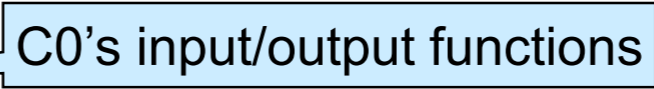
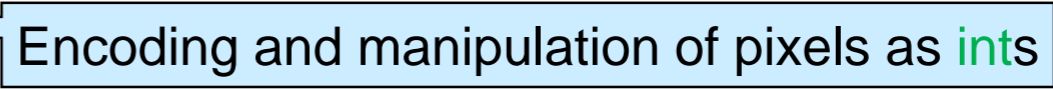

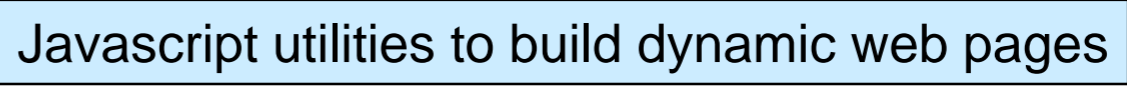


Libraries

Reusing Code

- All but the simplest programs reuse code already written
 - system code
 - `#use <conio>` 
 - simple code you wrote in the past
 - `pixel-int.c0` 
 - complex code somebody else wrote
 - `pixel.o0` 
 - `jquery.js` 
- Why?
 - Writing correct code is hard and time-consuming!
- These are **libraries**
 - They separate out code used across many applications from the applications themselves

Abstraction

- Libraries promote **abstraction**
 - Focus on **what** the library code does
 - e.g., print an integer to terminal using `printint`
 - not on **how** it does it
 - the many minute steps to turn a integer into terminal output
- Abstraction has lots of **benefits**
 - Hide inessential details
 - writing code is hard enough without also having to know how `printint` works
 - Make code more manageable
 - if we find a bug in `printint`, there is a single place where to fix it
 - Allow for transparent improvements
 - if we find a better way of printing, update the library not the applications

Computer science is all about abstraction!

What's a Library Anyway?

1. The interface

- Lists the functionalities the library exports and how to use them

```
void printint(int i);
```

Everything we need to use this functionality:

- name of the function
- number and type of arguments
- output type
- contracts

2. The implementation

- The code that implements them

```
void printint(int i) {
```

```
  ...
```

Complex low-level code

```
}
```

3. The documentation

- The explanation of what they do

"print i to standard output"

Human readable,
often in a web page or thick manual

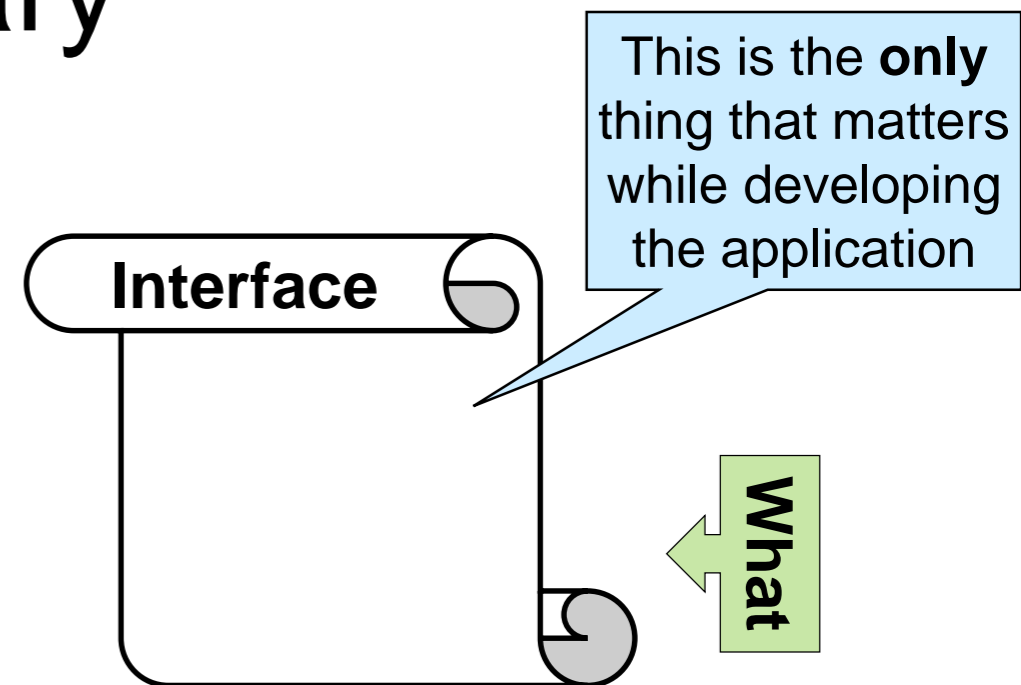
What

How

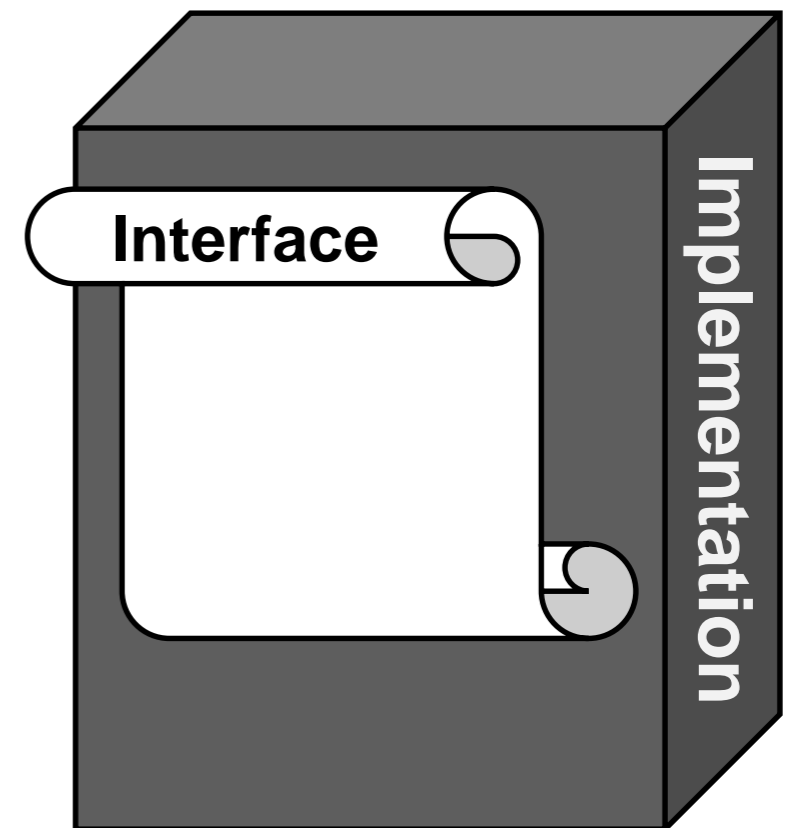
What

Using a Library

- When **writing** application code, we only use the functionalities listed in the interface
 - *No reliance on implementation*



- When **compiling** the application, we involve an implementation of the library
 - Needed for the application to run



- Implementation is a **black box**

Types of Libraries

- System libraries

- part of the programming language

`#use <conio>`

- No need to load any file to use them

```
Linux Terminal
# cc0 -d my-math-application.c0
```

- User-defined libraries

- written by users or downloaded from the Internet

`pixels.c0` or `pixels.o0`

- must be compiled with the application

```
Linux Terminal
# cc0 -d pixels.c0 my-image-application.c0
```

