

Warm-up: What to eat?

We are trying healthy by finding the optimal amount of food to purchase.

We can choose the amount of **stir-fry** (ounce) and **boba** (fluid ounces).

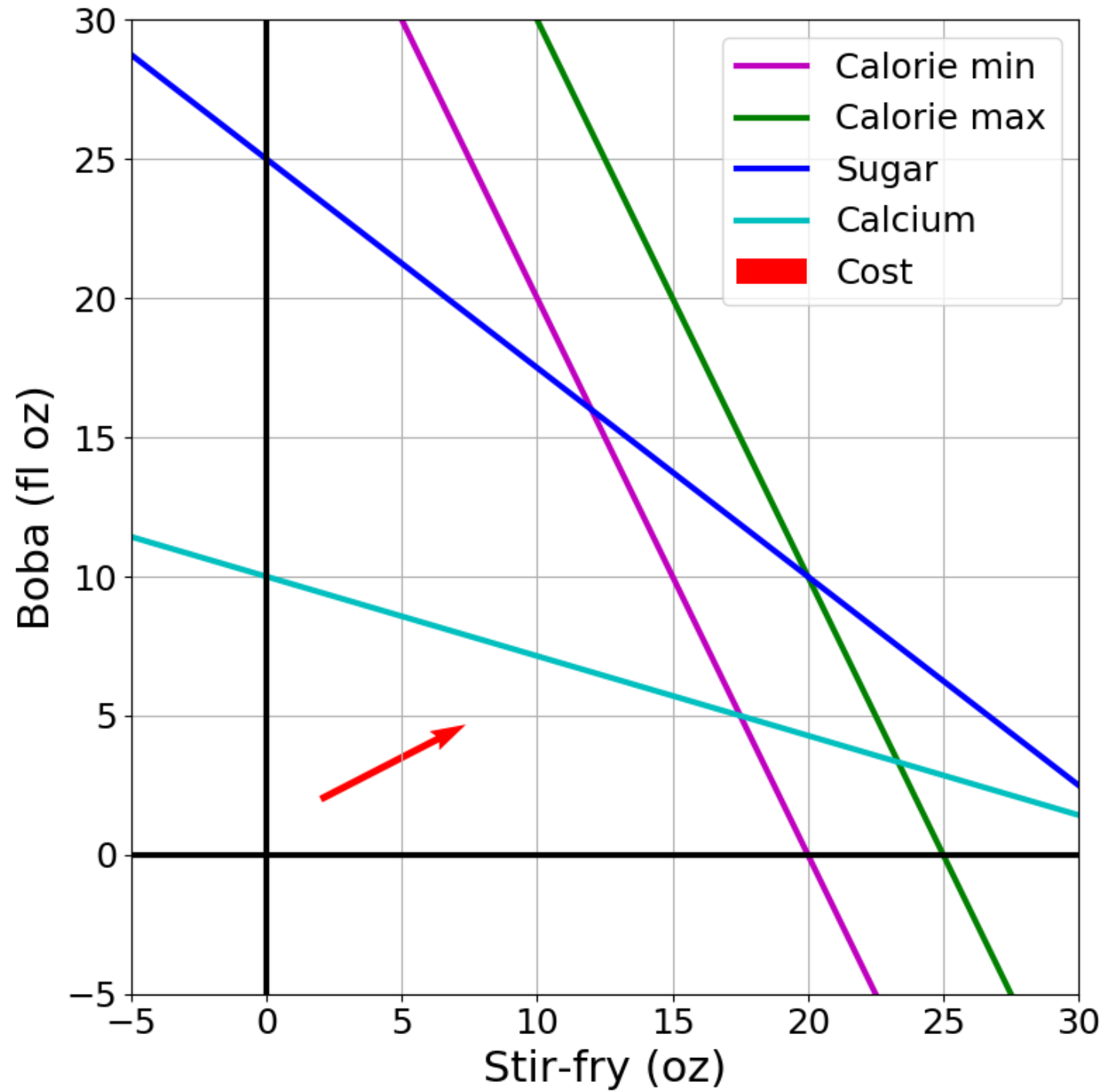
Healthy Squad Goals

- $2000 \leq \text{Calories} \leq 2500$
- $\text{Sugar} \leq 100 \text{ g}$
- $\text{Calcium} \geq 700 \text{ mg}$

Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
Boba (per fl oz)	0.5	50	4	70

What is the cheapest way to stay “healthy” with this menu?

How much **stir-fry** (ounce) and **boba** (fluid ounces) should we buy?



Announcements

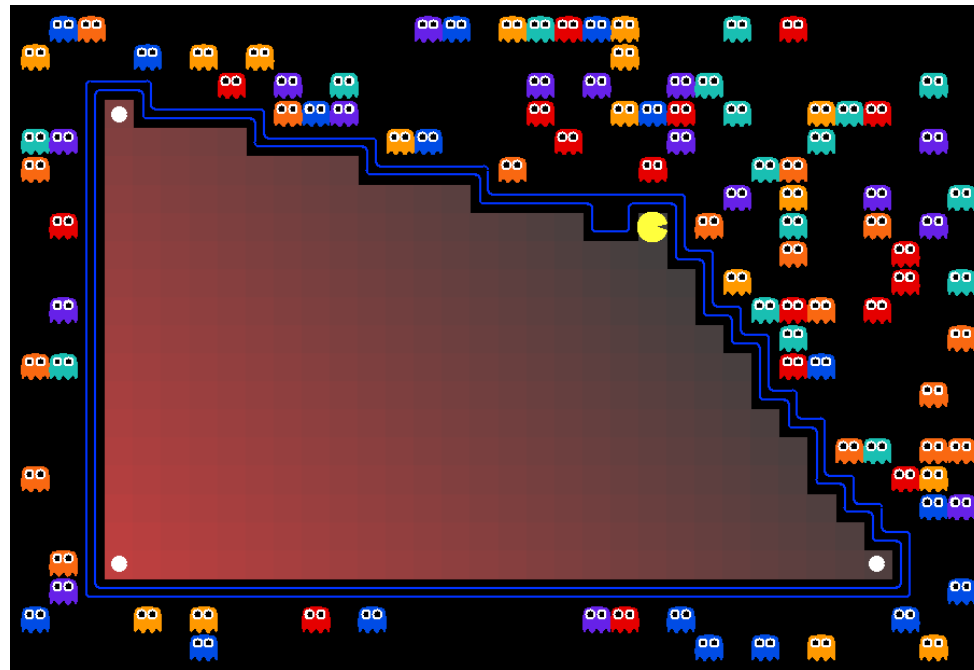
Assignments:

