Automated mechanism design

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General vs. specific mechanisms

- Mechanisms such as Clarke (VCG) mechanism are very general…
- … but will instantiate to something specific in any specific setting
  - This is what we care about

Example: Divorce arbitration

- Outcomes:   

- Each agent is of *high* type w.p. .2 and *low* type w.p. .8
  - Preferences of *high* type:
    - u(get the painting) = 11,000
    - u(museum) = 6,000
    - u(other gets the painting) = 1,000
    - u(burn) = 0
  - Preferences of *low* type:
    - u(get the painting) = 1,200
    - u(museum) = 1,100
    - u(other gets the painting) = 1,000
    - u(burn) = 0

Clarke (VCG) mechanism

	high	low
high	 Both pay 5,000	 Husband pays 200
low	 Wife pays 200	 Both pay 100

Expected sum of divorcees' utilities = 5,136

“Manual” mechanism design has yielded

- some positive results:
  - “Mechanism x achieves properties P in any setting that belongs to class C”
- some impossibility results:
  - “There is no mechanism that achieves properties P for all settings in class C”

Difficulties with manual mechanism design

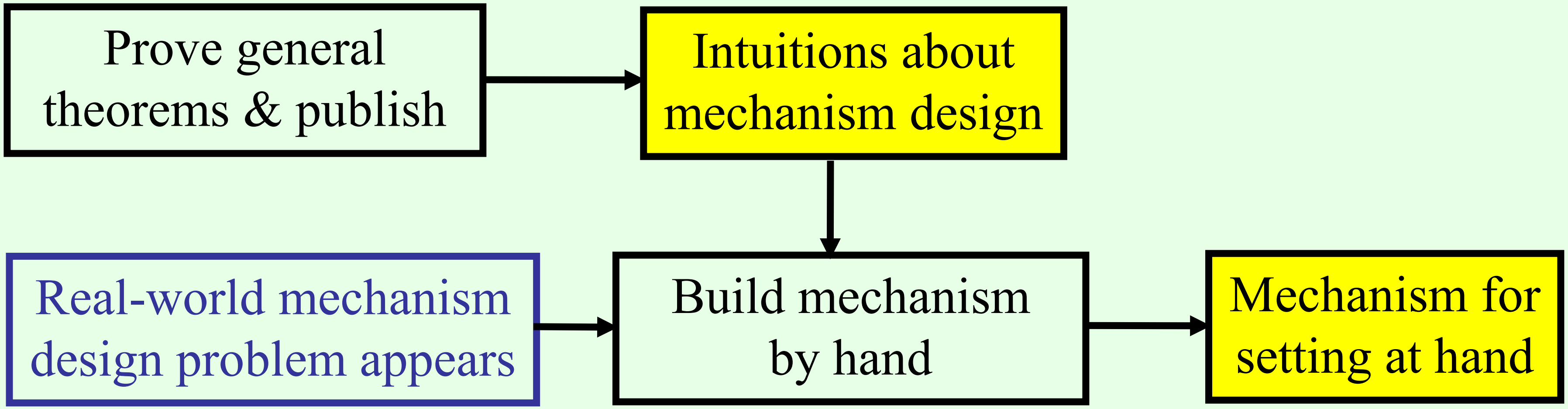
- Design problem instance comes along
  - Set of outcomes, agents, set of possible types for each agent, prior over types, …
- What if no canonical mechanism covers this instance?
  - Unusual objective, or payments not possible, or …
  - Impossibility results may exist for the general class of settings
    - But instance may have additional structure (restricted preferences or prior) so good mechanisms exist (but unknown)
- What if a canonical mechanism does cover the setting?
  - Can we use instance's structure to get higher objective value?
  - Can we get stronger nonmanipulability/participation properties?
- Manual design for every instance is prohibitively slow

Automated mechanism design (AMD)

