# Announcements

## Assignments:

- HW2 (written)
  - Due Tuesday 9/12, 10pm
- P1: Search
  - Due Monday 9/18, 10pm
  - Working in pairs is suggested but not required

## Polls

- Don’t worry if you miss a few
- Talk to us if you are **systematically** missing polls

# Announcements

## Recitation

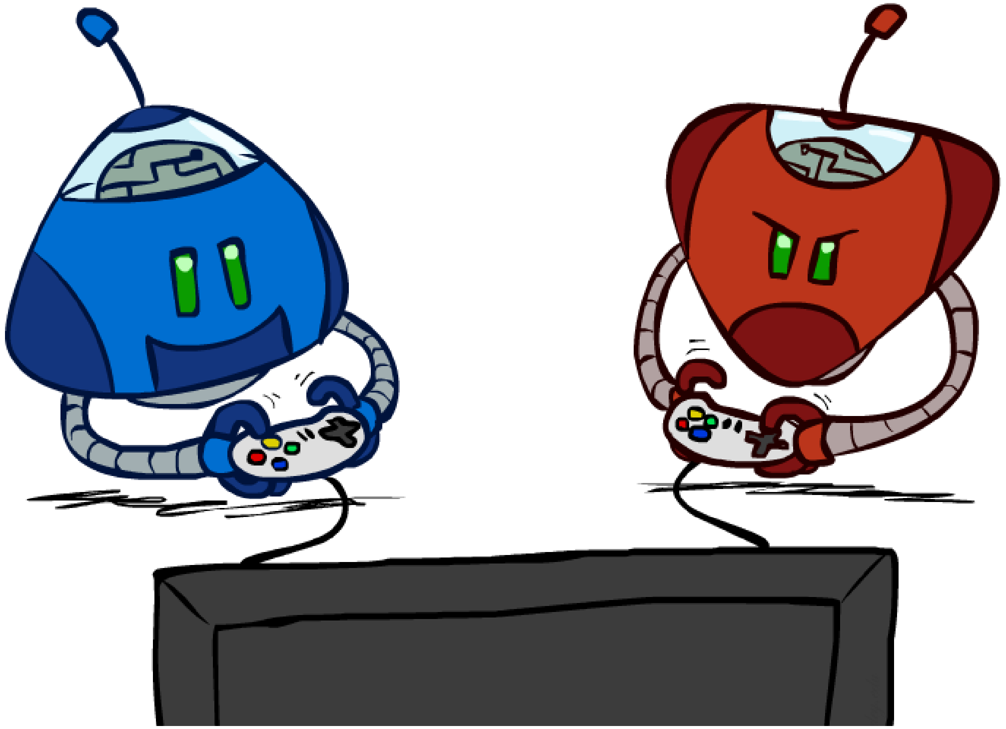
- Join any recitation you want this week
- Stay tuned to Piazza for post about informally changing section

## More coming on Piazza

- Recitation change form (probably end of next week)

# AI: Representation and Problem Solving

## Adversarial Search



Instructors: Vincent Conitzer and Aditi Raghunathan

Slide credits: CMU AI, http://ai.berkeley.edu

# Outline

History / Overview

Zero-Sum Games (Minimax)

Evaluation Functions

Search Efficiency (α-β Pruning)

Games of Chance (Expectimax)

# Game Playing State-of-the-Art

**Checkers:**

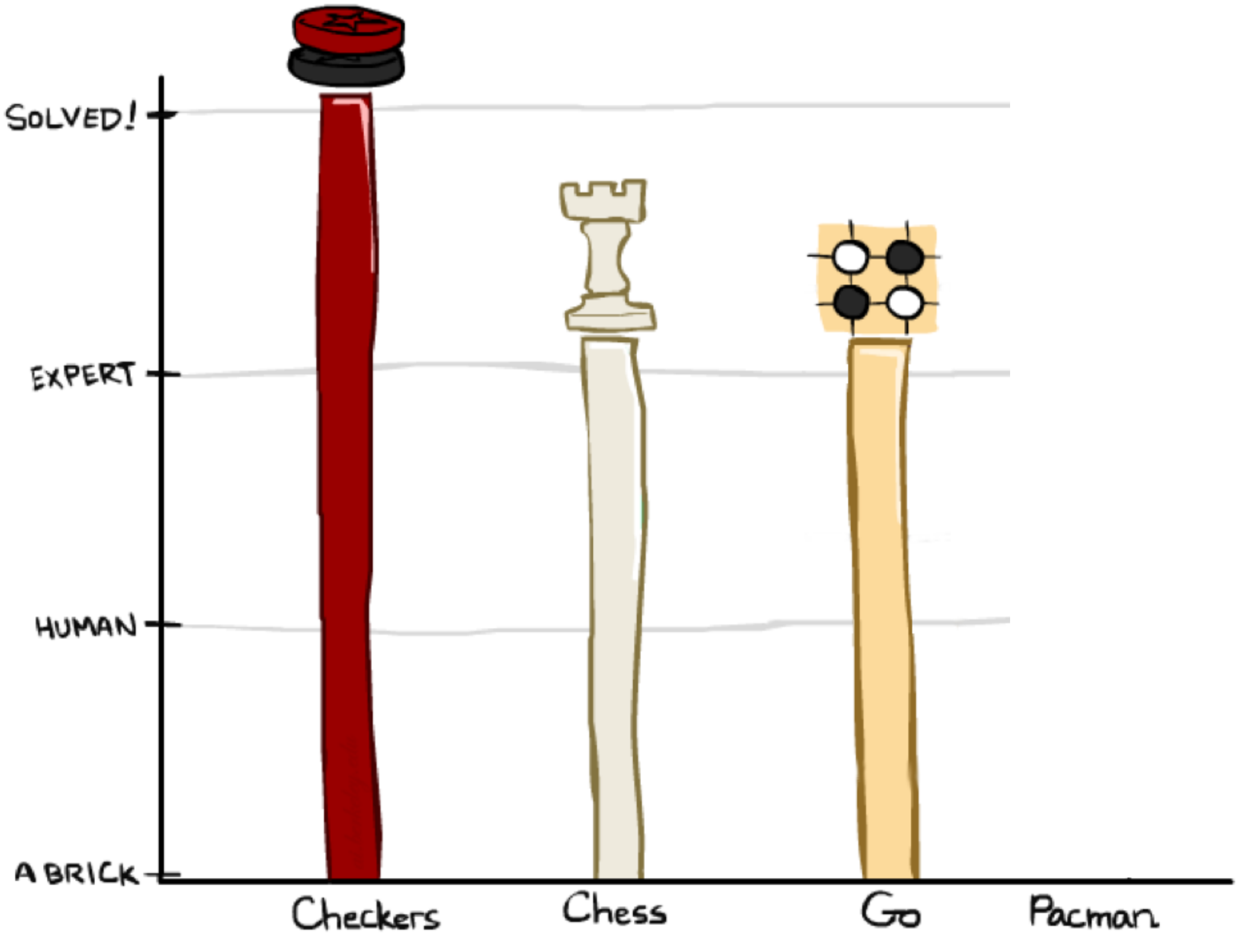
- 1950: First computer player.
- 1959: Samuel's self-taught program.
- 1994: First computer world champion: Chinook ended 40-year-reign of human champion Marion Tinsley using complete 8-piece endgame.
- 2007: Checkers solved! Endgame database of 39 trillion states

**Chess:**

- 1945-1960: Zuse, Wiener, Shannon, Turing, Newell & Simon, McCarthy.
- 1960s onward: gradual improvement under "standard model"
- 1997: special-purpose chess machine Deep Blue defeats human champion Gary Kasparov in a six-game match. Deep Blue examined 200M positions per second and extended some lines of search up to 40 ply. Current programs running on a PC rate > 3200 (vs 2870 for Magnus Carlsen).

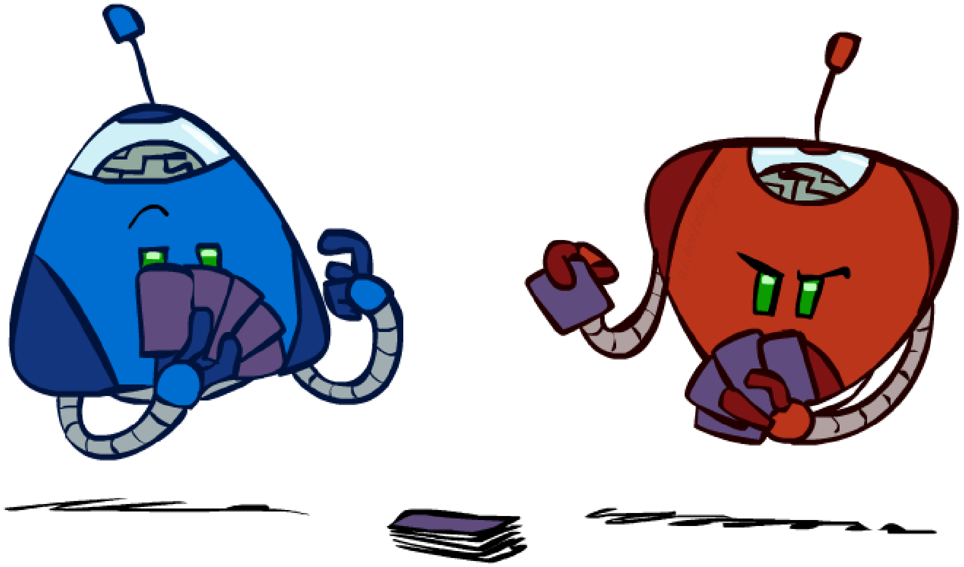
**Go:**

- 1968: Zobrist's program plays legal Go, barely (b>300!)
- 2005-2014: Monte Carlo tree search enables rapid advances: current programs beat strong amateurs, and professionals with a 3-4 stone handicap.
- 2015: AlphaGo from DeepMind beats Lee Sedol



# Types of Games

Many different kinds of games!

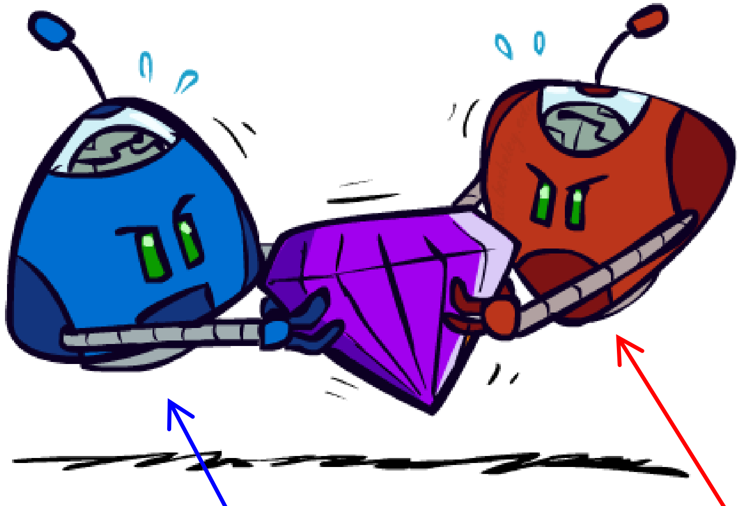


Axes:

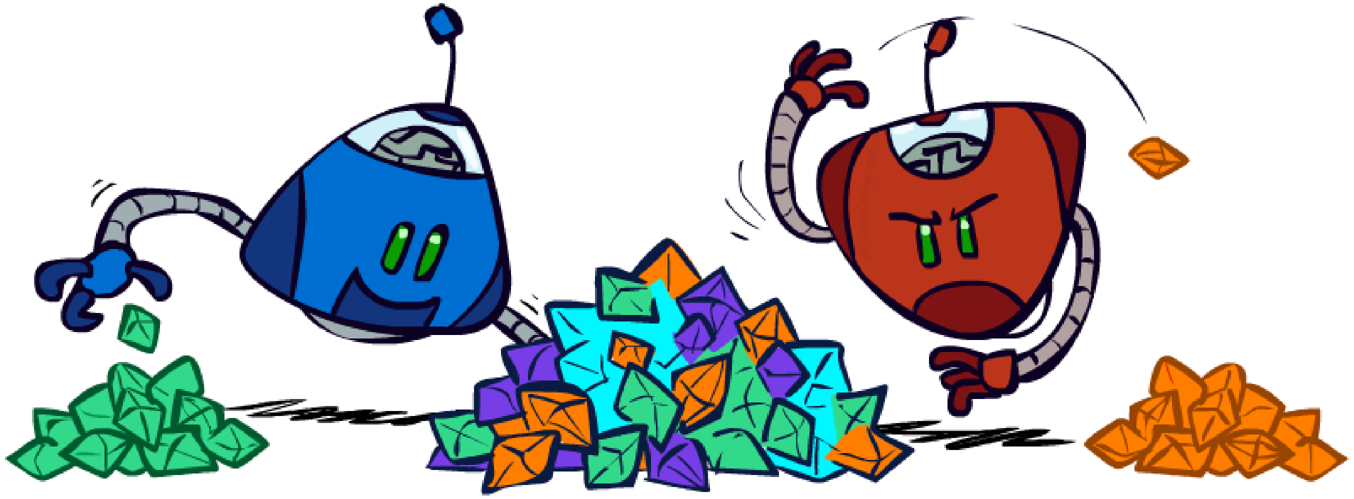
- Deterministic or stochastic?
- Perfect information (fully observable)?
- One, two, or more players?
- Turn-taking or simultaneous?
- Zero sum?

Want algorithms for calculating a ***contingent plan*** (a.k.a. strategy or policy) which recommends a move for every possible eventuality

# Zero-Sum Games



- Two-Player Zero-Sum Games
  - Agents have ***opposite*** utilities
  - Pure competition:
    - One ***maximizes***, the other ***minimizes***



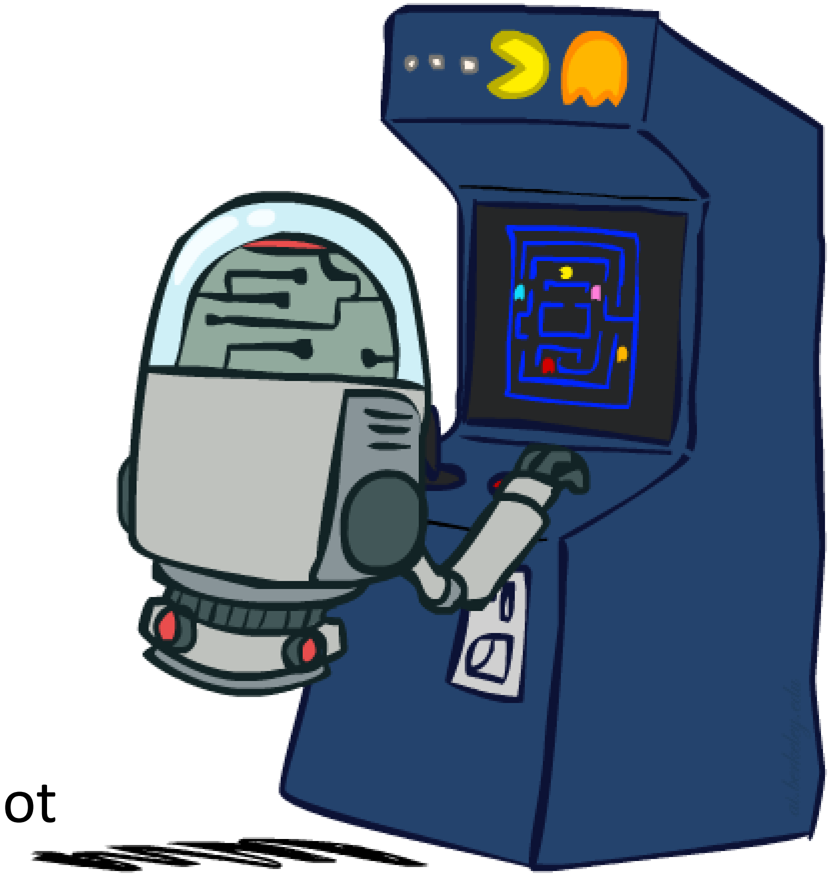
- General Games
  - Agents have ***independent*** utilities
  - Cooperation, indifference, competition, shifting alliances, and more are all possible