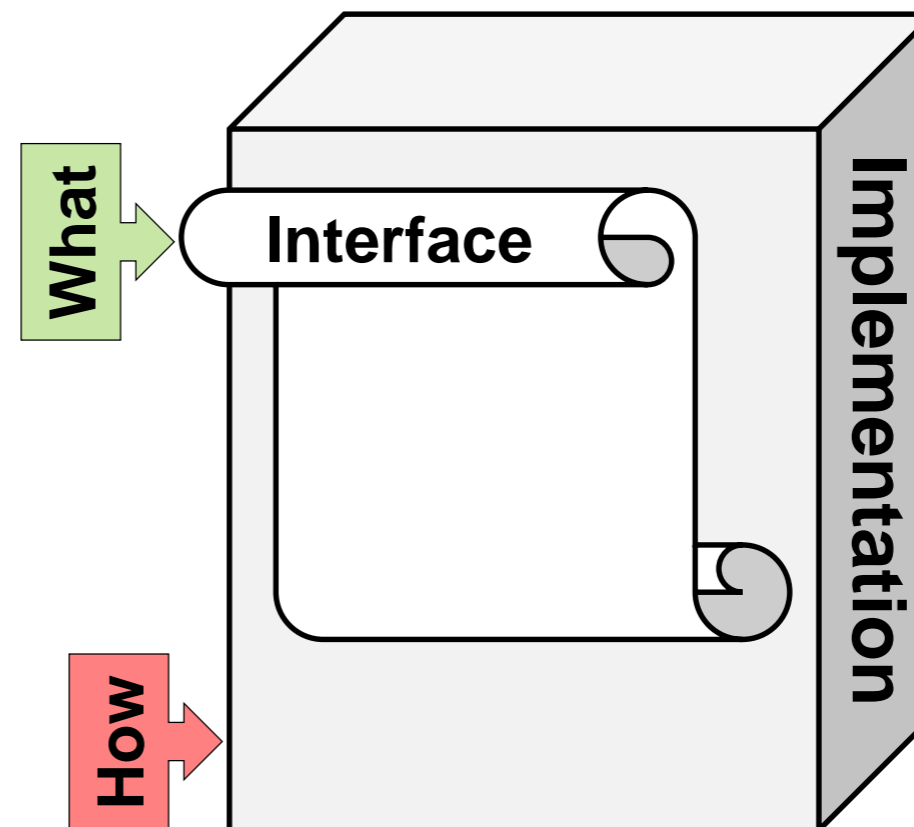
Also called an **API**

Application Programming Interface

Writing a Library

- When writing a library, we need to
 - decide on the interface
 - implement every functionality exported by the interface
 - Fill the black box
 - write lots of documentation

- In this class, we will be writing some of the system libraries that are native in other languages



Abstract Data Types

- A library that defines a **new type** and the ways to use it

E.g.,
pixels

- Defines the type `pixel_t` of pixels
 - The **only** way we shall refer to pixel in application
- Defines functions that manipulate pixels

```
int get_red(pixel_t p)
/*@ensures 0 <= \result && \result < 256; @*/ ;
int get_green(pixel_t p) ...
int get_blue(pixel_t p) ...
pixel_t make_pixel(int red, int green, int blue) ...
```

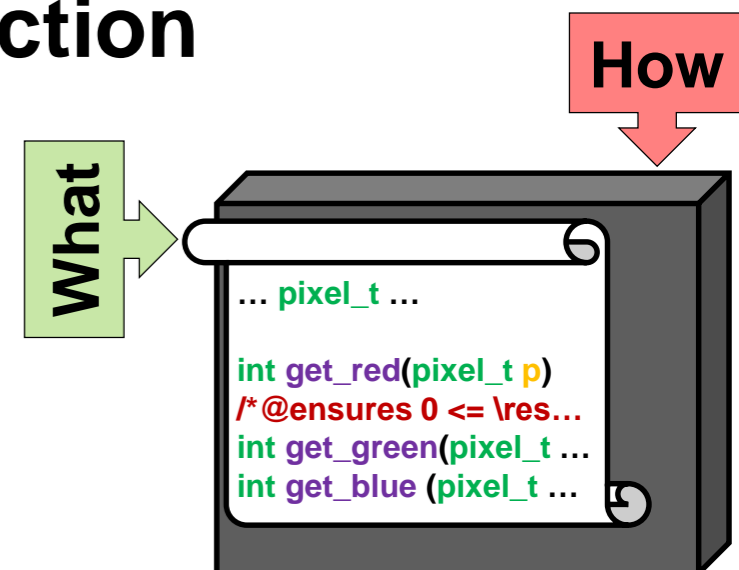
- The **only** operations we shall use to manipulate pixels
 - Except for functions we write using them

This is the pixel interface

```
... pixel_t ...
int get_red(pixel_t p)
/*@ensures 0 <= \res...
int get_green(pixel_t ...
int get_blue (pixel_t ...
```

- ADT's promote a very strong form of **abstraction**

- If the client only uses the interface, we can use *any correct implementation* and the application will work the same!



Self-Sorting Arrays

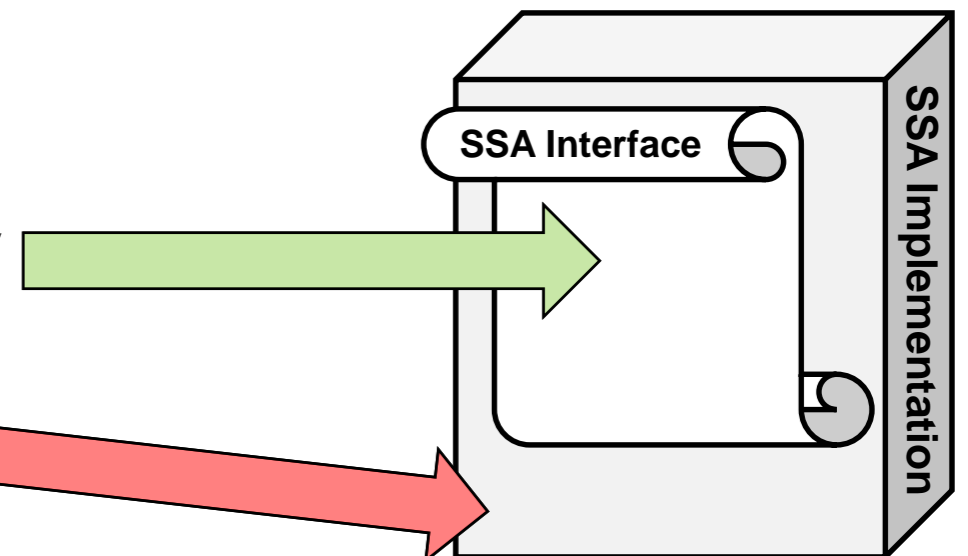
Writing Libraries

- In this course, we will be writing many libraries
- Case study to gain familiarity
 - and define important concepts

Self-sorting arrays (SSA)

- a toy data structure that works just like arrays of strings but
 - has a function that reports the length of the SSA
 - guarantees that its elements are sorted

- What we need to do:
 - A. Define the interface of the SSA library
 - B. Implement it



SSA Interface

Interface Contents

SSA Interface

```
// typedef _____ ssa_t;
```

1. A **type** for self-sorting arrays

ssa_t

- SSA's are a data structure
- We need a type to refer to them in code
 - define variables that can hold an SSA
 - define functions that manipulate them, ...

