15-281 Fall 2023 — Lecture Activity

Learning Objectives

- To practice computing HMM probabilities
- To run Predict and Observe steps in the forward algorithm

Q1. Hidden Markov Models

Below is the Weather HMM from class. We want to predict the weather given both a distribution for weather today given weather yesterday (transition model) and weather today given people using umbrella today (sensor model). We want to compute the weather on day 4 $P(X_4|e_1, e_2, e_3, e_4)$ assuming that we've witnessed umbrellas each day (all the evidence is True that there is an umbrella). Note that the states are the weather $X_i = W_i$ and the evidence is the umbrella $e_i = U_i$.

An HMM is defined by:

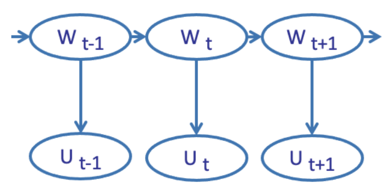
- Initial distribution: $P(X_1)$
- Transition model: $P(X_t \mid X_{t-1})$
- Sensor model: $P(E_t \mid X_t)$

W_{t-1}	$P(W_t \mid W_{t-1})$	
	sun	rain
sun	0.9	0.1
rain	0.3	0.7

W_t	$P(U_t \mid W_t)$	
	true	false
sun	0.2	0.8
rain	0.9	0.1

Given $P(X_1)$ = {sun:0.5, rain:0.5}

Compute $P(X_4\text{=sun} \mid e_4 = e_3 = e_2 = e_1 = \text{True})$



(a) OBSERVE! We are given the initial distribution $P(X_1)$. Now we need to Observe the evidence $e_1 = True$ and compute $P(X_1|e_1)$. We can derive the equation for $P(X_1|e_1)$ directly using Bayes Rule with the probabilities $P(e_1|X_1)$ and $P(X_1)$ or by computing the joint $P(X_1, e_1)$ and normalizing $Z = P(e_1)$. Write the equation below and then compute the probability table $P(X_1|e_1)$.

$P(X_1 \mid e_1) =$	**Table** $P(X_1 \mid e_1)$

(b) PREDICT! Now we have $P(X_1|e_1)$, and we want to Predict $P(X_2|e_1)$. We can do this by summing over X_1: $P(X_2|e_1) = \sum_{x \in X_1} P(X_2|x)P(x|e_1)$. Rewrite this equation below and then compute the probability table using your answer above and the HMM model tables.

$P(X_2 \mid e_1) =$	**Table** $P(X_2 \mid e_1)$

An HMM is defined by:

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- Sensor model: $P(E_t \mid X_t)$

W_{t-1}	$P(W_t \mid W_{t-1})$	
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sun	0.2	0.8
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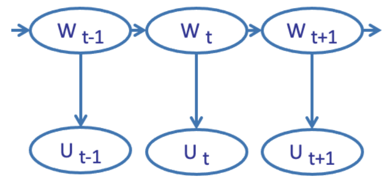
Given $P(X_1)$ = {sun:0.5, rain:0.5}
Compute $P(X_4\text{=sun} \mid e_4\text{=}e_3\text{=}e_2\text{=}e_1\text{=True})$



(c) OBSERVE! Now that we've predicted X_2, we can update the probability given new evidence $e_2 = True$. Use the Observation update to write the formula for $P(X_2|e_1, e_2)$ using $P(X_2|e_1)$ above and then solve.

$P(X_2|e_1, e_2) =$	**Table** $P(X_2|e_1, e_2)$

(d) PREDICT! Compute $P(X_3|e_1, e_2)$ using the transition probabilities and $P(X_2|e_1, e_2)$ above. Write this equation below and then compute the probability table.

$P(X_3|e_1, e_2) =$	**Table** $P(X_3|e_1, e_2)$

(e) OBSERVE! Now that we've predicted X_3, we can update the probability given new evidence $e_3 = True$. Use the Observation update to write the formula for $P(X_3|e_1, e_2, e_3)$ using $P(X_3|e_1, e_2)$ above and then solve.

$P(X_3|e_1, e_2, e_3) =$	**Table** $P(X_3|e_1, e_2, e_3)$

(f) PREDICT! Compute $P(X_4|e_1, e_2, e_3)$ using the transition probabilities and $P(X_3|e_1, e_2, e_3)$ above. Write the equation below and then compute the probability table.

$P(X_4|e_1, e_2, e_3) =$	**Table** $P(X_4|e_1, e_2, e_3)$

(g) OBSERVE! Finally, we can update the probability of X_4 given new evidence $e_4 = True$ (and the rest of the evidence). Use the Observation update rule to write the formula for $P(X_4|e_1, e_2, e_3, e_4)$ using $P(X_4|e_1, e_2, e_3)$ above and then solve for the new probability table.

$P(X_4|e_1, e_2, e_3, e_4) =$	**Table** $P(X_4|e_1, e_2, e_3, e_4)$