# “Standard” Games

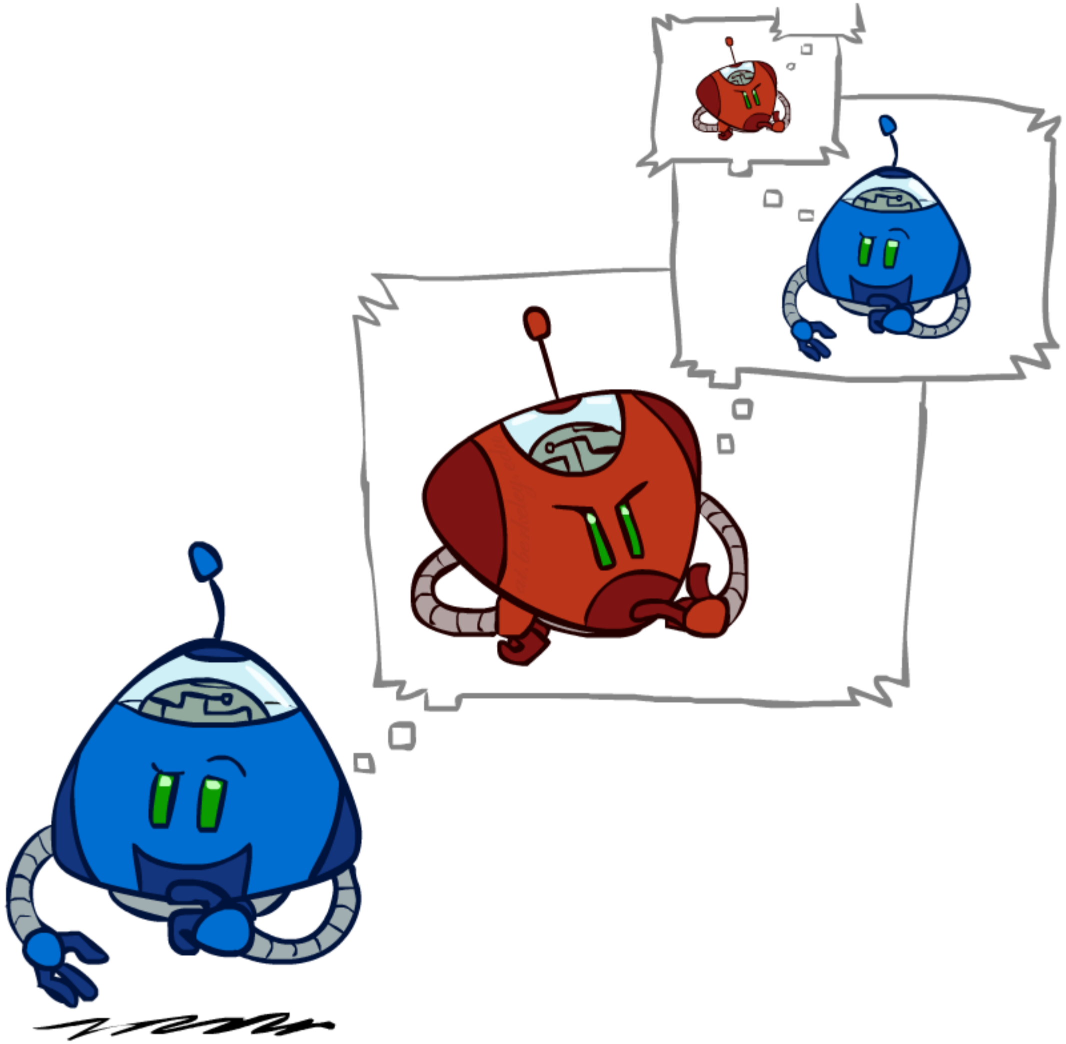
Standard games are deterministic, observable, two-player, turn-taking, zero-sum

Game formulation:

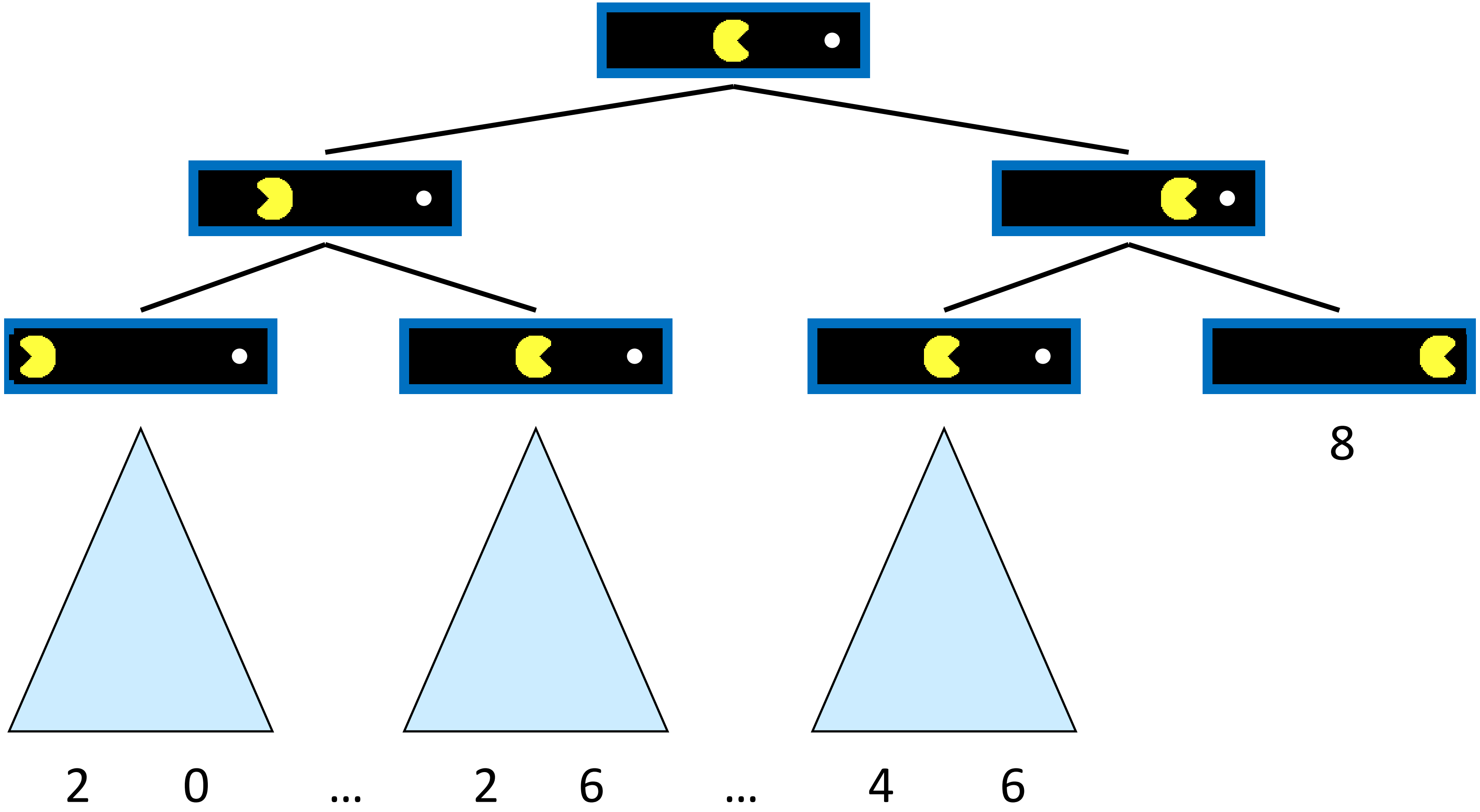
- Initial state: $s_0$
- Players: Player(s) indicates whose move it is
- Actions: Actions(s) for player on move
- Transition model: Result(s,a)
- Terminal test: Terminal-Test(s)
- Terminal values: Utility(s,p) for player p
  - Or just Utility(s) for player making the decision at root



# Adversarial Search



# Single-Agent Trees

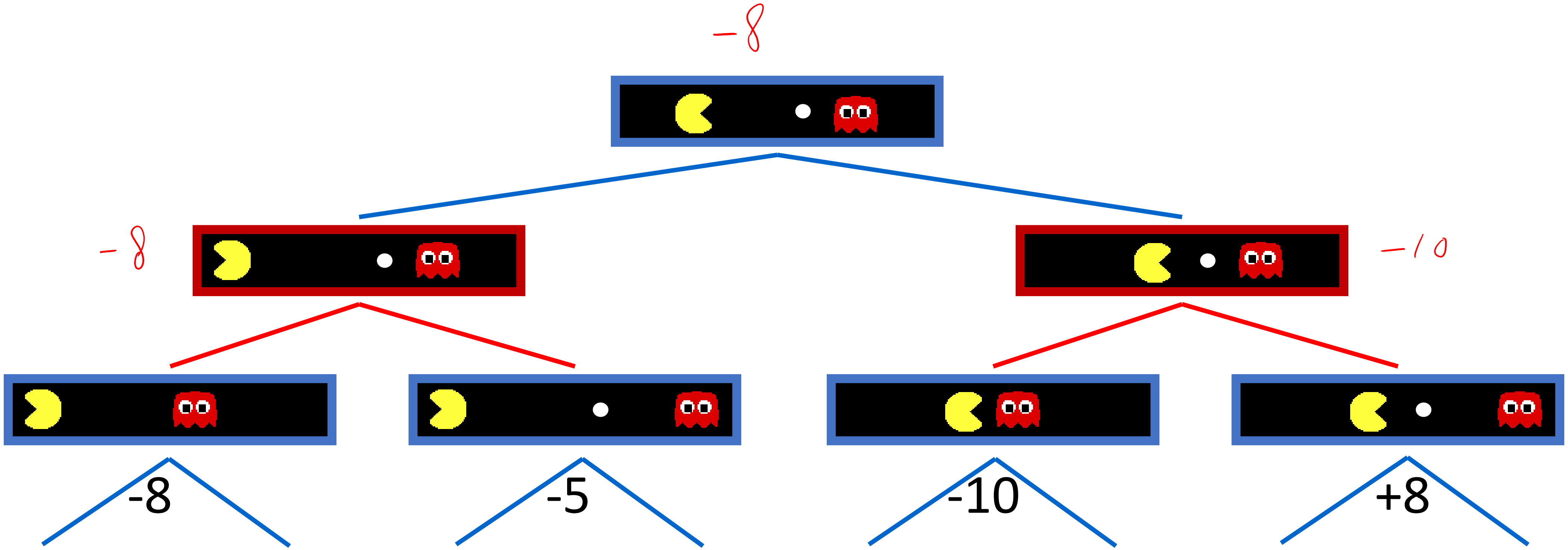


# Minimax

States

Actions

Values

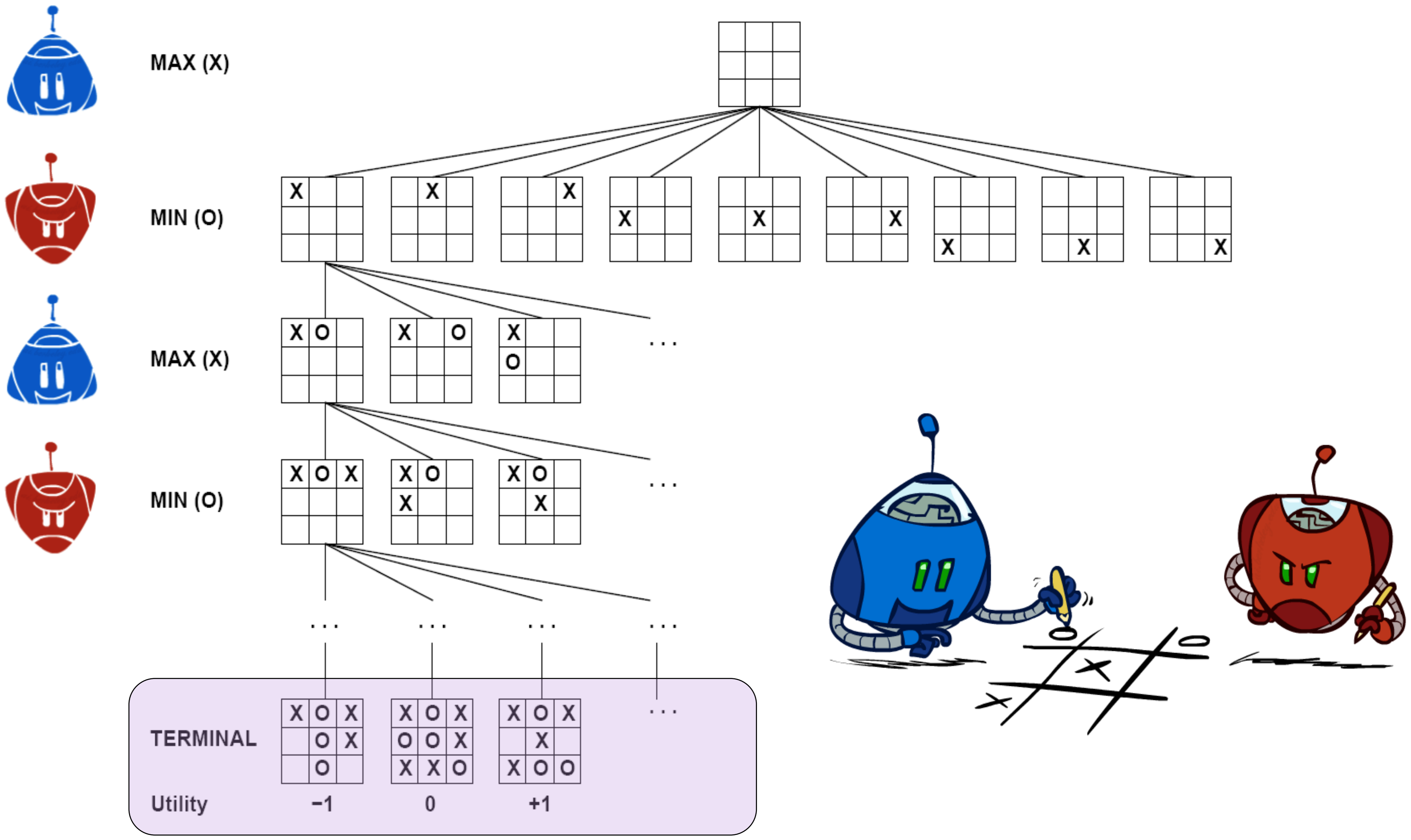


# Minimax

States

Actions

Values



# Minimax Code

```
def max_value(state):

    if state.is_leaf:
        return state.value
    # TODO Also handle depth limit

    best_value = -10000000

    for action in state.actions:
        next_state = state.result(action)

        next_value = min_value(next_state)

        if next_value > best_value:
            best_value = next_value

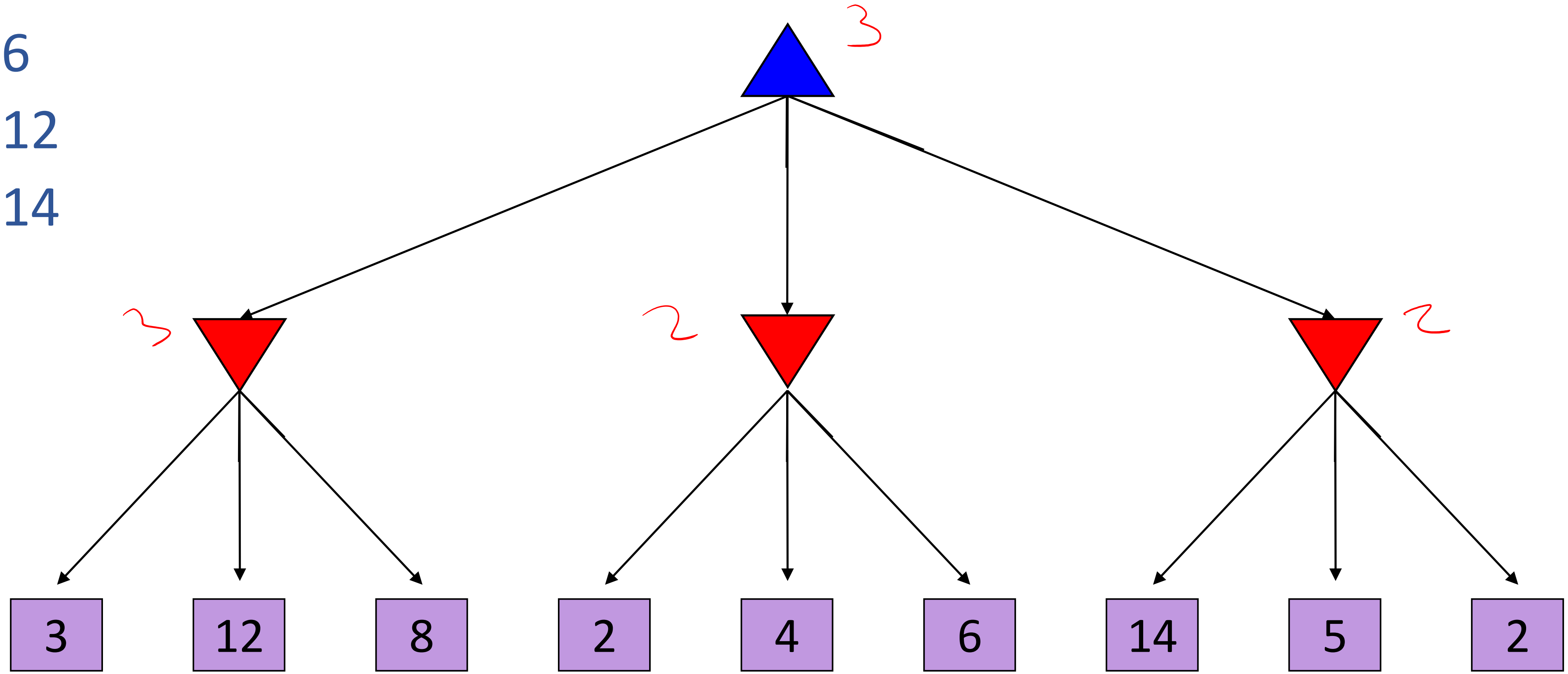
    return best_value

def min_value(state):
```

# Poll 1 (+ worksheet Poll 2 and 3 for Q1a/b)

What is the minimax value at the root?