- HW3 (online)
 - Due tonight (9/19), 10 pm
- HW4 (online)
 - Due 9/26, 10 pm
- P1: Search and Games due yesterday (9/18)!!
- P2: Linear/Integer Programming
 - Due 10/5, 10pm (1 week after the exam)

- Exam 1 Sep 28!

AI: Representation and Problem Solving

Linear Programming



Instructors: Vincent Conitzer and Aditi Raghunathan

Slide credits: CMU AI with drawings from <http://ai.berkeley.edu>

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JULES

This is the entire taco bell menu with calories - can you craft me an order that is about 1000 calories (give or take 100) that feels balanced and tasty?



CHATGPT

First, I'll need to extract the contents of the provided PDF to understand the Taco Bell menu and the calorie content of each item. Let's proceed with that.



CODE INTERPRETER

```
import PyPDF2

# Open the PDF file
with open("/mnt/data/premium.pdf",
"rb") as file:
    # Initialize PDF reader
    reader =
PyPDF2.PdfFileReader(file)
```

4:10



CHATGPT

Here's a crafted order totaling approximately 920 calories:

- Premium Roast Coffee - Hot
- Diet Pepsi® (20 fl oz)
- Diet Pepsi® (30 fl oz)
- Diet Dr Pepper® (20 fl oz)
- Brisk® Unsweetened No Lemon Iced Tea (20 fl oz)
- Diet Dr Pepper® (30 fl oz)
- Brisk® Unsweetened No Lemon Iced Tea (30 fl oz)
- Pepsi® Zero Sugar (20 fl oz)
- Mild Sauce Packet (V)
- Hot Sauce Packet (V)
- Breakfast Salsa Packet (V)
- Brisk® Unsweetened No Lemon Iced Tea (16 fl oz)
- Fire Sauce Packet (V)
- Diablo Sauce Packet (V)
- Mtn Dew® Zero Sugar (20 fl oz)
- Diet Dr Pepper® (16 fl oz)
- Diet Pepsi® (16 fl oz)
- Premium Roast Coffee - Iced
- Pepsi® Zero Sugar (16 fl oz)
- Premium Roast Coffee - Hot
- Mtn Dew® Zero Sugar (16 fl oz)
- Pepsi® Zero Sugar (30 fl oz)
- Premium Roast Coffee - Iced

Message



4:10

- Water
- Mad Tasty Yuzu Citrus Sparkling Water
- Mad Tasty Unicorn Tears Sparkling Water
- Mad Tasty Grapefruit Sparkling Water
- Mtn Dew® Baja Blast Zero Sugar (16 fl oz)
- Mtn Dew® Zero Sugar (30 fl oz)
- Mtn Dew® Baja Blast Zero Sugar (20 fl oz)
- Mad Tasty Watermelon Kiwi Sparkling Water
- Mtn Dew® Baja Blast Zero Sugar (30 fl oz)
- Red Sauce
- Vanilla Creamer
- Reduced-Fat Sour Cream
- G2 - Fruit Punch (16 fl oz)
- Hot Coffee with Creamer
- Hot Coffee with Creamer
- Iced Coffee with Creamer
- Iced Coffee with Creamer
- Black Beans (V)
- G2 - Fruit Punch (20 fl oz)
- Black Beans
- Brisk® Dragon Paradise Sparkling Iced Tea (16 fl oz)
- Nacho Cheese Sauce
- Vodka (80 proof)
- Tequila (80 proof)

