

1 Bayes' Nets Warm Up

1. Are the following statements true or false? If it is false, briefly justify why not.
 - (a) In a Bayes Net, a directed acyclic graph, each node can correspond to multiple random variables.

 - (b) Bayes Nets allow us to represent conditional independence relationships in a concise way.

 - (c) The number of entries in a conditional probability table in a Bayes Net is exponentially larger than in a joint probability table.

 - (d) In a Bayes Net with no edges, the variables are all dependent on each other.

2 Bayes' Nets

- Now let's look at some common types of queries we might be interested in answering using a Bayes' Net.

Let C = some variable we care about, D = some variable we don't care about (and we also don't know the value of), E = some variable we've observed to have the value e (i.e., current knowledge/evidence). (Suppose for now C, D, E are the only nodes in our BN.)

Express each of the following generic queries using probability notation, then write how we would compute it using the full joint distribution, $P(C, D, E)$. The first expression has been written for you as an example to get started.

- What's the probability of the outcome $C = c$ given we have observed $E = e$?

$$P(c | e) = \frac{P(c, e)}{P(e)} =$$

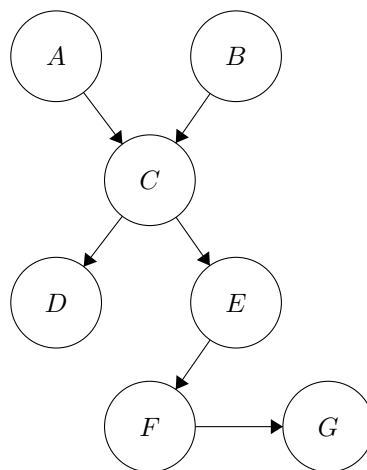
- What are the probabilities of all possible outcomes of C given we have observed $E = e$?

$$P(C | e) = \frac{P(C, e)}{P(e)} =$$

- Which value of C is least likely given we have observed $E = e$?

- Bayesian networks represent our beliefs about the conditional independence relationships between the variables in a system. The joint distribution is factored into conditional distributions, which are represented by the connections between nodes.

Consider the following Bayes net.



- Suppose all variables have domains of size 2. How many entries are in the full joint probability table $P(A, B, C, D, E, F, G)$?

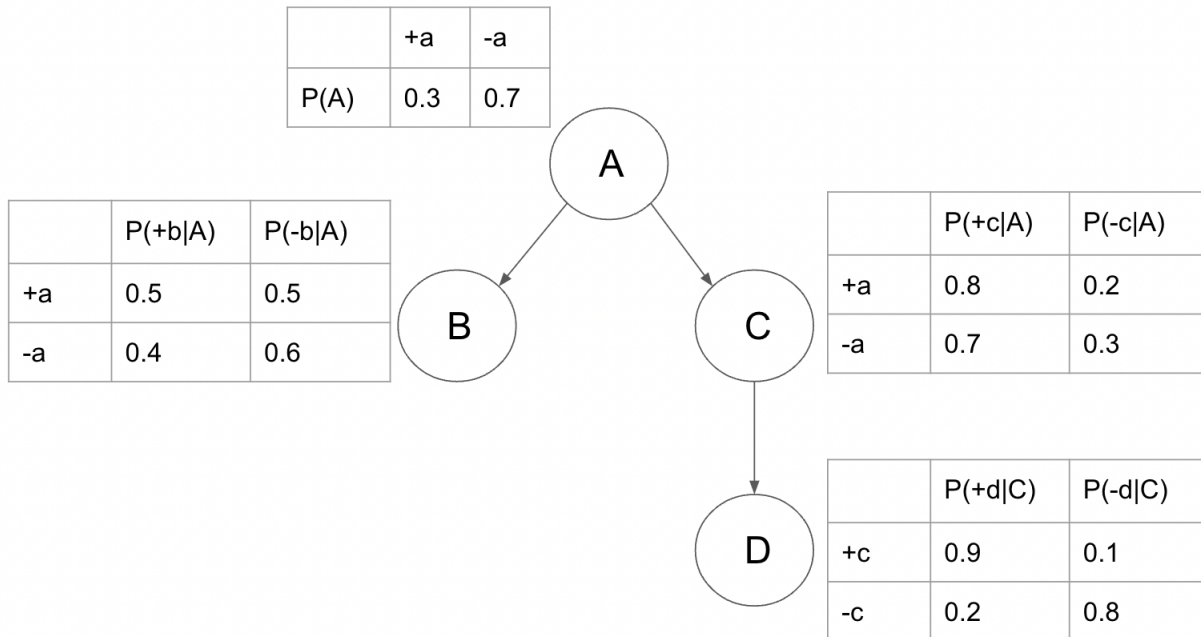
- (b) How many entries are in the conditional probability table at each of the individual nodes?