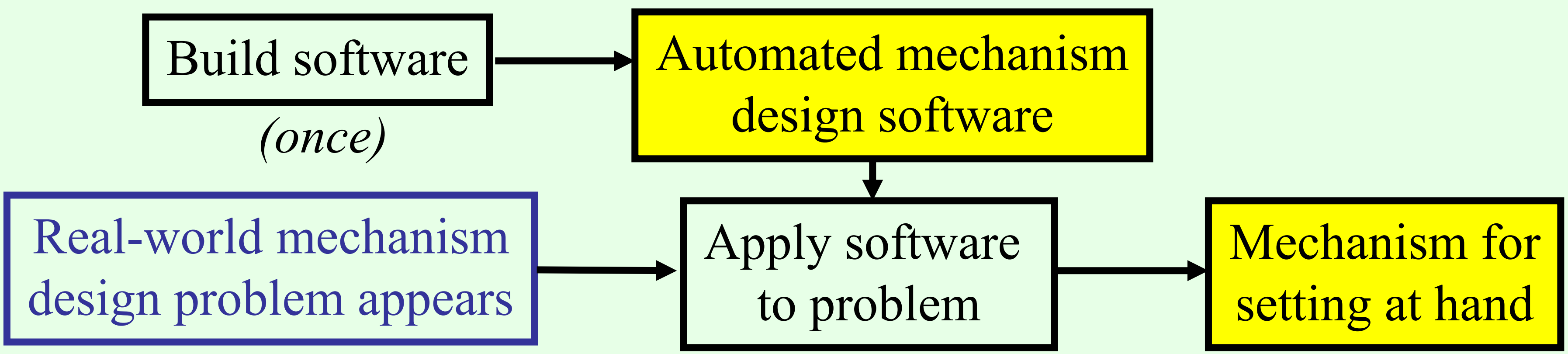
- Idea: Solve mechanism design as optimization problem automatically
- Create a mechanism for the specific setting at hand rather than a class of settings
- Advantages:
  - Can lead to greater value of designer's objective than known mechanisms
  - Sometimes circumvents economic impossibility results & always minimizes the pain implied by them
  - Can be used in new settings & for unusual objectives
  - Can yield stronger incentive compatibility & participation properties
  - Shifts the burden of design from human to machine