A) 2
B) 3
C) 6
D) 12
E) 14



# Poll 1

What is the minimax value at the root?

A) 2

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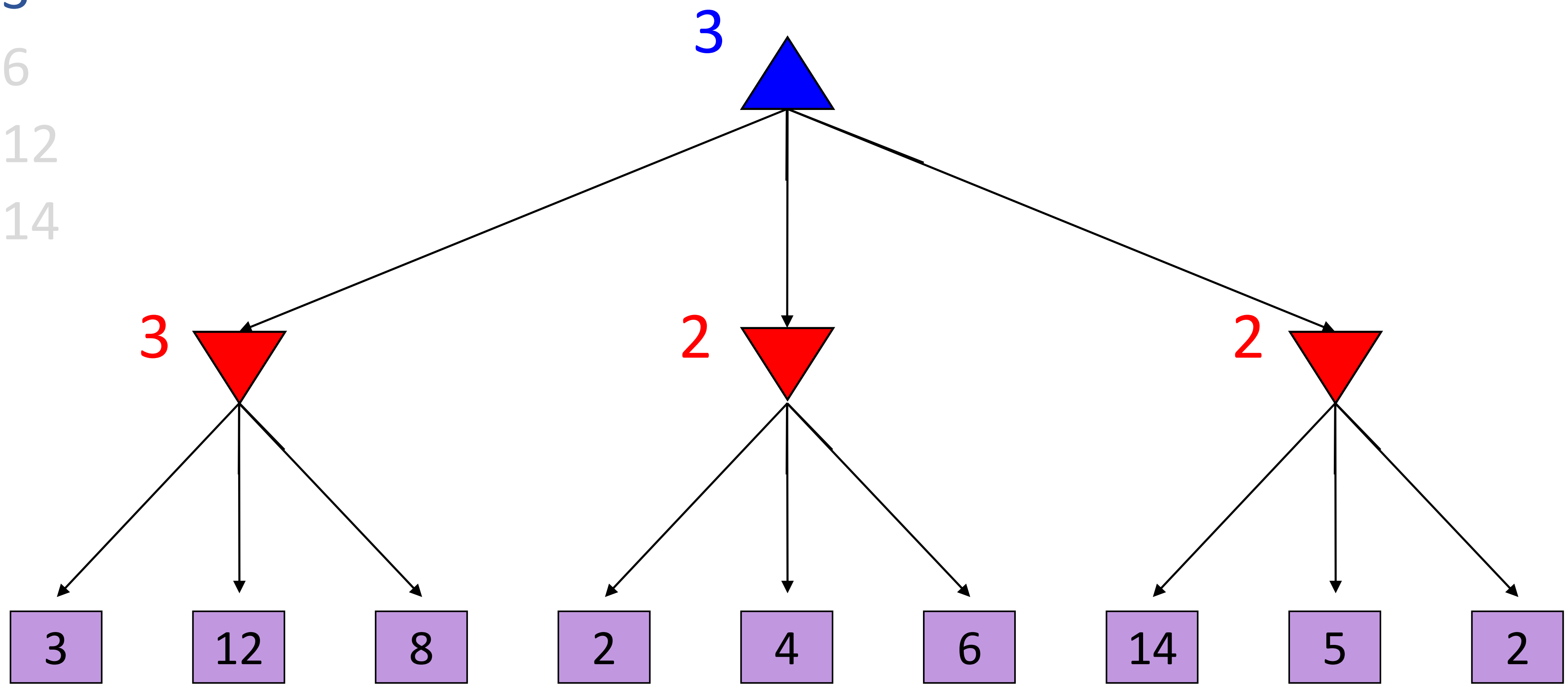

# Minimax Notation

```
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        if next_value > best_value:
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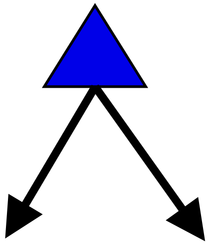
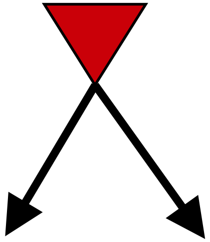
    return best_value

def min_value(state):
```

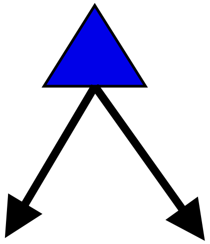




$$V(s) = \max_{a} V(s'),$$

where $s' = result(s, a)$

# Minimax Notation



$$V(s) = \max_{a} V(s'),$$

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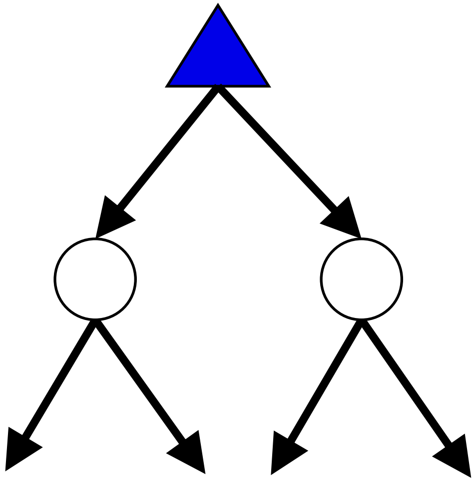


$$\hat{a} = \operatorname{argmax}_{a} V(s'),$$

where $s' = result(s, a)$

# Generic Game Tree Pseudocode

```
function minimax_decision( state )
    return argmax $_{\text{a in state.actions}}$ value( state.result(a) )

function value( state )
    if state.is_leaf
        return state.value

    if state.player is MAX
        return max $_{\text{a in state.actions}}$ value( state.result(a) )

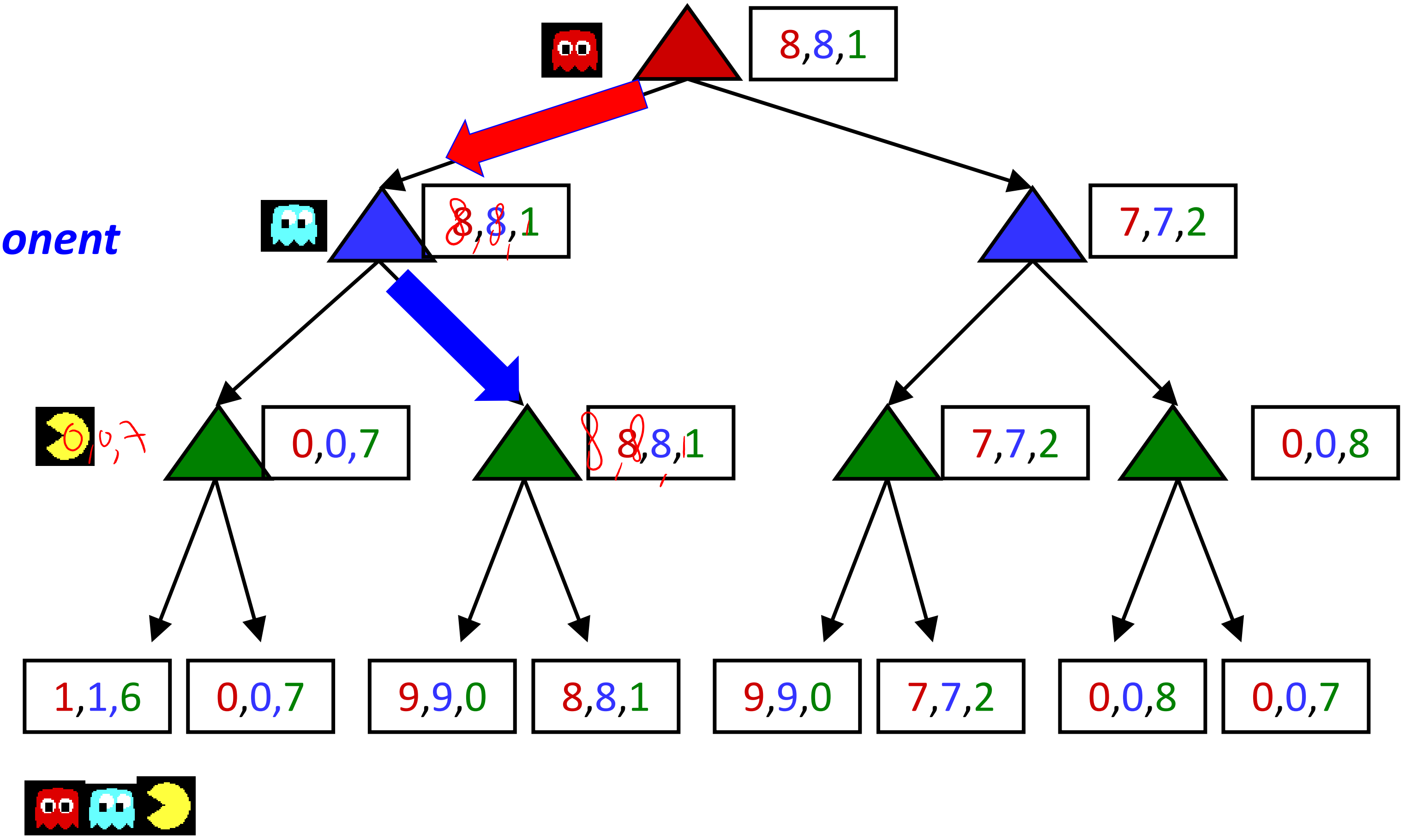
    if state.player is MIN
        return min $_{\text{a in state.actions}}$ value( state.result(a) )
```

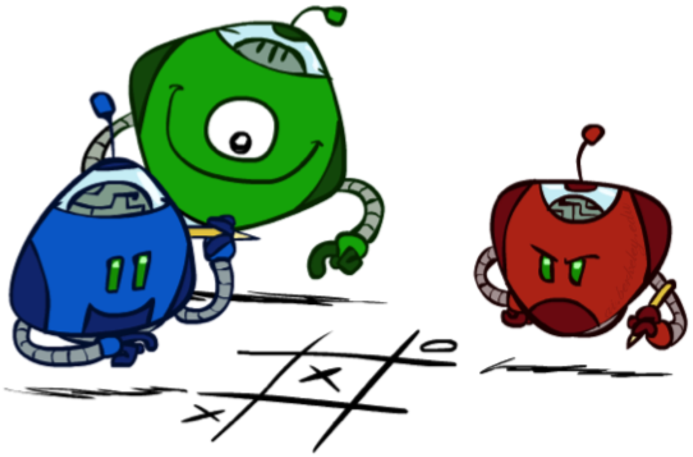
# Generalized minimax (better name: backward induction)

## What if the game is not zero-sum, or has multiple players?

### Generalization of minimax:

- Terminals have ***utility tuples***
- Node values are also utility tuples
- ***Each player maximizes its own component***
- Can give rise to cooperation and competition dynamically…





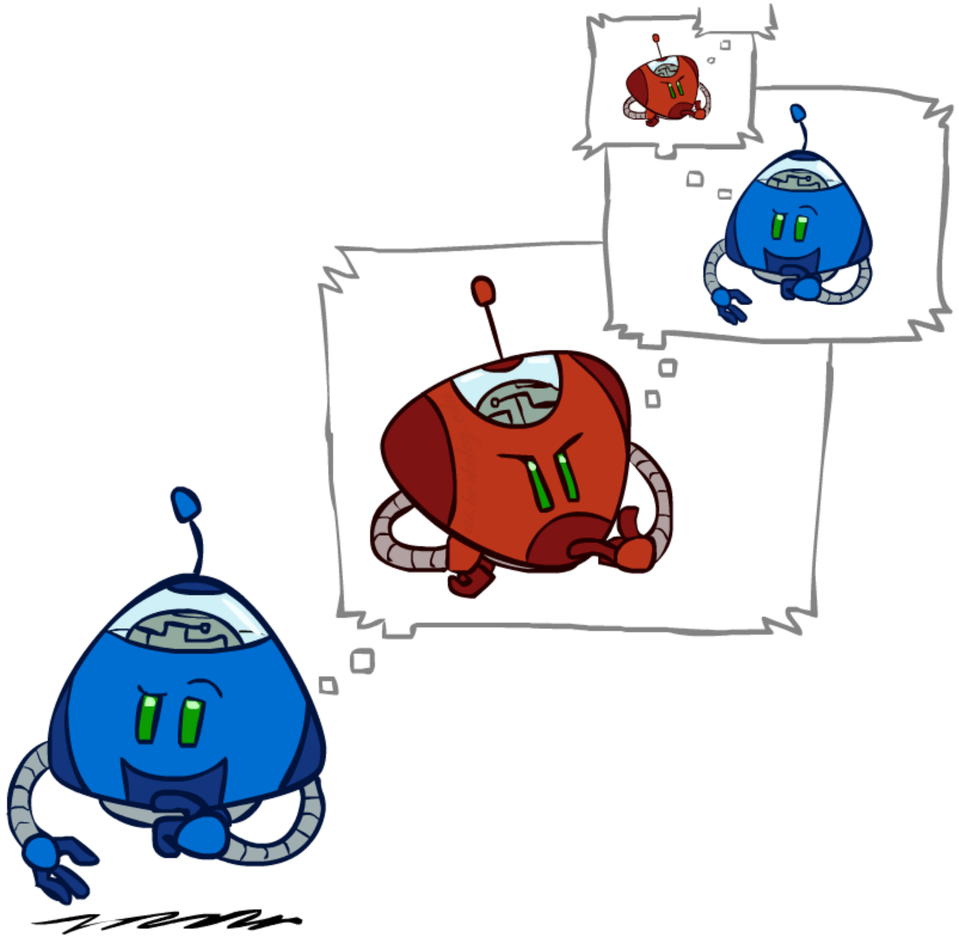
# Minimax Efficiency

## How efficient is minimax?

- Just like (exhaustive) DFS
- Time: $O(b^m)$
- Space: $O(bm)$

## Example: For chess, $b \approx 35$, $m \approx 100$

- Exact solution is completely infeasible
- Humans can't do this either, so how do we play chess?
- Bounded rationality – Herbert Simon



# Resource Limits

# Resource Limits

Problem: In realistic games, cannot search to leaves!

