Kidney exchanges

(largely follows Abraham, Blum, Sandholm 2007 paper)

Vincent Conitzer

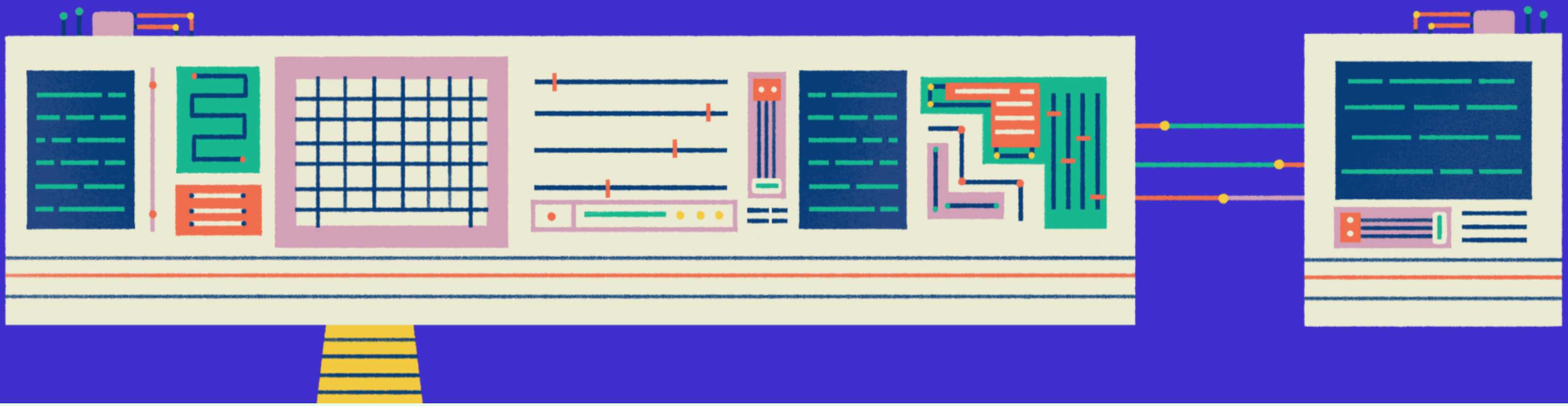
Prescription AI

This series explores the promise of AI to personalize, democratize, and advance medicine—and the dangers of letting machines make decisions.

THE BOTPERATING TABLE

How AI changed organ donation in the US

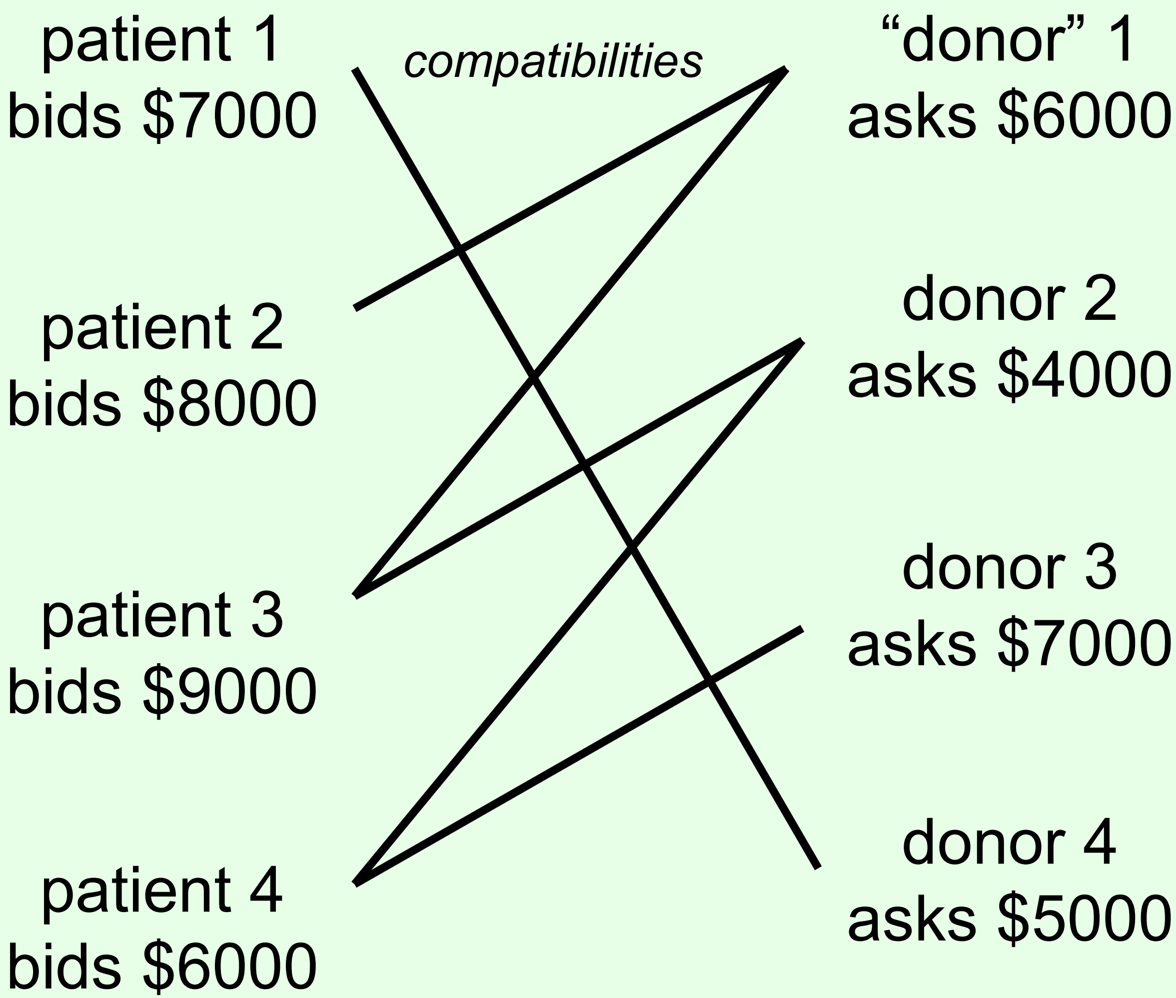
By Corinne Purtill • September 10, 2018



Kidney transplants

- Kidneys filter waste from blood
- Kidney failure results in death in months
- Dialysis: regularly get blood filtered in hospital using external machine
  - Low quality of life
- Preferred option: kidney transplant
  - Cadaver kidneys
  - Donation from live person (better)
- Must be compatible
- Shortage of kidneys…

