15784 Midterm practice

Your name:

Please read instructions carefully. Do not worry if you cannot finish everything. Do not write down disorganized answers in the hope of getting partial credit; it's better to do a few questions completely right. Please write your answers down clearly (think before you write). You can use extra pages. Good luck!

Problem 1: Modified Rock-Paper-Scissors.

Consider the following modified version of Rock-Paper-Scissors, where losing with Paper to Scissors is considered doubly humiliating:

	Rock	Paper	Scissors
Rock	0,0	-1,1	1,-1
Paper	1,-1	0,0	-2,2
Scissors	-1,1	2,-2	0,0

a. Wright argues that in every equilibrium of this game, every pure strategy must receive positive probability from both players. Is Wright right or wrong? Explain why.

b. Based on your answer in **a**, compute a Nash equilibrium of this game. Is it the unique equilibrium? Why (not)?

Problem 2: A game with a hidden coin flip.

In this problem, you will solve a simple game (reminiscent of "Liar's Dice" if you happen to know it). Player 1 flips a coin and sees the result; player 2 does not see the result. Heads is a "winning" coin flip, Tails is a "losing" coin flip. Player 1 makes a claim about the coin flip to player 2, either claiming to have flipped Heads, or claiming to have flipped Tails. Player 2 can choose to Dispute the claim, or to Accept it.

If player 2 chooses to Dispute, player 1 must show the coin. (Of course, player 1 cannot change the result of the coin flip.) If player 1 lied, player 2 wins; if player 1 told the truth, player 1 wins. If player 2 chooses to Accept, then whatever player 1 claimed stands (regardless of what she actually flipped), and player 2 must flip the coin to compete with that claim.

For example, suppose player 1 flips Tails, but then claims to have flipped Heads. If player 2 Disputes, player 2 wins, because player 1 lied. If player 2 Accepts, then player 1's claim of Heads stands (and the fact that she actually flipped Tails becomes irrelevant), and player 2 must flip the coin to compete with Heads. If player 2 flips Tails, then he loses, because Tails is worse than Heads. If player 2 flips Heads, we have a tie.

Suppose the utility for winning is 1, the utility for losing is -1, and the utility for a tie is 0 (it's a zero-sum game). Give the extensive form of the game, convert it to normal (matrix) form (explaining what the strategies mean and calculating the expected utilities), and solve for the equilibrium of this game. (*Hint:* the normal form should be 4×4.)

Problem 3: The distancing dilemma.

During a pandemic, two people are approaching each other on a narrow trail. They can't come close to each other. On both sides of the trail there is mud. If one person goes and stands in the mud, they can pass each other. (If they both go into the mud on their respective sides, they can also pass each other.) So, initially, both players have a choice between Mud (M) and Trail (T). If you choose T and the other M, you'll get a utility of 2. If you choose M (in which case you'll definitely get to pass), you'll get a utility of 1 – you'll pass but have muddy shoes.

However, if *both* choose T, the situation hasn't resolved yet – they'll still be stuck. Realizing that this has happened, they will get another chance where, again, they can choose M or T. The payoffs will be the same as above, except if they *again* both choose T, they'll have to give up and turn around, for a utility of 0. (Let's say otherwise an officer will come and tell both of them to go home, for causing trouble.) So there are 2 rounds of the game in this case, and never more.

a. Draw the above game in extensive form. Hint: While this is (at most) a 2-round game in which in both rounds the players choose simultaneously, only one person can move at a node in an extensive-form game. Thus, for each of the rounds, you'll have to sequentialize the moves by the players, but you can make them effectively simultaneous within each round by not letting the other person learn what the first person did in the same round (but after the first round completes, they'll know everything that happened up to that point). So the tree will be four moves deep on the side where in the first round they both choose T.

b. Give the normal form of this game. Note that it should be a 4×4 game because the strategy must specify an action at every information set (even if that information set is not reachable given the other part of the strategy). There should be a fair amount of repetition in the matrix.

c. Give a pure-strategy subgame-perfect Nash equilibrium in which player 1 obtains utility 2. The strategies should be ones from your game in **b**.

d. Solve for a symmetric subgame-perfect Nash equilibrium of this game (which will involve randomization). (An equilibrium is symmetric if the row and column player use the same strategy.) Hint: you can do this by a sort of backward induction: first solve for an equilibrium of the second round (after both choose T), and then replace this subgame by the values of that equilibrium, and solve the first round.

Problem 4: Disarmament.

Consider the following game.

	L	C	R
U	2,2	0,0	0,3
D	3,0	1,1	1,0

First assume both players are playing simultaneously (as usual).
a. Solve for all the Nash equilibria of this game. Use iterated strict dominance as much as possible to remove as many strategies as possible, and explain which strict dominance relations you are using.