| Variable | A | B | C | D | E | F | G |
|-----------|-----|-----|-----|-----|-----|-----|-----|
| # Entries | | | | | | | |

- (c) How many CPT entries are needed to represent this Bayes net in total?
- (d) How do we represent the full joint probability using the conditional probability tables of each variable?
- (e) Give an expression for the probability $P(D, E \mid C)$. Use as many conditional independence relations as possible from the Bayes Net.
- (f) Now give an expression for the probability $P(A, B \mid C)$. Still use as many conditional independence relations as possible from the Bayes Net.

3 Bayes Net Calculations

Consider the following Bayes Net:

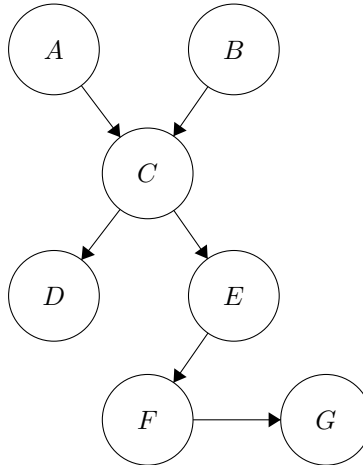


Calculate the following probabilities based on the given Bayes Net:

1. $P(+a, -b, -c, +d)$
2. $P(+c)$
3. $P(-d \mid -c)$
4. $P(-b, -c, +d)$
5. $P(+a \mid -b, -c, +d)$

4 Bayes Net Independence

Consider the Bayes net from the Question 2 (included below again for convenience). We will be considering whether or not certain combinations of variables will be conditionally independent given other variables.



1. Are A and B conditionally independent given C ?
2. Are A and B conditionally independent given D ?
3. Are A and D conditionally independent given C ?
4. Are D and G conditionally independent given E ?
5. Are D and G conditionally independent given C ?

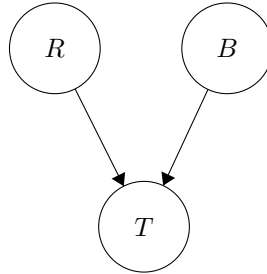
5 Pinky's Bayes' Net

Pinky the Penguin loves spending time at CMU. Some of her favorite activities include working out (W), cooking (C), and teaching 281 (T). She has seen that her happiness level (H) is directly affected by whether or not she teaches 281 (T) and whether or not she is able to cook her meals (C). If Pinky is happy (H), she has the energy to be able to work out (W). Overall, Pinky feels that working out has allowed her to live a healthy lifestyle (Y).

1. Using the variables defined above W , C , H , T , and Y , draw the Bayes net representing Pinky's life from the information given in the description above.
2. Can you find a conditional independence relationship between variables? How about a group of three variables such that two variables are NOT conditionally independent given the third?

6 Common Effect

Recall the common effect example from lecture: both raining (R) and a ballgame (B) can cause traffic (T).



In this example $P(R = 1) = 0.7$, $P(B = 1) = 0.2$, and the probability table of T given R and B is below:

| R | B | $P(T = 1 R, B)$ | $P(T = 0 R, B)$ |
|-----|-----|-----------------|-----------------|
| 1 | 1 | 0.9 | 0.1 |
| 1 | 0 | 0.5 | 0.5 |
| 0 | 1 | 0.5 | 0.5 |
| 0 | 0 | 0.1 | 0.9 |

We will now answer a few queries on this Bayes Net to determine the probability of a ballgame given different sets of knowledge.

1. What is the factorization of the joint distribution? What is the probability that it is not raining and there is no ballgame but there is still traffic?
2. What is the probability of a ballgame if we have not observed any of the variables?
3. What is the probability of a ballgame given that there is traffic? How does it compare to the probability from the previous question? Is ballgame independent of traffic?
4. What is the probability of a ballgame given that there is traffic but no rain? How does it compare to the probability from the previous questions? Is ballgame conditionally independent of rain given traffic?