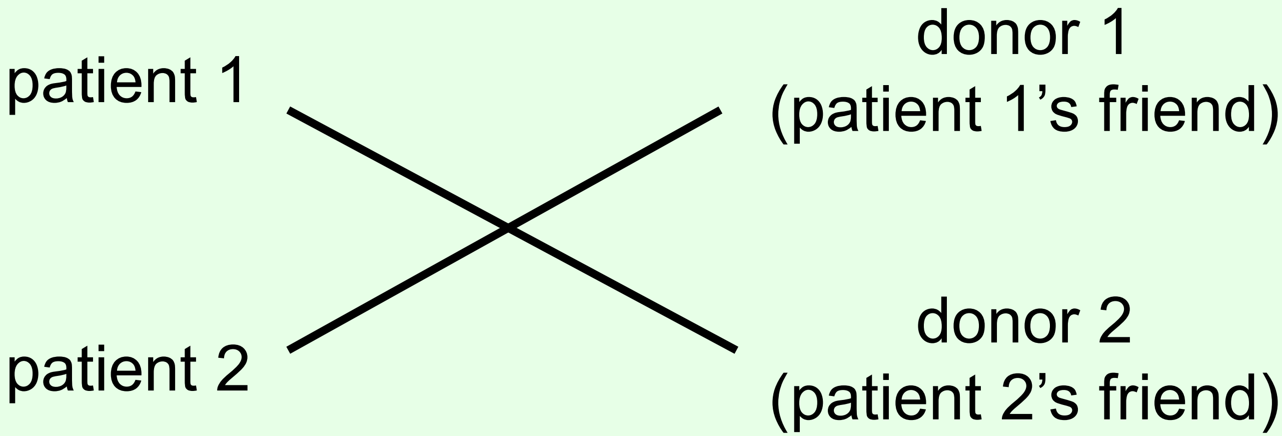
An imaginary kidney exchange with money



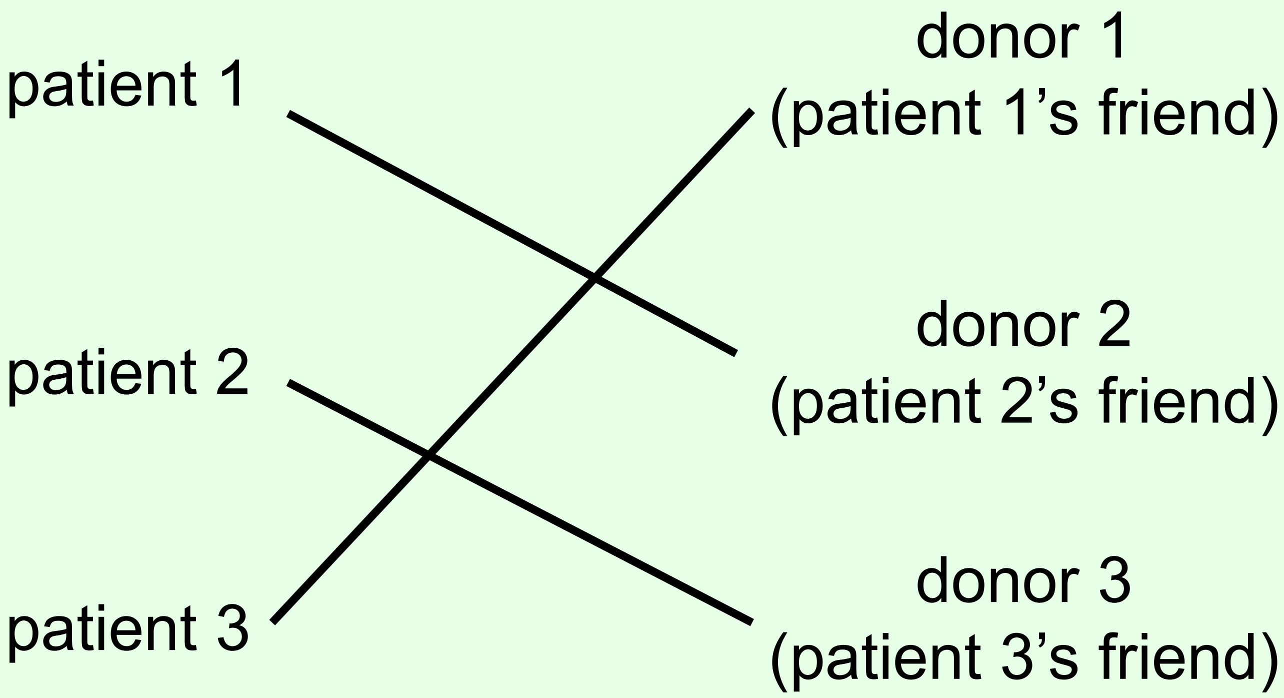
Selling kidneys is illegal!

- Large international black market
  - Desperate people on both ends…
- What can we do legally?