Classical vs. automated mechanism design

Classical



Automated



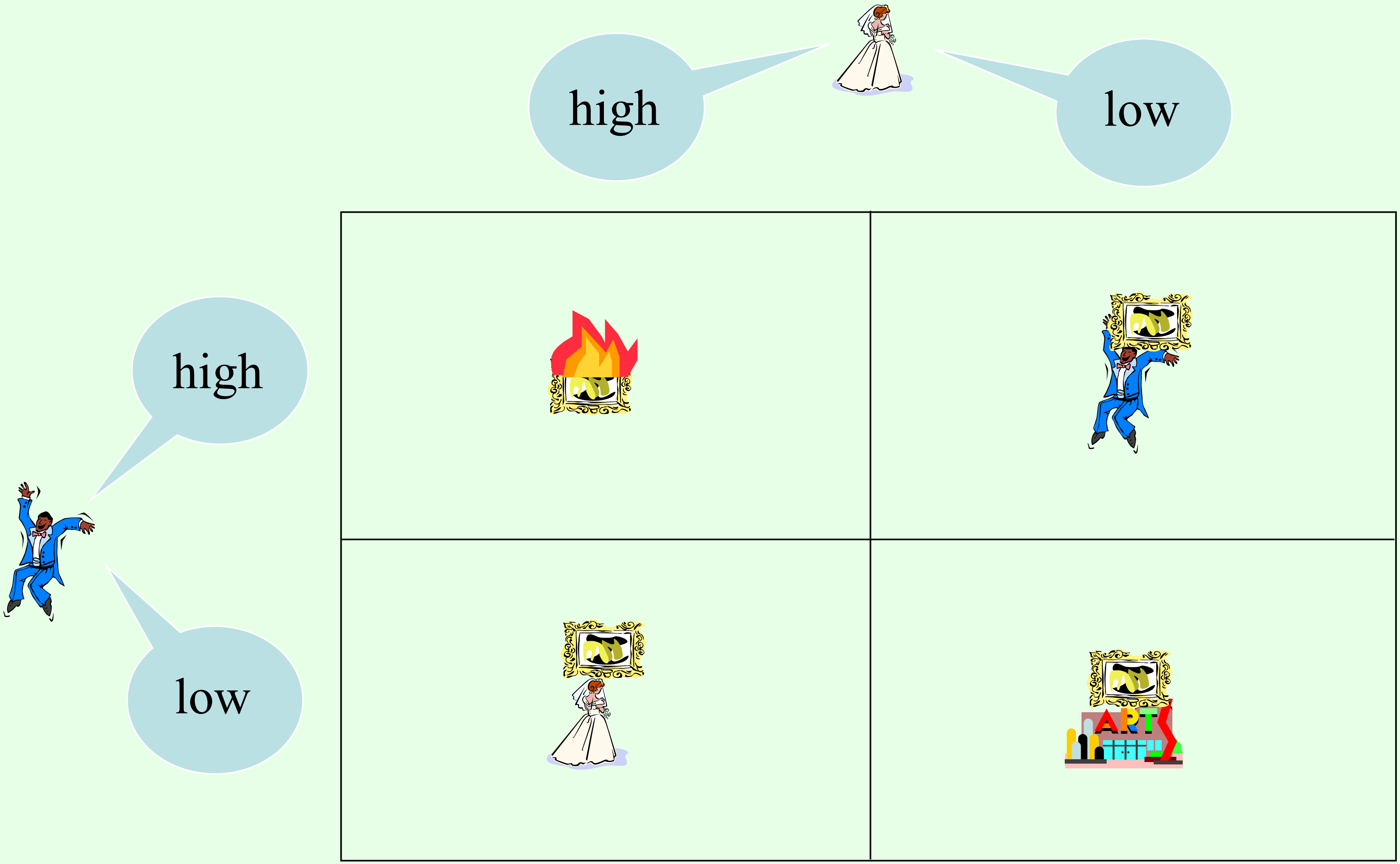
Input

- Instance is given by
  - Set of possible *outcomes*
  - Set of *agents*
    - For each agent
      - set of possible *types*
      - *probability distribution* over these types
  - *Objective function*
    - Gives a value for each outcome for each combination of agents' types
    - E.g. social welfare, payment maximization
  - Restrictions on the mechanism
    - Are payments allowed?
    - Is randomization over outcomes allowed?
    - What versions of incentive compatibility (IC) & individual rationality (IR) are used?

Output

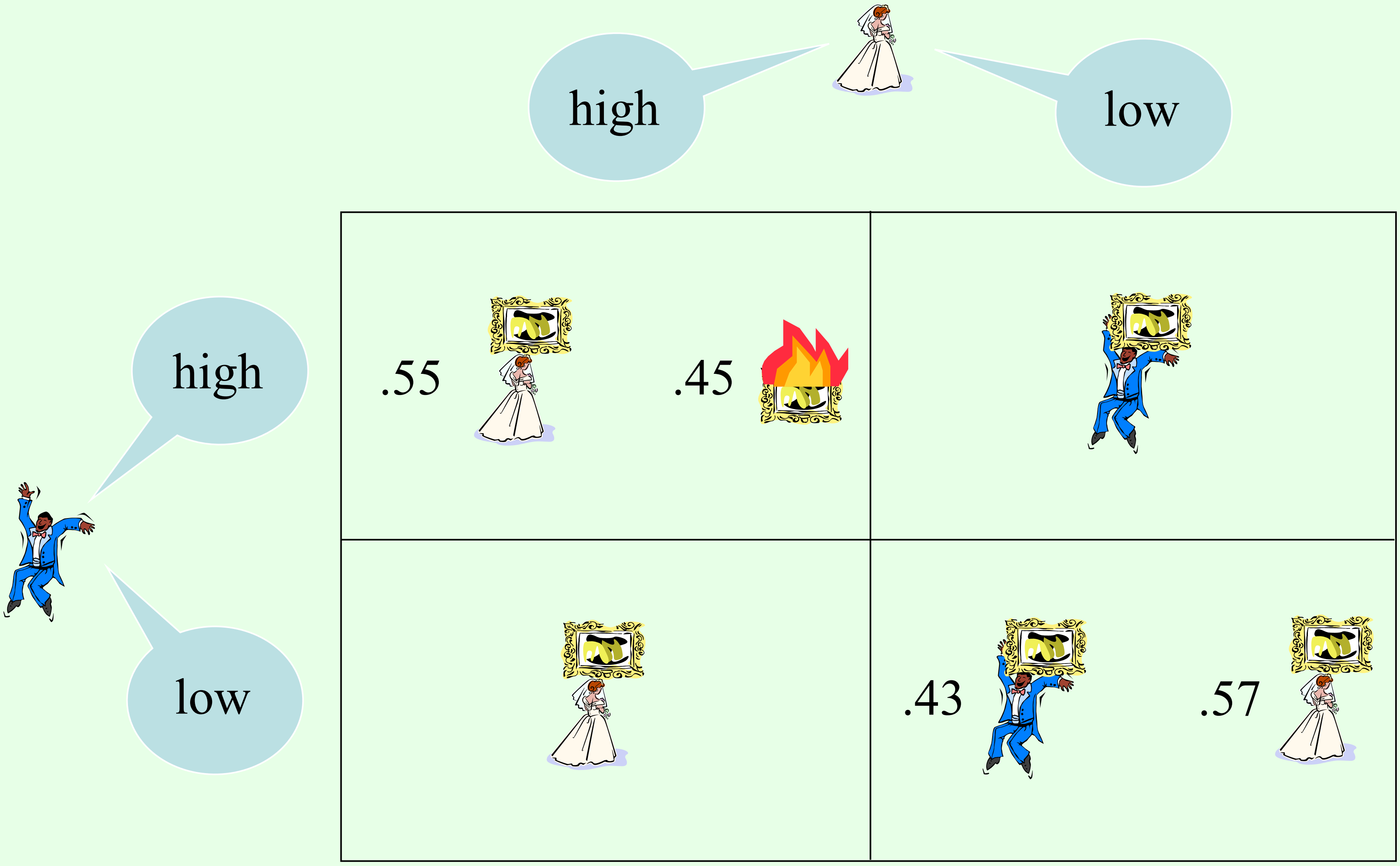
- *Mechanism*
  - A mechanism maps combinations of agents' revealed types to outcomes
    - *Randomized mechanism* maps to probability distributions over outcomes
    - Also specifies payments by agents (if payments allowed)
- *…* which
  - satisfies the IR and IC constraints
  - maximizes the expectation of the objective function