Convention: types exported by a library end in `_t`

● We do **not** want the client to learn the details of this type

- This type is **abstract** for the client: *just a name*
- We define it as a **pseudo-typedef**

A commented-out **typedef** with underscores

```
// typedef _____ ssa_t;
```

Another convention

- The implementation will contain the actual definition of **ssa_t**
 - **Concrete** type of SSA's

What

How

```
// typedef _____ ssa_t;  
  
// Operations
```

2. The **operations** provided by the library to manipulate SSA's

- What should these be?
 - SSA's are just fancy arrays
 - We will need SSA versions of the standard operations on arrays
 - ❑ create a new array
 - ❑ read a value from an array index
 - ❑ replace the value at an array index
 - Plus *a function that returns the length*
 - ❑ not just in contracts, but in regular code

Interface Contents

2. The **operations** provided by the library to manipulate SSA's

- Creating a new SSA

Number of elements

Newly
created
SSA

```
ssa_t ssa_new(int size); // akin to alloc_array(string, size)
```

This is a **function prototype**:
a function definition without a body

- Reading the value at an index of an SSA

```
string ssa_get(ssa_t A, int i); // akin to ... A[i] ...
```

Recall that our SSAs
contain strings
(we'll learn later how to generalize)

- Replacing the value at an index of an SSA

```
void ssa_set(ssa_t A, int i, string x); // akin to A[i] = x
```

➤ unlike regular arrays, this may rearrange the contents of A to keep it sorted

- Returning the length of an SSA

```
int ssa_len(ssa_t A); // akin to \length(A) but better
```

➤ unlike regular array, this can be used anywhere in code

Interface Contents

SSA Interface

```
// typedef _____ ssa_t;  
  
int ssa_len(ssa_t A);  
  
ssa_t ssa_new(int size) ;  
  
string ssa_get(ssa_t A, int i);  
  
void ssa_set(ssa_t A, int i, string x);
```

3. The **contracts** of each operation

- The client needs to be able to write safe code

- Provide arguments that satisfy the preconditions of each function
- Use the functions' postconditions to reason about follow-up code

- Reading the value at an index of an SSA

- Same contracts as native A[i]

- `/*@requires 0 <= i && i < \length(A); @*/`

- So:

```
string ssa_get(ssa_t A, int i) // akin to ... A[i] ...  
/*@requires 0 <= i && i < \length(A); @*/ ;
```

Can this be right?

Interface Contents

SSA Interface

```
// typedef _____ ssa_t;  
int ssa_len(ssa_t A);  
  
ssa_t ssa_new(int size) ;  
  
string ssa_get(ssa_t A, int i);  
  
void ssa_set(ssa_t A, int i, string x);
```

3. The **contracts** of each operation

```
string ssa_get(ssa_t A, int i) // akin to ... A[i] ...  
/* @requires 0 <= i && i < \length(A); @*/ ;
```

Can this be right?

- **\length** is defined only for C0 arrays
 - But SSAs are not C0 arrays
 - ❑ length can be retrieved
 - ❑ elements stay sorted
 - ❑ should be manipulated only with operations in the SSA interface
- We can however use **ssa_len**

```
string ssa_get(ssa_t A, int i) // akin to ... A[i] ...  
/* @requires 0 <= i && i < ssa_len(A); @*/ ;
```


Interface Contents

3. The **contracts** of each operation

- With `ssa_len`, we can give a meaningful precondition to `ssa_get`
- and to `ssa_set`
- and a postcondition to `ssa_new`

SSA Interface

```
// typedef _____ ssa_t;

int ssa_len(ssa_t A)
/* @ensures \result >= 0; @*/ ;

ssa_t ssa_new(int size)
/* @requires 0 <= size; @*/
/* @ensures ssa_len(\result) == size; @*/ ;

string ssa_get(ssa_t A, int i)
/* @requires 0 <= i && i < ssa_len(A); @*/ ;

void ssa_set(ssa_t A, int i, string x)
/* @requires 0 <= i && i < ssa_len(A); @*/ ;
```

Interface Contents

- But what kind of type can `ssa_t` be?

- An array? **x**

- No way to get the length of an array in C0

- An `int`, `bool` or `char`? **x**

- No way to represent arbitrarily many strings

- A `string`? **x**

- Let's not go there ...

- A struct? **x**

- Structs cannot be passed as function arguments directly

- Then, `ssa_t` must be a **pointer**

- Update the pseudo-`typedef` to reflect this

- Disallow NULL as a valid `ssa_t`

- ❑ Every operation that takes an `ssa_t` has a NULL-check as a precondition

- ❑ Every operation that returns an `ssa_t` has a NULL-check as a postcondition

SSA Interface

```
// typedef ____* ssa_t;

int ssa_len(ssa_t A)
/* @requires A != NULL; @*/
/* @ensures \result >= 0; @*/;

ssa_t ssa_new(int size)
/* @requires 0 <= size; @*/
/* @ensures \result != NULL; @*/
/* @ensures ssa_len(\result) == size; @*/;

string ssa_get(ssa_t A, int i)
/* @requires A != NULL; @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;

void ssa_set(ssa_t A, int i, string x)
/* @requires A != NULL; @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;
```

We never use NULL for an empty data structure

Interface Contents

SSA Interface

```
// typedef _____ * ssa_t;

int ssa_len(ssa_t A)
/* @requires A != NULL;          @*/
/* @ensures \result >= 0;       @*/ ;

ssa_t ssa_new(int size)
/* @requires 0 <= size;          @*/
/* @ensures \result != NULL;    @*/
/* @ensures ssa_len(\result) == size; @*/ ;

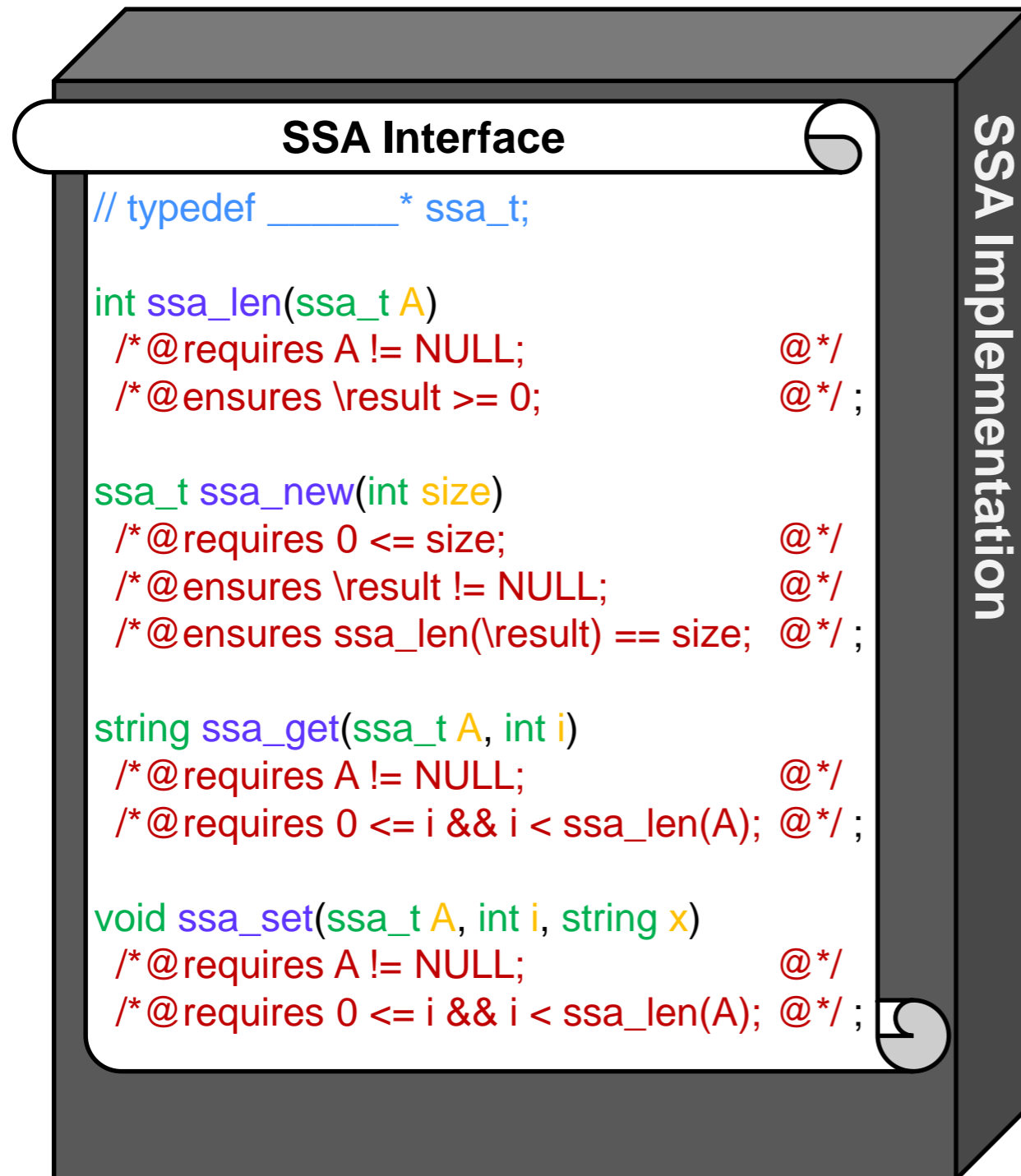
string ssa_get(ssa_t A, int i)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/ ;

void ssa_set(ssa_t A, int i, string x)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/ ;
```