Solution 1: Bounded lookahead

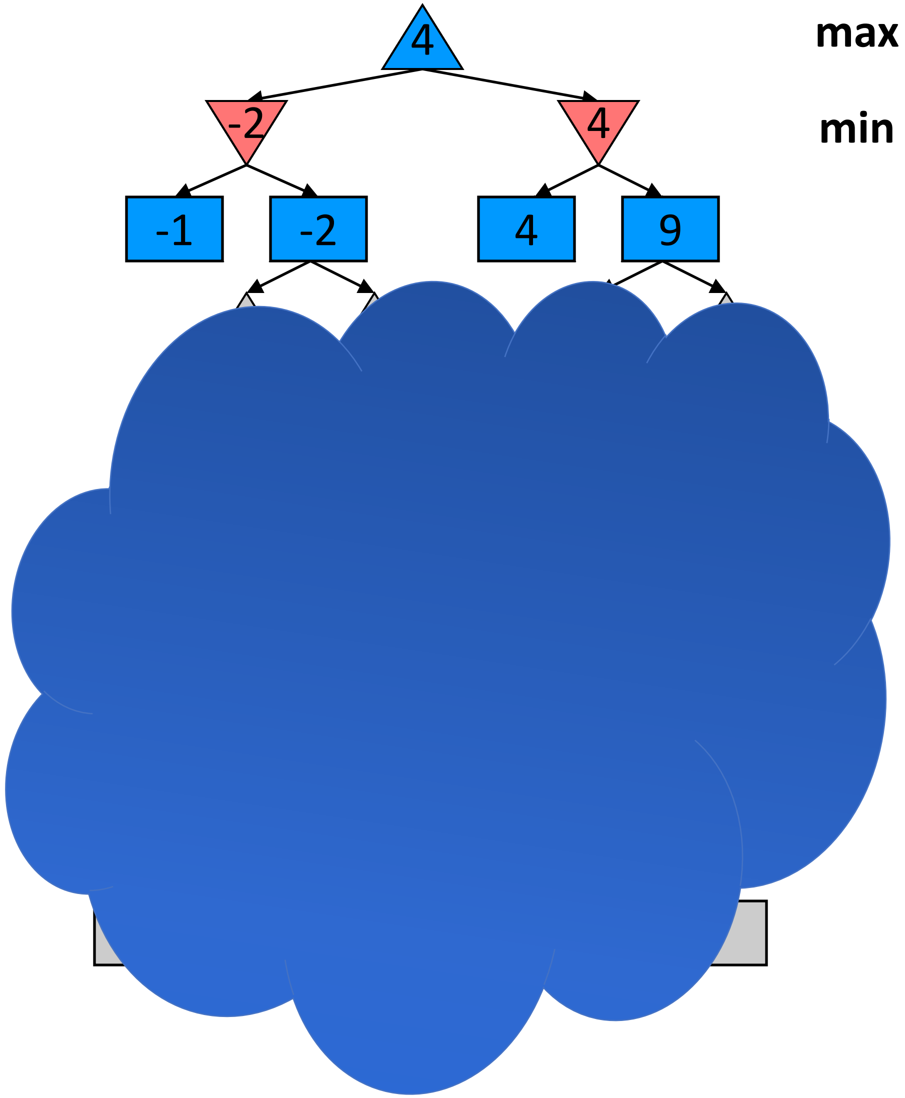
- Search only to a preset ***depth limit*** or ***horizon***
- Use an ***evaluation function*** for non-terminal positions

Guarantee of optimal play is gone

More plies make a BIG difference

Example:

- Suppose we have 100 seconds, can explore 10K nodes / sec
- So can check 1M nodes per move
- For chess, b=~35 so reaches about depth 4 – not so good



# Depth Matters

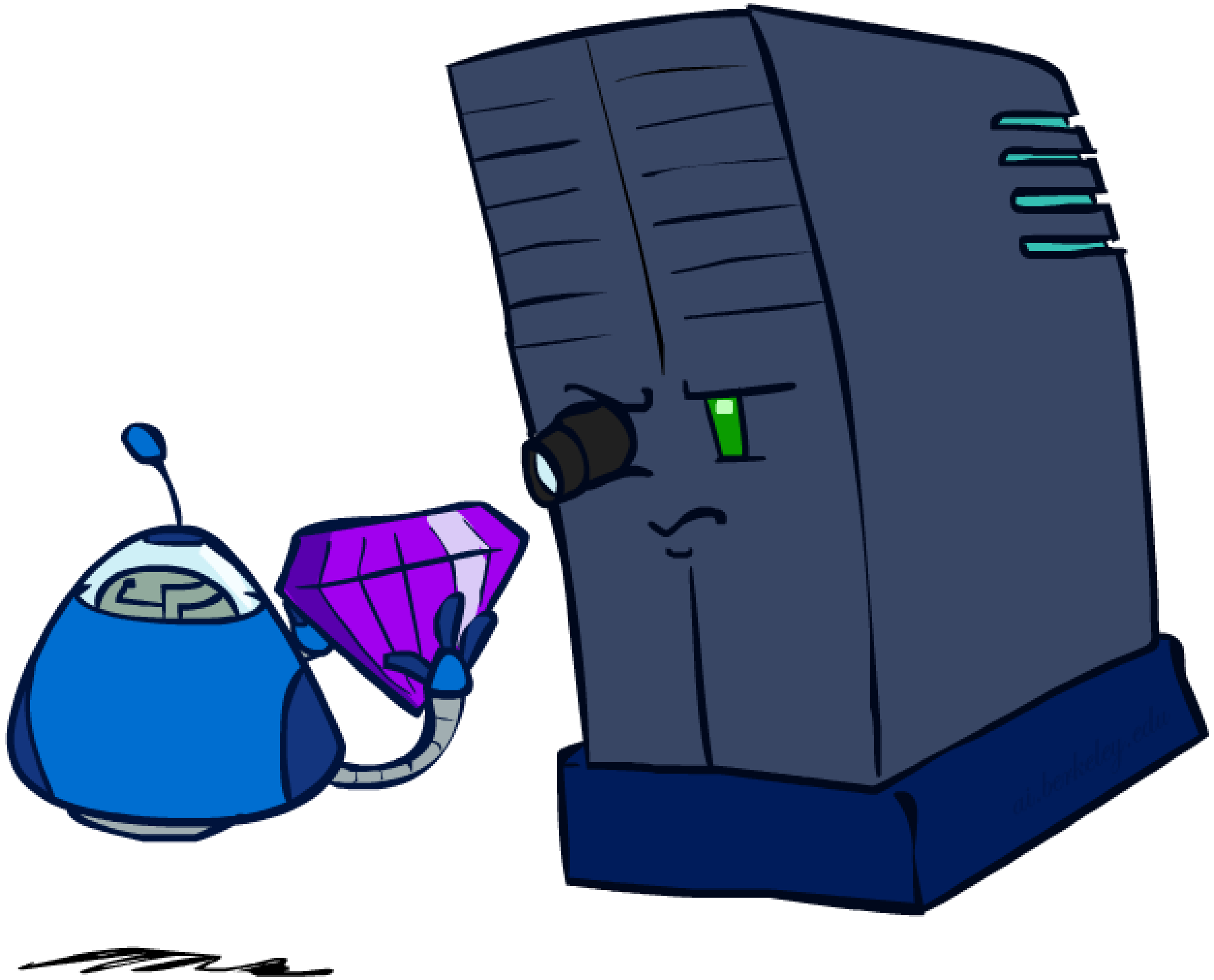
Evaluation functions are always imperfect

Deeper search => better play (usually)

Or, deeper search gives same quality of play with a less accurate evaluation function

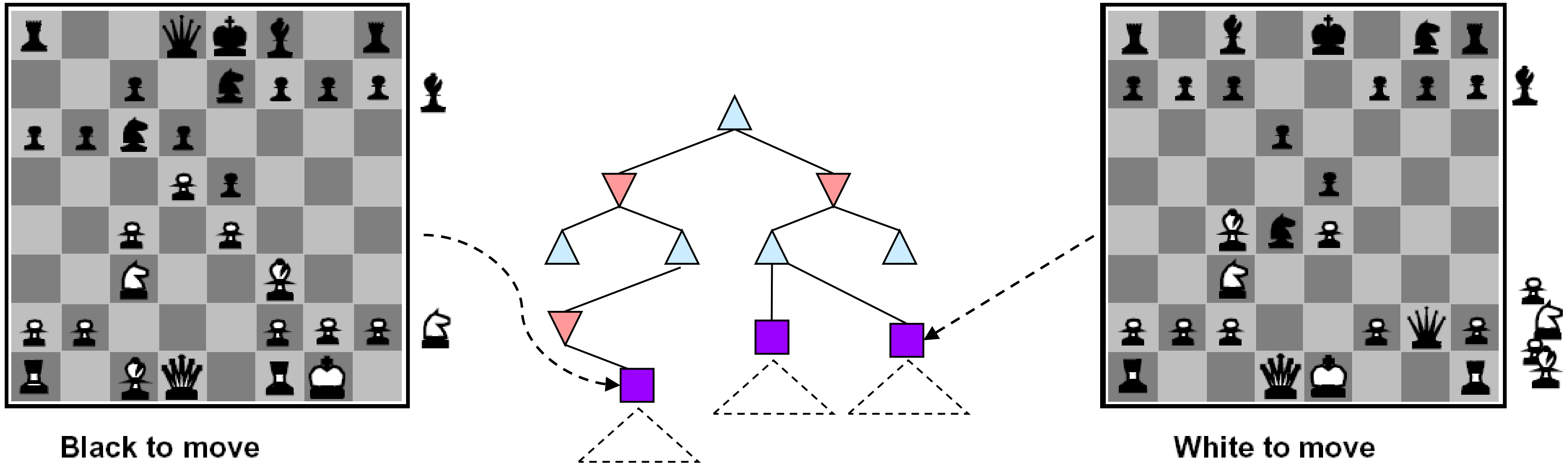
An important example of the tradeoff between complexity of features and complexity of computation

# Evaluation Functions



# Evaluation Functions

Evaluation functions score non-terminals in depth-limited search



**Black to move**

**White slightly better**

**White to move**

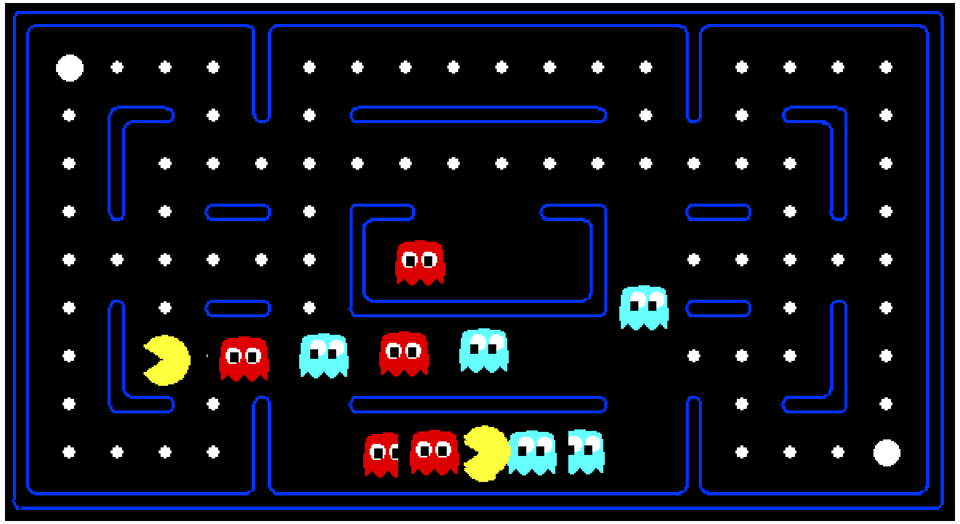
**Black winning**

Ideal function: returns the actual minimax value of the position

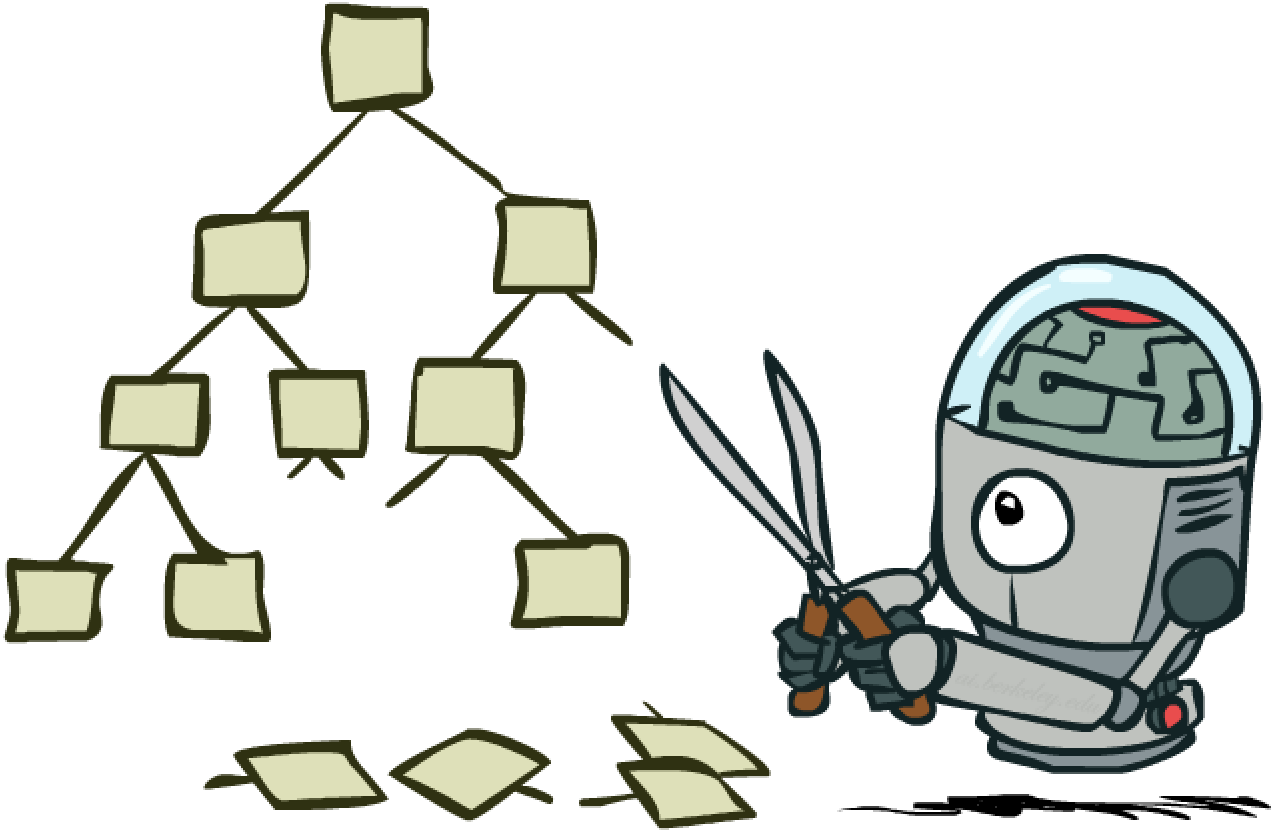
In practice: typically weighted linear sum of features:

- EVAL(s) = $w_1 f_1(s) + w_2 f_2(s) + .... + w_n f_n(s)$
- E.g., $w_1 = 9$, $f_1(s)$ = (num white queens – num black queens), etc.