Optimal BNE incentive compatible deterministic mechanism without payments for maximizing sum of divorcees' utilities



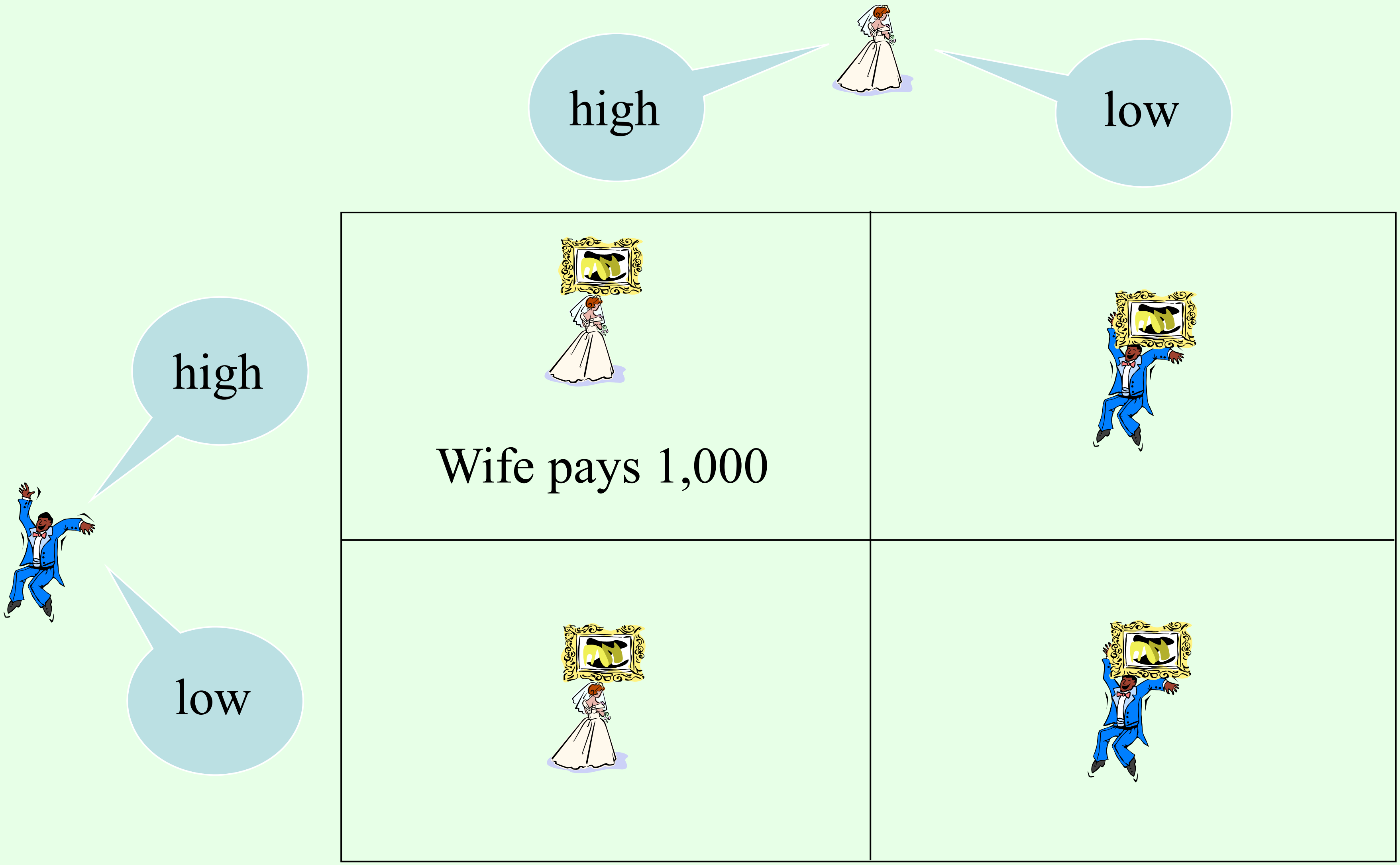
Expected sum of divorcees' utilities = 5,248