Message



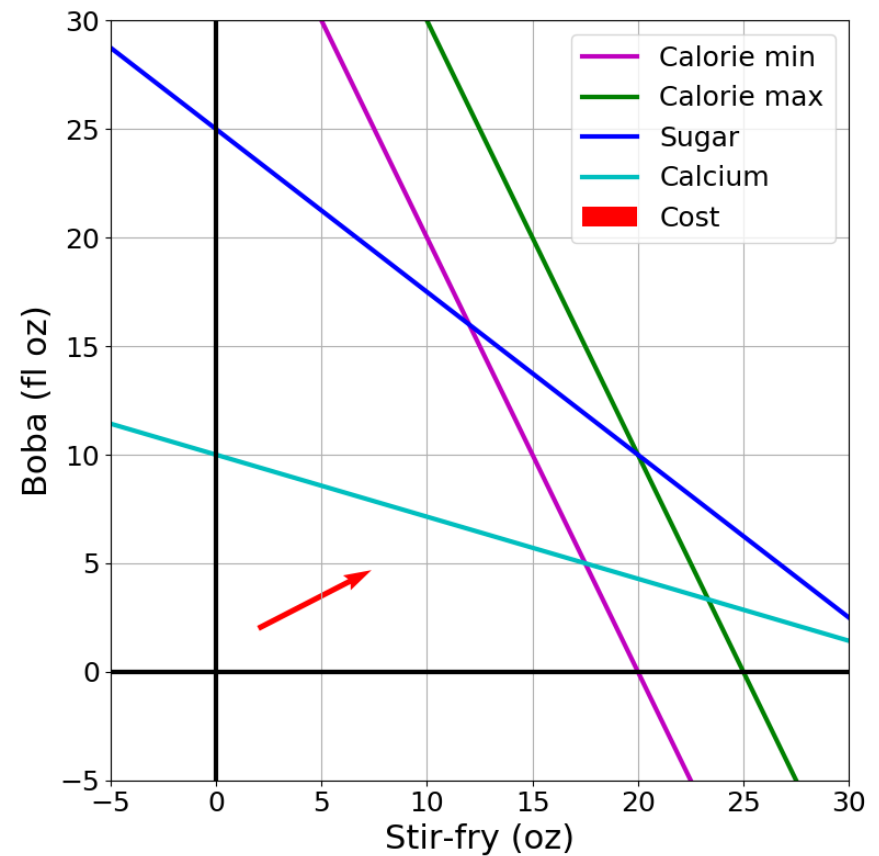
Optimization

Problem
Description

Optimization
Representation

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

Graphical Representation



Constraint Satisfaction Problems

Map coloring



Any x
s.t. x satisfies constraints



Notation Alert!

Optimization Formulation

Diet Problem

Any x

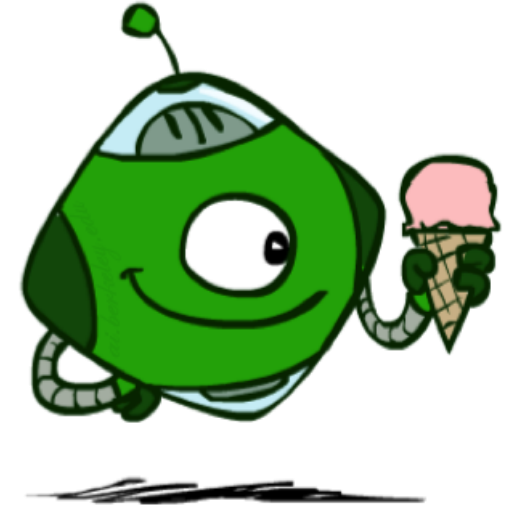
$$1s + .5b \\ 1x_1 + .5x_2$$

s.t. x satisfies constraints

$$2000 \leq 100s + 50b \leq 2500 \\ 3s + 4b \leq 100 \\ 20s + 70b \geq 700 \\ s \geq 0 \\ b \geq 0$$

$$\vec{x} = (x_1, x_2)^T$$

$$x_1 = s \\ x_2 = b$$



Healthy Squad Goals

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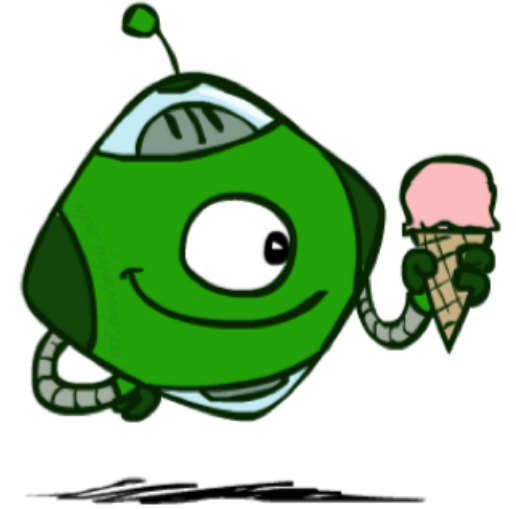
Notation Alert!

Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
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Optimization Formulation

Diet Problem

$$\begin{array}{lll} \min_x & cost(\mathbf{x}) & \text{Objective} \\ \text{s.t.} & \mathbf{x} \text{ satisfies constraints} & \end{array}$$



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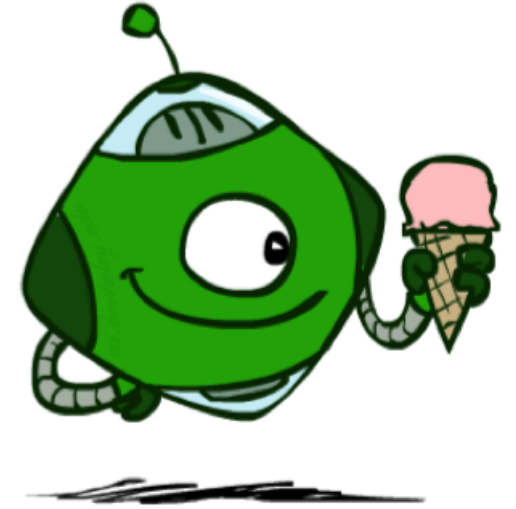
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Notation Alert!

Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_{\mathbf{x}} & \text{cost}(\mathbf{x}) \\ \text{s.t.} & \text{calories}(\mathbf{x}) \text{ contained} \\ & \text{sugar}(\mathbf{x}) \leq \text{limit} \\ & \text{calcium}(\mathbf{x}) \geq \text{limit} \end{array}$$



Healthy Squad Goals

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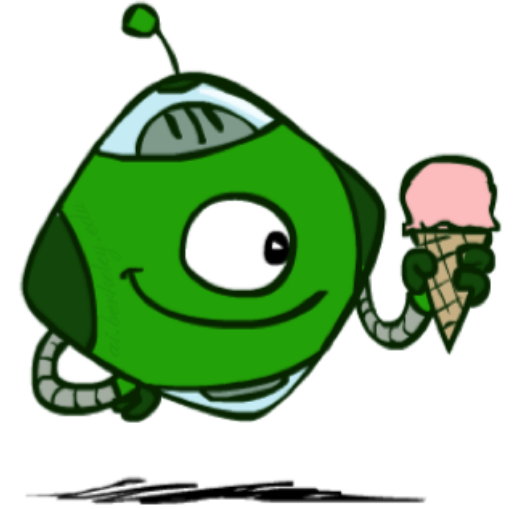
Food	Cost	Calories	Sugar	Calcium
Stir-fry (per oz)	1	100	3	20
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Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_{x_1, x_2} & 1 x_1 + 0.5 x_2 \\ \text{s.t.} & 100 x_1 + 50 x_2 \geq 2000 \\ & 100 x_1 + 50 x_2 \leq 2500 \\ & 3 x_1 + 4 x_2 \leq 100 \\ & 20 x_1 + 70 x_2 \geq 700 \end{array}$$

-100x₁ - 50x₂ ≤ -2000



Healthy Squad Goals

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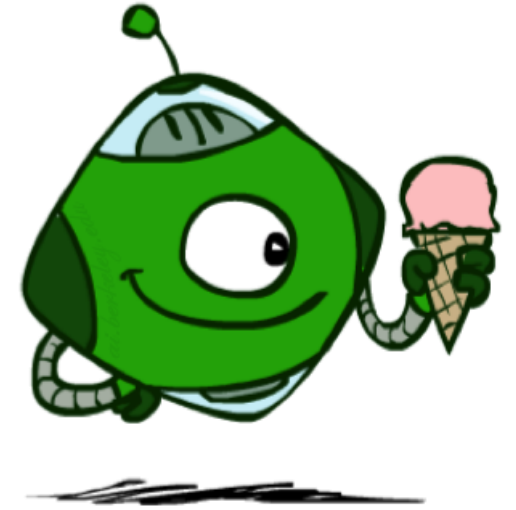
Optimization Formulation

Diet Problem

$$\begin{aligned} \min_{x_1, x_2} \quad & c_1 x_1 + c_2 x_2 \\ \text{s.t.} \quad & a_{1,1} x_1 + a_{1,2} x_2 \geq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \geq b_4 \end{aligned}$$

$$A = \begin{array}{cc} & \begin{array}{c} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{array}$$

$$b = \begin{array}{c} \text{Limit} \\ \begin{bmatrix} 2000 \\ 2500 \\ 100 \\ 700 \end{bmatrix} \end{array} \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array}$$



Cost

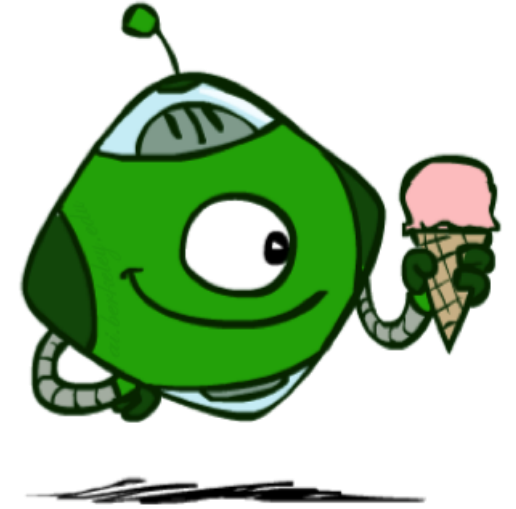
$$c = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

Notation Alert!

Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & a_{1,1} x_1 + a_{1,2} x_2 \geq b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & a_{4,1} x_1 + a_{4,2} x_2 \geq b_4 \end{array}$$



Cost

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

Limit

$$\mathbf{b} = \begin{bmatrix} 2000 \\ 2500 \\ 100 \\ 700 \end{bmatrix} \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array}$$

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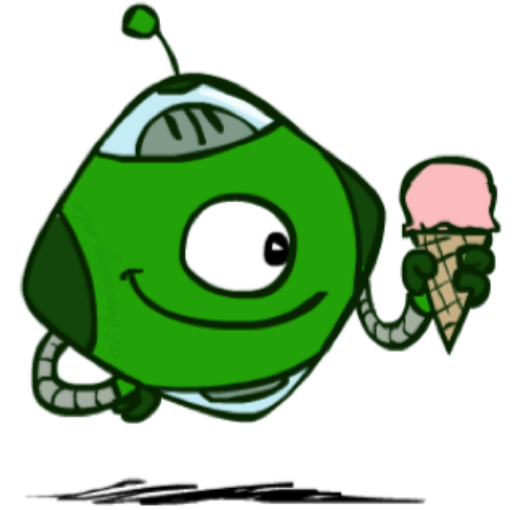
Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & -a_{1,1} x_1 - a_{1,2} x_2 \leq -b_1 \\ & a_{2,1} x_1 + a_{2,2} x_2 \leq b_2 \\ & a_{3,1} x_1 + a_{3,2} x_2 \leq b_3 \\ & -a_{4,1} x_1 - a_{4,2} x_2 \leq -b_4 \end{array}$$

$$A = \begin{array}{cc} & \begin{array}{l} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} 100 & 50 \\ 100 & 50 \\ 3 & 4 \\ 20 & 70 \end{bmatrix} \end{array}$$