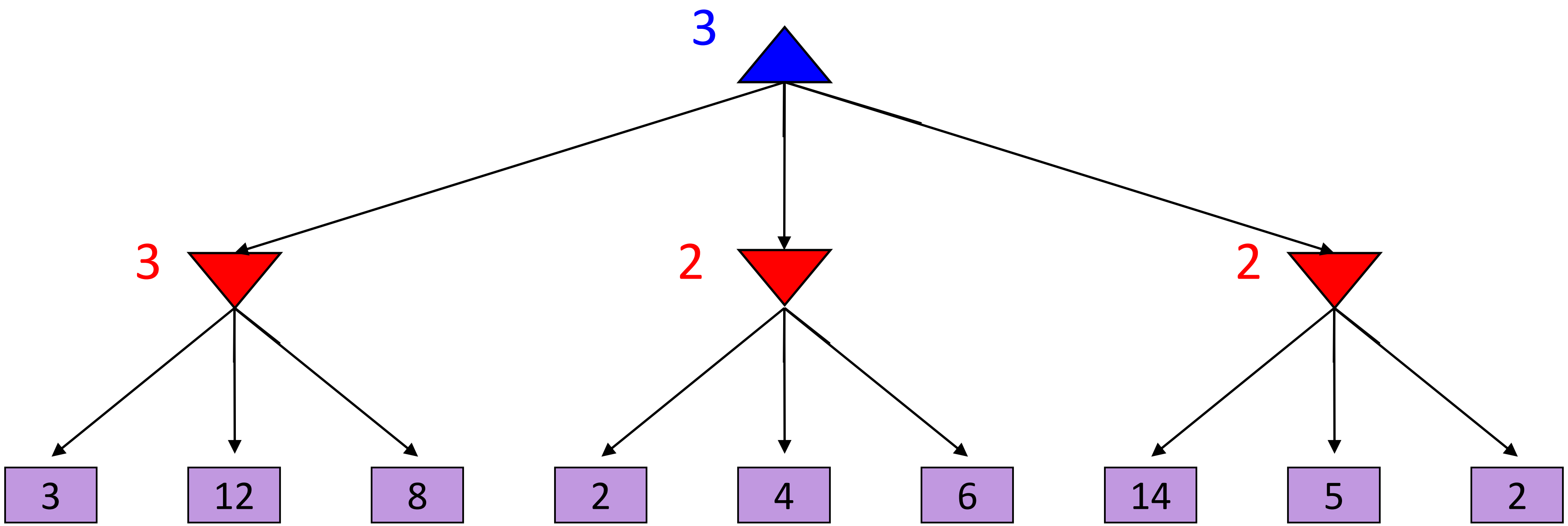
# Evaluation for Pacman



# Game Tree Pruning

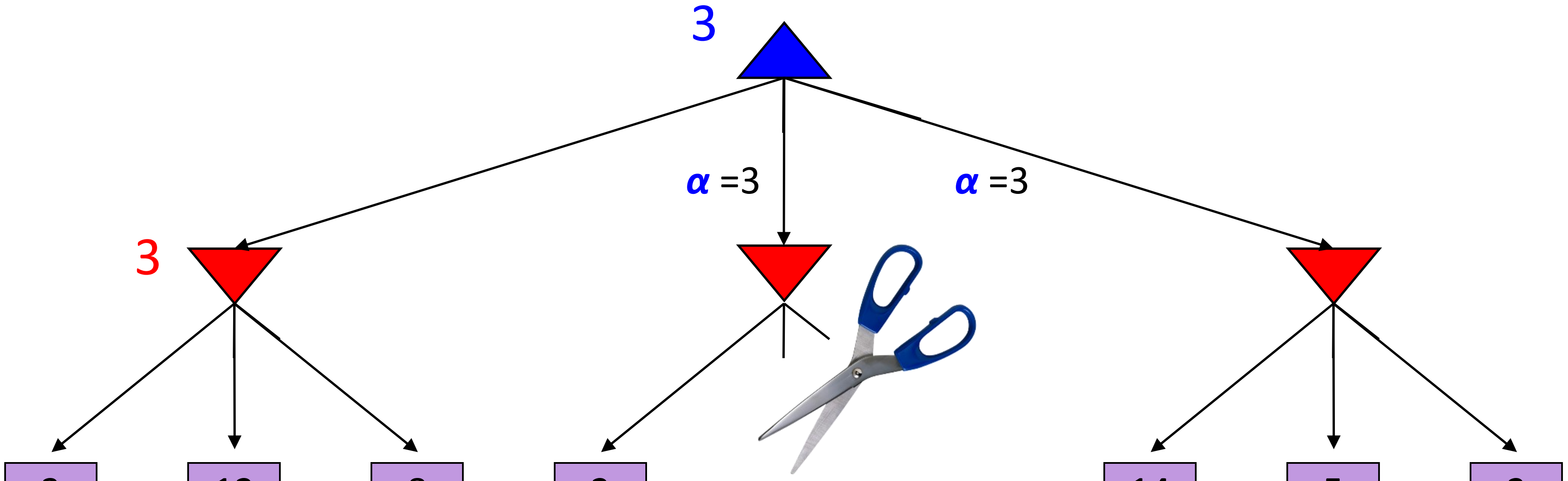


# Minimax Example



# Alpha-Beta Example

***α*** = best option so far from any MAX node on this path



***The order of generation matters***: more pruning is possible if good moves come first

# Alpha-Beta Implementation

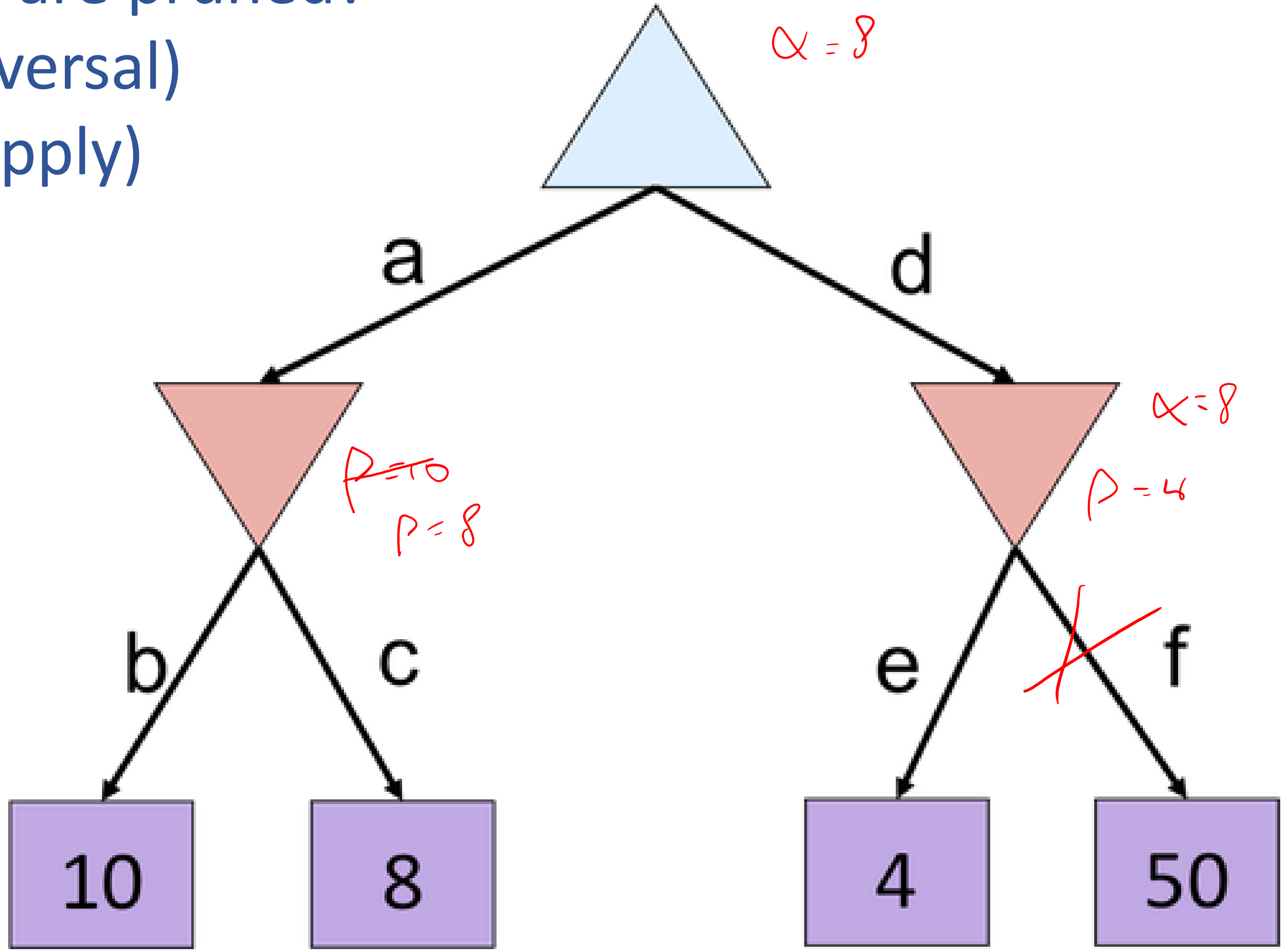
α: MAX's best option on path to root
β: MIN's best option on path to root

```
def max-value(state, α, β):
    initialize v = -∞
    for each successor of state:
        v = max(v, value(successor, α, β))
        if v ≥ β
            return v
        α = max(α, v)
    return v
```

```
def min-value(state , α, β):
    initialize v = +∞
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

# On your own

Which branches are pruned?
(Left to right traversal)
(Select all that apply)



# Poll 4

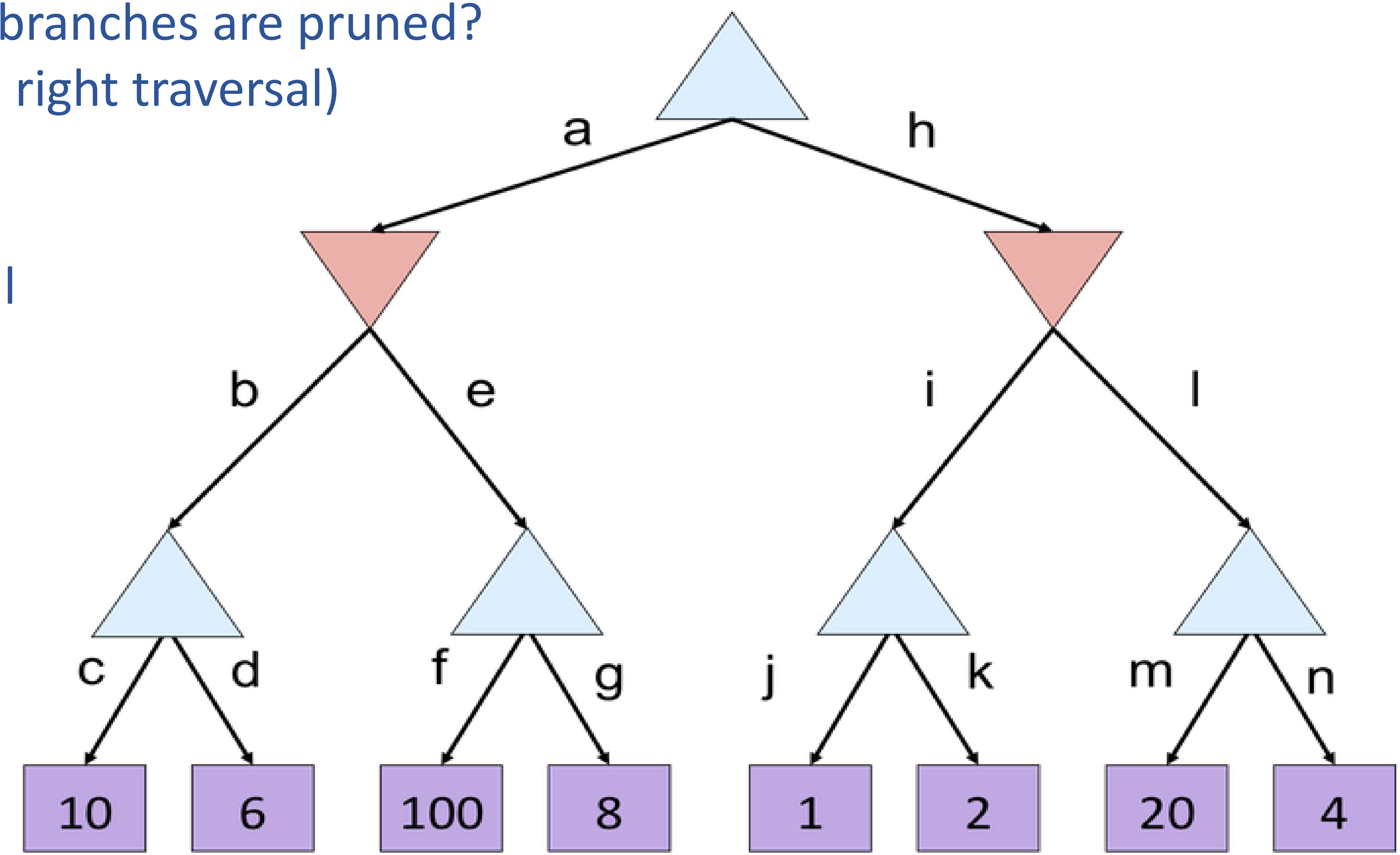
Which branches are pruned?
(Left to right traversal)

A) e, l

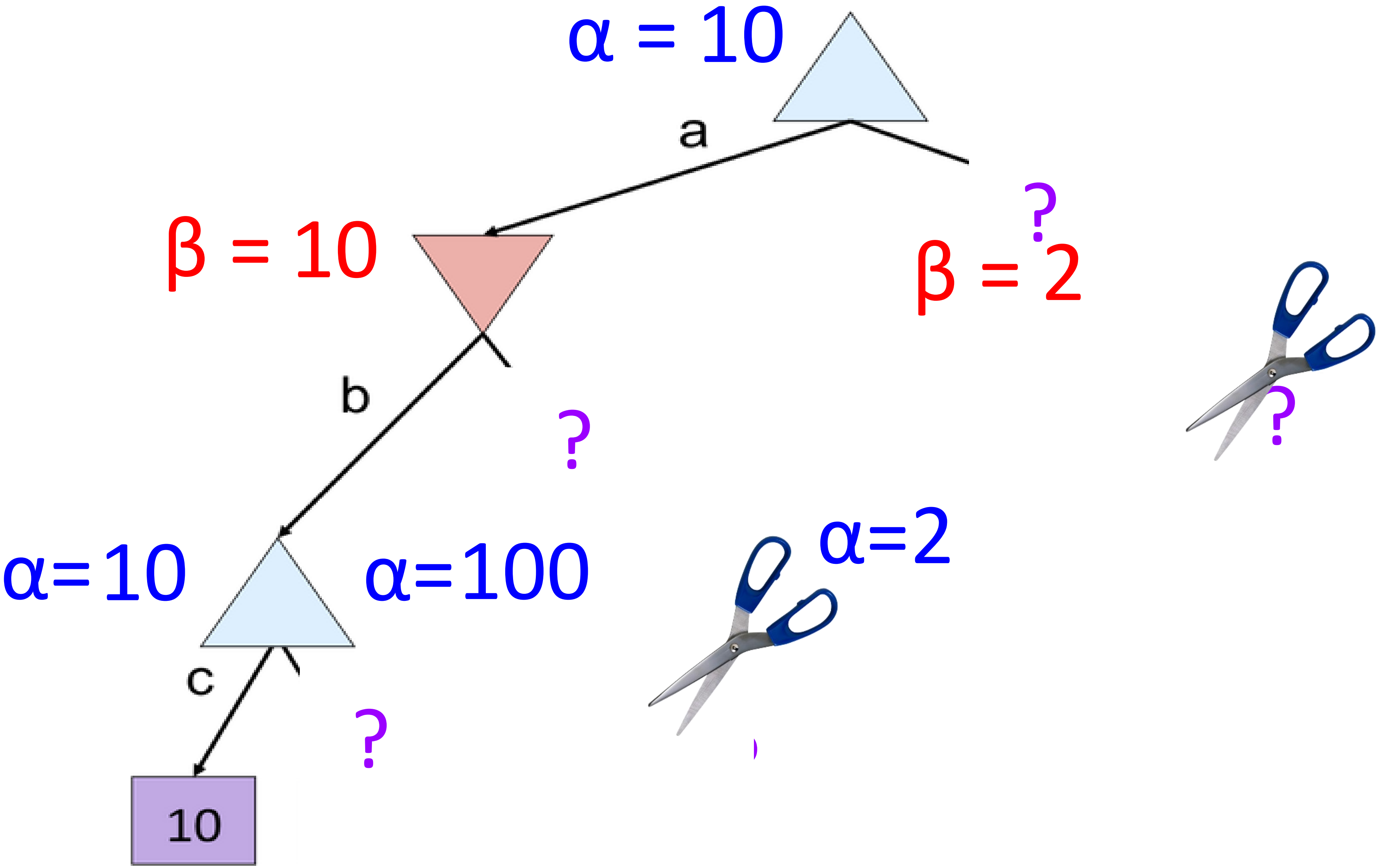
B) g, l

C) g, k, l

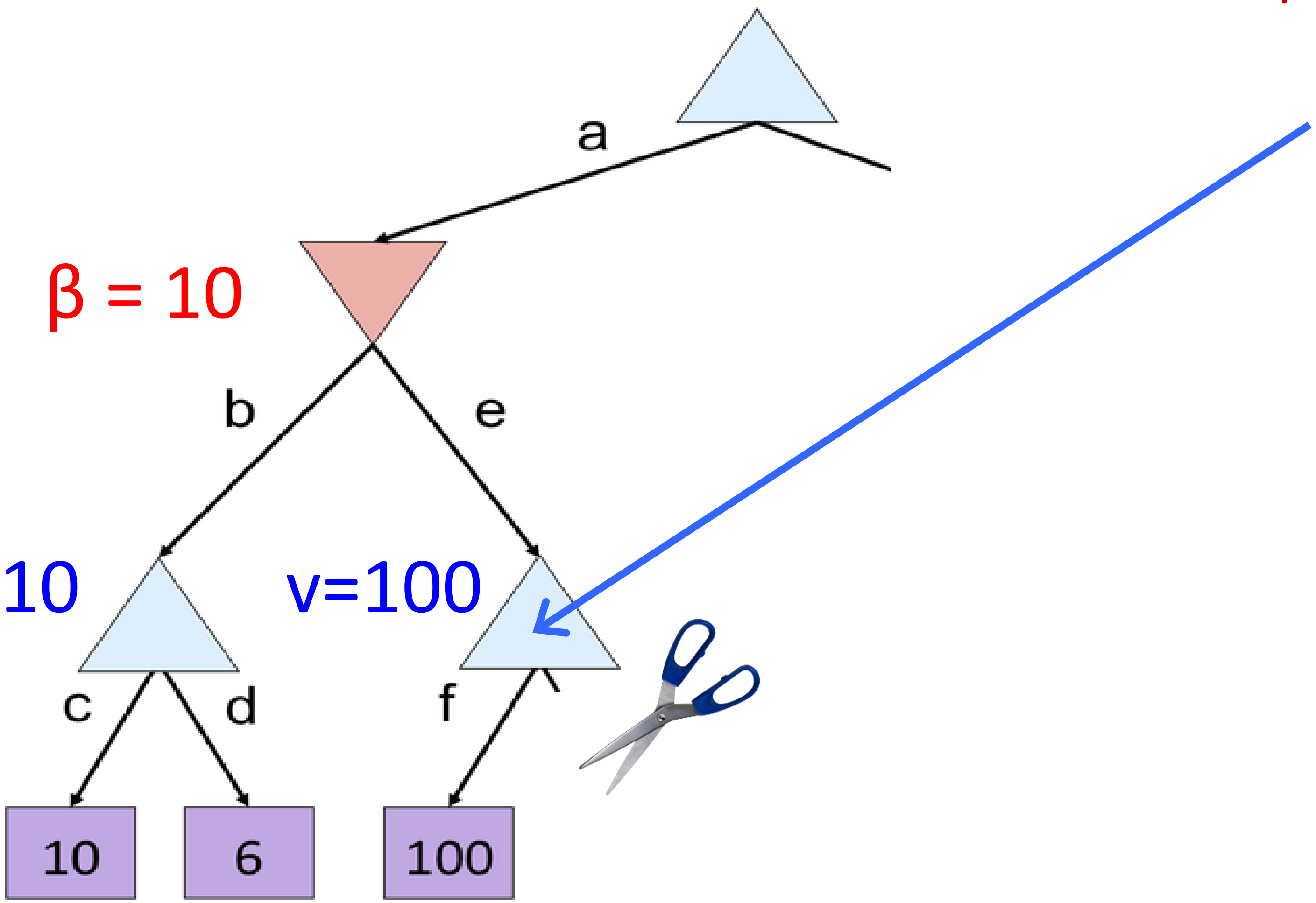
D) g, n



# Poll 4



# Alpha-Beta Code



α: MAX's best option on path to root

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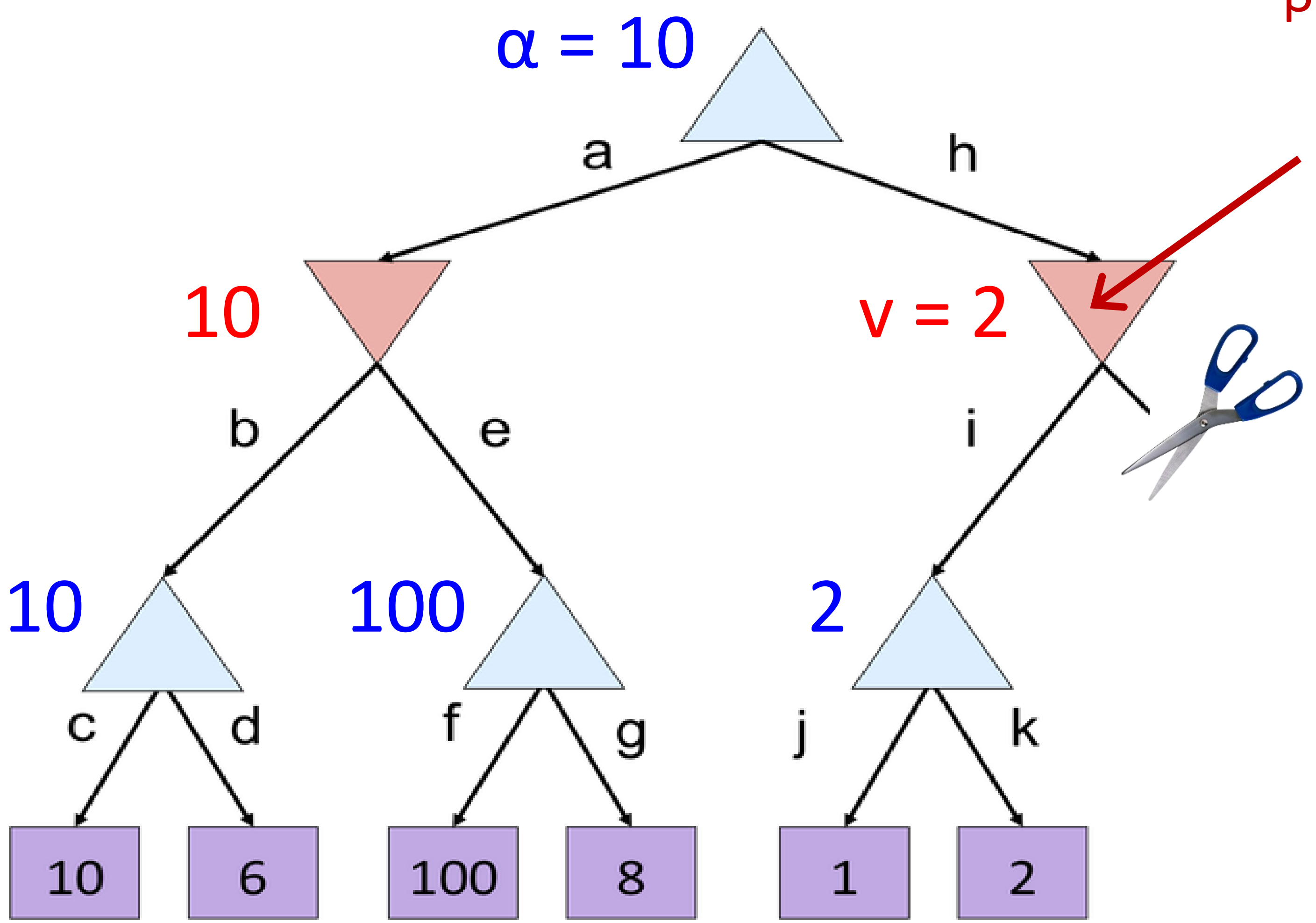
# Alpha-Beta Code



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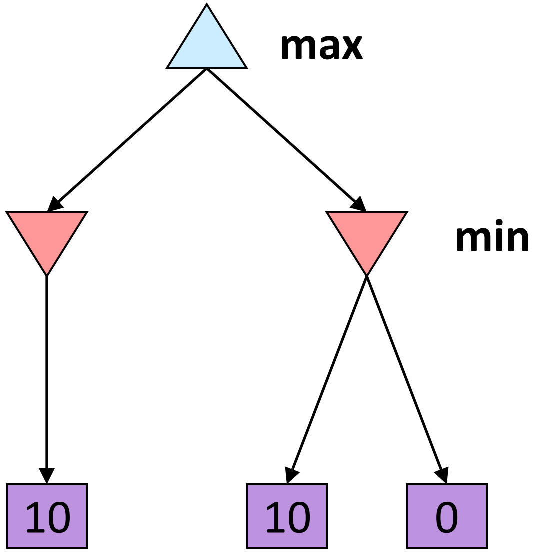
    for each successor of state:
        v = min(v, value(successor, α, β))
        if v ≤ α
            return v
        β = min(β, v)
    return v
```