Optimal BNE incentive compatible *randomized* mechanism without payments for maximizing sum of divorcees' utilities



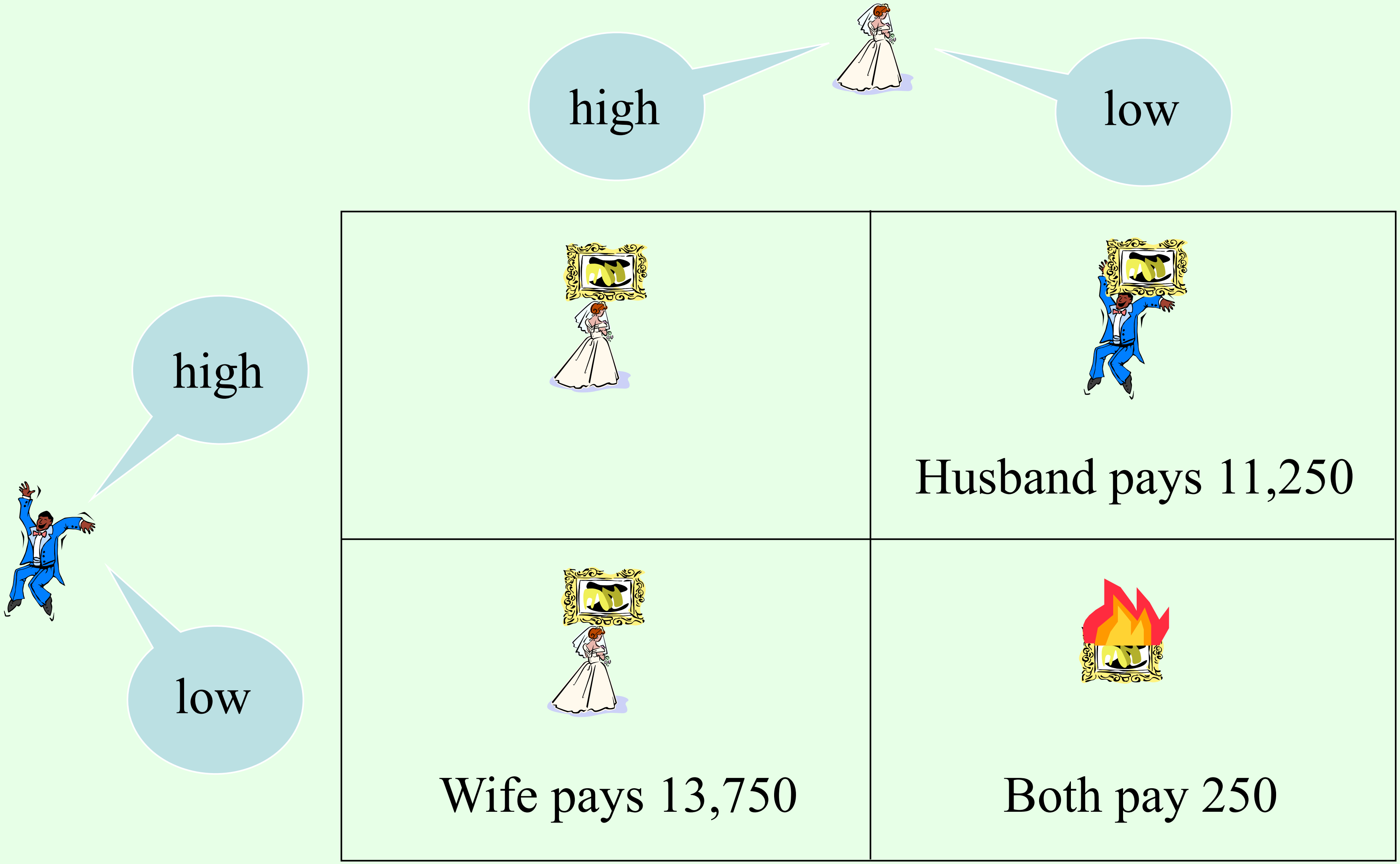
Expected sum of divorcees' utilities = 5,510

