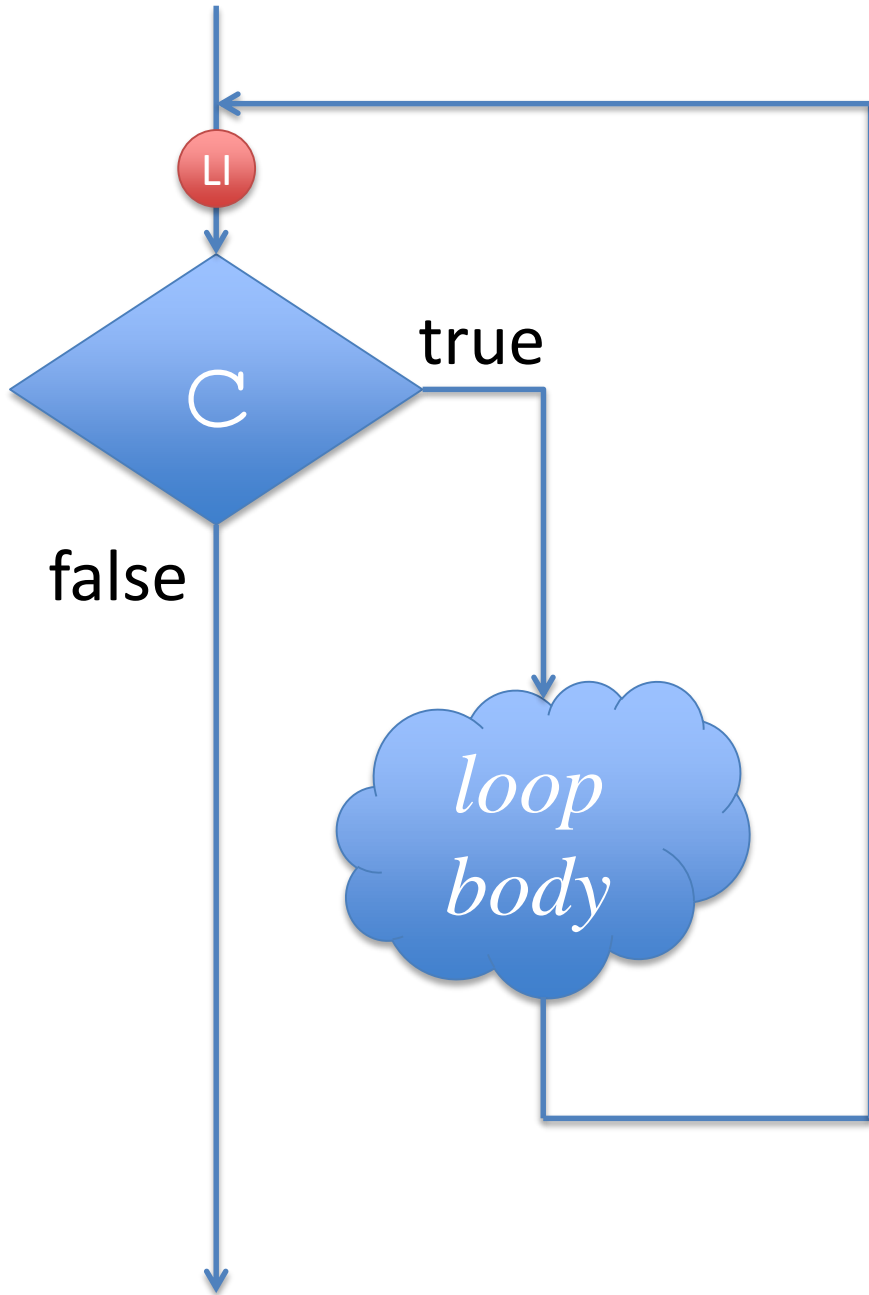


```
while (C) {  
    loop body  
}
```

Loop Invariant

A boolean condition that is checked *immediately before every evaluation of the loop guard*.





```
while (C)  
//@loop_invariant LI;  
{  
    loop body  
}
```