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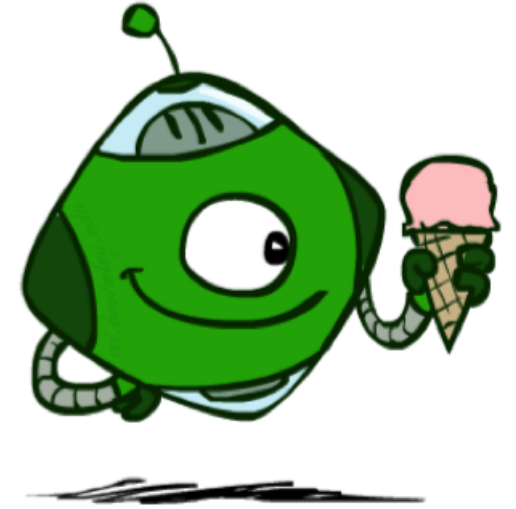
Cost

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Optimization Formulation

Diet Problem

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Cost

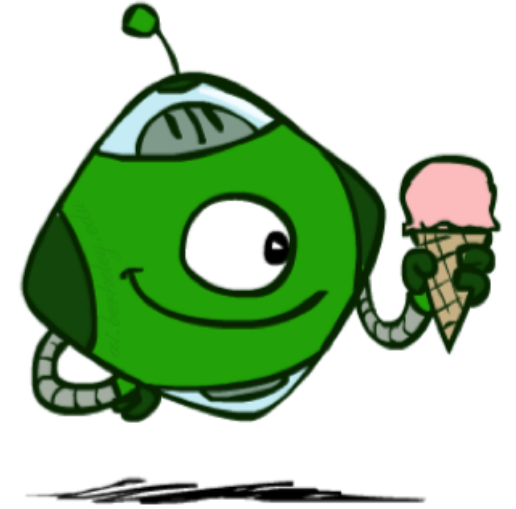
$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

$$A = \begin{array}{cc} & \begin{array}{c} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix} & \end{array} \quad \mathbf{b} = \begin{array}{c} \text{Limit} \\ \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix} \end{array} \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array}$$

Optimization Formulation

Diet Problem

$$\begin{array}{ll} \min_x & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$



Cost

$$\mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

Limit

$$\mathbf{A} = \begin{array}{cc} & \begin{array}{c} \text{Stir-fry} \\ \text{Boba} \end{array} \\ \begin{bmatrix} -100 & -50 \\ 100 & 50 \\ 3 & 4 \\ -20 & -70 \end{bmatrix} & \end{array} \quad \mathbf{b} = \begin{array}{c} \begin{bmatrix} -2000 \\ 2500 \\ 100 \\ -700 \end{bmatrix} \\ \begin{array}{l} \text{Calorie min} \\ \text{Calorie max} \\ \text{Sugar} \\ \text{Calcium} \end{array} \end{array}$$

Notation Alert!

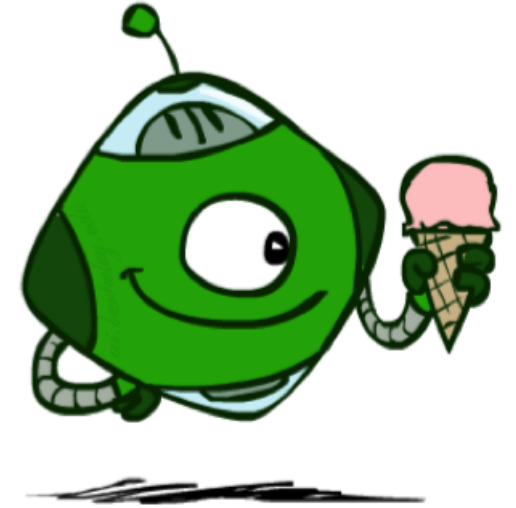
Poll 1

What has to increase to add more nutrition constraints?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \end{array}$$

Select all that apply

- A) length \mathbf{x}
- B) length \mathbf{c}
- C) height \mathbf{A}
- D) width \mathbf{A}
- E) length \mathbf{b}



Poll 1

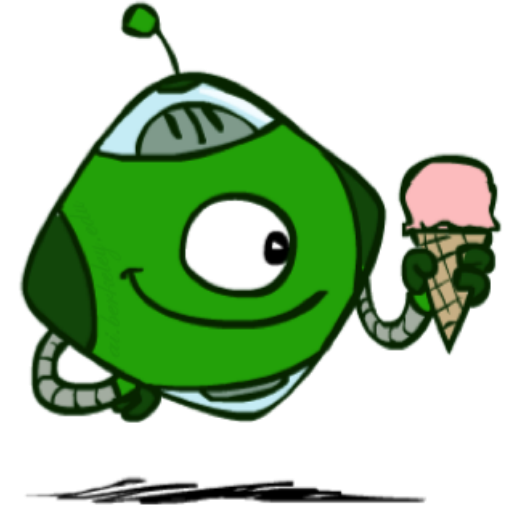
What has to increase to add more nutrition constraints?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

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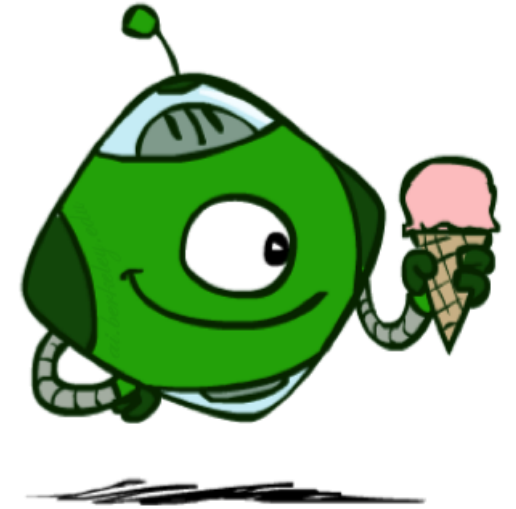
Poll 2