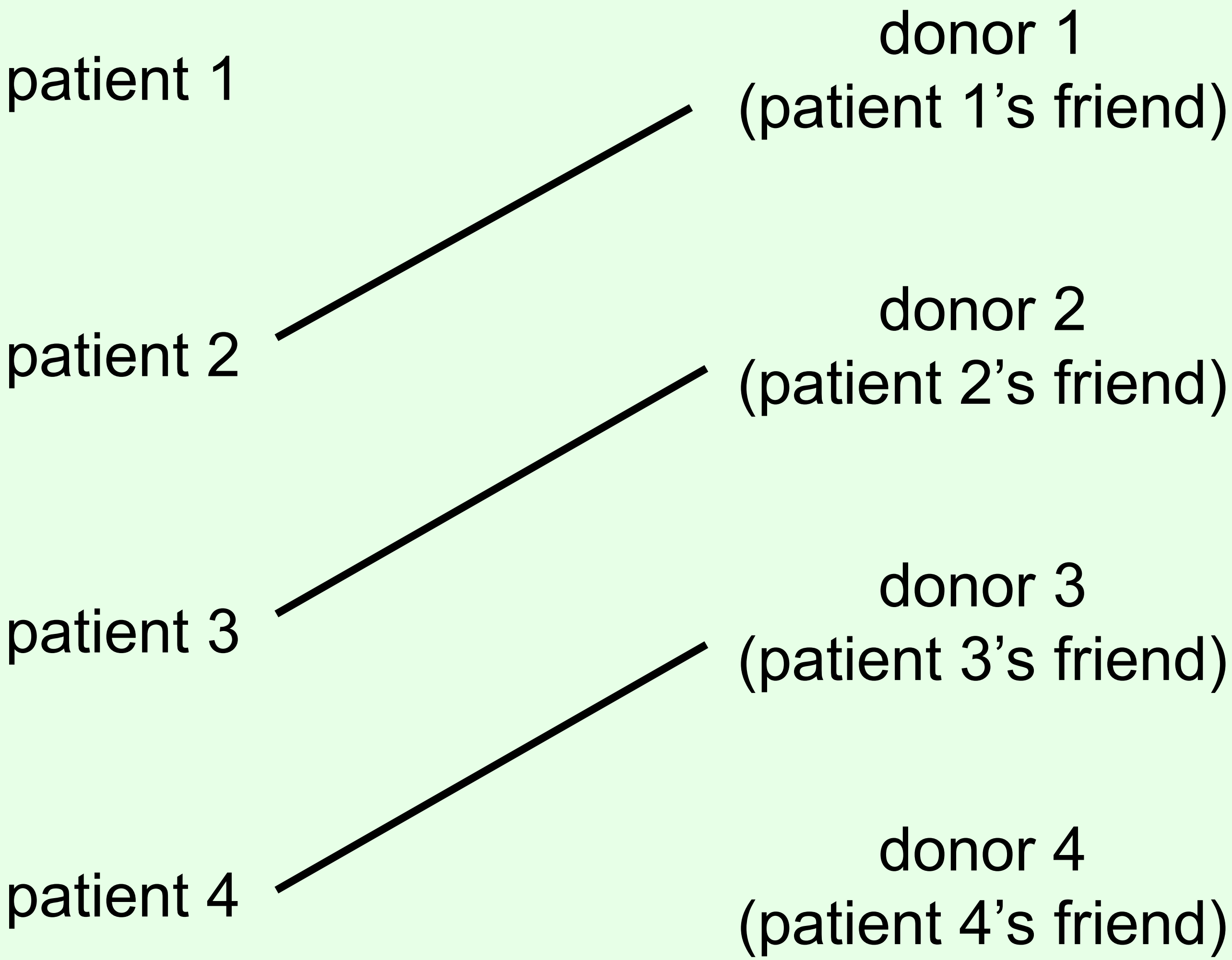
Kidney exchange



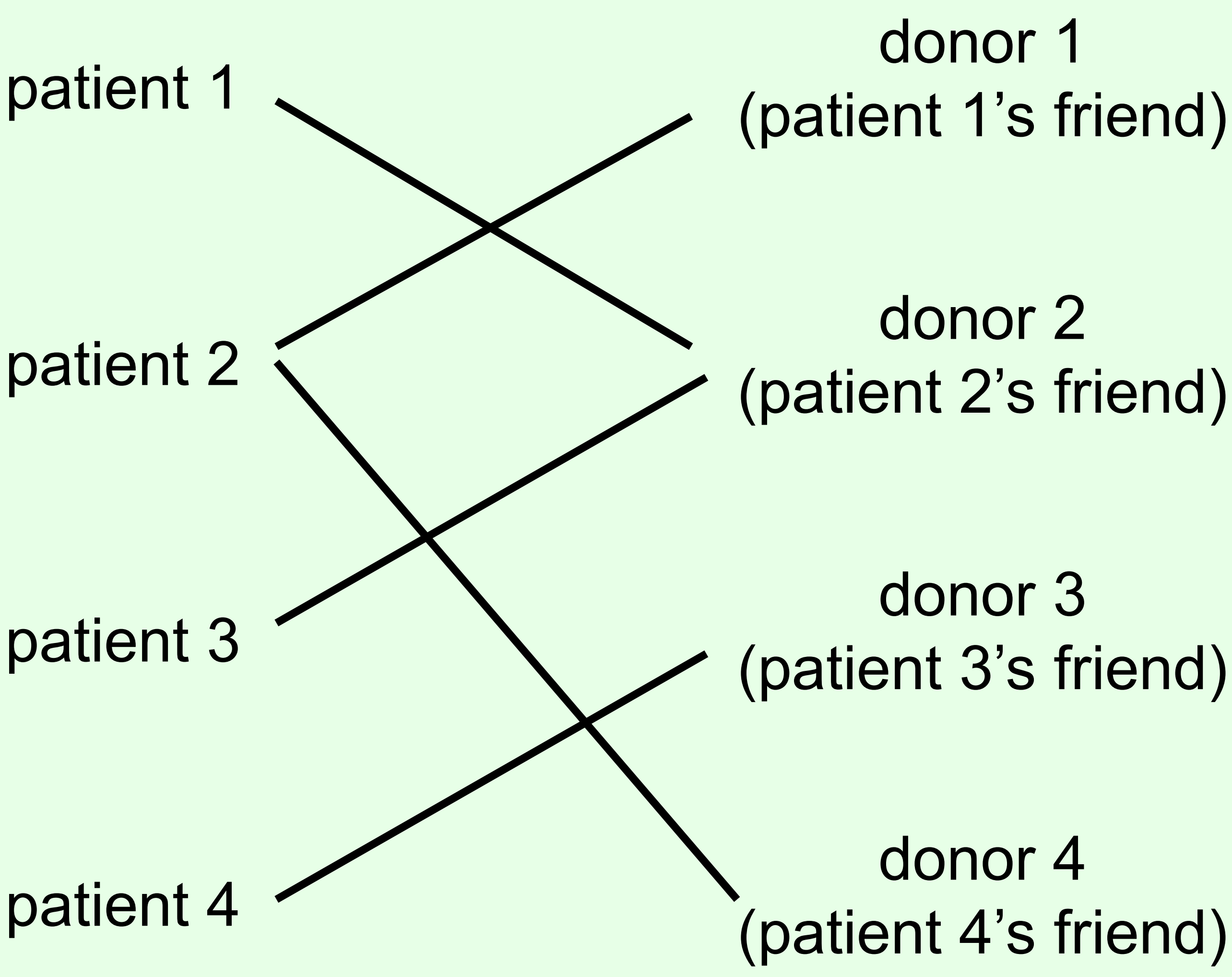
Kidney exchange (3-cycle)