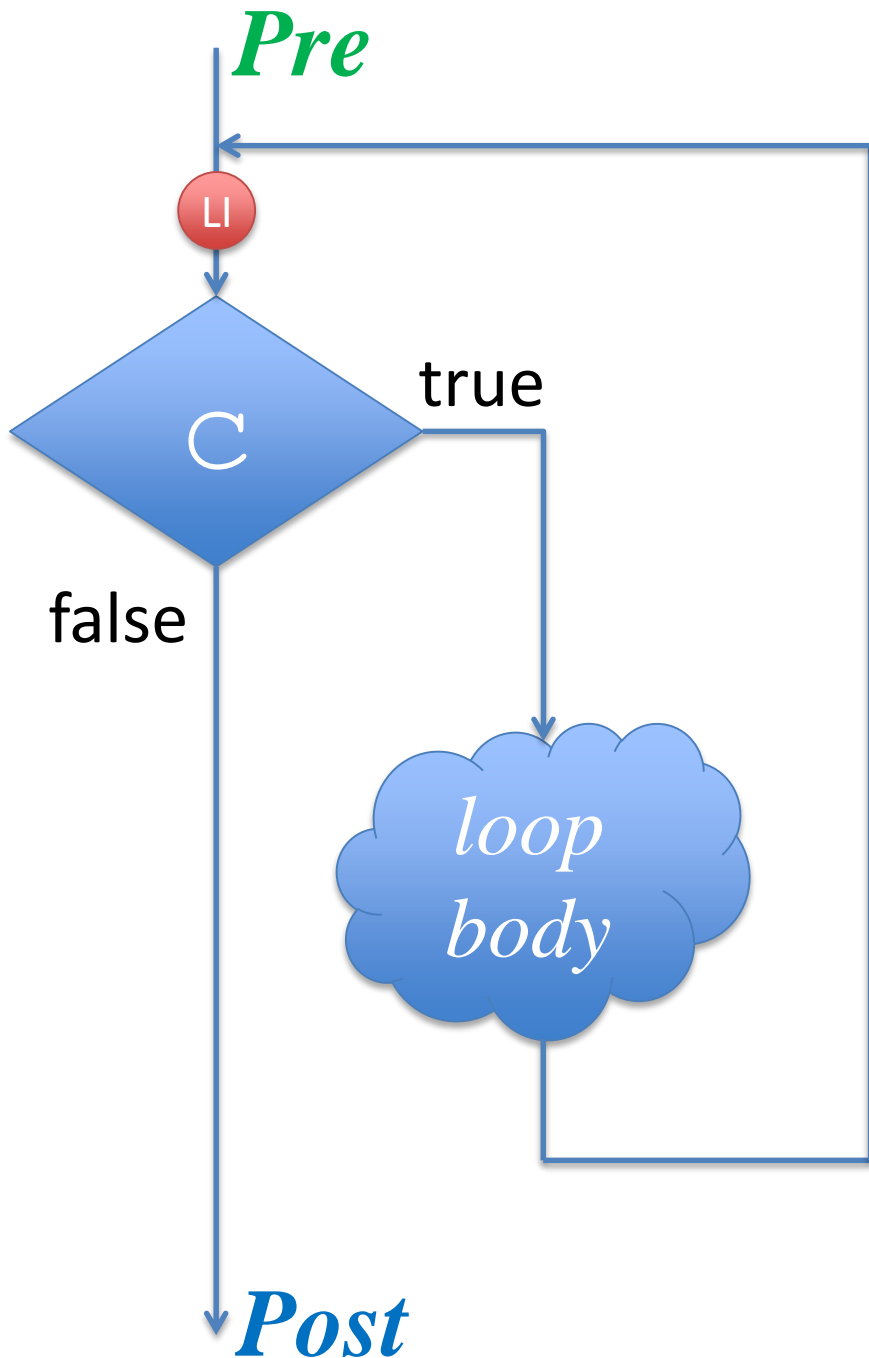
Loop Invariant

A boolean condition that is checked *immediately before every evaluation of the loop guard*.