What has to increase to add more menu items?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{A}\mathbf{x} \leq \mathbf{b} \end{array}$$

Select all that apply

- A) length \mathbf{x}
- B) length \mathbf{c}
- C) height \mathbf{A}
- D) width \mathbf{A}
- E) length \mathbf{b}



Poll 2

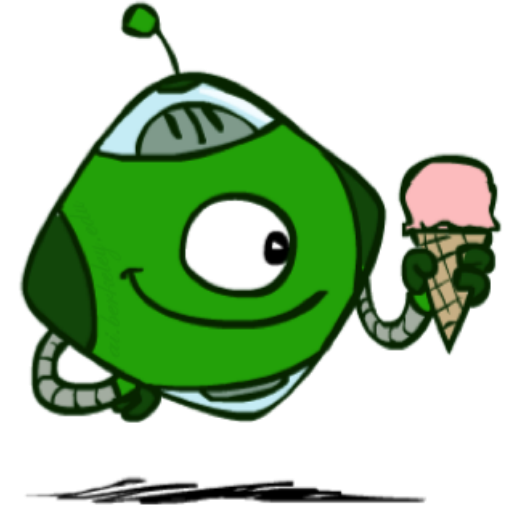
What has to increase to add more nutrition constraints?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}$$

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Poll 3

$N = \# \text{var}$ $M = \# \text{constraints}$

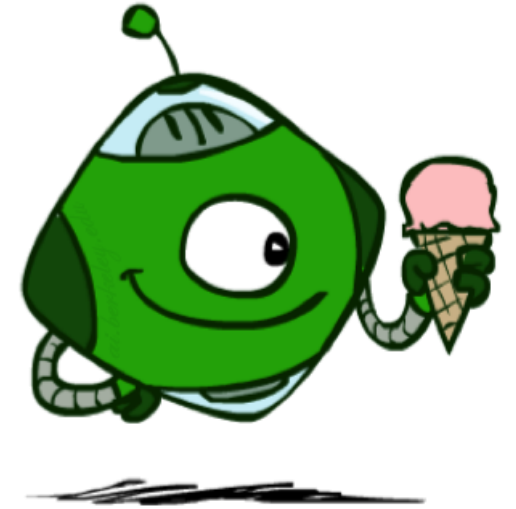
If $A \in \mathbb{R}^{M \times N}$, which of the following also equals N ?

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

Select all that apply

- A) length \mathbf{x}
- B) length \mathbf{c}
- C) length \mathbf{b}

Notation Alert!



Linear Programming

Linear objective with linear constraints

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & A\mathbf{x} \leq \mathbf{b} \end{array}$$

As opposed to general optimization

$$\begin{array}{ll} \min. & f_0(\mathbf{x}) \\ \text{s.t.} & f_i(\mathbf{x}) \leq 0, \quad i = 1 \dots M \\ & \mathbf{a}_i^T \mathbf{x} = \mathbf{b}_i, \quad i = 1 \dots P \end{array}$$

Linear Programming

Different formulations

Inequality form

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} \leq \mathbf{b} \end{array}$$

General form

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} + \mathbf{d} \\ \text{s.t.} & \mathbf{Gx} \leq \mathbf{h} \\ & \mathbf{Ax} = \mathbf{b} \end{array}$$

Standard form

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

Important to pay attention to form!

Linear Programming

Different formulations

Inequality form

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} \leq \mathbf{b} \end{array}$$

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Standard form

$$\begin{array}{ll} \min. & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} = \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{array}$$

Can switch between formulations!