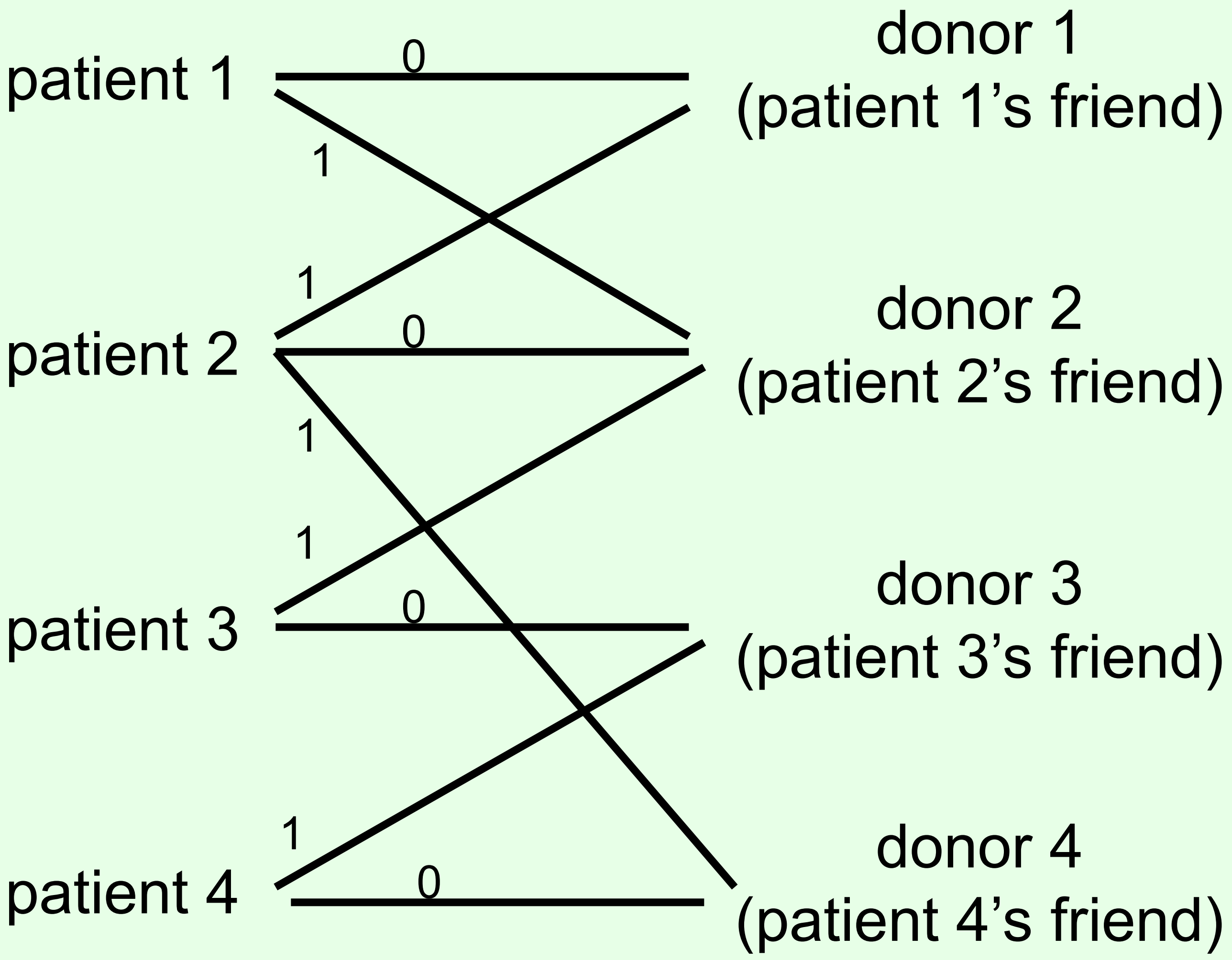
Another example



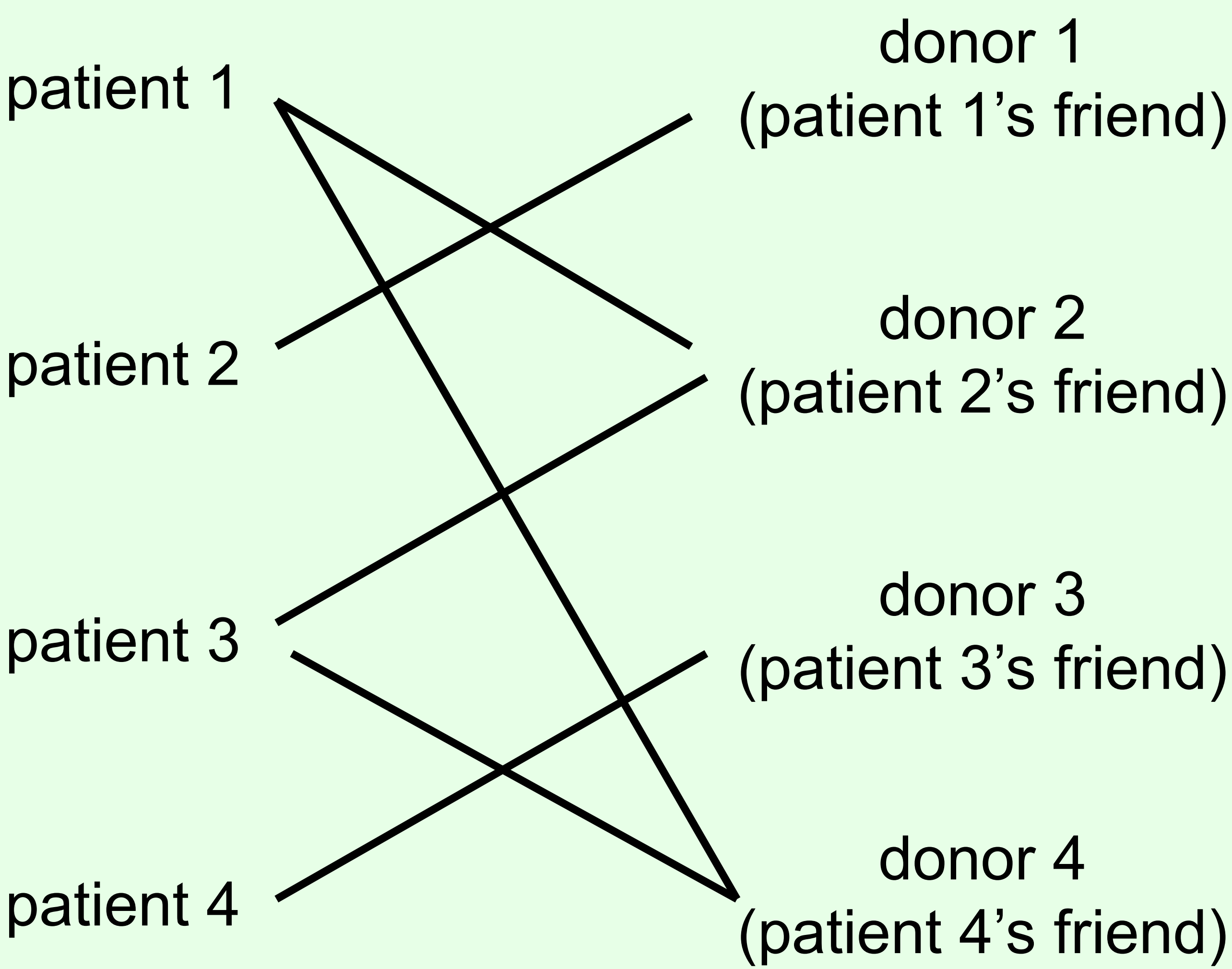
More complex example



Solving kidney exchange as maximum weighted bipartite matching



Which solution is better?