- It is **true** even if the loop runs 0 times (i.e., is skipped)
- It is **true** immediately before each evaluation of the loop guard, including the last evaluation if the loop terminates
- It is **true** immediately after the loop terminates, if the loop terminates

Proving the Correctness of a function with one loop

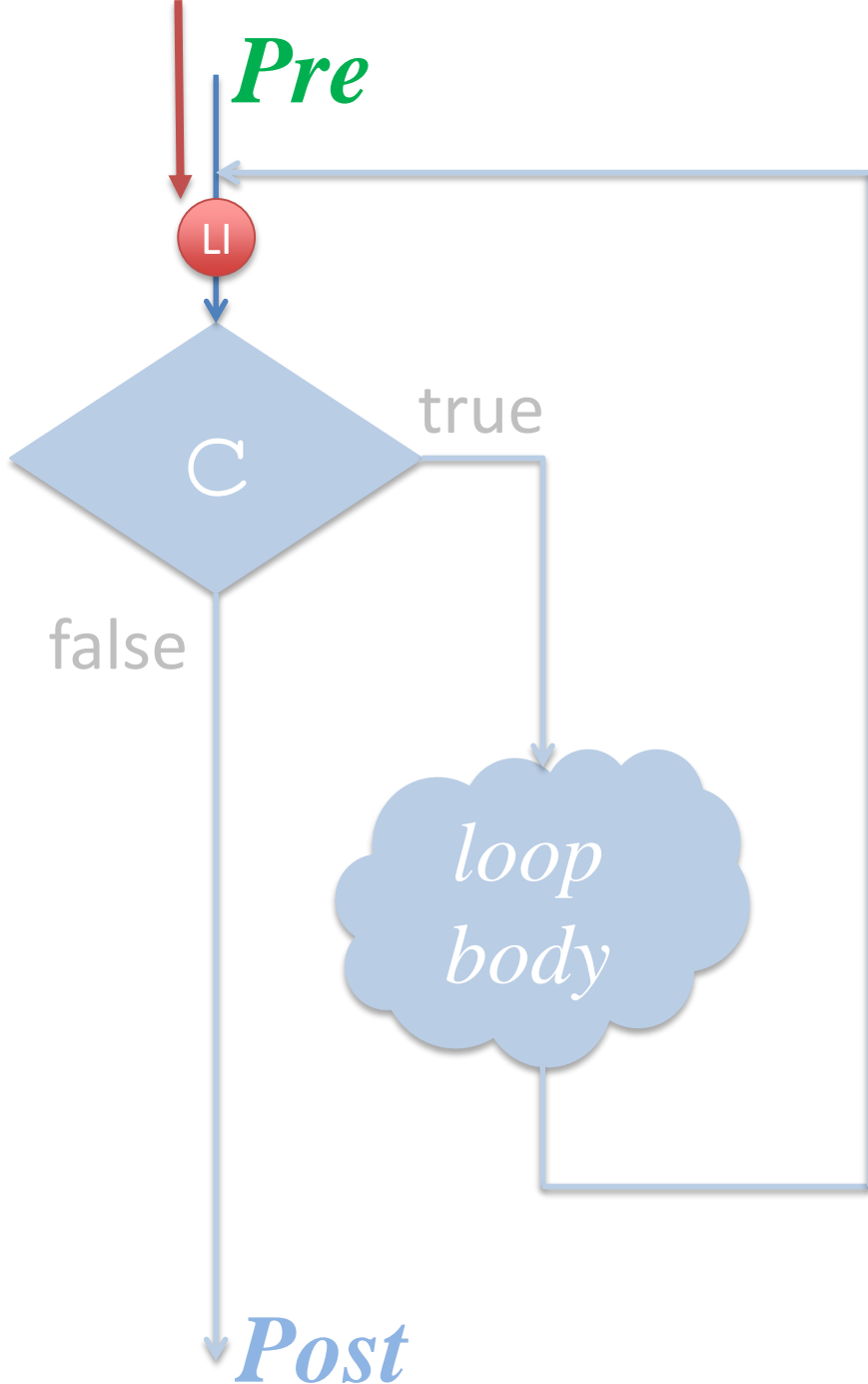
Correctness: if **preconditions** hold,
then **postconditions** must hold



```
//@requires Pre;  
//@ensures Post;
```

...