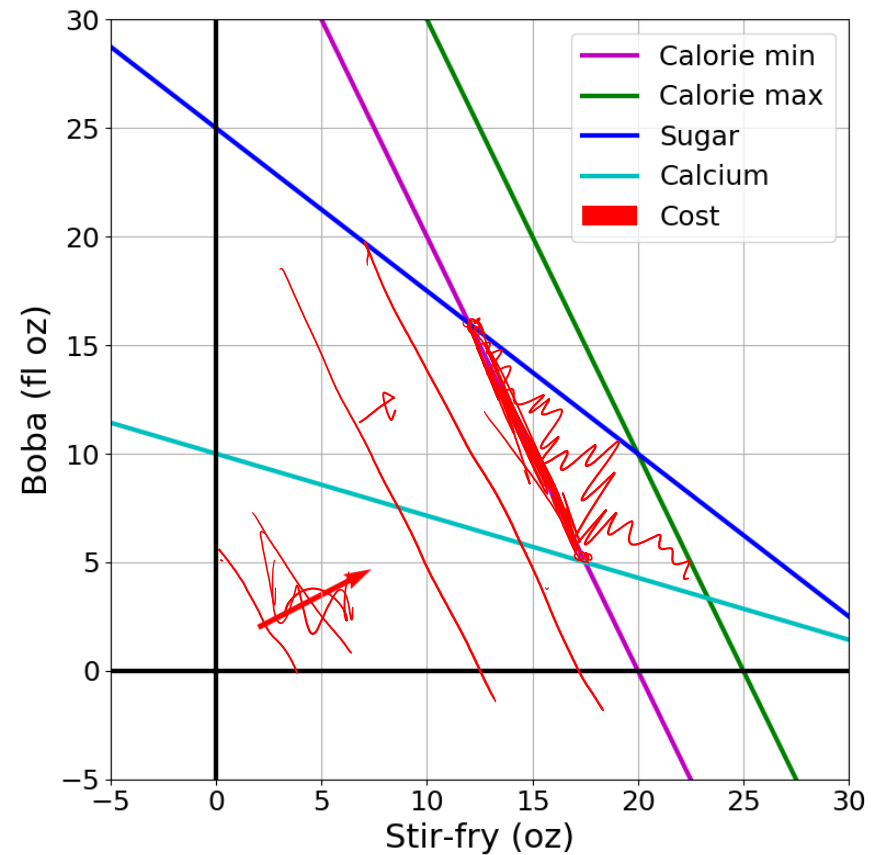
Optimization

Problem
Description

Optimization
Representation

$$\begin{array}{ll} \min_{\mathbf{x}} & \mathbf{c}^T \mathbf{x} \\ \text{s.t.} & \mathbf{Ax} \leq \mathbf{b} \end{array}$$

Graphical Representation



Graphics Representation

Geometry / Algebra I Quiz

What shape does this inequality represent?

$$a_1 x_1 + a_2 x_2 \leq b_1$$

Graphics Representation

Geometry / Algebra I Quiz

What shape does this inequality represent?

$$a_1 x_1 + a_2 x_2 = b_1$$

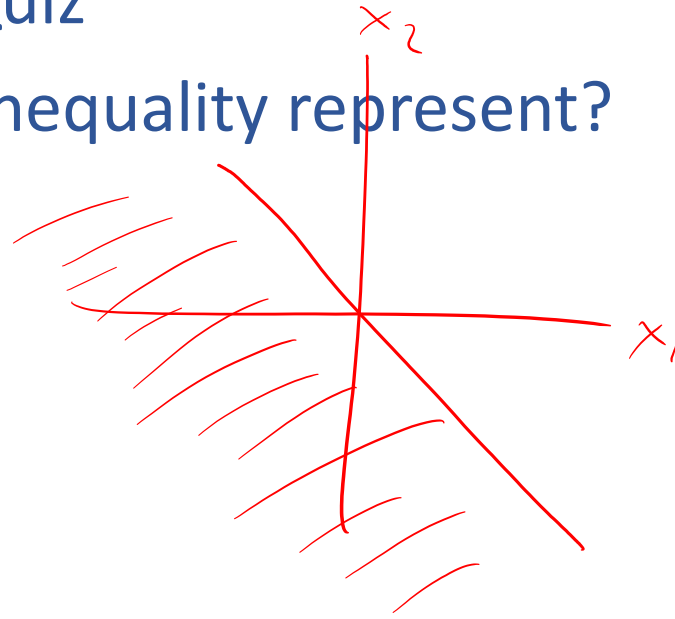
$$a_1 x_1 + a_2 x_2 \leq b_1$$

$$a_{1,1} x_1 + a_{1,2} x_2 \leq b_1$$

$$a_{2,1} x_1 + a_{2,2} x_2 \leq b_2$$

$$a_{3,1} x_1 + a_{3,2} x_2 \leq b_3$$

$$a_{4,1} x_1 + a_{4,2} x_2 \leq b_4$$



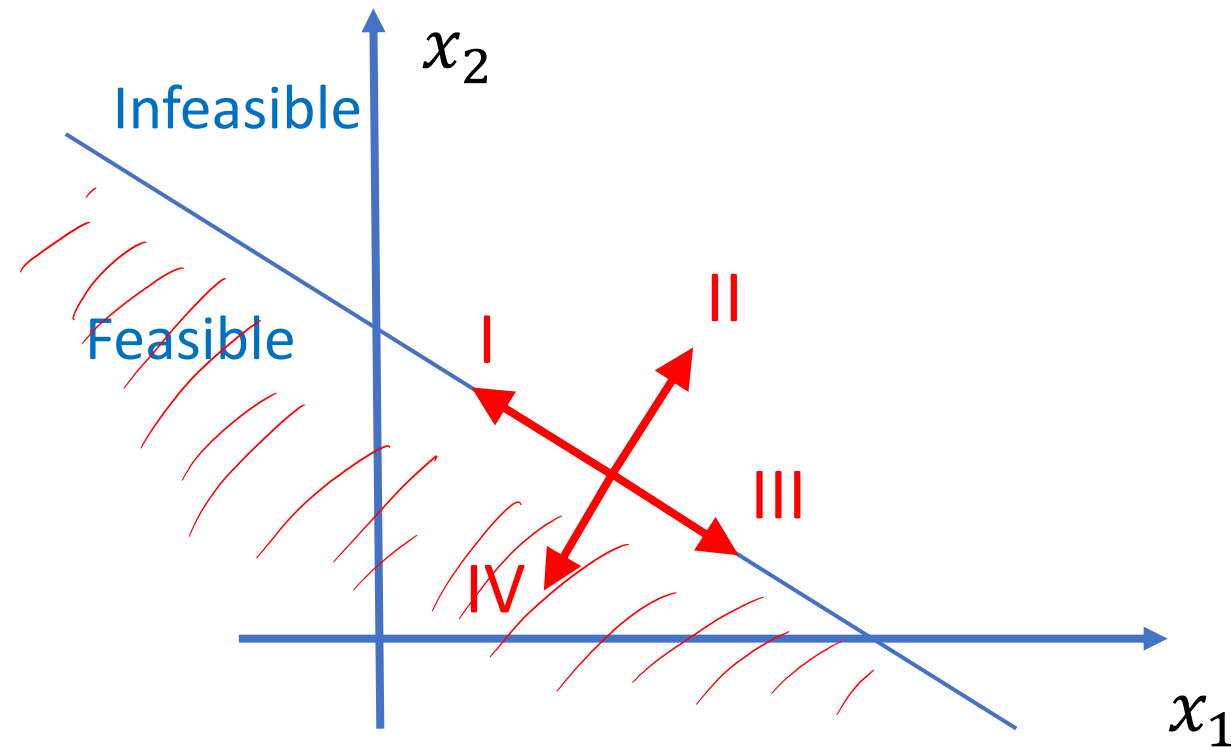
Poll 4

What is the relationship between the half plane:

$$a_1 x_1 + a_2 x_2 \leq b_1$$

and the vector:

