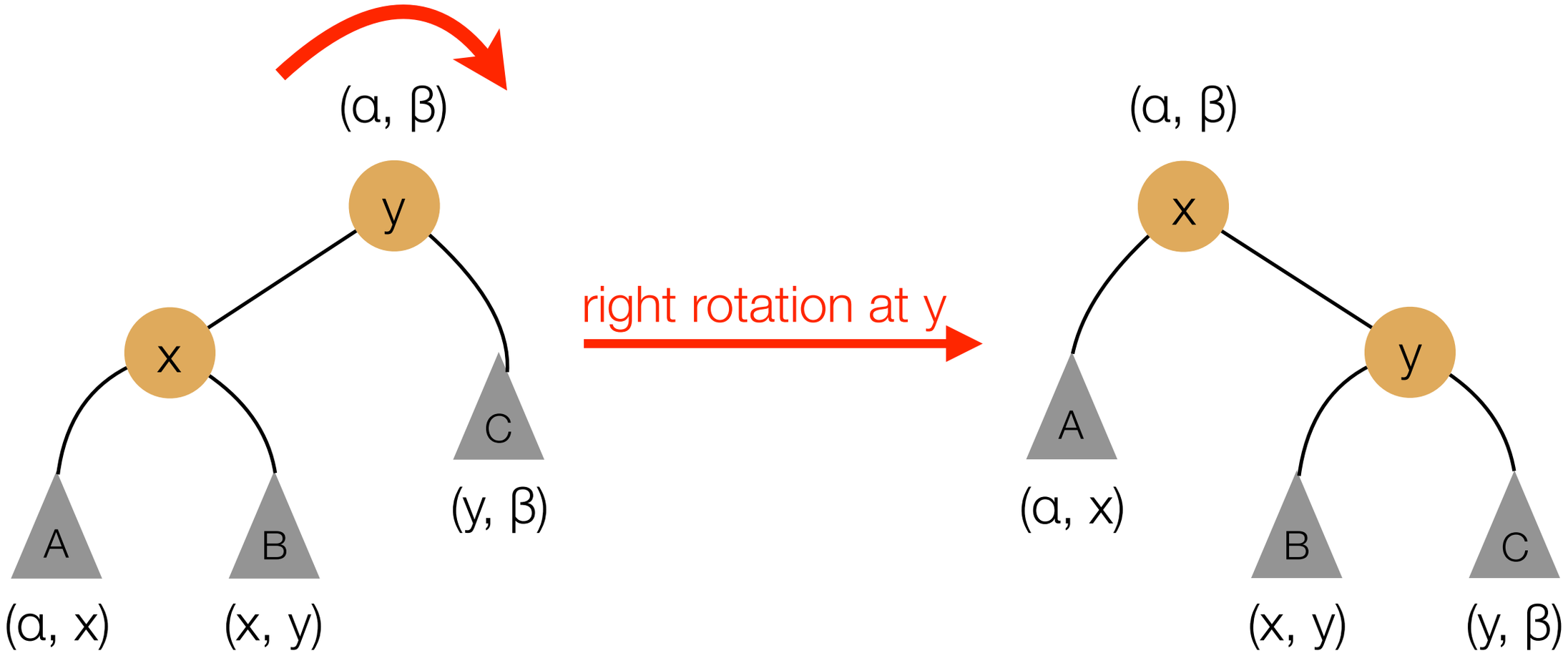


A < x < B < y < C

A < x < B < y < C

Rotation maintains order invariant.

Right rotation



A < x < B < y < C

A < x < B < y < C

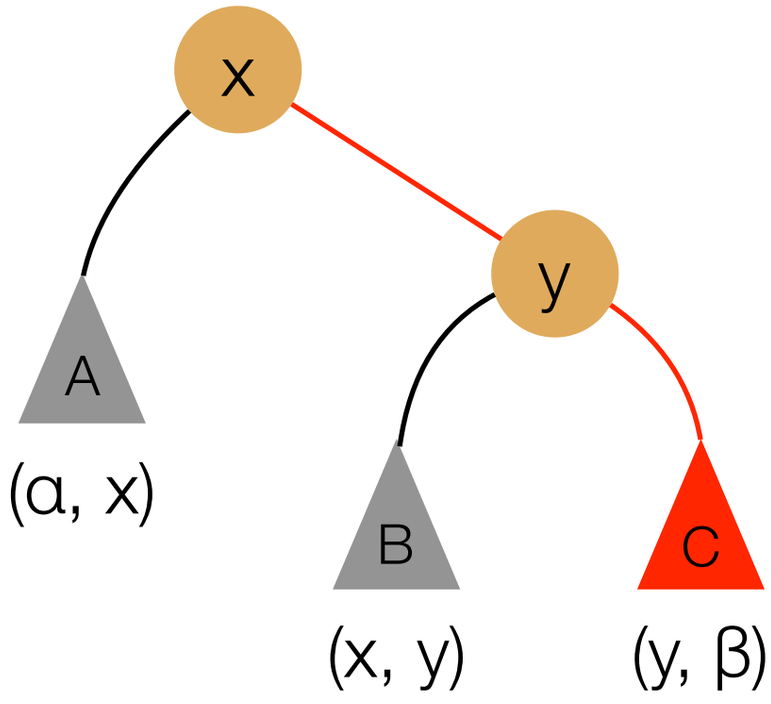
Rotation maintains order invariant.