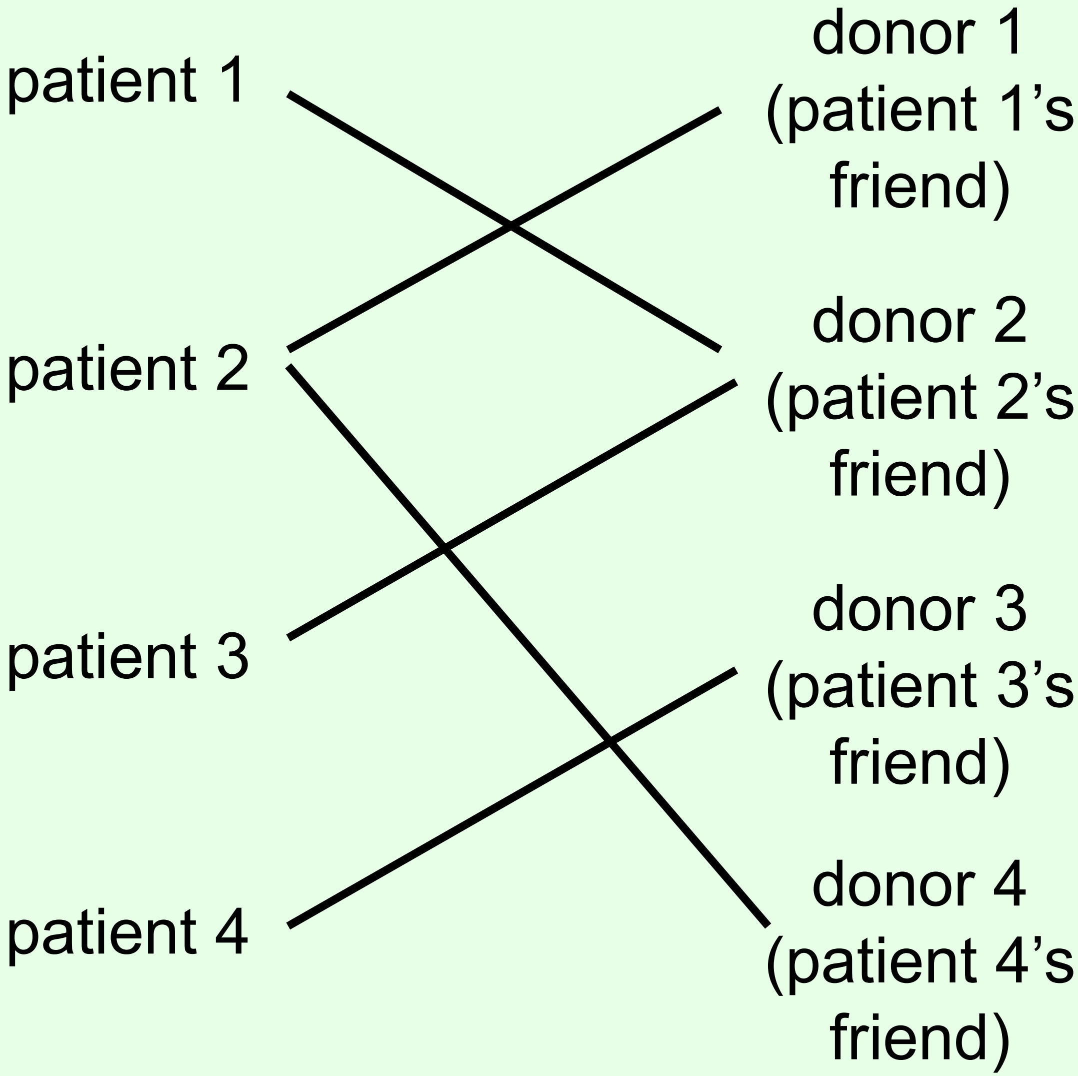


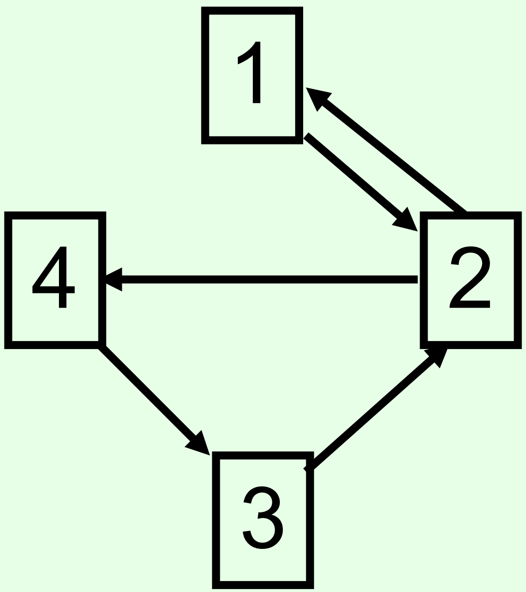
Long cycles are impractical

- All patients in a cycle must be operated on simultaneously
  - Otherwise donor can wait for friend to receive kidney, then back out
  - Contracts to donate an organ not binding
- If last-minute test reveals incompatibility, whole thing falls apart

