Computational Microeconomics - Practice Final

Problem 1: True or False (24 points).

Label each of the following statements as true or false. You are not required to give any explanation.

1. The Japanese auction and the second-price sealed-bid auction are strategically equivalent even when valuations are interdependent.
2. In the combinatorial auction winner determination problem, if we allow bids to be partially accepted, this makes winner determination easier computationally.
3. Any valuation function can be expressed in the OR language.
4. The Borda rule (where an alternative gets one point for being ranked second-to-last, two points for being ranked third-to-last, ...) is strategy-proof, i.e., it cannot be manipulated by misrepresenting one's preferences.
5. Risk aversion is inconsistent with maximizing expected utility.
6. When playing a two-player zero-sum game, a maximin strategy is not always a Nash equilibrium strategy.
7. We know how to compute a Nash equilibrium of a two-player zero-sum game using linear programming.
8. We know how to compute a Nash equilibrium of a two-player general-sum game using linear programming.
9. Committing to a pure strategy before the other player moves is never disadvantageous.
10. In some games, people do not play an equilibrium right away, but over time (repeated play) they get close to playing an equilibrium.
11. In some games, people play very differently from what a simple game-theoretic analysis would suggest.
12. If you have a non-direct revelation mechanism and know its solution (e.g., equilibrium), you can always turn this into an incentive compatible direct-revelation mechanism (where agents report their valuations/types directly and are incentivized to report truthfully).

Problem 2: Planning to go to one or more restaurants (20 points).

We have some set of people who want to go to some set of restaurants. For each person i and each restaurant r, i has a value v_{ir} for going to that restaurant. Also, for every two people i and j, person i has a value of w_{ij} for going to the same restaurant as j (note w_{ij} is not necessarily equal to w_{ji}). An agent i's valuation is the sum of that agent's applicable v_{ir} and w_{ij} (you can get only one of your v_{ir} but potentially multiple of your w_{ij}). We wish to determine who should go to which restaurants, so as to maximize the sum of the agents' valuations. Every agent must go to a single restaurant. Note that not everyone needs to go to the same restaurant (though they can).

For example, consider three agents Alice, Bob, and Carol, who are considering whether to go to a French, Indian, or Mexican restaurant. Alice likes French ($v_{AF} = 10$) and to be with Bob ($w_{AB} = 8$). Bob likes Indian ($v_{BI} = 12$) and to be with Carol ($w_{BC} = 7$). Carol likes Mexican ($v_{CM} = 11$) and to be with Alice ($w_{CA} = 9$). Nobody likes anything or anyone else, i.e., all the other v_{ir} and w_{ij} are zero.

a. What is the optimal solution for this example?

b. Compute the Clarke mechanism (GVA) payments of all the agents in the example. (Here, some payments may be negative because some agents may contribute to the welfare of others by being present.)

c. Give an integer program for computing the optimal solution in general (for arbitrary v_{ir} and w_{ij}; your integer program doesn't have to compute the Clarke payments, just the optimal solution). You can write it either mathematically or in the modeling language (but if write it mathematically, be very precise in your use of $\forall$ and be clear about which variables you are summing over—the modeling language of course forces you to do so). You don't need to enter the data from the above example.

Problem 3: Modified Rock-Paper-Scissors (20 points).

Consider the following modified version of Rock-Paper-Scissors, where losing with Paper to Scissors is considered doubly humiliating:

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-2,2
Scissors	-1,1	2,-2	0,0

a. Wright argues that in every equilibrium of this game, every pure strategy must receive positive probability from both players. Is Wright right or wrong? Explain why.

b. Based on your answer in **a**, compute a Nash equilibrium of this game. Is it the unique equilibrium? Why (not)?

Problem 4: A game with a hidden coin flip (20 points).

In this problem, you will solve a simple game (reminiscent of "Liar's Dice" if you happen to know it). Player 1 flips a coin and sees the result; player 2 does not see the result. Heads is a "winning" coin flip, Tails is a "losing" coin flip. Player 1 makes a claim about the coin flip to player 2, either claiming to have flipped Heads, or claiming to have flipped Tails. Player 2 can choose to Dispute the claim, or to Accept it.

If player 2 chooses to Dispute, player 1 must show the coin. (Of course, player 1 cannot change the result of the coin flip.) If player 1 lied, player 2 wins; if player 1 told the truth, player 1 wins. If player 2 chooses to Accept, then whatever player 1 claimed stands (regardless of what she actually flipped), and player 2 must flip the coin to compete with that claim.