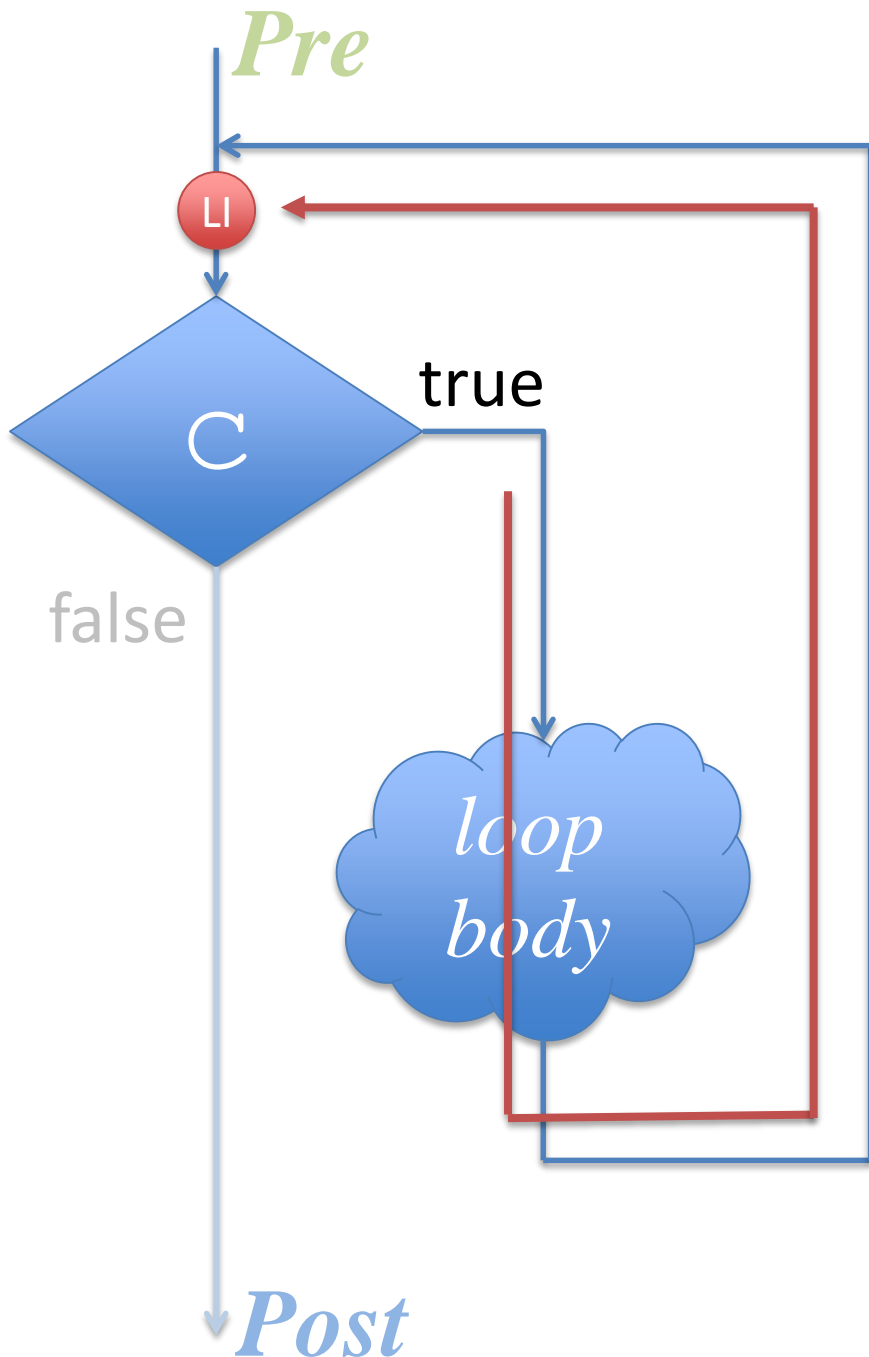
```
while (C)  
  //@loop_invariant LI;  
  {  
    loop body  
  }
```



Showing *LI* valid – 1

INIT

Show that the loop invariant *LI* is true immediately before the first evaluation of the loop guard *C*.