Restoring height invariant

- While rotations maintain the ordering invariant, one rotation alone may not restore the height invariant.
- It turns out that a sequence of **one to two rotations** is sufficient to restore the height invariant.
- Relying on the fact that we rotate at the **lowest** node that violates the height invariant, there are **4 violation cases** and **corresponding rotation patterns** to restore the height invariant.

Cases 1 and 2: insert into right subtree

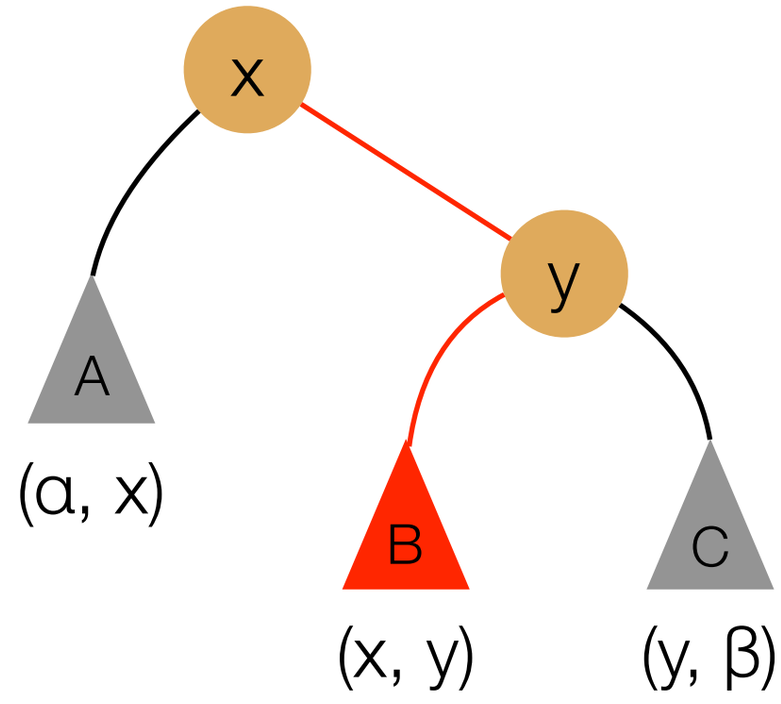
Case 1: (α, β)



A < x < B < y < C

right subtree, right child thereof

Case 2: (α, β)