- Require each cycle has length at most k

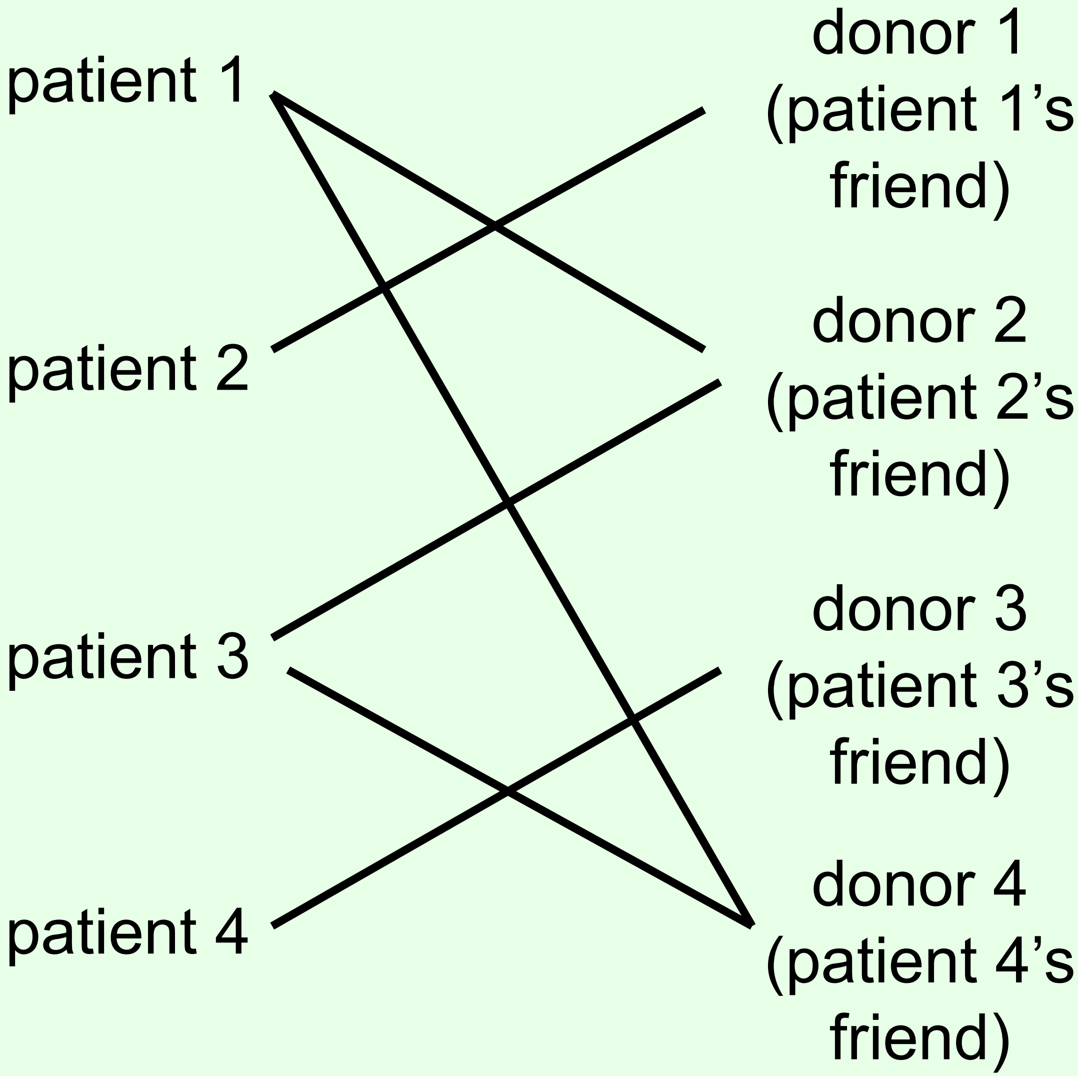
Different representation

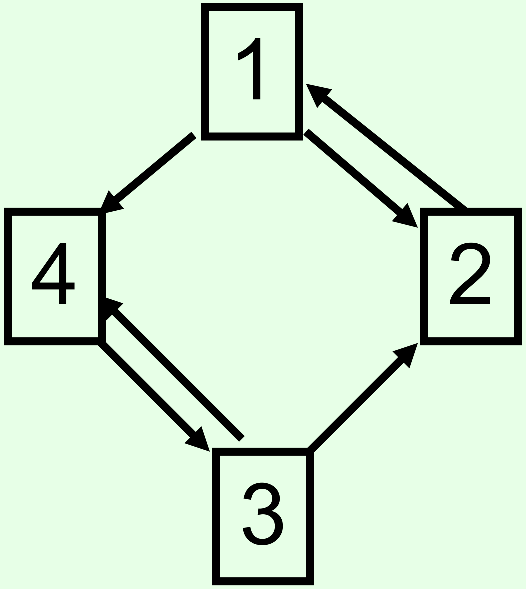




edge from i to j = patient i wants donor j's kidney

Different representation

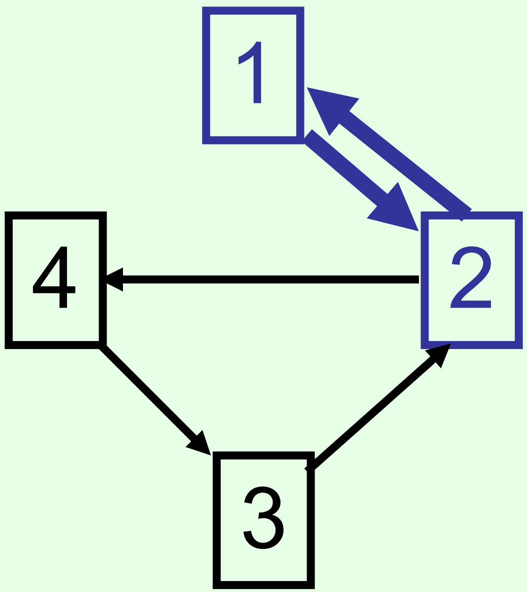