What

Client Application

Using a library



```
SSA Interface
// typedef _____* ssa_t;

int ssa_len(ssa_t A)
/* @requires A != NULL;          @*/
/* @ensures \result >= 0;       @*/;

ssa_t ssa_new(int size)
/* @requires 0 <= size;          @*/
/* @ensures \result != NULL;    @*/
/* @ensures ssa_len(\result) == size; @*/;

string ssa_get(ssa_t A, int i)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;

void ssa_set(ssa_t A, int i, string x)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;

SSA Implementation
```

- The client only knows **what** the library does
 - the library interface
 - the library documentation
- The client does not know **how** it does it
 - treat the implementation as a black box
 - even if its code is available
 - it may change!

Searching an SSA

- Client code that uses binary search to check if a value is in an SSA

➤ This is OK because SSAs are sorted!

```
bool is_in(string x, ssa_t A, int n)
//@requires A != NULL;
//@requires n == ssa_len(A);
{
    int lo = 0;
    int hi = n;
    while (lo < hi)
    //@loop_invariant 0 <= lo && lo <= hi && hi <= n;
    {
        int mid = lo + (hi - lo) / 2;
        //@assert lo <= mid && mid < hi;
        string a = ssa_get(A, mid);
        int cmp = string_compare(a, x);
        if (cmp == 0) return true;
        if (cmp < 0) {
            lo = mid + 1;
        } else { //@assert cmp > 0;
            hi = mid;
        }
    }
    return false;
}
```

Precondition of
ssa_len and ssa_get

- All array operations are replaced with functions from the SSA interface

returns <0 if a “less than” x, 0 if equal, >0 otherwise

- **Safety** is supported by loop invariant and assertion
- For **correctness**, we would need to implement SSA versions of `gt_seg` and `lt_seg`

SSA Interface

```
// typedef _____* ssa_t;

int ssa_len(ssa_t A)
/*@requires A != NULL; @*/
/*@ensures \result >= 0; @*/;

ssa_t ssa_new(int size)
/*@requires 0 <= size; @*/
/*@ensures \result != NULL; @*/
/*@ensures ssa_len(\result) == size; @*/;

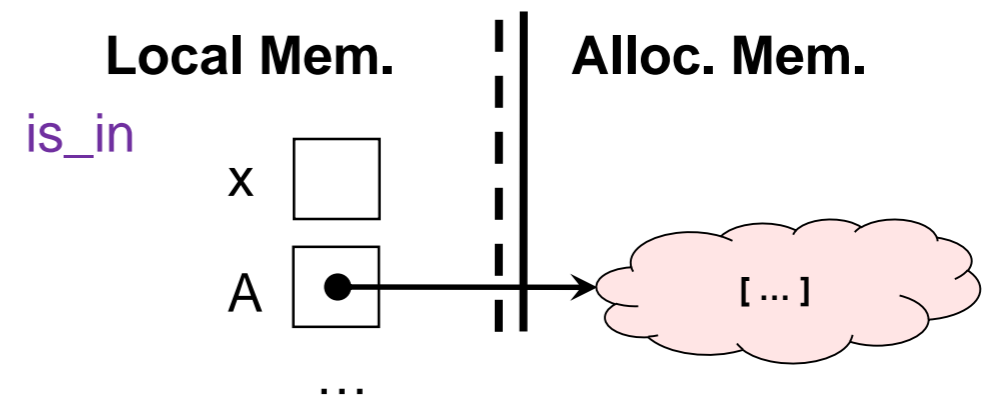
string ssa_get(ssa_t A, int i)
/*@requires A != NULL; @*/
/*@requires 0 <= i && i < ssa_len(A); @*/;

void ssa_set(ssa_t A, int i, string x)
/*@requires A != NULL; @*/
/*@requires 0 <= i && i < ssa_len(A); @*/;
```

Searching an SSA

- Client view of memory

```
bool is_in(string x, ssa_t A, int n)
//@requires n == ssa_len(A);
{
  int lo = 0;
  int hi = n;
  while (lo < hi)
  //@loop_invariant 0 <= lo && lo <= hi && hi <= n;
  {
    int mid = lo + (hi - lo) / 2;
    //@assert lo <= mid && mid < hi;
    string a = ssa_get(A, mid);
    int cmp = string_compare(a, x);
    if (cmp == 0) return true;
    if (cmp < 0) {
      lo = mid + 1;
    } else { //@assert cmp > 0;
      hi = mid;
    }
  }
  return false;
}
```