Optimal BNE incentive compatible randomized mechanism *with payments* for maximizing sum of divorcees' utilities



Expected sum of divorcees' utilities = 5,688

Optimal BNE incentive compatible randomized mechanism with payments for *maximizing arbitrator's revenue*



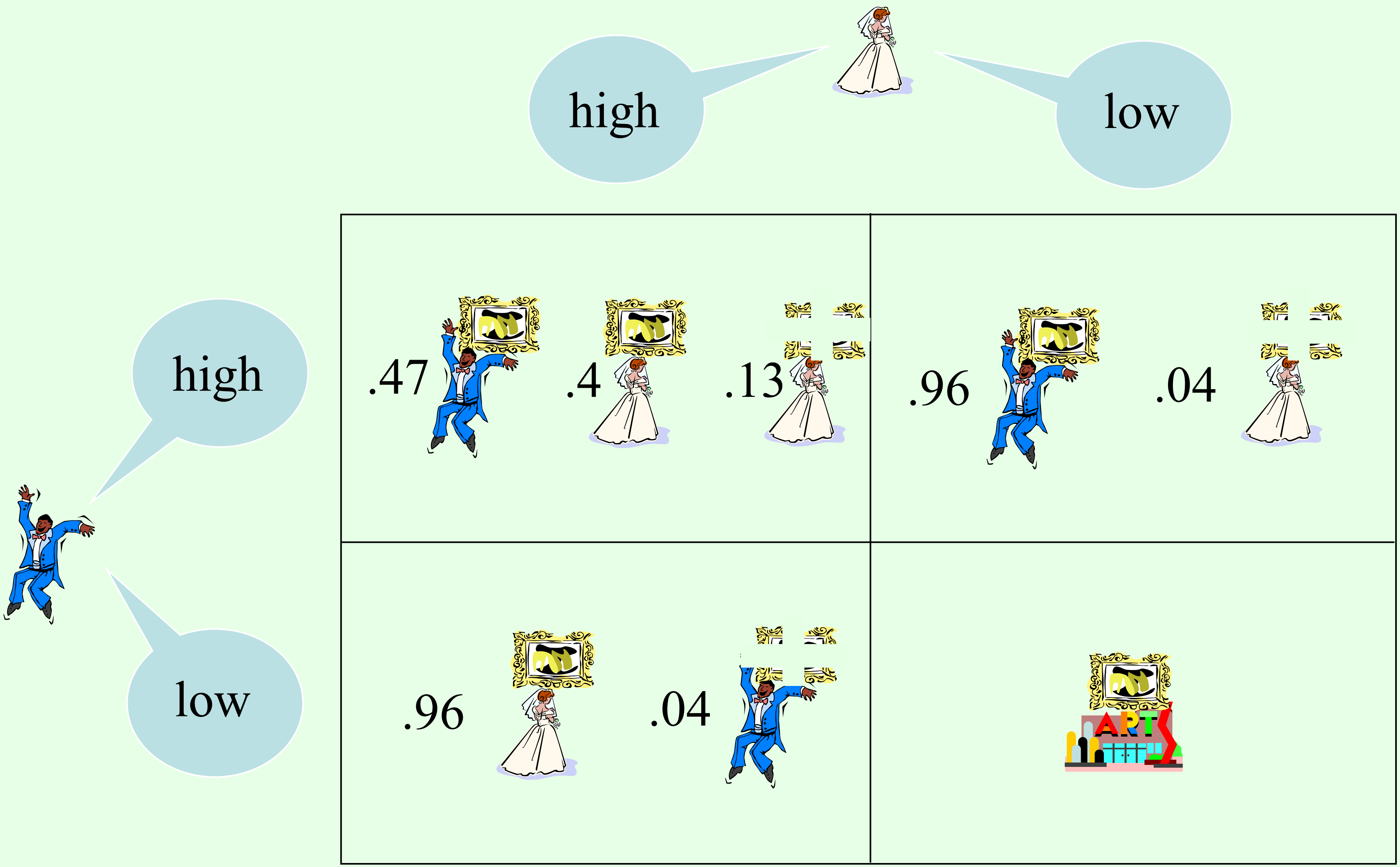
Expected sum of divorcees' utilities = 0 Arbitrator expects 4,320