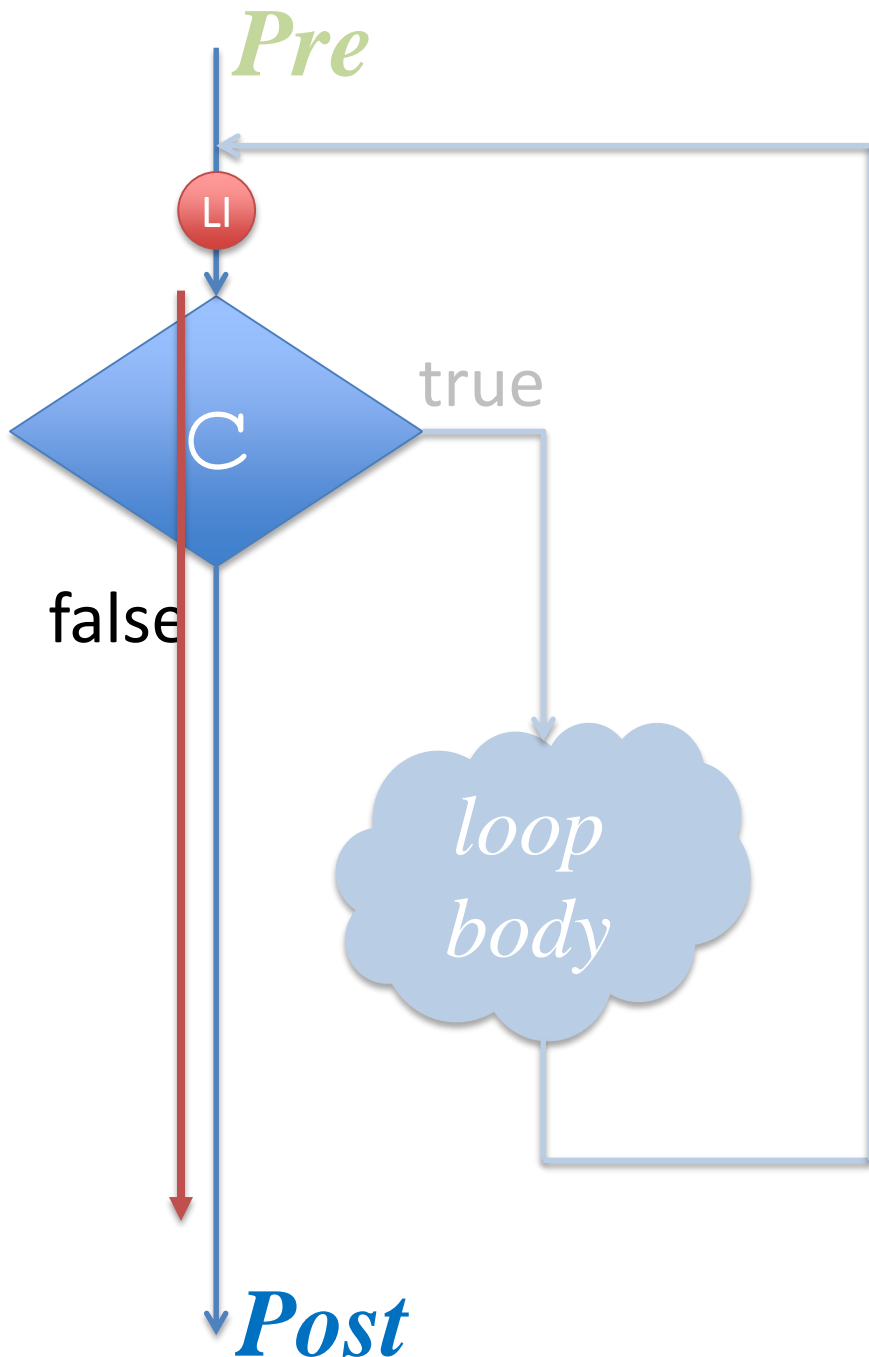


Showing ***LI*** valid – 2

PRESERVATION

Show that:

if the loop invariant ***LI*** is true immediately before the evaluation of the loop guard ***C***,
then ***LI*** is true immediately before the next evaluation of the loop guard ***C***.