# Alpha-Beta Pruning Properties

Theorem: This pruning has ***no effect*** on minimax value computed for the root!

Good child ordering improves effectiveness of pruning

- Iterative deepening helps with this



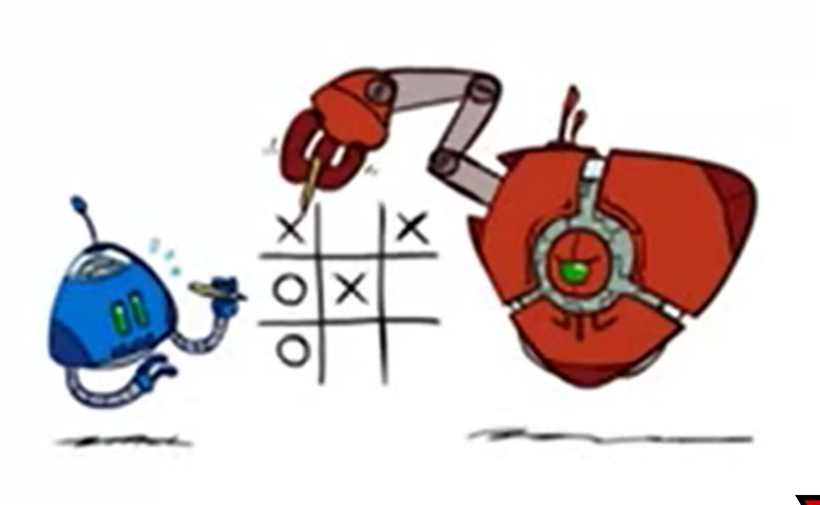
With "perfect ordering":

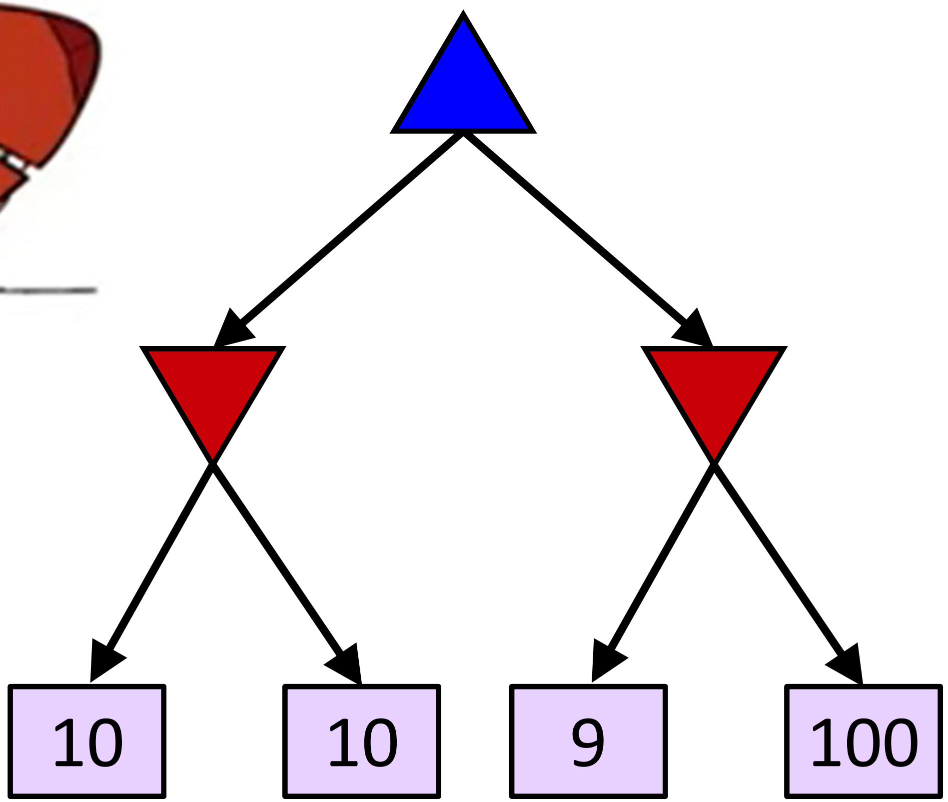
- Time complexity drops to O($b^{m/2}$)
- Doubles solvable depth!
- 1M nodes/move => depth=8, respectable

This is a simple example of metareasoning (computing about what to compute)

# Modeling Assumptions

Know your opponent





# Modeling Assumptions

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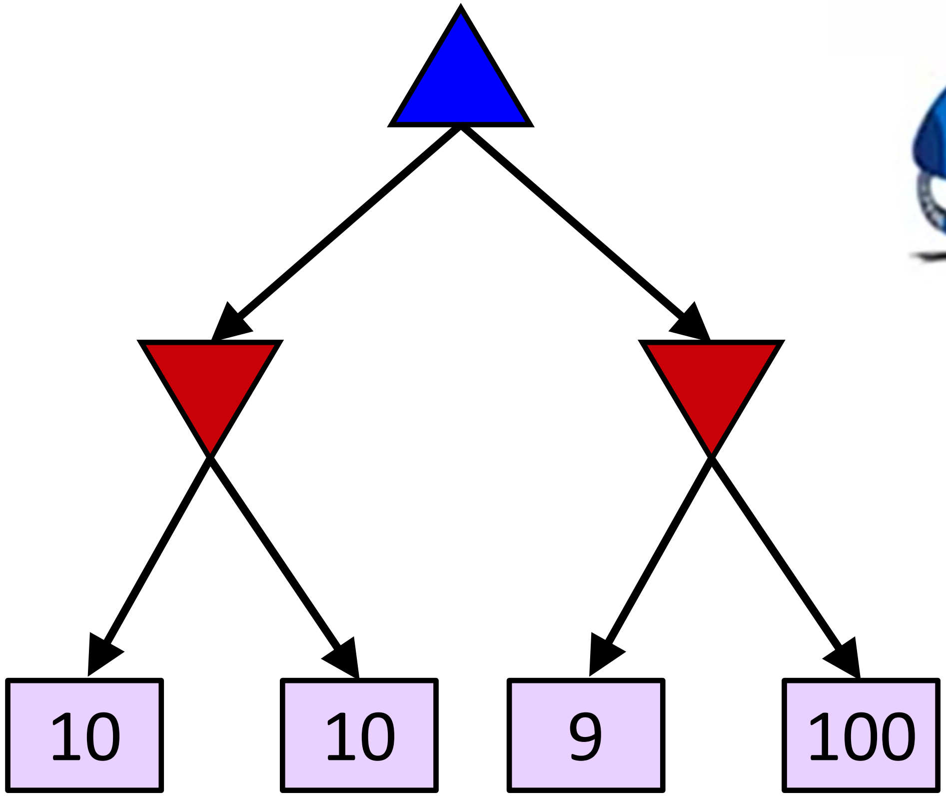


# Modeling Assumptions

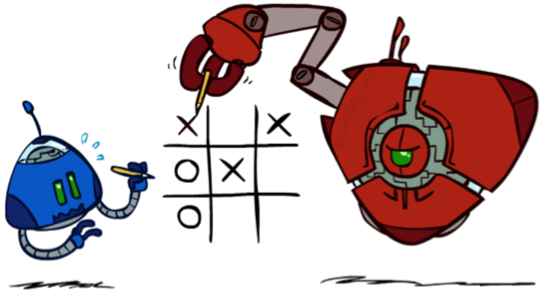
## Dangerous Pessimism

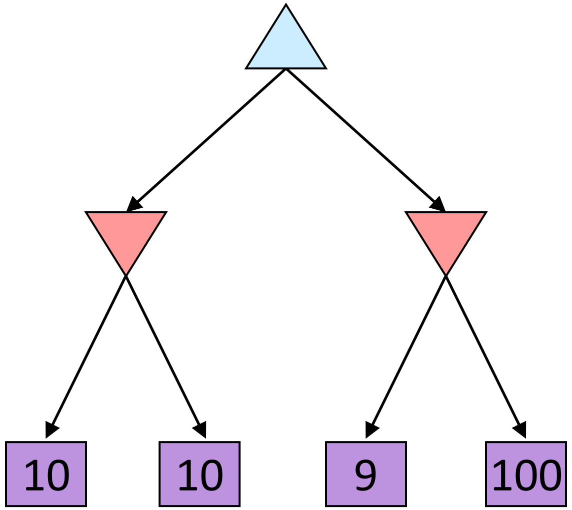
Assuming the worst case when it’s not likely