Modified divorce arbitration example

- Outcomes:

- Each agent is of *high* type with probability 0.2 and of *low* type with probability 0.8
  - Preferences of *high* type:
    - u(get the painting) = 100
    - u(other gets the painting) = 0
    - u(museum) = 40
    - u(get the pieces) = -9
    - u(other gets the pieces) = -10
  - Preferences of *low* type:
    - u(get the painting) = 2
    - u(other gets the painting) = 0
    - u(museum) = 1.5
    - u(get the pieces) = -9
    - u(other gets the pieces) = -10

Optimal *dominant-strategies* incentive compatible randomized mechanism for maximizing expected sum of utilities



How do we set up the optimization?

- Use linear programming
- Variables:
  - $p(o \mid \theta_1, \ldots, \theta_n)$ = probability that outcome o is chosen given types $\theta_1, \ldots, \theta_n$
  - (maybe) $\pi_i(\theta_1, \ldots, \theta_n)$ = i's payment given types $\theta_1, \ldots, \theta_n$
- Strategy-proofness constraints: for all $i, \theta_1, \ldots \theta_n, \theta_i'$:

$$\Sigma_o p(o \mid \theta_1, \ldots, \theta_n) u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n) \geq$$
$$\Sigma_o p(o \mid \theta_1, \ldots, \theta_i', \ldots, \theta_n) u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_i', \ldots, \theta_n)$$

- Individual-rationality constraints: for all $i, \theta_1, \ldots \theta_n$:

$$\Sigma_o p(o \mid \theta_1, \ldots, \theta_n) u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n) \geq 0$$

- Objective (e.g. sum of utilities)

$$\Sigma_{\theta_1, \ldots, \theta_n} p(\theta_1, \ldots, \theta_n) \Sigma_i (\Sigma_o p(o \mid \theta_1, \ldots, \theta_n) u_i(\theta_i, o) + \pi_i(\theta_1, \ldots, \theta_n))$$

- Also works for BNE incentive compatibility, ex-interim individual rationality notions, other objectives, etc.
- For deterministic mechanisms, use mixed integer programming (probabilities in $\{0, 1\}$)
  - Typically designing the optimal deterministic mechanism is NP-hard

Computational complexity of automatically designing deterministic mechanisms

- Many different variants
  - Objective to maximize: Social welfare/revenue/designer's agenda for outcome
  - Payments allowed/not allowed
  - IR constraint: ex interim IR/ex post IR/no IR
  - IC constraint: Dominant strategies/Bayes-Nash equilibrium
- The above already gives 3 * 2 * 3 * 2 = 36 variants
- Approach: Prove hardness for the case of only 1 type-reporting agent
  - results imply hardness in more general settings

DSE & BNE incentive compatibility constraints coincide when there is only 1 (reporting) agent

Dominant strategies:

Reporting truthfully is optimal for *any* types the others report

	t_{21}	t_{22}
t_{11}	o_5	o_9
t_{12}	o_3	o_2

$$u_1(t_{11},o_5) \geq u_1(t_{11},o_3)$$

AND

$$u_1(t_{11},o_9) \geq u_1(t_{11},o_2)$$

Bayes-Nash equilibrium:

Reporting truthfully is optimal *in expectation* over the other agents' (true) types

	t_{21}	t_{22}
t_{11}	o_5	o_9
t_{12}	o_3	o_2

$$P(t_{21})u_1(t_{11},o_5) + P(t_{22})u_1(t_{11},o_9) \geq P(t_{21})u_1(t_{11},o_3) + P(t_{22})u_1(t_{11},o_2)$$

With only 1 reporting agent, the constraints are the same

	t_{21}
t_{11}	o_5
t_{11}	o_3

$$u_1(t_{11},o_5) \geq u_1(t_{11},o_3)$$

is equivalent to

$$P(t_{21})u_1(t_{11},o_5) \geq P(t_{21})u_1(t_{11},o_3)$$

Ex post and *ex interim* individual rationality constraints coincide when there is only 1 (reporting) agent