In what follows, we consider commitment. A commitment is always observed by the other player before moving. The follower always breaks ties in the leader's favor.
b. Suppose the row player can commit to a mixed strategy before the column player moves. To which mixed strategy will the row player commit? Why?

c. Suppose the column player can commit to a mixed strategy before the row player moves. To which mixed strategy will the column player commit? Why?

d. Now suppose the game is played as follows. First, the column player can remove one column from the game (commit not to play that column). Then, after observing which column has been removed, the row player can commit to one row (commit to a pure strategy). Finally, the column player chooses one of the remaining columns. What will happen in the equilibrium, and why?

e. Finally, suppose the game is played as follows. The row player can choose one of the *columns* to "disable" (the column player will not be able to play that column anymore), and in addition, the row player can commit to a mixed strategy. What will the row player do?

f. Formulate the last problem as a mixed integer linear program. It should extend the linear program we saw in class for finding an optimal mixed strategy to commit to (for the one in class, there was no option to disable). Specifically, given a game with rows $r \in R$, columns $c \in C$, utilities $u_1(r,c)$ for the row player and $u_2(r,c)$ for the column player, a budget k where the row player is allowed to remove up to k of the columns, and an "intended" column c^*, give a mixed integer linear program with variables $p_r \in [0,1]$ (for the probabilities that the row player places on the rows) and $x_c \in \{0,1\}$ (where $x_c = 1$ means column c has been removed), that computes the optimal solution for the row player among solutions that result in the column player playing c^*.

Problem 5: Signals about the boss.

Spite and Tardy are two employees in an office. Every day, their boss (who is not a strategic player in this game) chooses either to wear light or dark clothing, each with probability 50%. The employees also have to choose whether to wear light or dark; if an employee chooses the same color as the boss, the boss likes that employee a bit more. That employee will receive 1 unit of utility for being liked by the boss. However, Spite mostly wants to get Tardy in trouble. If Tardy chooses a color that does not match the boss, then Spite gets 2 units of utility for that. Tardy does not care whether Spite gets in trouble.

Spite always gets to the office early, and can see the boss coming. So, Spite can observe the boss' clothing color, and *could* choose to simply pick the same color. (Both Spite and Tardy have clothes in their office to change into.) However, Tardy always gets to the office late, and at that point, the boss will be in a closed office, so Tardy cannot see the boss's color. But, Tardy can see Spite's color. So Tardy could choose to match Spite's color. The risk for Tardy is that Spite is deliberately trying to mislead Tardy. What will happen in equilibrium? To illustrate the game, here are some example outcomes:

The boss wears light. Spite sees this, and chooses to wear dark (perhaps to mislead Tardy). Then Tardy sees that Spite is wearing dark (but does not see what the boss is wearing), and chooses to wear dark as well. Spite gets a utility of 2 for Tardy getting in trouble, and Tardy gets a utility of 0. (Neither employee is liked by the boss.)

The boss wears light. Spite sees this, and chooses to wear dark (perhaps to mislead Tardy). Then Tardy sees that Spite is wearing dark (but does not see what the boss is wearing), and chooses to wear light (perhaps worried that Spite is trying to mislead Tardy). Spite gets a utility of 0 (Tardy does not get in trouble *and* Spite doesn't match the boss), and Tardy gets a utility of 1 for matching the boss.

The boss wears light. Spite sees this, and chooses to wear light. Then Tardy sees that Spite is wearing light (but does not see what the boss is wearing), and chooses to wear dark (perhaps worried that Spite is trying to mislead Tardy). Spite gets a utility of $2 + 1 = 3$ for Tardy getting in trouble *and* matching the boss' color, and Tardy gets a utility of 0.

a. Model the above game as an extensive-form game. It should start with a move by Nature (the boss's clothing color). Be sure to get the information sets right.

b. Convert the game to normal form. Each player should have 4 pure strategies. Give these strategies names and explain what they mean.

c. One strategy σ_i *very weakly dominates* another strategy σ_i' if against any (pure) opponent strategy s_{-i}, $u_i(\sigma_i, s_{-i}) \geq u_i(\sigma_i', s_{-i})$. (The difference from regular weak dominance is that t here is no requirement that one of the inequalities is strict.) For each player, two pure strategies are very weakly dominated. Identify these strategies, and remove them from the game. You should also identify a strategy that very weakly dominates these removed strategies (and you are not allowed to just say that they very weakly dominate each other; one of them should be removed before the other).

d. You should have a 2×2 game left. Write the remaining game here, and solve for all of its Nash equilibria.