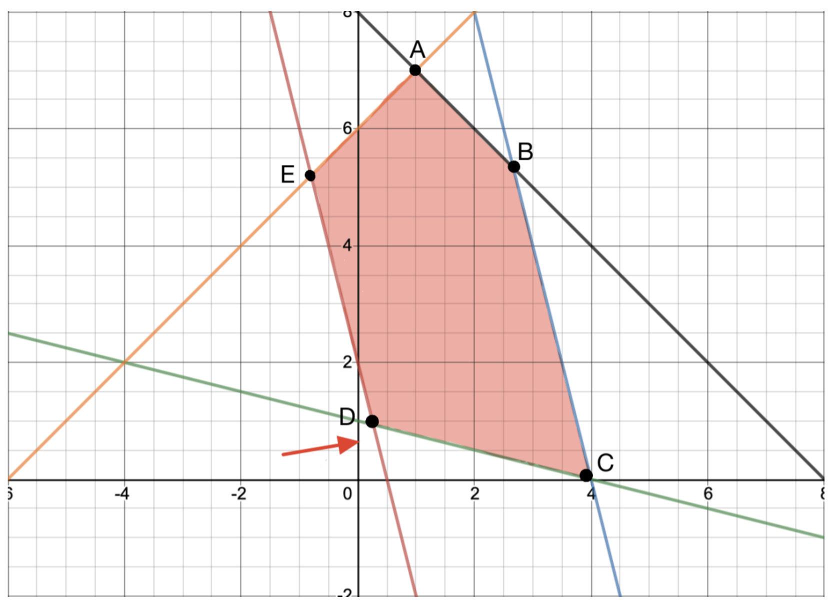
1 Algorithms for Solving Linear Programming

In lecture, we went through two algorithms for solving linear programming programs - vertex enumeration and the hill climbing algorithm.

Consider this linear programming problem. The goal is to minimize the cost, and the cost vector (red) is perpendicular to the blue and red lines.



1. Briefly describe both algorithms and explain how they differ. (hint: use terms such as vertices, intersections and neighbors).

2. Run the hill climbing algorithm starting from point B. Now try running the algorithm starting from point C. How do their solutions differ?

2 Cargo Plane: Linear Programming Formulation

A cargo plane has three compartments for storing cargo: front, center and rear. These compartments have the following limits on both weight and space:

Compartment	**Weight capacity (tons)**	**Space capacity (cubic metres)**
Front	10	6800
Centre	16	8700
Rear	8	5300

The following four cargoes are available for shipment on the next flight:

Cargo	**Weight (tons)**	**Volume (cubic metres/ton)**	**Profit ($/ton)**
C1	18	480	310
C2	15	650	380
C3	23	580	350
C4	12	390	285

Any proportion of these cargos can be accepted. The objective is to determine how much of each cargo C1, C2, C3 and C4 should be accepted and how to distribute each among the compartments so that the total profit for the flight is maximised. **Formulate** the above problem as a linear program (what is the objective and the constraints?). Think about the assumptions you are making when formulating this problem as a linear program.

If you were to put this linear program into standard form, what would be the dimensions of $A, \mathbf{b}, \mathbf{c}, \mathbf{x}$?

Now consider a simpler problem. There is a cargo plane with a single compartment with limit on 20 tons weight and 2400 cubic meters limit on space. You want to use this cargo plane to transport boxes of oranges and pineapples to sell in a market overseas.

Your goal is to maximize the number of gold pieces under following constraints:

- The market only allows each person to sell 14 boxes.
- 1 box of oranges has weight 1 ton and volume of 100 cubic meters.
- 1 box of pineapples has weight 2 tons and volume of 300 cubic meters.
- You earn 5 gold pieces for 1 box of oranges.
- You earn 12 gold pieces for 1 box of pineapples.

We will now formulate and solve the LP.

1. Write the LP in inequality form.
2. Graph the constraints, cost vector, and at least 3 cost contours. Indicate the feasible region.
3. What is the **optimal number** of boxes of oranges and pineapples? How much gold does this earn?

3 CSP as IP

Alice, Bob, and Charles want to study in Gates, which has 9 floors. To maximize productivity, we need to assign each of them to a separate floor without violating the following constraints:

- Alice only has access to floors 4 through 8
- Bob only has access to floors 3 through 7
- Charles must be at least 2 floors higher than Bob
- Alice must be at least 1 floor higher than Bob
- Alice must be at least 1 floor lower than Charles

Our goal is to assign Alice, Bob, and Charles to the highest possible floors.

1. Formulate the problem as a CSP.

2. Formulate the problem as an IP problem.

4 Baymax's Factory

Baymax and the 281 TAs have opened a factory to produce special medicine and bandages. These are really difficult to produce and require the collaboration of robots and humans.

To produce an ounce of medicine, it takes 0.2 hours of human labor and 4 hours of robot labor. To produce an inch of bandage, it takes 0.5 hours of human labor and 2 hours of robot labor. An ounce of medicine sells for $30 and an inch of bandages sells for $30. Medicine and bandages can be sold in fractions of an ounce or inch.

We want to maximize our profit so we can buy gifts for all the students. However, the TAs are really busy so they can only devote 90 human hours. In addition, Baymax can only devote 800 robot hours because he has other obligations to tend to. How can we maximize our profit?

1. Is this a linear, mixed or integer programming problem? Formulate and solve it.

2. Now suppose the items can only be sold in whole units (by ounce/inch). Is this a linear, mixed, or integer programming problem? Perform branch and bound for one branch level. You do not have to evaluate; writing out the constraints will suffice.

3. Now assume medicine can be sold in fractions but bandages can only be sold in whole units. What kind of a programming problem would this be, and how would our evaluation process differ from the problem type in part b?

4. How many optimal solutions can a LP have? How about IP?

5 4-Queens

Recall the 4-Queens problem. The goal is to place 4 chess queens on a 4x4 chess board such at no two queens are in the same row, column and diagonal.

Formulate the 4-Queens problem as an integer programming problem.