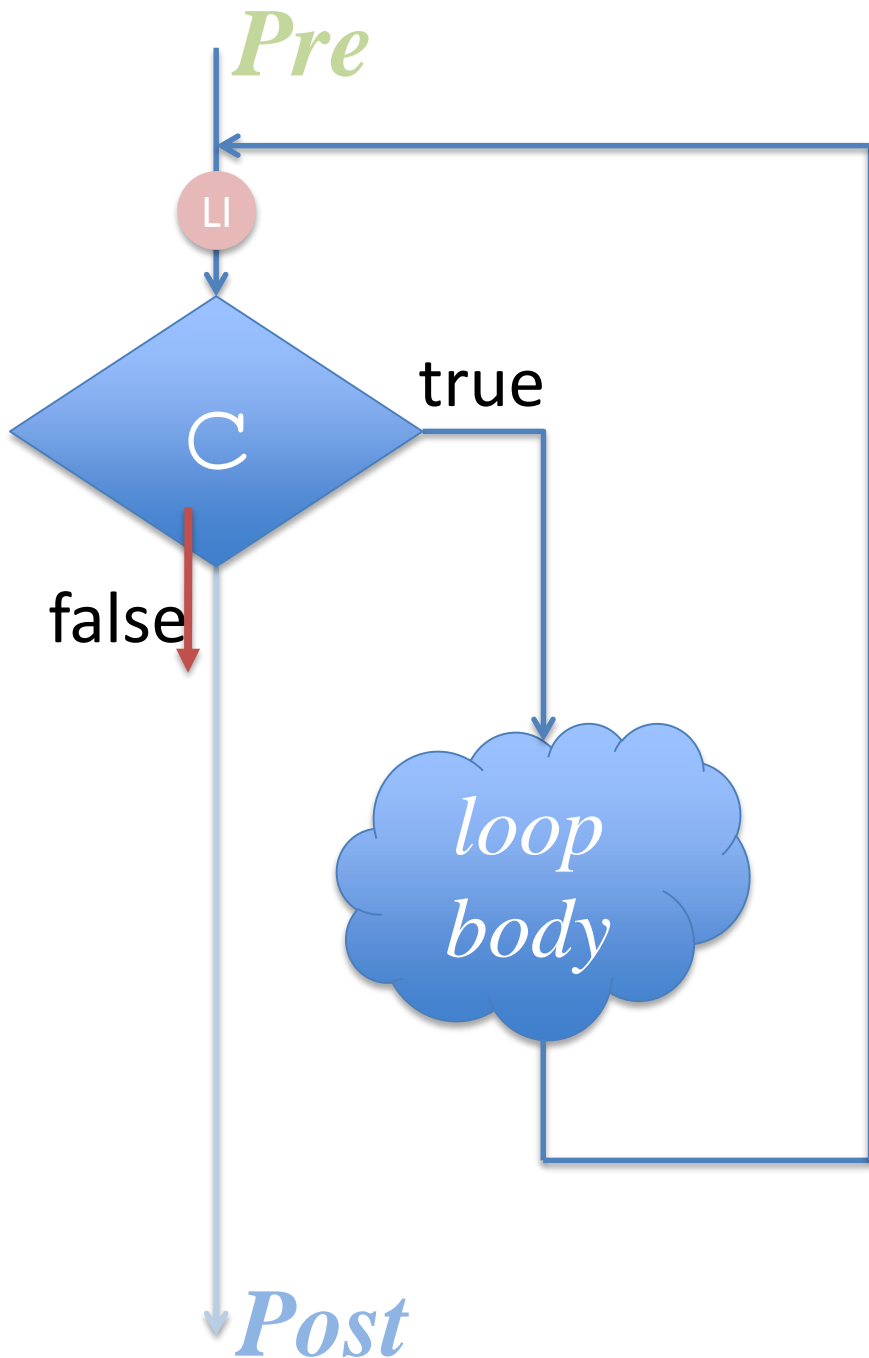


EXIT

*If loop invariant is valid,
show that:*

the logical conjunction
of the loop invariant ***LI***
and the negation of the
loop guard ***C*** implies the
desired postcondition
Post.

$$\mathbf{LI} \wedge \sim \mathbf{C} \rightarrow \mathbf{Post}$$



TERMINATION

Show that the loop will always terminate (i.e., that *C* must eventually be false)

Correctness of a function with one loop

- Show that ***LI*** is valid
 - **INIT**: ***LI*** holds initially
 - **PRES**: ***LI*** is preserved by an arbitrary iteration
- **EXIT**: ***LI*** $\wedge \sim \textcolor{purple}{C} \rightarrow \textcolor{blue}{Post}$
- **TERM**: loop terminates