For example, suppose player 1 flips Tails, but then claims to have flipped Heads. If player 2 Disputes, player 2 wins, because player 1 lied. If player 2 Accepts, then player 1's claim of Heads stands (and the fact that she actually flipped Tails becomes irrelevant), and player 2 must flip the coin to compete with Heads. If player 2 flips Tails, then he loses, because Tails is worse than Heads. If player 2 flips Heads, we have a tie.

Suppose the utility for winning is 1, the utility for losing is -1, and the utility for a tie is 0 (it's a zero-sum game). Give the extensive form of the game, convert it to normal (matrix) form (explaining what the strategies mean and calculating the expected utilities), and solve for the equilibrium of this game. (*Hint:* the normal form should be 4×4 and the whole process should be quite similar to a game you have seen before.)

Problem 5: Poor generalizations of the Vickrey auction (20 points). What we call the Generalized Vickrey Auction is a nice way to generalize the Vickrey auction to combinatorial auctions. But, there are many other ways to generalize the Vickrey auction to combinatorial auctions. We will consider some not-so-clever ways to generalize it here. We will call these *Poorly Generalized Vickrey Auctions (PGVAs)*. All of these allocate the items efficiently, but they determine the payments differently. For each of them, you must show one or more bad properties of this auction, by giving some examples. Your examples should consist of specific bids with specific numbers (they can be quite simple). You can assume all bidders are single-minded for this question.

PGVA #1: A bidder who wins bundle S pays the value of the highest other bid on a subbundle $S' \subseteq S$. For example, if there are only two bids, $(\{A\}, 5)$ and $(\{A, B\}, 10)$, the second bidder wins and pays 5 because $\{A\} \subseteq \{A, B\}$ (the first bidder bids on a subset of what the second bidder bids on). Show that this auction is not strategy-proof, that is, sometimes a bidder is better off bidding something other than her true valuation.

PGVA #2: A bidder who wins bundle S pays the value of the highest other bid on a superbundle $S' \supseteq S$. For example, if there are only two bids, $(\{A\}, 5)$ and $(\{A, B\}, 10)$, the second bidder wins and pays 0 because $\{A\} \not\supseteq \{A, B\}$ (the first bidder does not bid on a superset of what the second bidder bids on). Show that this auction does not satisfy voluntary participation, that is, sometimes a bidder ends up with negative utility. Also show that this auction is not strategy-proof, that is, sometimes a bidder is better off bidding something other than her true valuation.

PGVA #3: A bidder who wins bundle S pays the value of the highest other bid on an intersecting bundle S' (where $S' \cap S \neq \emptyset$). For example, if there are only two bids, $(\{A\}, 5)$ and $(\{A, B\}, 10)$, the second bidder wins and pays 5 because $\{A\} \cap \{A, B\} \neq \emptyset$ (the first bidder bids on a bundle that overlaps with what the second bidder bids on). Show that this auction does not satisfy voluntary participation, that is, sometimes a bidder ends up with negative utility. Also show that this auction is not strategy-proof, that is, sometimes a bidder is better off bidding something other than her true valuation.

Problem 6: Expected revenue of a Vickrey auction with a reserve price & optimizing the reserve price (20 points).

Recall that a Vickrey auction with a reserve price r works as follows. Let b_1 and b_2 be the highest and second-highest bids, respectively.

- If $b_1 \geq b_2 \geq r$, then the highest bidder wins and pays b_2.
- If $b_1 \geq r > b_2$, then the highest bidder wins and pays r.
- If $r > b_1$, nobody wins.

A Vickrey auction with reserve price is still strategy-proof, so you can assume bidders bid truthfully. Suppose all bidders have their valuation drawn (independently) from the uniform distribution over $[0, 1]$. Calculate the expected revenue as a function of r. Then, find the value of r that maximizes expected revenue for the auctioneer.

Note: I will give you half credit for this question if you solve it with only two bidders, and full credit if you solve it with n bidders.

Hint: As we did in class, consider the probability density function of the revenue. Above r, this probability density function is exactly the same as if there were no reserve price (the probability of getting any particular revenue above r is the same as if there were no reserve price). Specifically, as we calculated in class, the density function of revenue for the Vickrey auction is

$$f(x) = n(n-1)(x^{n-2} - x^{n-1})$$

(resulting in an expected revenue of

$$\int_0^1 x f(x) dx = \frac{n-1}{n+1}$$

for the no-reserve price case). So, the above-r component of the expected revenue can be calculated similarly. In our setting, it is not possible to get a revenue below r (other than 0). But, there is a significant probability of getting revenue exactly r. What is it? You need to combine this with the first (above-r) part to calculate the total expected revenue. Then you can optimize with respect to r. (You have actually seen the answer before, but you still need to do the math...)