## Dangerous Optimism

Assuming chance when the world is adversarial

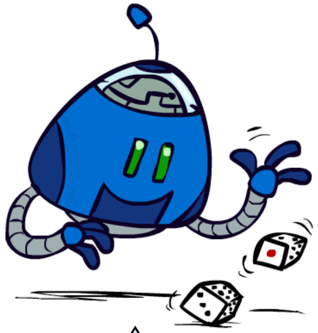
# Chance outcomes in trees

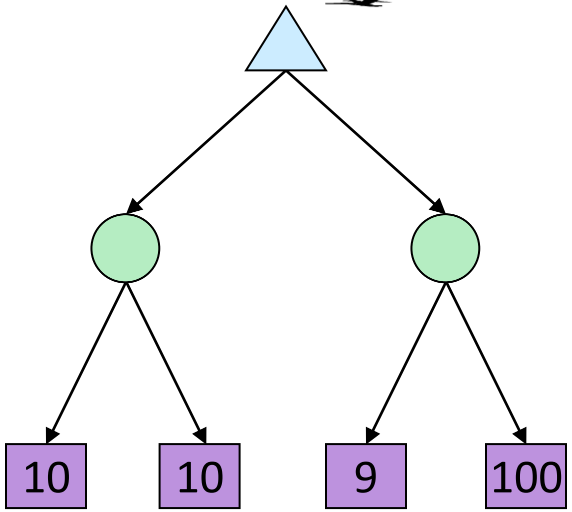




Tictactoe, chess

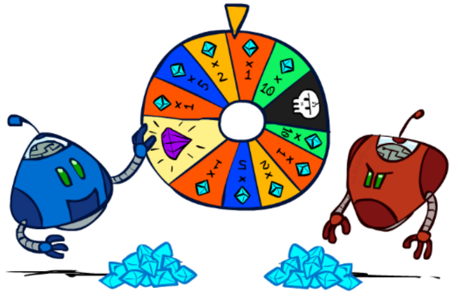
***Minimax***

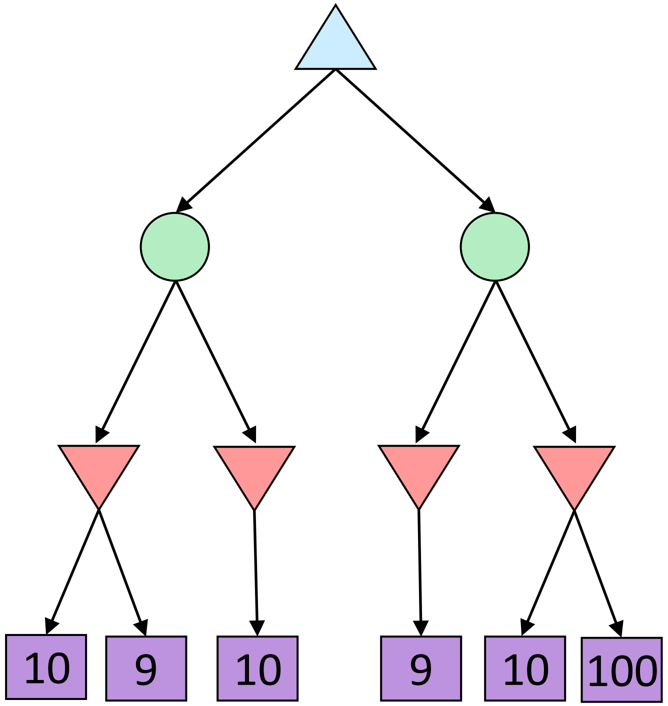




Tetris, investing

***Expectimax***

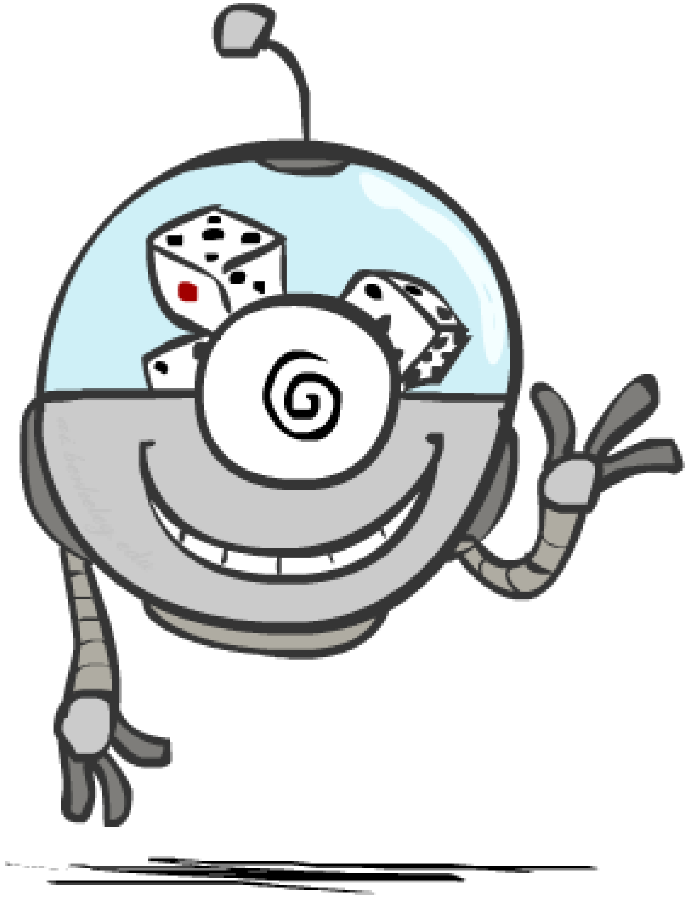




Backgammon, Monopoly

***Expectiminimax***

# Probabilities



# Probabilities

A random variable represents an event whose outcome is unknown

A probability distribution is an assignment of weights to outcomes

Example: Traffic on freeway

- Random variable: T = whether there’s traffic
- Outcomes: T in {none, light, heavy}
- Distribution:

  P(T=none) = 0.25, P(T=light) = 0.50, P(T=heavy) = 0.25

Probabilities over all possible outcomes sum to one

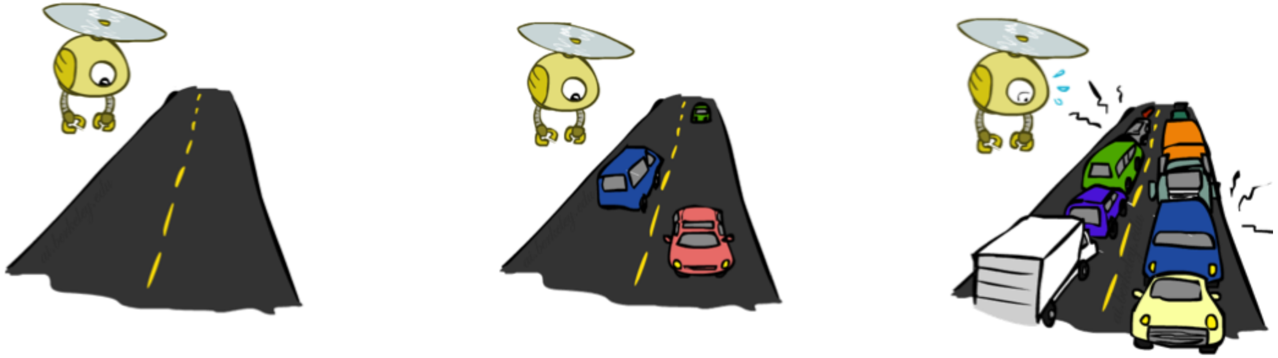
# Expected Value

Expected value of a function of a random variable:

Average the values of each outcome,
weighted by the probability of that outcome

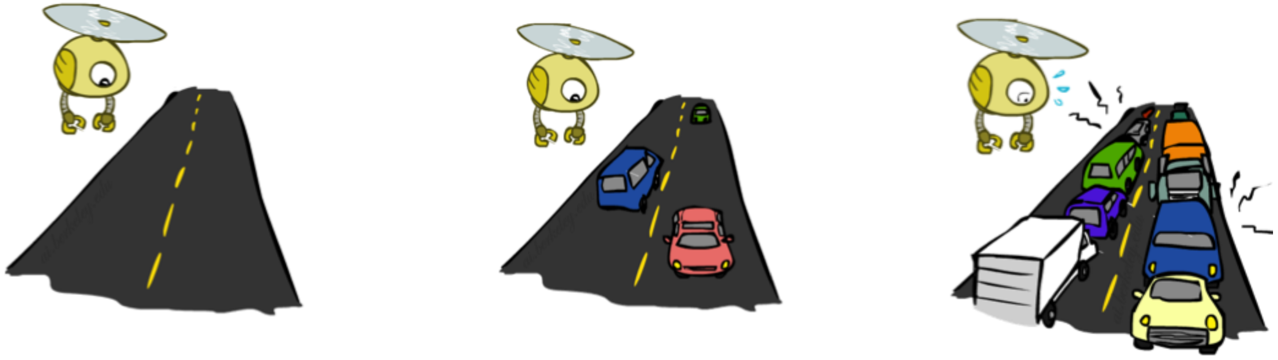
Example: How long to get to the airport?

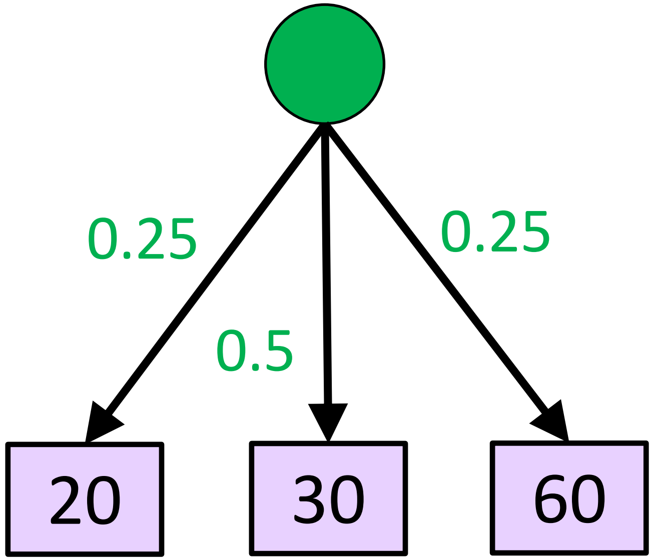
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Time: | 20 min | | 30 min | | 60 min | | |
| | x | + | x | + | x | ⇨ | 35 min |
| Probability: | 0.25 | | 0.50 | | 0.25 | | |



# Expectations

| | | | | | |
|---|---|---|---|---|---|
| Time: | 20 min | | 30 min | | 60 min |
| | x | + | x | + | x |
| Probability: | 0.25 | | 0.50 | | 0.25 |





## Max node notation

$$V(s) = \max_a V(s'),$$

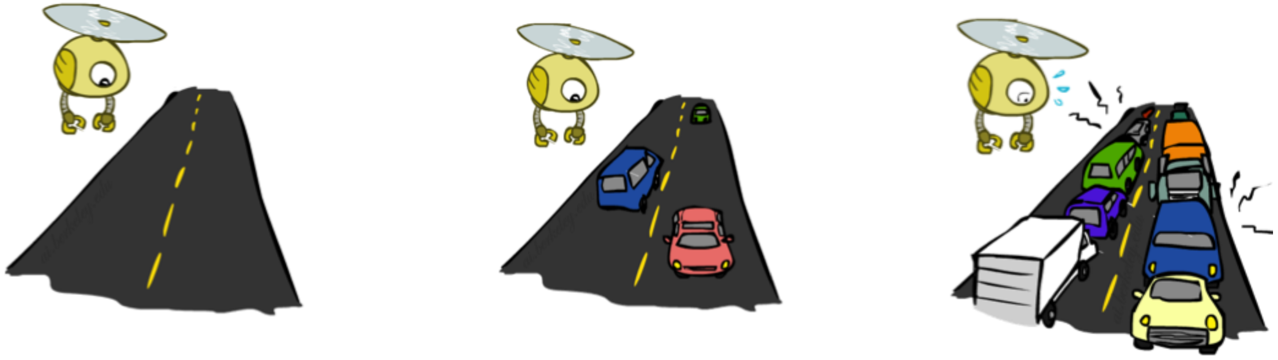
where $s' = result(s, a)$

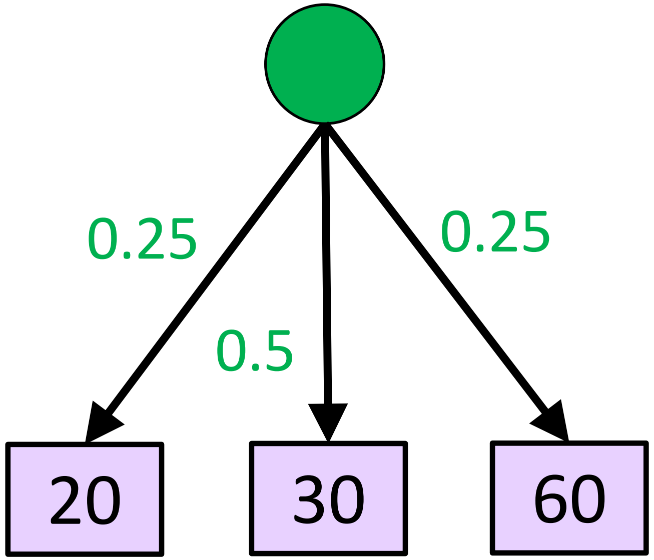
## Chance node notation

$$V(s) =$$

# Expectations

| | | | | | |
|---|---|---|---|---|---|
| Time: | 20 min | | 30 min | | 60 min |
| | x | + | x | + | x |
| Probability: | 0.25 | | 0.50 | | 0.25 |





## Max node notation

$$V(s) = \max_{a} V(s'),$$

where $s' = result(s, a)$

## Chance node notation

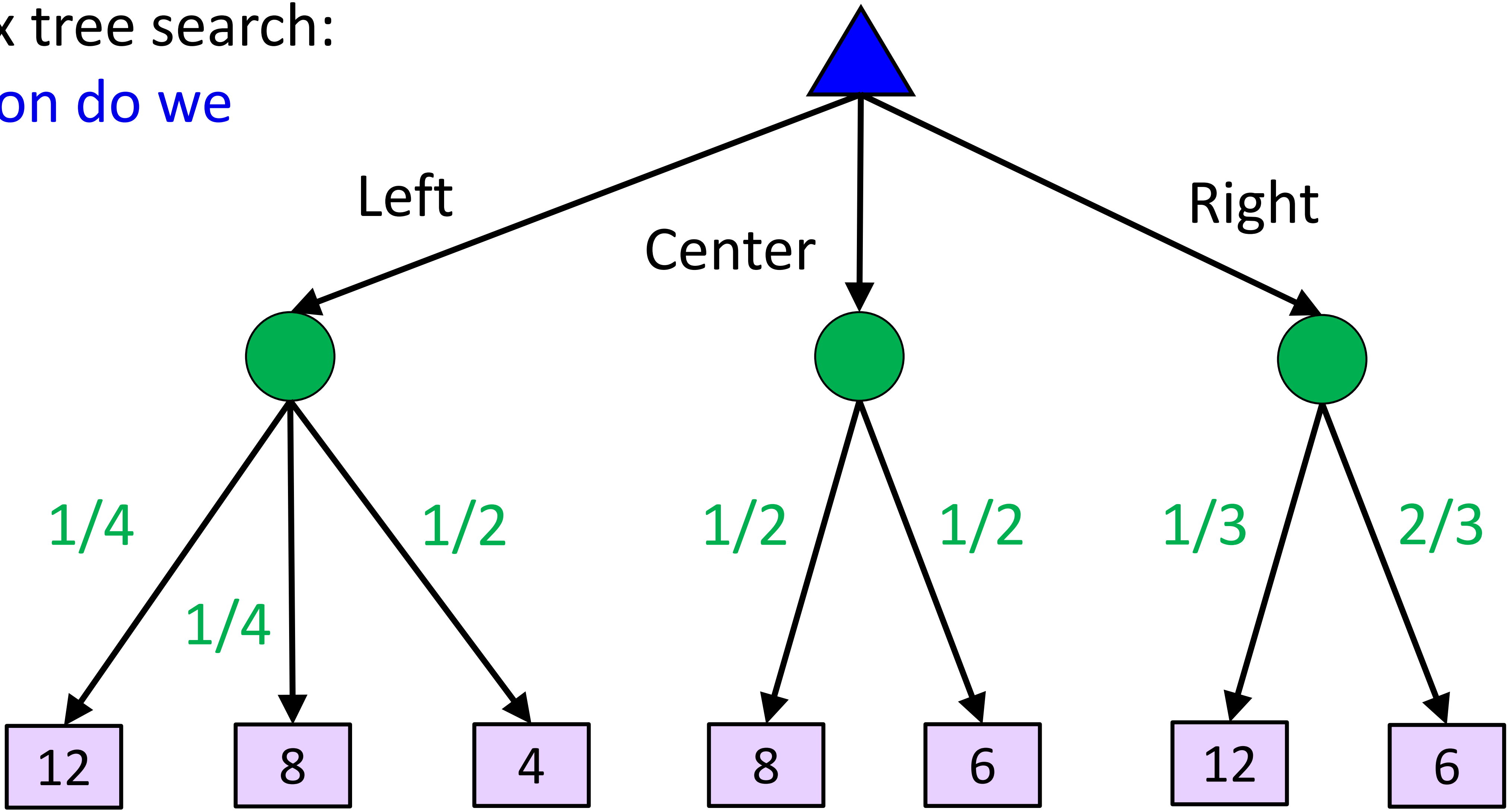
$$V(s) = \sum_{s'} P(s')\, V(s')$$

# On your own…

Expectimax tree search:

Which action do we choose?