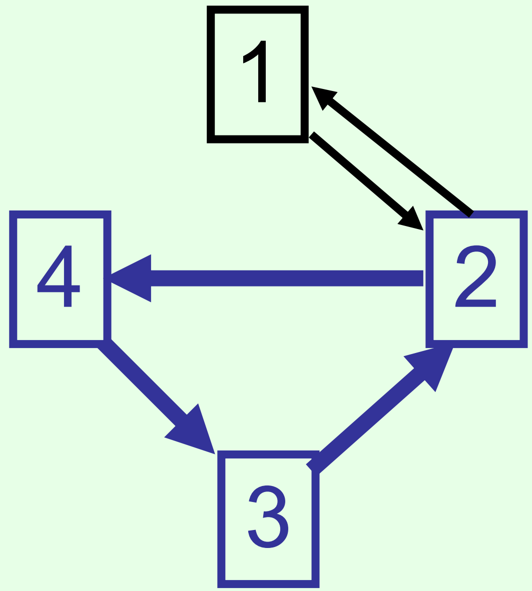
edge from i to j = patient i wants donor j's kidney

Market clearing problem

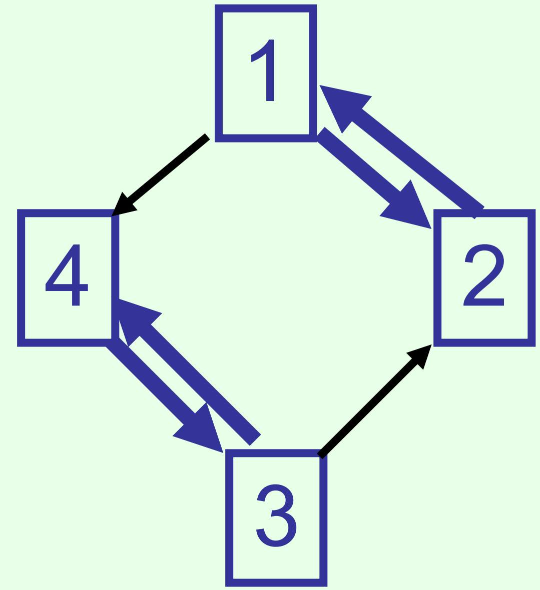
- Try to cover as many vertices as possible with (vertex-)disjoint cycles of length at most k