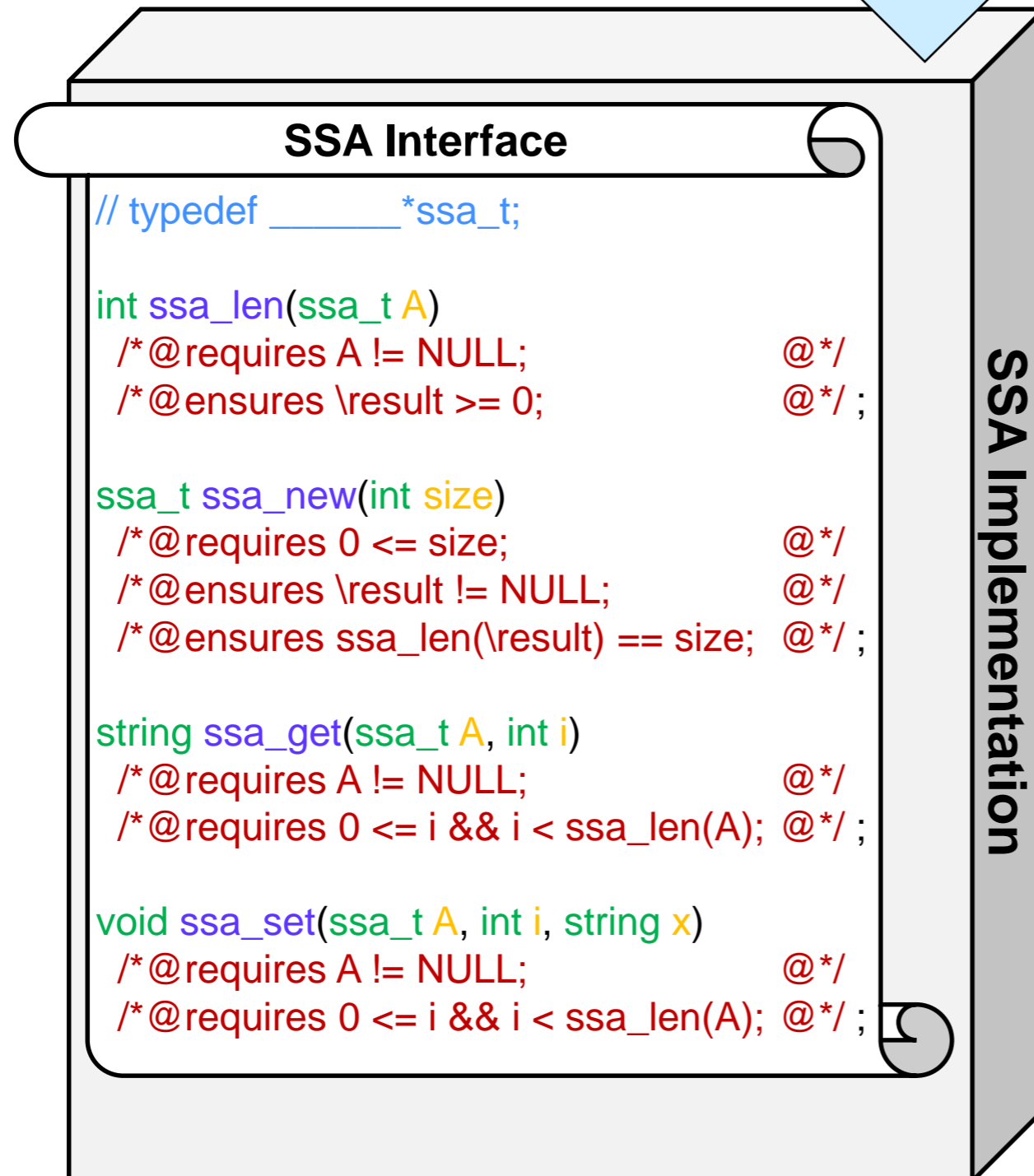


- The client has no knowledge of how A is represented in memory

SSA Implementation

Implementing SSAs

Now we've got to fill the box



- Define the type `ssa_t`
 - **Concrete** type
- Write code for every function
- Make sure it is safe and correct

How

Concrete Type

SSA Interface

```
// typedef _____* ssa_t;  
  
// ...
```

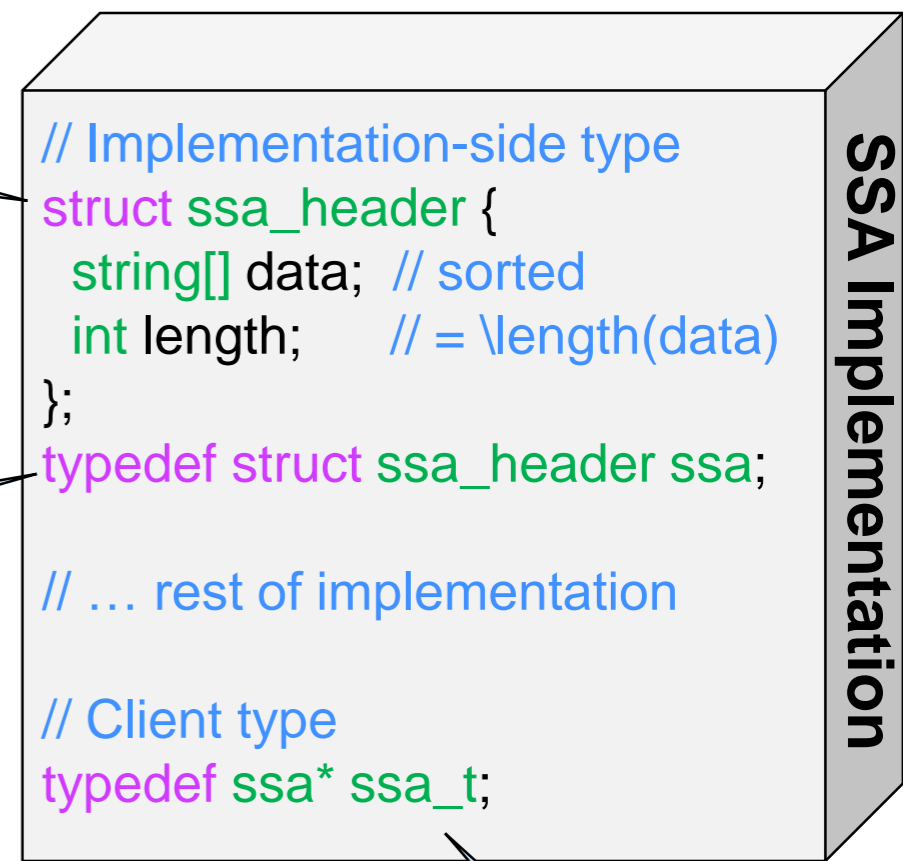
- Store elements in a C0 array, but keep track of the length
 - Package them together in a struct

This is the **concrete** implementation type

- Define an internal nickname for it
 - So that the code is succinct and readable
 - It's convenient that it **not** be a pointer

Internal nickname

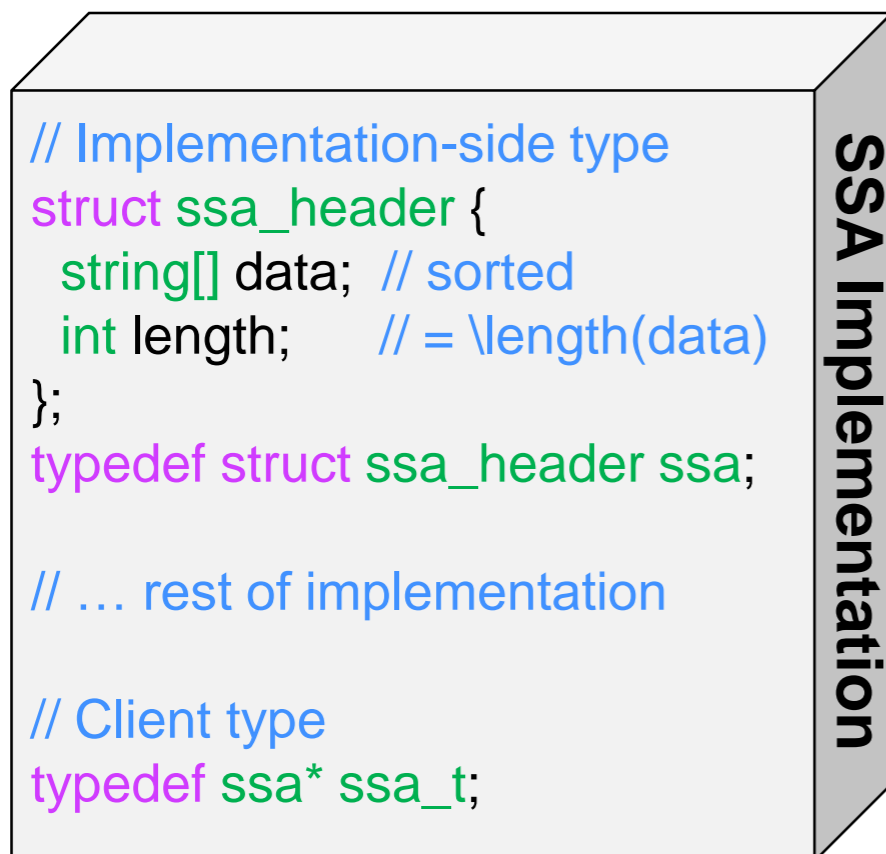
- Define the abstract type exported to the client
 - This is what connects the concrete implementation type with the exported abstract type