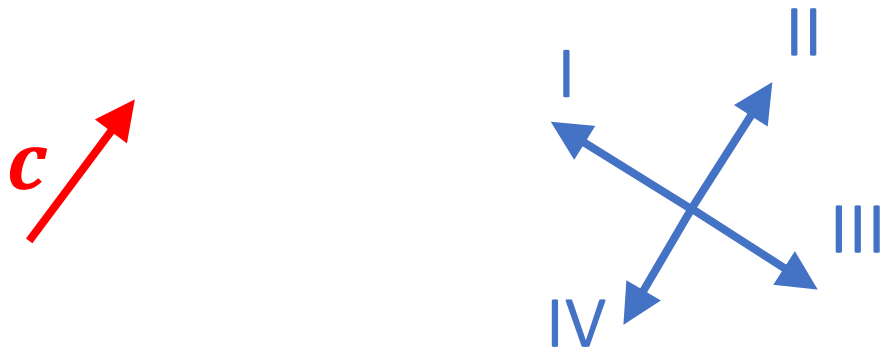
$$[a_1, a_2]^T$$



Poll 4

Given the cost vector $[c_1, c_2]^T$ and initial point $\mathbf{x}^{(0)}$,

Which unit vector step $\Delta \mathbf{x}$ will cause $\mathbf{x}^{(1)} = \mathbf{x}^{(0)} + \Delta \mathbf{x}$ to have the lowest cost $\mathbf{c}^T \mathbf{x}^{(1)}$?



Notation Alert!

Cost Contours

Given the cost vector $[c_1, c_2]^T$ where will

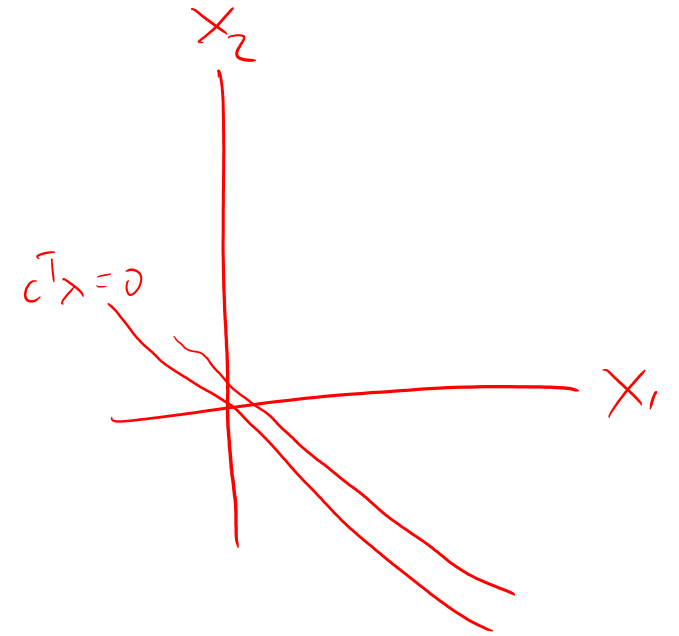
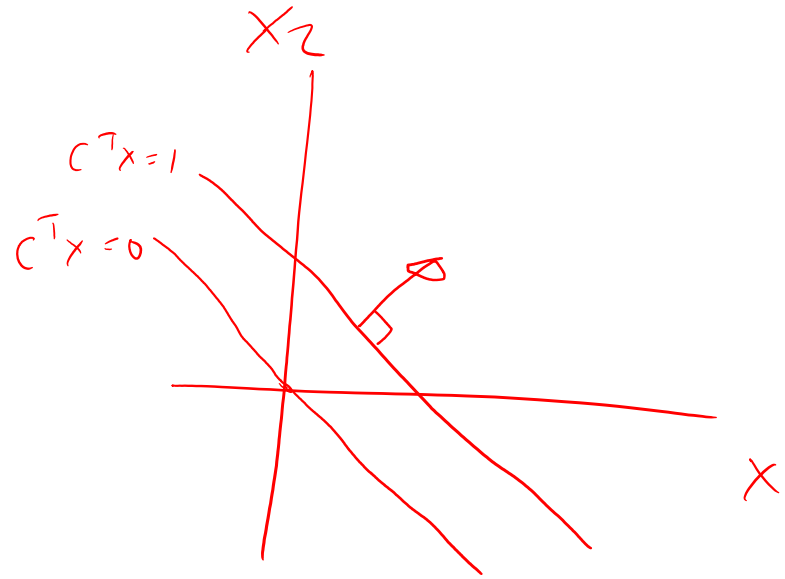
$$c^T x = 0 ?$$

$$c^T x = 1 ?$$

$$c^T x = 2 ?$$

$$c^T x = -1 ?$$

$$c^T x = -2 ?$$



Poll 5

As the magnitude of \mathbf{c} increases, the distance between the contours lines of the objective $\mathbf{c}^T \mathbf{x}$:

A) Increases

B) Decreases