k=2



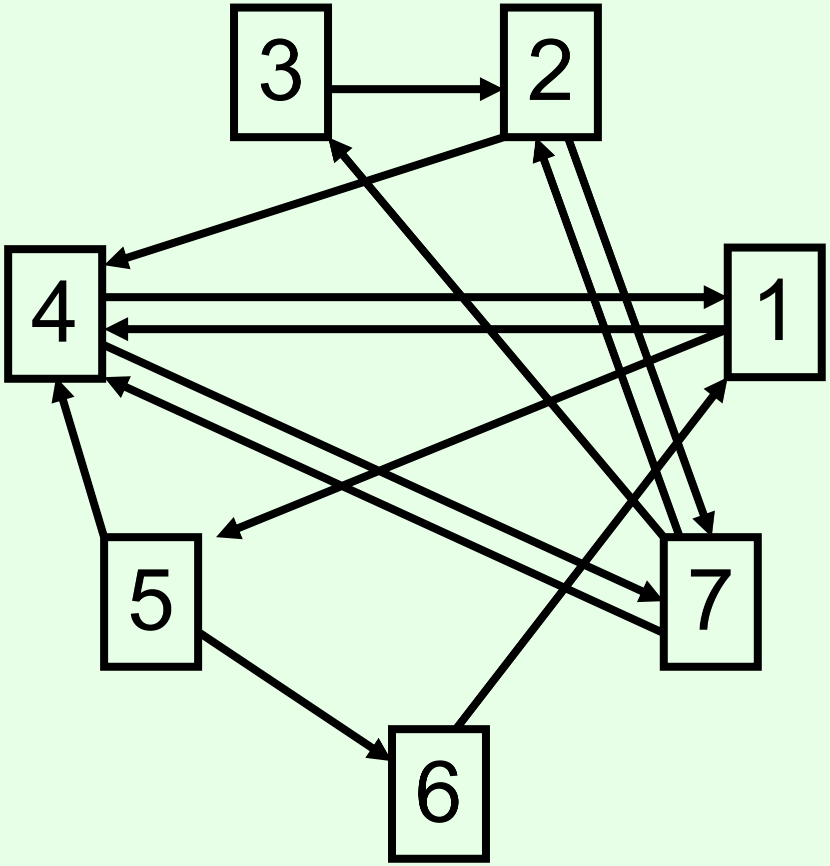
k=3



k=2,3

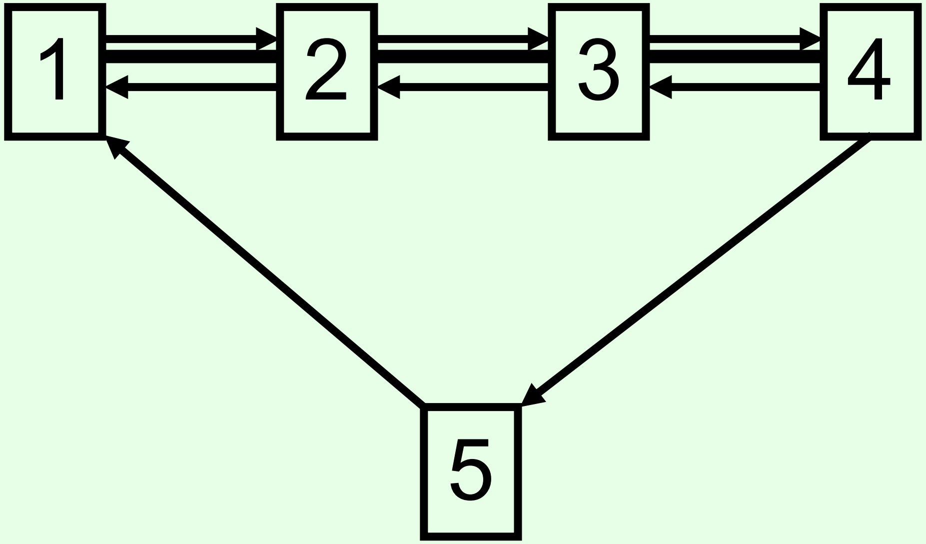
Market clearing problem

- Try to cover as many vertices as possible with (vertex-)disjoint cycles of length at most k



Special case: k=2

- If edges go in both directions, replace by undirected edge
- Remove other edges



- Maximum matching problem!

Complexity

- k = 2: in P by maximum matching
- k = number of vertices (no constraint): in P by maximum weighted bipartite matching
- k = 3, 4, 5, ….: NP-hard!

An integer programming formulation

- For each edge from i to j, make a binary variable x_{ij}
  - 1 if i gets j's kidney, 0 otherwise
- maximize $\Sigma_{ij} x_{ij}$
- subject to:
- for every i: $\Sigma_j x_{ij} = \Sigma_j x_{ji}$
  - (number of kidneys received by i = number of kidneys given by i)
- for every j: $\Sigma_i x_{ij} \leq 1$
  - (j gives at most 1 kidney)
- for every path $i_1\ i_2 \ldots i_k\ i_{k+1}$ with $i_1 \neq i_{k+1}$: $\Sigma_{1 \leq j \leq k} x_{i_j i_{j+1}} \leq k-1$
  - (no path of length k that doesn't end up where it started, hence no cycles greater than k)

Another integer programming formulation
(turns out better)

- For each cycle c of length at most k, make a binary variable x_c
  - 1 if all edges on this cycle are used, 0 otherwise
- maximize $\Sigma_c |c| x_c$
- subject to:
- for every vertex i: $\Sigma_{c:\ i \text{ in } c} x_c \leq 1$
  - (every vertex in at most one used cycle)

Program size

- Even for small k, number of paths/cycles is too large in reasonably large exchanges
- Solution: generate constraints/variables on the fly during solving
  - Constraint/column generation

Another integer program (not in paper)

- Say an "event" is a set of simultaneous operations
- Denote events by $t = 1, \dots, T$ (how big should T be?)
- For each edge from i to j, for each t, make a binary variable x_{ijt}
  - 1 if i gets j's kidney in event t, 0 otherwise
- maximize $\Sigma_{i,j,t} x_{ijt}$
- subject to:
- for every i, t: $\Sigma_j x_{ijt} = \Sigma_j x_{jit}$
  - (number of kidneys received by i in event t = number of kidneys given by i in event t)
- for every j: $\Sigma_{i,t} x_{ijt} \leq 1$
  - (j gives at most 1 kidney overall)
- for every t: $\Sigma_{i,j} x_{ijt} \leq k$
  - (at most k operations per event)

Other applications

- Barter exchanges: agents want to swap items without paying money
- Peerflix (DVDs)
- Read It Swap It (books)
- Intervac (holiday houses)
- National odd shoe exchange
  - People with different foot sizes
  - Amputees

Modeling

- What assumptions have we implicitly made in modeling a kidney exchange?
- What problems might come up that we haven't thought about?
- What additional aspects could one model to get even better results?