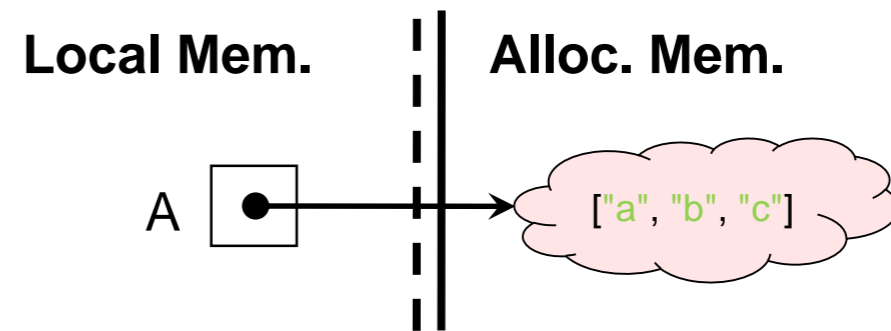
Abstract
client type

Client vs. Implementation View

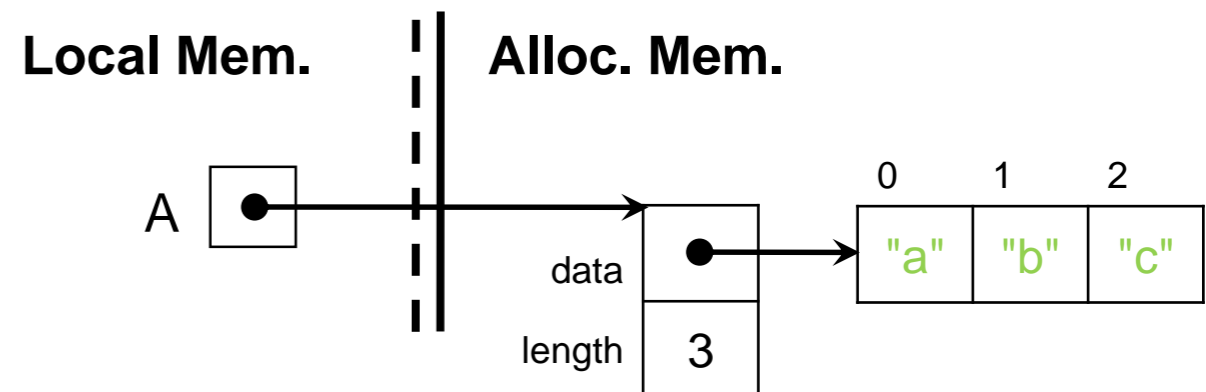
An SSA containing "a", "b" and "c"



- Client view



- Implementation view



Implementing `ssa_get`

SSA Interface

```
string ssa_get(ssa_t A, int i)
/*@requires A != NULL; @*/
/*@requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
  string[] data; // sorted
  int length;   // = \length(data)
};
typedef struct ssa_header ssa;

string ssa_get(ssa* A, int i)
/*@requires A != NULL;
/*@requires 0 <= i && i < ssa_len(A);
{
  return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

- Simply return the i -th element of the underlying array

`return A->data[i]`

- Is this safe? We need to check

- $A \neq \text{NULL}$

- By 1st precondition



- $i \leq 0$

- By 2nd precondition (first conjunct)



- $i < \text{length}(A \rightarrow \text{data})$

- We know that $i < \text{ssa_len}(A)$

- but we don't know how $\text{ssa_len}(A)$ and $\text{length}(A \rightarrow \text{data})$ are related

- **Not supported!**



Let's also write `ssa_len`

SSA Interface

```
int ssa_len(ssa_t A)
/*@requires A != NULL;          @*/
/*@ensures \result >= 0;       @*/;

string ssa_get(ssa_t A, int i)
/*@requires A != NULL;          @*/
/*@requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
  string[] data; // sorted
  int length;   // = \length(data)
};
typedef struct ssa_header ssa;
```

```
int ssa_len(ssa* A)
/*@requires A != NULL;
/*@ensures \result >= 0;
{
  return A->length;
}
```

```
string ssa_get(ssa* A, int i)
/*@requires A != NULL;
/*@requires 0 <= i && i < ssa_len(A);
{
  return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

- Simply return the length field
- Is this safe? We need to check
 - **A != NULL**
 - By precondition ✓
 - Does this help us with `ssa_get`?
 - No useful postcondition ✗
 - Peeking at the code of `ssa_len` would be operational reasoning!

Let's also write `ssa_len`

SSA Interface

```
int ssa_len(ssa_t A)
/*@requires A != NULL;          @*/
/*@ensures \result >= 0;      @*/;

string ssa_get(ssa_t A, int i)
/*@requires A != NULL;          @*/
/*@requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
  string[] data; // sorted
  int length;    // = \length(data)
};
typedef struct ssa_header ssa;

int ssa_len(ssa* A)
/*@requires A != NULL;
/*@ensures \result >= 0;
/*@ensures \result == \length(A->data);
{
  return A->length;
}

string ssa_get(ssa* A, int i)
/*@requires A != NULL;
/*@requires 0 <= i && i < ssa_len(A);
{
  return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

- Add a *useful* postcondition

`\result == \length(A->data)`

- Is this safe? We need to check

- `A != NULL`

- By precondition



- Is `ssa_len` correct?

- No relation between `A->length` and `\length(A->data)`



Innocent mistake: define `ssa_new` as

```
ssa* ssa_new(int size) {
  ssa* A = alloc(ssa);
  A->length = size;
  A->data = alloc_array(string, size+1);
  return A;
}
```

Let's also write `ssa_len`

SSA Interface

```
int ssa_len(ssa_t A)
/*@requires A != NULL;          @*/
/*@ensures \result >= 0;       @*/;

string ssa_get(ssa_t A, int i)
/*@requires A != NULL;          @*/
/*@requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
  string[] data; // sorted
  int length;    // = \length(data)
};
typedef struct ssa_header ssa;

int ssa_len(ssa* A)
/*@requires A != NULL;
/*@requires A->length == \length(A->data);
/*@ensures \result >= 0;
/*@ensures \result == \length(A->data);
{
  return A->length;
}

string ssa_get(ssa* A, int i)
/*@requires A != NULL;
/*@requires 0 <= i && i < ssa_len(A);
{
  return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