A: Left
B: Center
C: Right
D: Eight



# On your own…

Expectimax tree search:

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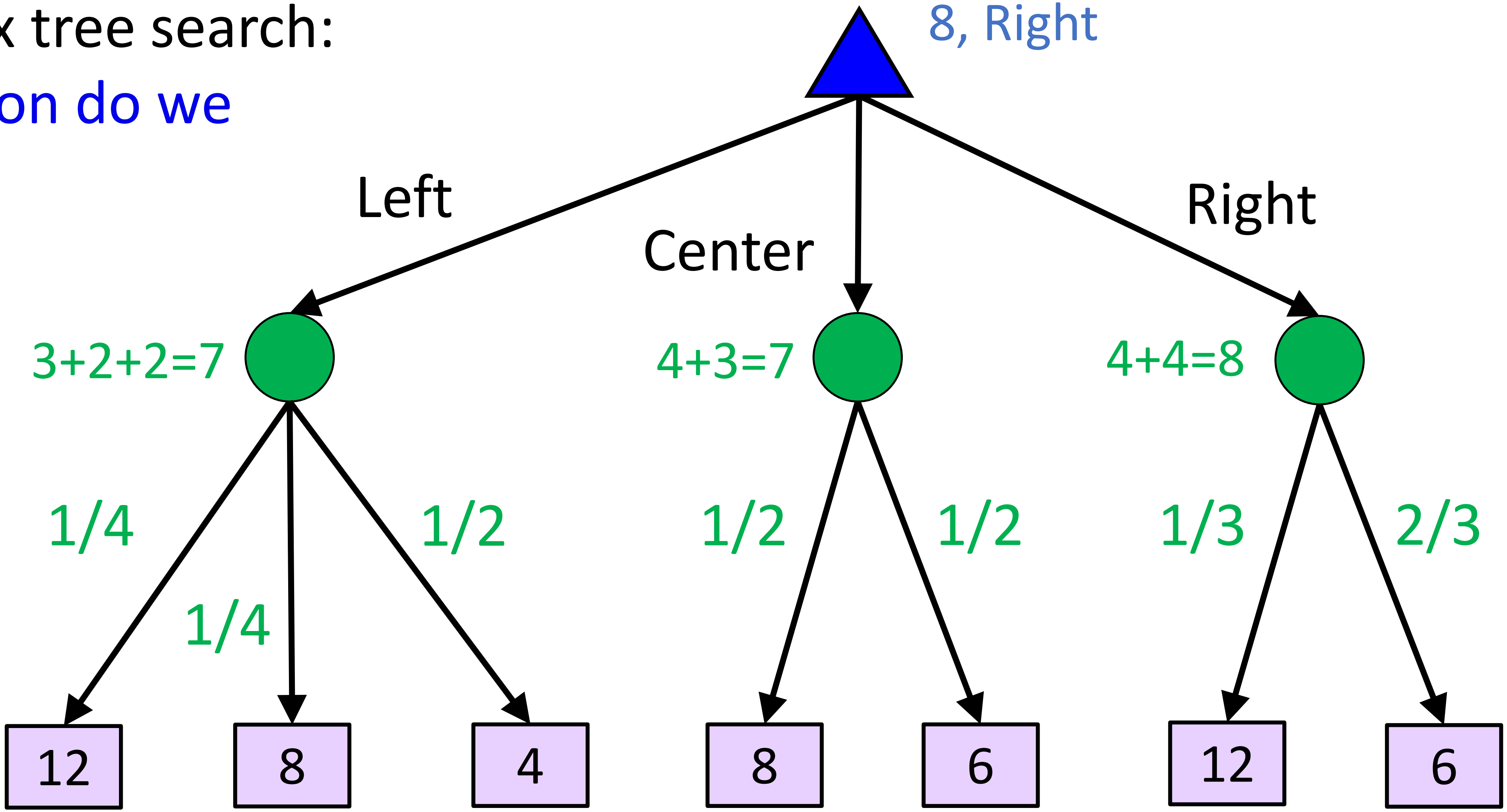


# Expectimax Code

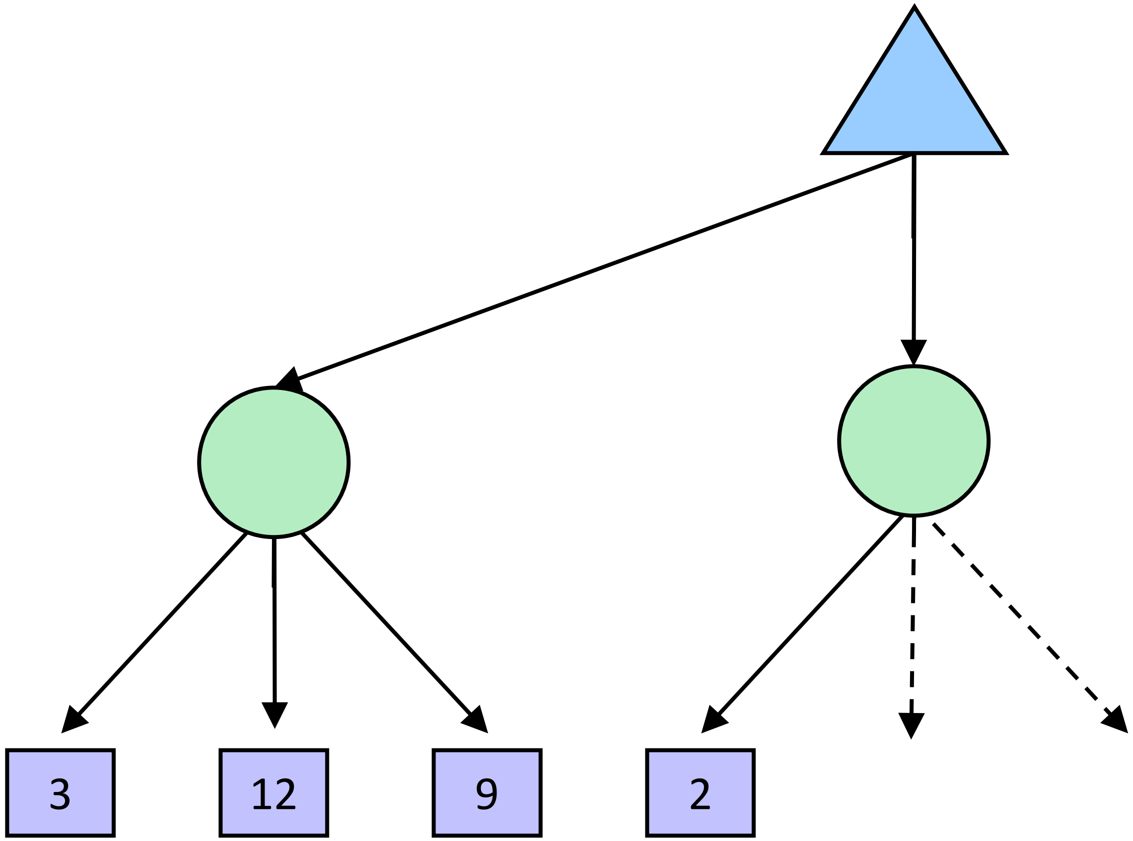
```
function value( state )
    if state.is_leaf
        return state.value

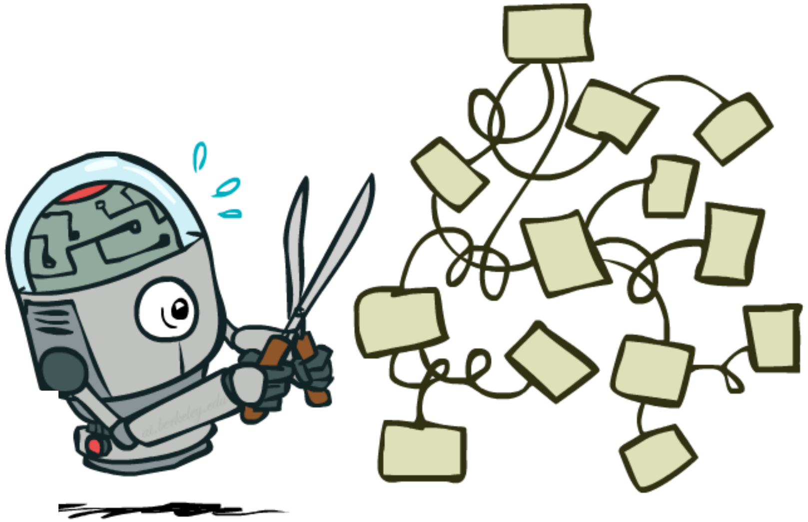
    if state.player is MAX
        return $\max_{a \text{ in state.actions}}$ value( state.result(a) )

    if state.player is MIN
        return $\min_{a \text{ in state.actions}}$ value( state.result(a) )

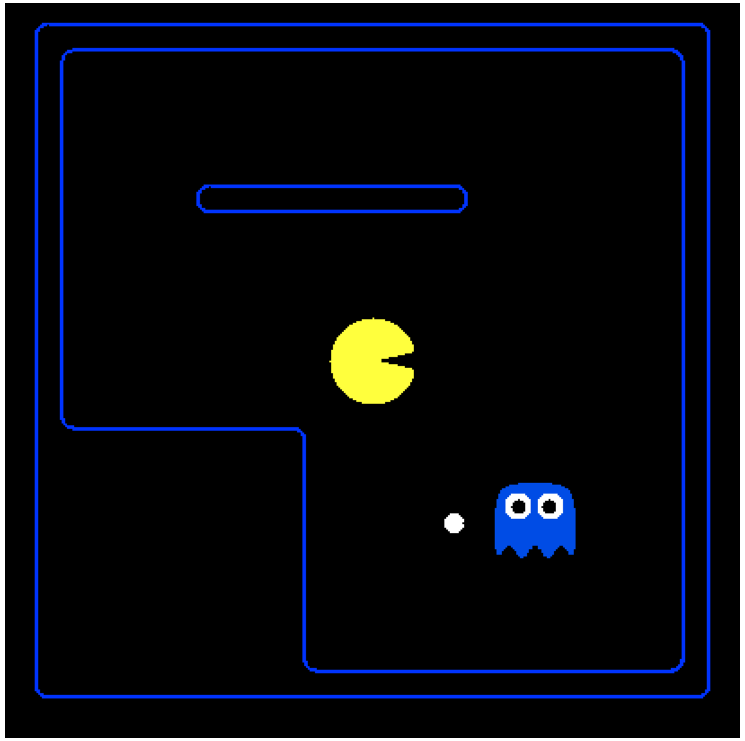
    if state.player is CHANCE
        return $\text{sum}_{s \text{ in state.next\_states}}$ P( s ) * value( s )
```

# Expectimax Pruning?





# Modeling Assumptions



| | Minimax Ghost | Random Ghost |
|---|---|---|
| Minimax Pacman | | |
| Expectimax Pacman | | |

Results from playing 5 games

# Activity sheet

Q1c – practice alpha-beta pruning *on your own*

Q2 – apply minimax and evaluation functions (heuristics) to Connect 4

# Summary

## Games require decisions when optimality is impossible

- Bounded-depth search and approximate evaluation functions

## Games force efficient use of computation

- Alpha-beta pruning

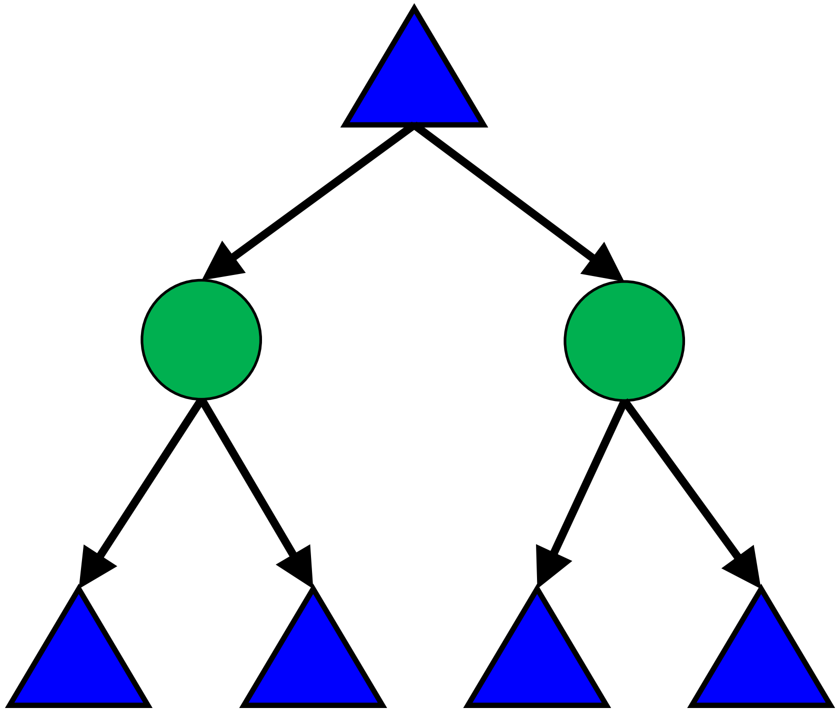
## Game playing has produced important research ideas

- Reinforcement learning (checkers)
- Iterative deepening (chess)
- Monte Carlo tree search (Go)
- Solution methods for partial-information games in economics (poker)

## Video games present much greater challenges – lots to do!

- $b = 10^{500}$, $|S| = 10^{4000}$, $m = 10{,}000$

# Preview: MDP/Reinforcement Learning Notation



$$V(s) = \max_a \sum_{s'} P(s')\, V(s')$$

# Preview: MDP/Reinforcement Learning Notation

Standard expectimax:

$$V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$$

Bellman equations:

$$V(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]$$

Value iteration:

$$V_{k+1}(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V_k(s')], \qquad \forall\, s$$

Q-iteration:

$$Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$$

Policy extraction:

$$\pi_V(s) = \operatorname*{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \qquad \forall\, s$$

Policy evaluation:

$$V_{k+1}^{\pi}(s) = \sum_{s'} P(s'|s,\pi(s))[R(s,\pi(s),s') + \gamma V_k^{\pi}(s')], \qquad \forall\, s$$

Policy improvement:

$$\pi_{new}(s) = \operatorname*{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \qquad \forall\, s$$

# Preview: MDP/Reinforcement Learning Notation

Standard expectimax: $V(s) = \max_a \sum_{s'} P(s'|s,a)V(s')$

Bellman equations: $V(s) = \max_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')]$

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Q-iteration: $Q_{k+1}(s,a) = \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma \max_{a'} Q_k(s',a')], \quad \forall\, s,a$

Policy extraction: $\pi_V(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V(s')], \quad \forall\, s$

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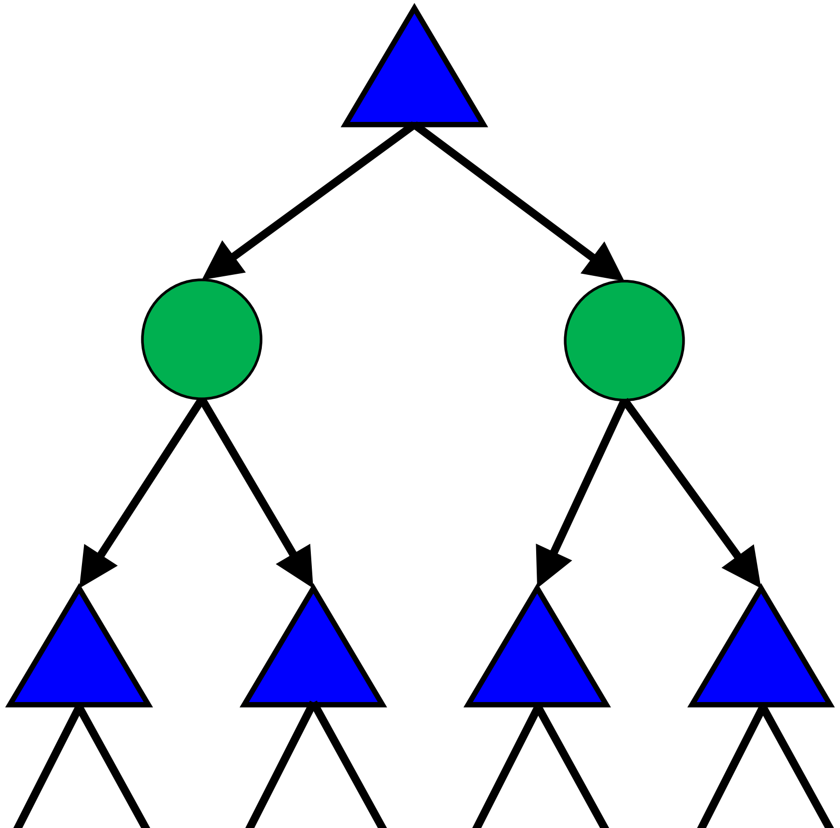
Policy improvement: $\pi_{new}(s) = \operatorname{argmax}_a \sum_{s'} P(s'|s,a)[R(s,a,s') + \gamma V^{\pi_{old}}(s')], \quad \forall\, s$

# Why Expectimax?

Pretty great model for an agent in the world

Choose the action that has the: highest expected value



# Bonus Question

Let’s say you know that your opponent is actually running a depth 1 minimax, using the result 80% of the time, and moving randomly otherwise

Question: What tree search should you use?

A: Minimax

B: Expectimax

C: Something completely different