Learning Objectives

- To graph and solve a linear programming problem given a set of constraints
- To run the branch and bound algorithm to find IP solutions

Q1. Linear Programming

Below is a 2D linear programming (LP) problem. Rewrite it in inequality form with the matrices A, b, and c. Graph the constraints and the cost vector and find the solution.

$$\min_{x,y} -x - 3y$$

$-1.4x - y \le -4.58$ (orange)	$1.56x - y \le 3.42$ (green)
$1.9x + y \le 12.16$ (red)	$-0.44x + y \le 4.21$ (blue)

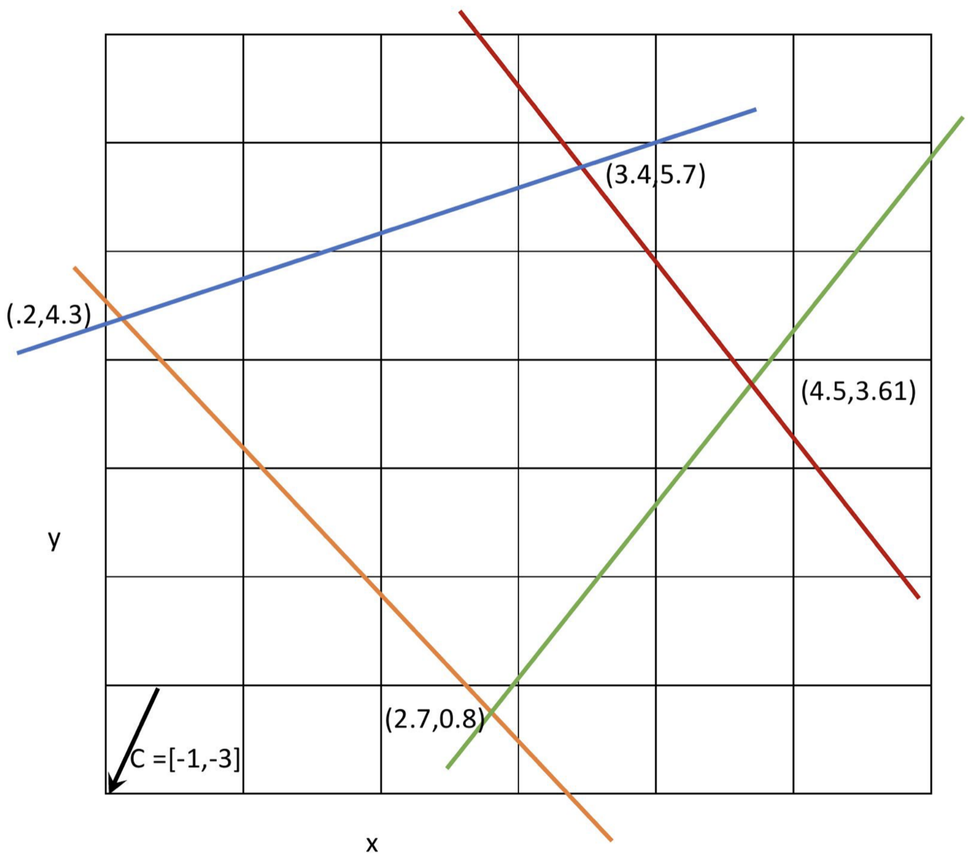

Figure 1: Plot of 2D Constraints. The solution is (3.4, 5.7).

Q2. Branch and Bound

Now consider the problem as an IP problem. Using the branch and bound algorithm described in lecture to find the integer solution. Remember to have your priority queue ordered by the objective value of each LP problem.

What is the IP solution?

We add the original LP to the PQ with priority -20.5.

we pull it off the queue, we chose to branch on the x (you could have branched on y instead). We create two new LP problems; one that adds the constraint $x \leq 3$ and the other that adds the constraint $x \geq 4$.

The PQ now has:

- Original+[$x \leq 3$] leads to solution (3,5.53) and has priority -19.59
- Original+[$x \geq 4$] leads to solution (4,4.56) and has priority -17.68

We pull off the Original+[$x \leq 3$] solution. It has an integer solution for x and non-integer for y, so we branch on y. We create two new problems; one adds the constraint $y \leq 5$ and the other adds the constraint $y \geq 6$.

The PQ now has:

- Original+[$x \leq 3$]+[$y \leq 5$] leads to solution (3,5) and has priority -18
- Original+[$x \geq 4$] leads to solution (4,4.56) and has priority -17.68
- Original+[$x \leq 3$]+[$y \geq 6$] leads to solution [not feasible] and has priority +infinity

We pull off the Original+[$x \leq 3$]+[$y \leq 5$] solution because it has the highest priority (lowest cost). It has an integer solution so the IP solution is (3,5).