

Homework 5: Bayesian games and Generalized Vickrey Auction (due Dec. 1 before 5pm)

Please read the rules for assignments on the course web page (<http://www.cs.cmu.edu/~15326-f23/>). Use Piazza for questions and Gradescope to turn this in.

1. Funding a project.

There is a project that needs funding. Two agents can fund the project. Each agent has two choices: 1. Pay 1 to fund the project; 2. Pay 0. Depending on funding, the project may succeed or fail. If the total amount of money paid by the agents is 2, then the project will succeed with probability 1. If the total amount is 1, then the project will succeed with probability $1/3$. If the total amount is 0, then the project will succeed with probability 0.

Each agent i ($i \in \{1, 2\}$) has a valuation v_i for a successful project. (Failed projects are worthless.) This valuation does not depend on whether the agent contributed or not (the agent can make use of the project regardless). Each valuation is drawn uniformly at random from $[0, 4]$.

For example, if agent 1 draws valuation 2 and pays 0, and agent 2 draws valuation 3 and pays 1, then:

- The probability of a successful project is $1/3$;
- Agent 1's expected utility is $(1/3) \cdot 2 - 0 = 2/3$;
- Agent 2's expected utility is $(1/3) \cdot 3 - 1 = 0$.

(a) Show that the following strategy is a Bayes-Nash equilibrium of this game (if both players use it): pay 1 if and only if your valuation is at least 2. (Hint: show that if the player's valuation is 2, then she is indifferent between the two actions, given that the other player uses this strategy.)

(b) More generally, suppose that each player's valuation is drawn (independently) from some arbitrary distribution with cumulative density function F (but keep everything else the same). Show that the following is a Bayes-Nash equilibrium of this game (if both players use it): pay if and only if your valuation is at least x , where x is the solution to $F(x) \cdot x - 2x + 3 = 0$.

2. Generalized Vickrey Auction (= Clarke mechanism on combinatorial auctions).

Consider the following 3 bids in a combinatorial auction with 3 items (with free disposal):

$(\{a, b\}, 10)$ XOR $(\{c\}, 4)$
 $(\{a, b\}, 6)$ XOR $(\{b, c\}, 9)$
 $(\{a\}, 3)$ XOR $(\{a, b, c\}, 11)$

Solve the winner determination problem, and compute the GVA (Clarke) payments for the winning bidders. (Remember to remove each bidder's *entire* bid when calculating her payment.)