- Add it as a precondition
 - $A \rightarrow \text{length} == \backslash \text{length}(A \rightarrow \text{data})$
- Is this safe? We need to check
 - $A \neq \text{NULL}$ ✓
 - By precondition
- Is `ssa_len` correct?
 - $A \rightarrow \text{length} == \backslash \text{length}(A \rightarrow \text{data})$
 - By new precondition
 - $\backslash \text{result} == A \rightarrow \text{length}$
 - By code
 - $\backslash \text{result} == \backslash \text{length}(A \rightarrow \text{data})$ ✓
 - By previous two

Back to `ssa_get`

SSA Interface

```
int ssa_len(ssa_t A)
/* @requires A != NULL;          @*/
/* @ensures \result >= 0;       @*/;

string ssa_get(ssa_t A, int i)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
  string[] data; // sorted
  int length;    // = \length(data)
};
typedef struct ssa_header ssa;

int ssa_len(ssa* A)
/* @requires A != NULL;
/* @requires A->length == \length(A->data);
/* @ensures \result >= 0;
/* @ensures \result == \length(A->data);
{
  return A->length;
}

string ssa_get(ssa* A, int i)
/* @requires A != NULL;
/* @requires 0 <= i && i < ssa_len(A);
{
  return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

- Is the code for `ssa_get` safe?
 - The new postcondition of `ssa_len` takes care of the remaining safety check
 - `i < \length(A->data)` ✓
 - But `ssa_len` has a new precondition
 - `A->length == \length(A->data)`
 - we need to have a reason for why it is satisfied
 - but we don't
 - **Not supported!** ✗

Back to `ssa_get`

SSA Interface

```
int ssa_len(ssa_t A)
/* @requires A != NULL;          @*/
/* @ensures \result >= 0;       @*/;

string ssa_get(ssa_t A, int i)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
    string[] data; // sorted
    int length;   // = \length(data)
};
typedef struct ssa_header ssa;

int ssa_len(ssa* A)
/* @requires A != NULL;
/* @requires A->length == \length(A->data);
/* @ensures \result >= 0;
/* @ensures \result == \length(A->data);
{
    return A->length;
}

string ssa_get(ssa* A, int i)
/* @requires A != NULL;
/* @requires A->length == \length(A->data);
/* @requires 0 <= i && i < ssa_len(A);
{
    return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

● Is the code for `ssa_get` safe?

○ Add

`A->length == \length(A->data)`

as a precondition to `ssa_get` to support the safety of `ssa_len`

➤ `A->length == \length(A->data)`

□ By new precondition



Representation Invariants

Where are we?

SSA Interface

```
int ssa_len(ssa_t A)
/* @requires A != NULL;          @*/
/* @ensures \result >= 0;       @*/;

string ssa_get(ssa_t A, int i)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
    string[] data; // sorted
    int length;   // = \length(data)
};
typedef struct ssa_header ssa;

int ssa_len(ssa* A)
/*@requires A != NULL;
/*@requires A->length == \length(A->data);
/*@ensures \result >= 0;
/*@ensures \result == \length(A->data);
{
    return A->length;
}

string ssa_get(ssa* A, int i)
/*@requires A != NULL;
/*@requires A->length == \length(A->data);
/*@requires 0 <= i && i < ssa_len(A);
{
    return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

- All our code is safe
- Both functions have preconditions
 - A != NULL
 - A->length == \length(A->data)
 - ssa_set will need them too
 - and ssa_new will have them as postconditions

Where are we?

SSA Interface

```
int ssa_len(ssa_t A)
/* @requires A != NULL;          @*/
/* @ensures \result >= 0;       @*/;

string ssa_get(ssa_t A, int i)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
    string[] data; // sorted
    int length;    // = \length(data)
};
typedef struct ssa_header ssa;

int ssa_len(ssa* A)
/*@requires A != NULL;
/*@requires A->length == \length(A->data);
/*@ensures \result >= 0;
/*@ensures \result == \length(A->data);
{
    return A->length;
}

string ssa_get(ssa* A, int i)
/*@requires A != NULL;
/*@requires A->length == \length(A->data);
/*@requires 0 <= i && i < ssa_len(A);
{
    return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

- They are fundamental properties an **ssa*** must obey to be the representation of a valid SSA
 - NULL is not a valid SSA
 - The length field must be equal to the length of the array field data
- These are **invariants** of our representation:
 - Preconditions of every library function that takes an SSA as a parameter
 - Postcondition of every library function that returns or modifies an SSA

Representation Invariants

SSA Interface

```
int ssa_len(ssa_t A)
/* @requires A != NULL;          @*/
/* @ensures \result >= 0;       @*/;

string ssa_get(ssa_t A, int i)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
    string[] data; // sorted
    int length;    // = \length(data)
};
typedef struct ssa_header ssa;

int ssa_len(ssa* A)
/*@requires A != NULL;
/*@requires A->length == \length(A->data);
/*@requires is_sorted(A->data, 0, A->length);
/*@ensures \result >= 0;
/*@ensures \result == \length(A->data);
{
    return A->length;
}

string ssa_get(ssa* A, int i)
/*@requires A != NULL;
/*@requires A->length == \length(A->data);
/*@requires is_sorted(A->data, 0, A->length);
/*@requires 0 <= i && i < ssa_len(A);
{
    return A->data[i];
}
```

SSA Implementation

● Representation invariants

- Preconditions of every library function that takes an SSA as a parameter
- Postcondition of every library function that returns or modifies an SSA

○ Also called **data structure invariants**

● Do `ssa*` have other representation invariants?

- Yes! `A->data` should be sorted
`is_sorted(A->data, 0, A->length)`

Representation Invariants

SSA Interface

```
int ssa_len(ssa_t A)
/*@requires A != NULL;          @*/
/*@ensures \result >= 0;      @*/;

string ssa_get(ssa_t A, int i)
/*@requires A != NULL;          @*/
/*@requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
  string[] data; // sorted
  int length;    // = \length(data)
};
typedef struct ssa_header ssa;

bool is_ssa (ssa* A) { ... }

int ssa_len(ssa* A)
/*@requires is_ssa(A);
/*@ensures \result >= 0;
/*@ensures \result == \length(A->data);
{
  return A->length;
}

string ssa_get(ssa* A, int i)
/*@requires is_ssa(A);
/*@requires 0 <= i && i < ssa_len(A);
{
  return A->data[i];
}

// ... rest of implementation
```

SSA Implementation

- Factor them out into a single function that checks that they are satisfied

is_ssa

- Representation invariant function

Convention: name of representation invariant functions start with `is_`

Representation Invariants

SSA Interface

```
ssa_t ssa_new(int size)
/*@requires 0 <= size; @*/
/*@ensures \result != NULL; @*/
/*@ensures ssa_len(\result) == size; @*/;

void ssa_set(ssa_t A, int i, string x)
/*@requires A != NULL; @*/
/*@requires 0 <= i && i < ssa_len(A); @*/;
```

```
struct ssa_header {
  string[] data; // sorted
  int length; // = \length(data)
};
typedef struct ssa_header ssa;

bool is_ssa (ssa* A) { ... }

ssa* ssa_new(int size)
/*@requires size >= 0;
/*@ensures is_ssa(\result);
/*@ensures ssa_len(\result) == size;
{
  ssa* A = alloc(ssa);
  A->data = alloc_array(string, size);
  A->length = size;
  return A;
}

void ssa_set(ssa* A, int i, string x)
/*@requires is_ssa(A);
/*@requires 0 <= i && i < ssa_len(A);
/*@ensures is_ssa(A);
{ /* left as exercise */ }
```

SSA Implementation

- The remaining functions

- Precondition of every `ssa*` parameter

Defining the internal type `ssa` not to be a pointer allows simpler allocations

- Postcondition of

- every returned `ssa*`
- every modified `ssa*` parameter

The representation Invariant Function

```
struct ssa_header {
    string[] data; // sorted
    int length;   // = \length(data)
};
typedef struct ssa_header ssa;

// Representation invariant
bool is_ssa (ssa* A) {
    return A != NULL
        && A->length == \length(A->data)
        && is_sorted(A->data, 0, A->length);
}

// ... rest of implementation
```

SSA Implementation

Let's write it!