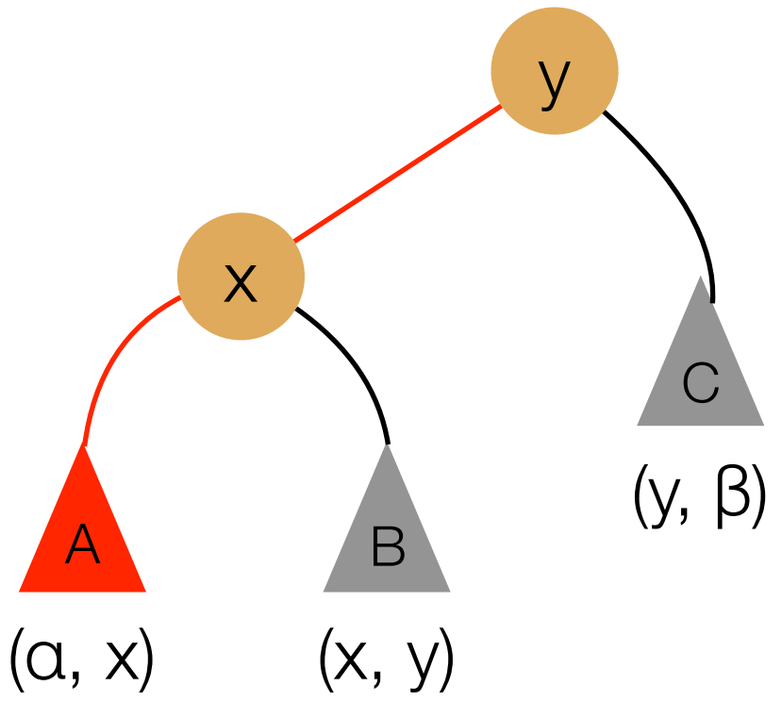


A < x < B < y < C

right subtree, left child thereof

Cases 3 and 4: insert into left subtree

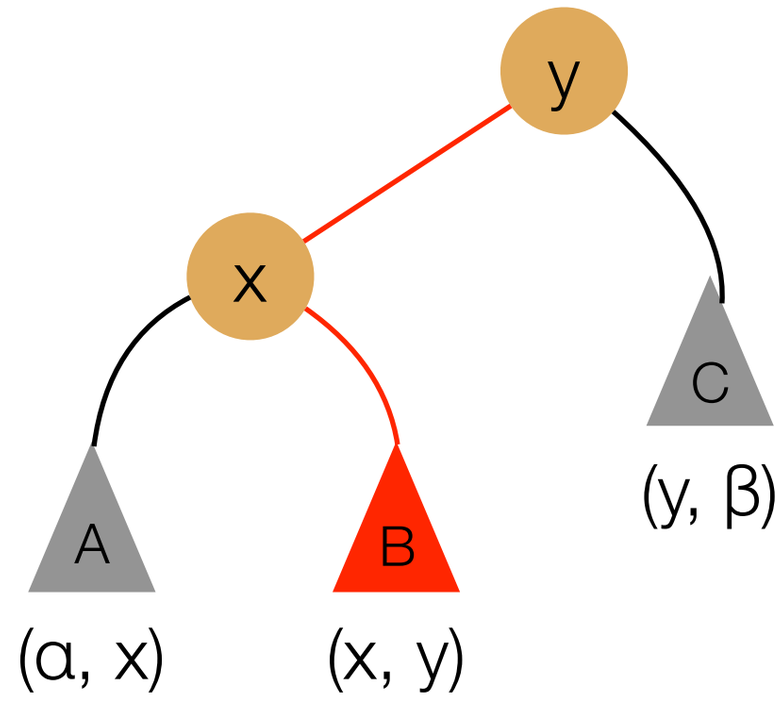
Case 3:



A < x < B < y < C

left subtree, left child thereof

Case 4:



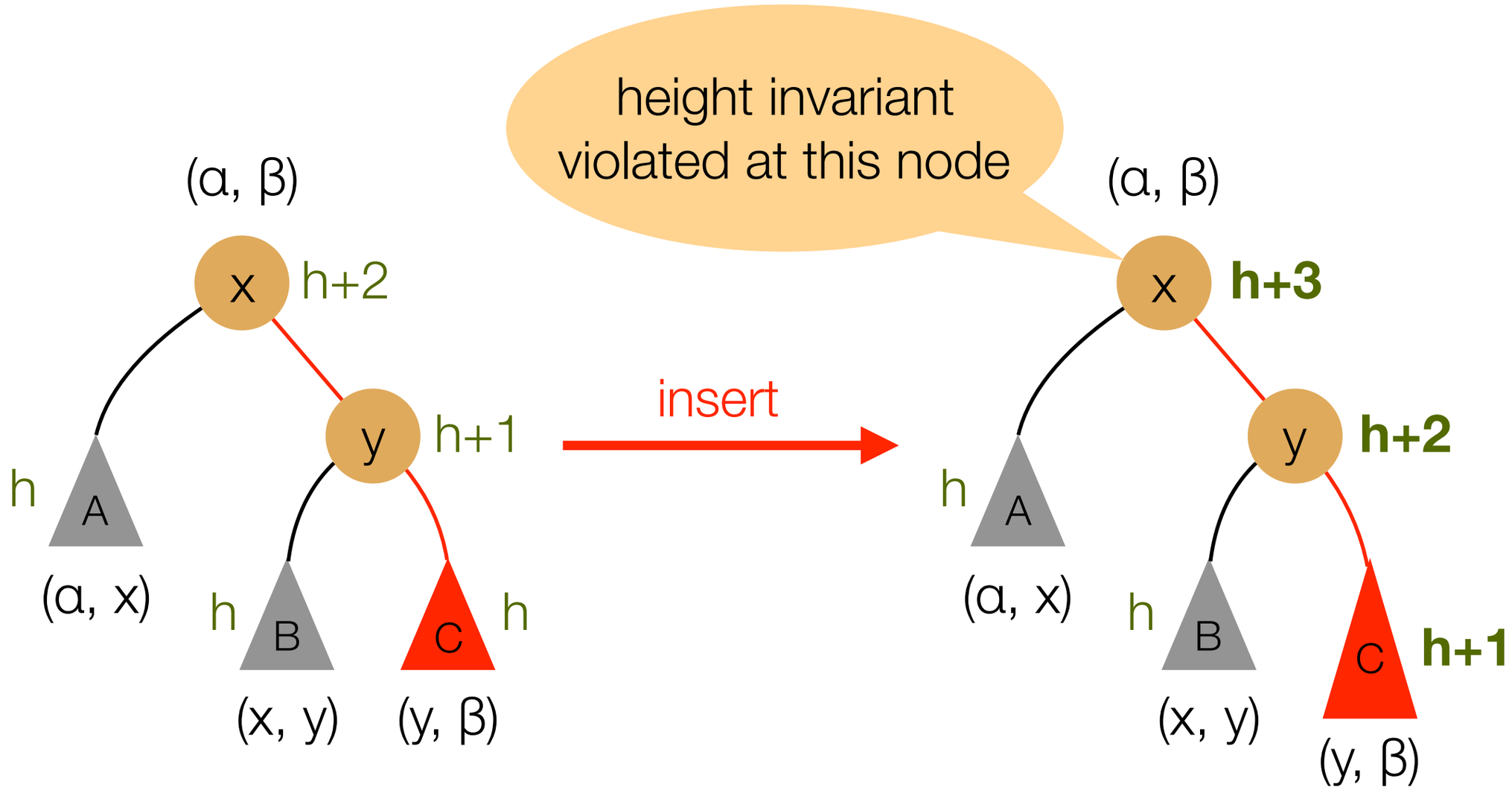
A < x < B < y < C

left subtree, right child thereof

(Symmetric to Cases 1 and 2)

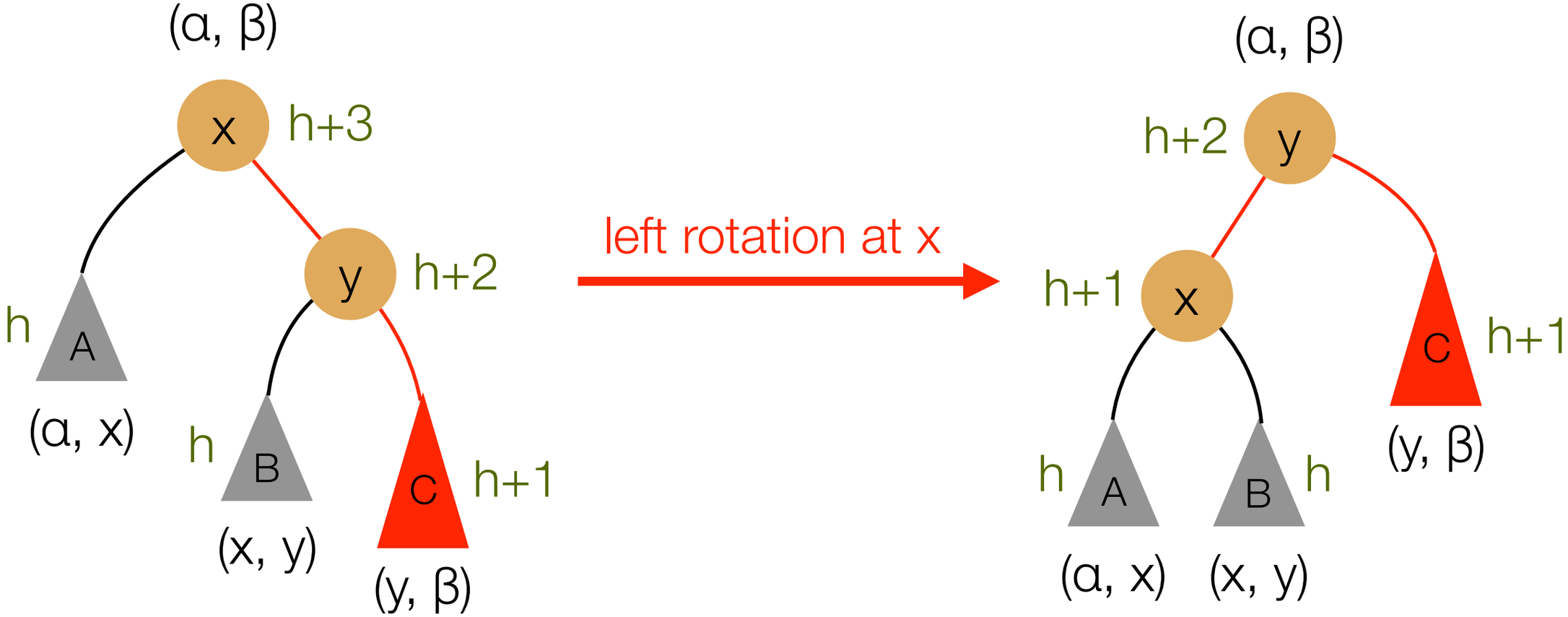
Rotation pattern for Case 1

If we are unlucky, insertion breaks the height invariant:



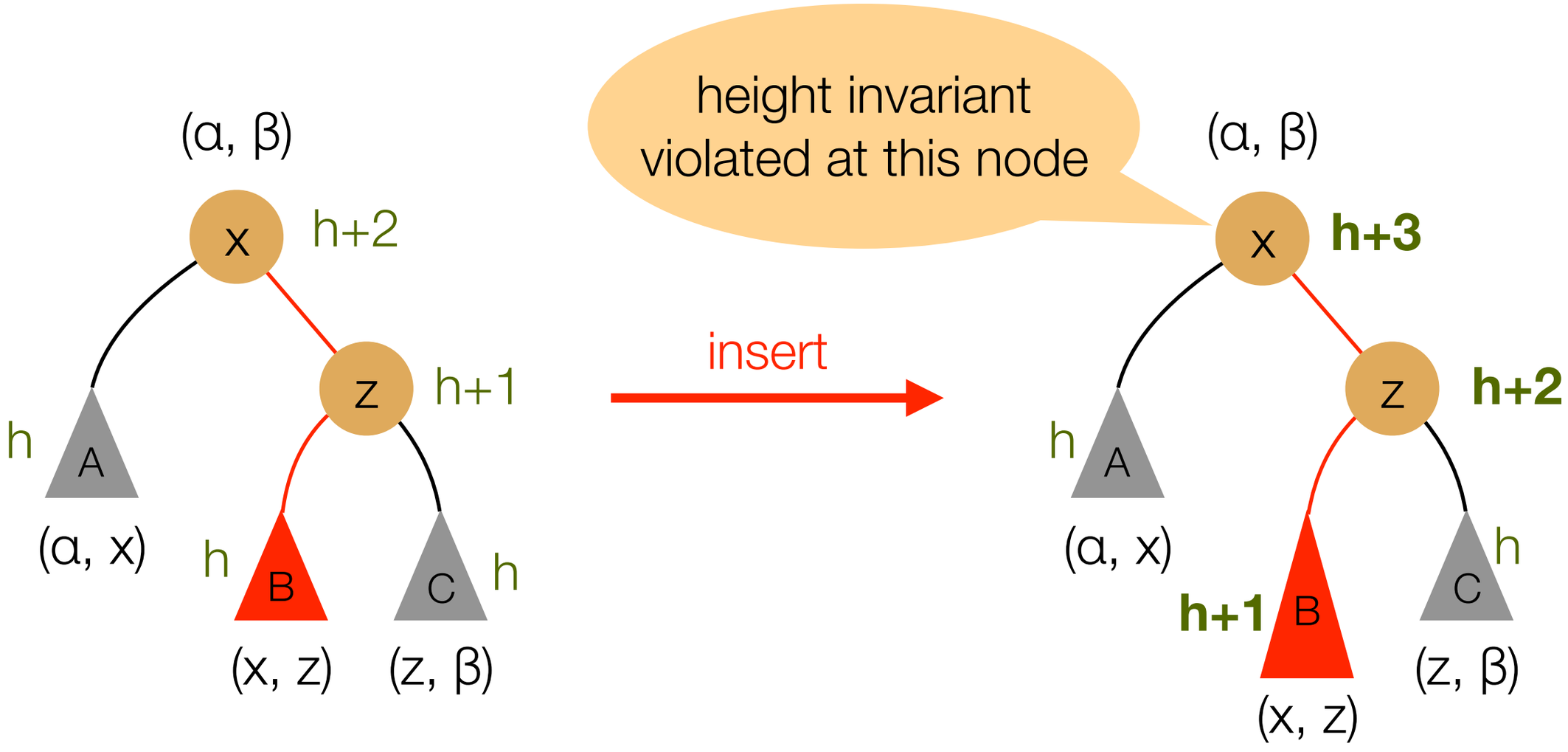
Rotation pattern for Case 1

Height invariant is restored by a **left rotation at x**:



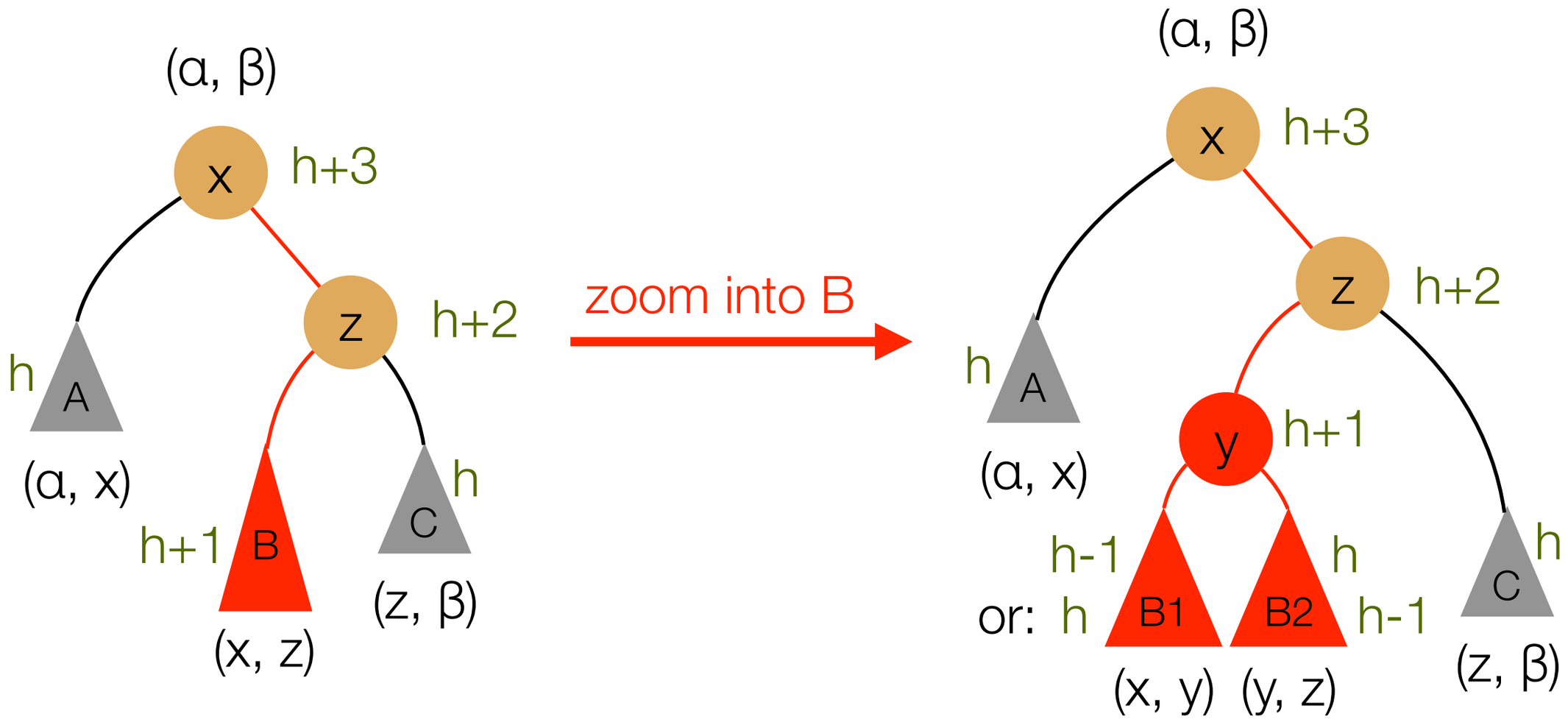
Rotation pattern for Case 2

If we are unlucky, insertion breaks the height invariant:



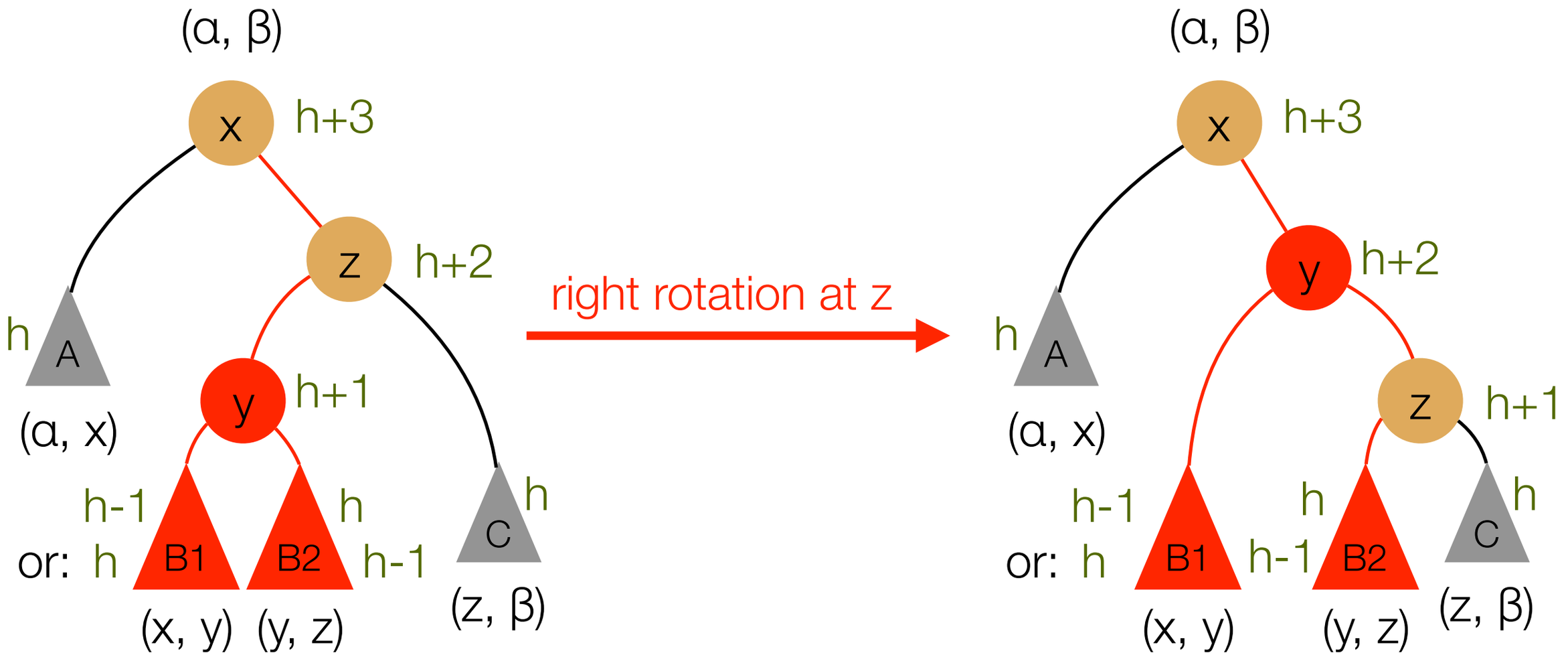
Rotation pattern for Case 2

Let's zoom into B:



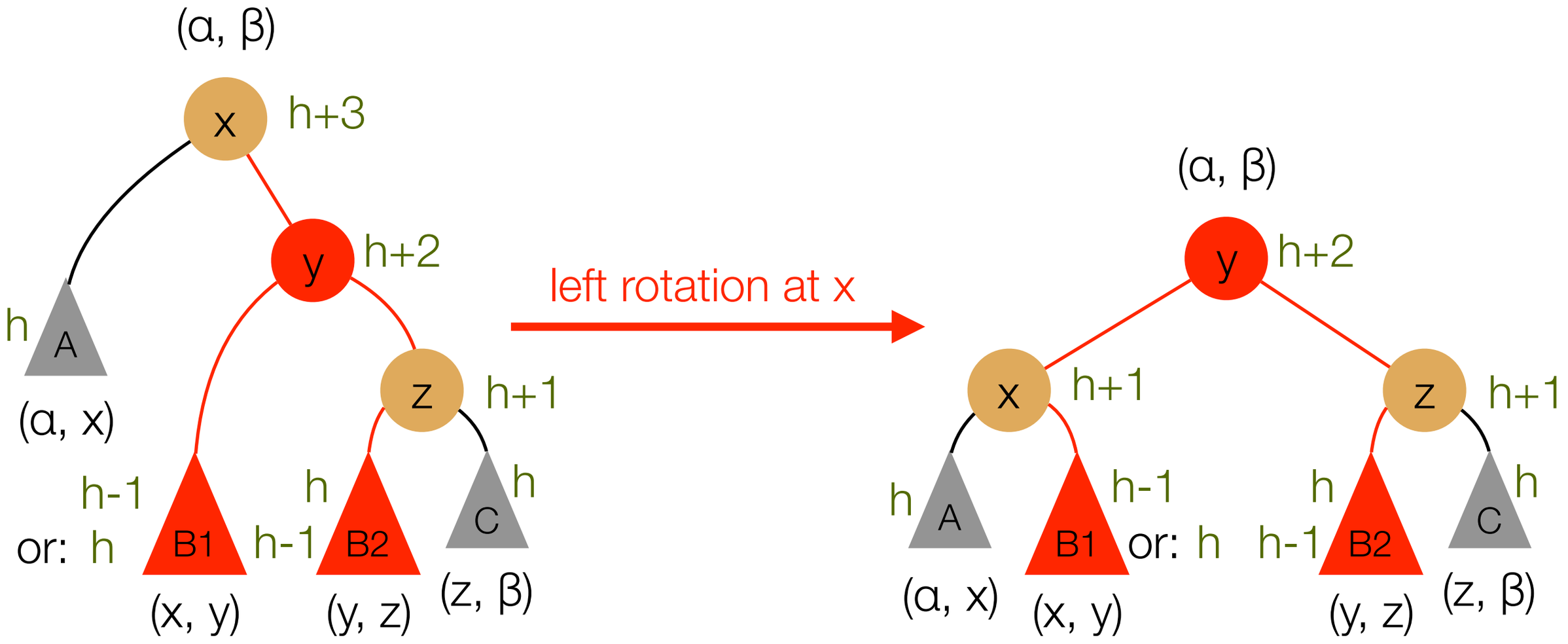
Rotation pattern for Case 2

Height invariant is restored by a **right rotation at z**, followed by a left rotation at x:



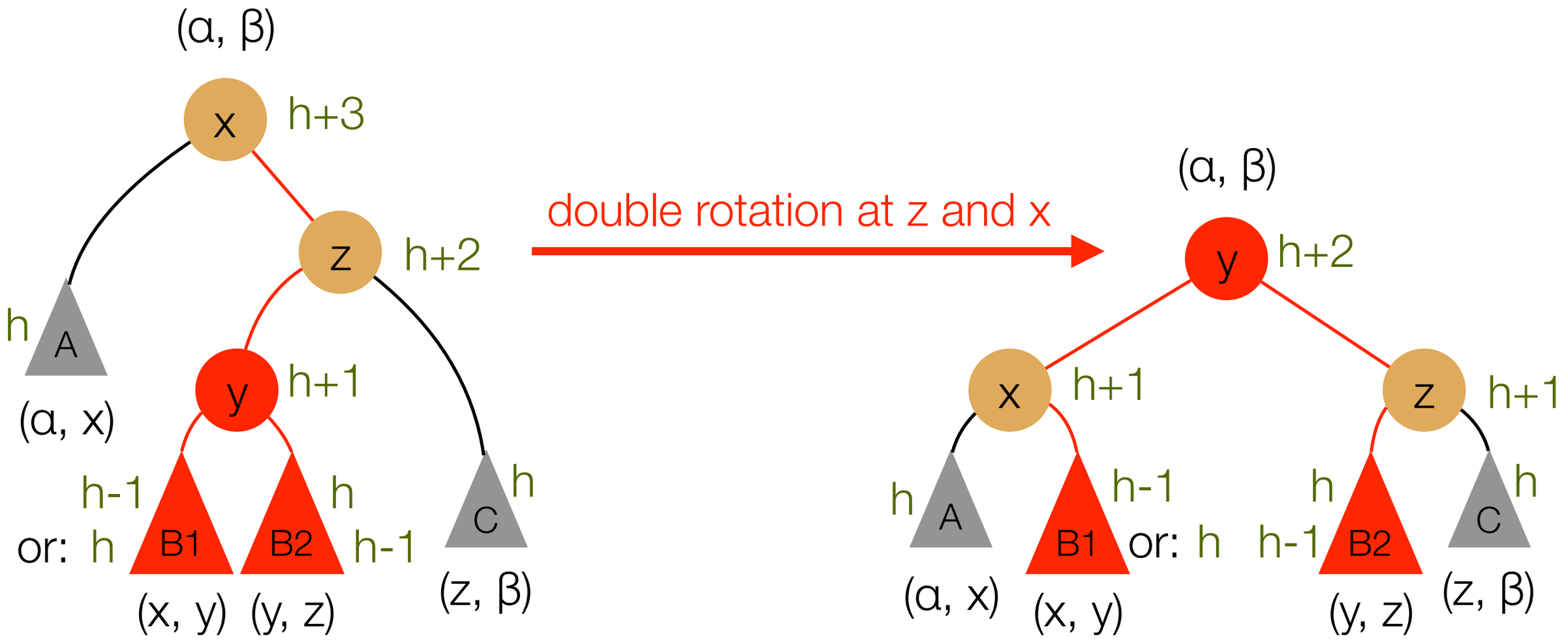
Rotation pattern for Case 2

Height invariant is restored by a right rotation at z, **followed by a left rotation at x**:



Rotation pattern for Case 2

Alternatively, we can view the two individual rotations as a combined **double rotation at z and x**:



Rotation patterns for Cases 3 and 4

- The rotation patterns for Cases 3 and 4 are symmetric to the ones for Cases 1 and 2, respectively.