Ex post:

Participating never hurts (for *any* types of the other agents)

	t_{21}	t_{22}
t_{11}	o_5	o_9
t_{12}	o_3	o_2

$u_1(t_{11},o_5) \geq 0$

AND

$u_1(t_{11},o_9) \geq 0$

Ex interim:

Participating does not hurt *in expectation* over the other agents' (true) types

	t_{21}	t_{22}
t_{11}	o_5	o_9
t_{12}	o_3	o_2

$P(t_{21})u_1(t_{11},o_5) +$

$P(t_{22})u_1(t_{11},o_9) \geq 0$

With only 1 reporting agent, the constraints are the same

	t_{21}
t_{11}	o_5
t_{11}	o_3

$u_1(t_{11},o_5) \geq 0$

is equivalent to

$P(t_{21})u_1(t_{11},o_5) \geq 0$

How hard is designing an optimal *deterministic* mechanism?

NP-complete (even with 1 reporting agent):	**Solvable in polynomial time (for any constant number of agents):**
1. Maximizing social welfare (no payments) 2. Designer's own utility over outcomes (no payments) 3. General (linear) objective that doesn't regard payments 4. Expected revenue	1. Maximizing social welfare (not regarding the payments) (VCG)

1 and 3 hold even with no IR constraints

AMD can create optimal (expected-revenue maximizing) combinatorial auctions

- Instance 1
  - 2 items, 2 bidders, 4 types each (LL, LH, HL, HH)
  - H=utility 2 for that item, L=utility 1
  - But: utility 6 for getting both items if type HH (complementarity)
  - Uniform prior over types
  - Optimal *ex-interim* IR, BNE mechanism (0 = item is burned):
  - Payment rule not shown
  - Expected revenue: 3.94 (VCG: 2.69)
- Instance 2
  - 2 items, 3 bidders
  - Complementarity and substitutability
  - Took 5.9 seconds
  - Uses randomization

	LL	LH	HL	HH
LL	0,0	0,2	2,0	2,2
LH	0,1	1,2	2,1	2,2
HL	1,0	1,2	2,1	2,2
HH	1,1	1,1	1,1	1,1

Optimal mechanisms for a public good

- AMD can design optimal mechanisms for public goods, taking money burning into account as a loss
- Bridge building instance
  - Agent 1: High type (prob .6) values bridge at 10. Low: values at 1
  - Agent 2: High type (prob .4) values bridge at 11. Low: values at 2
  - Bridge costs 6 to build
- Optimal mechanism (*ex-post* IR, BNE):

Outcome rule

	Low	High
Low	Don't build	Build
High	Build	Build

Payment rule

	Low	High
Low	0, 0	0, 6
High	4, 2	.67, 5.33

- There is no general mechanism that achieves budget balance, *ex-post* efficiency, and *ex-post* IR [Myerson-Satterthwaite 83]
- However, for this instance, AMD found such a mechanism

Combinatorial public goods problems

- AMD for interrelated public goods
- Example: building a bridge and/or a boat
  - 2 agents each uniform from types: {None, Bridge, Boat, Either}
    - Type indicates which of the two would be useful to the agent
    - If something is built that is useful to you, you get 2, otherwise 0
  - Boat costs 1 to build, bridge 3
- Optimal mechanism (*ex-post* IR, dominant strategies):

Outcome rule

(P(none), P(boat), P(bridge), P(both))

	None	Boat	Bridge	Either
None	(1,0,0,0)	(0,1,0,0)	(1,0,0,0)	(0,1,0,0)
Boat	(.5,.5,0,0)	(0,1,0,0)	(0,.5,0,.5)	(0,1,0,0)
Bridge	(1,0,0,0)	(0,1,0,0)	(0,0,1,0)	(0,0,1,0)
Either	(.5,.5,0,0)	(0,1,0,0)	(0,0,1,0)	(0,1,0,0)

- Again, no money burning, but outcome not always efficient
  - E.g., sometimes nothing is built while boat should have been

Additional & future directions

- Scalability is a major concern
  - Can sometimes create more concise LP formulations
    - Sometimes, some constraints are implied by others
  - In restricted domains faster algorithms sometimes exist
    - Can sometimes make use of partial characterizations of the optimal mechanism
- Automatically generated mechanisms can be complex/hard to understand
  - Can we make automatically designed mechanisms more intuitive?
- Using AMD to create conjectures about general mechanisms