- 1st attempt: simply copy the contracts it stands for
- **Problem:** `\length` can only be used in contracts



The representation Invariant Function

```
struct ssa_header {
    string[] data; // sorted
    int length;   // = \length(data)
};
typedef struct ssa_header ssa;

// Representation invariant
bool is_ssa (ssa* A)
//@requires A->length == \length(A->data);
{
    return A != NULL
        && is_sorted(A->data, 0, A->length);
}

// ... rest of implementation
```

SSA Implementation

Let's write it!

- 2nd attempt: move that part in the precondition of `is_ssa`
 - **Problem:** this is unsafe!
 - A may be NULL
 - NULL checked only *after* the precondition



The representation Invariant Function

```
struct ssa_header {
    string[] data; // sorted
    int length;   // = \length(data)
};
typedef struct ssa_header ssa;

// Representation invariant
bool is_array_expected_length(string[] A, int len) {
    //@assert \length(A) == len;
    return true;
}

bool is_ssa (ssa* A) {
    return A != NULL
        && is_array_expected_length(A->data, A->length)
        && is_sorted(A->data, 0, A->length);
}

// ... rest of implementation
```

SSA Implementation

Let's write it!

- 3rd attempt: move it into a helper function



The representation Invariant Function

```
struct ssa_header {
    string[] data; // sorted
    int length;   // = \length(data)
};
typedef struct ssa_header ssa;

// Representation invariant
bool is_ssa (ssa* A) {
    if (A == NULL) return false;
    //@assert A->length == \length(A->data);
    return is_sorted(A->data, 0, A->length);
}

// ... rest of implementation
```

SSA Implementation

Let's write it!

- Alternative 3rd attempt: turn it into an **//@assert** in **is_ssa**



Things to Note

- The representation invariant function `is_ssa` is **NOT part of interface**
 - Clients are allowed to manipulate SSA's **only using the interface functions**
 - If the library is correct, `is_ssa(A)` will always return true
 - Client must ensure the safety of library calls according to the interface
 - ❑ `A != NULL` only
 - Providing `is_ssa` to clients would encourage them to bypass the interface
 - ❑ use `is_ssa` to test if hacks are successful
 - The representation invariant function is an implementation device to ensure the safety and correctness of the library code
 - Used while developing the library
 - Every function that takes an SSA A must have `//@requires is_ssa(A);`
 - Every function that modifies an input SSA A must have `//@ensures is_ssa(A);`
 - Every function that returns an SSA must have `//@ensures is_ssa(\result);`

Things to Note

- The contracts in the interface and the implementation are **different**

Interface	Implementation
<pre>void ssa_set(ssa_t A, int i, string x) /* @requires A != NULL; @*/ /* @requires 0 <= i && i < ssa_len(A); @*/;</pre>	<pre>void ssa_set(ssa* A, int i, string x) //@requires is_ssa(A); //@requires 0 <= i && i < ssa_len(A); //@ensures is_ssa(A);</pre>

- The implementation contracts are more detailed
 - `is_ssa(A)` checks `A != NULL`
 - The implementation contains more information, so it needs to check more things
 - There is no point having `//@ensures A != NULL` in the interface
 - `ssa_set` is called with a *copy* of the address of A
 - when returning, the original has not changed, even if `ssa_set` modified its copy of A
 - ❑ If original A was not NULL when calling `ssa_set`, it will not be NULL when returning from it

Overall Implementation

```
// Implementation-side type
struct ssa_header {
    string[] data; // sorted
    int length;    // = \length(data)
};
typedef struct ssa_header ssa;

// Representation invariant
bool is_sorted(string* A, int lo, int hi) {
    /* left as exercise */
}

bool is_ssa(ssa* A) {
    if (A == NULL) return false;
    //@assert A->length == \length(A->data);
    return is_sorted(A->data, 0, A->length);
}

// Implementation of interface functions
int ssa_len(ssa* A)
//@requires is_ssa(A);
//@ensures \result >= 0;
//@ensures \result == \length(A->data);
{
    return A->length;
}

ssa* ssa_new(int size)
//@requires size >= 0;
//@ensures is_ssa(\result);
//@ensures ssa_len(\result) == size;
{
    ssa* A = alloc(ssa);
    A->data = alloc_array(string, size);
    A->length = size;
    return A;
}

string ssa_get(ssa* A, int i)
//@requires is_ssa(A);
//@requires 0 <= i && i < ssa_len(A);
{
    return A->data[i];
}

void ssa_set(ssa* A, int i, string x)
//@requires is_ssa(A);
//@requires 0 <= i && i < ssa_len(A);
//@ensures is_ssa(A);
{ /* left as exercise */
}

// Client type
typedef ssa* ssa_t;
```

SSA Implementation

- By convention, we put the interface after the implementation in the same file

SSA Interface

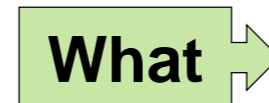
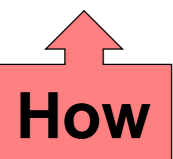
```
// typedef _____* ssa_t;

int ssa_len(ssa_t A)
/* @requires A != NULL;          @*/
/* @ensures \result >= 0;       @*/;

ssa_t ssa_new(int size)
/* @requires 0 <= size;          @*/
/* @ensures \result != NULL;    @*/
/* @ensures ssa_len(\result) == size; @*/;

string ssa_get(ssa_t A, int i)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;

void ssa_set(ssa_t A, int i, string x)
/* @requires A != NULL;          @*/
/* @requires 0 <= i && i < ssa_len(A); @*/;
```



Structure of a C0 Library File

```
/****** IMPLEMENTATION *****/
// Implementation-side type
struct ssa_header {
  ...
};
typedef struct ssa_header ssa;

// Representation invariant
bool is_ssa(ssa* A) {
  ...
}

// Implementation of interface functions
int ssa_len(ssa* A) { ... }

...

// Client type
typedef ssa* ssa_t;

/****** LIBRARY INTERFACE *****/
// typedef _____* ssa_t;

int ssa_len(ssa_t A)
/*@requires A != NULL;          @*/
/*@ensures \result >= 0;      @*/;

...
```

Implementation

Interface

- Implementation
 - Concrete type definition
 - Representation invariant function
 - Implementation of interface functions
 - Client type definition
- Interface
 - Abstract type name
 - Pseudo-**typedef**
 - Prototype of exported functions

We will revisit this

Compiling a Library in a C0 Application

- Library file contains both implementation and interface
- When compiling, library files come **before** application files
 - The application uses library interface types and functions
 - They need to be defined first
 - this happens in the library

