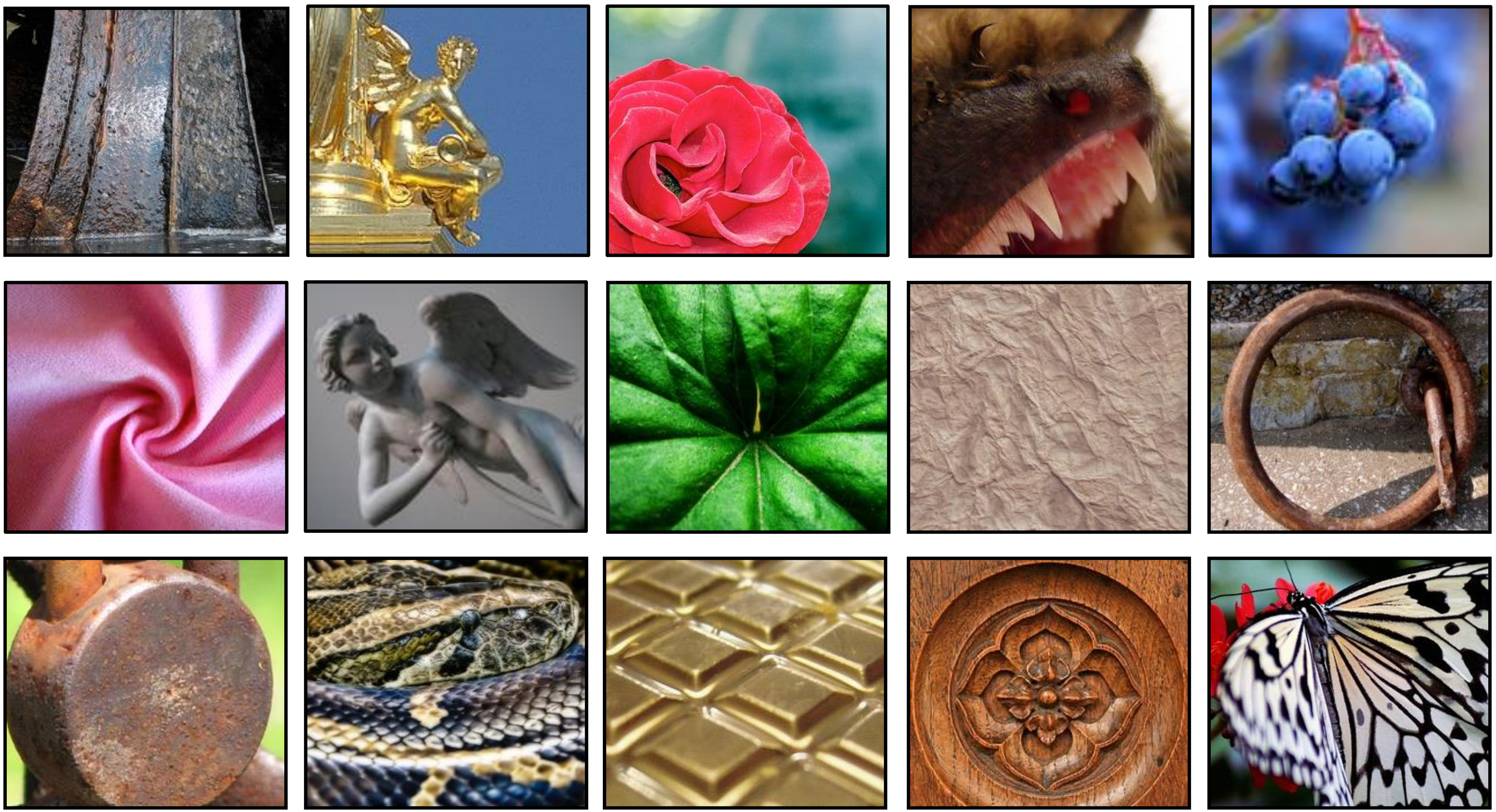
Radiometry and reflectance



http://www.cs.cmu.edu/~16385/

16-385 Computer Vision
Spring 2019, Lecture 14

Course announcements

- Lot's of grades posted during spring break.
  - Mid-semester grades.
  - Comments for HW1.
  - Grades and comments for HW2.

- Homework 4 will be posted tonight and will be due on March 3rd.
  - It's another shorter homework, based on material from this and the next lecture.

- Talk tomorrow: Jun-Yan Zhu, "Learning to synthesize images," noon-1 pm, GHC 6115

Overview of today’s lecture

- Measuring light and radiometry.
- Reflectance and BRDF.
- Light sources.

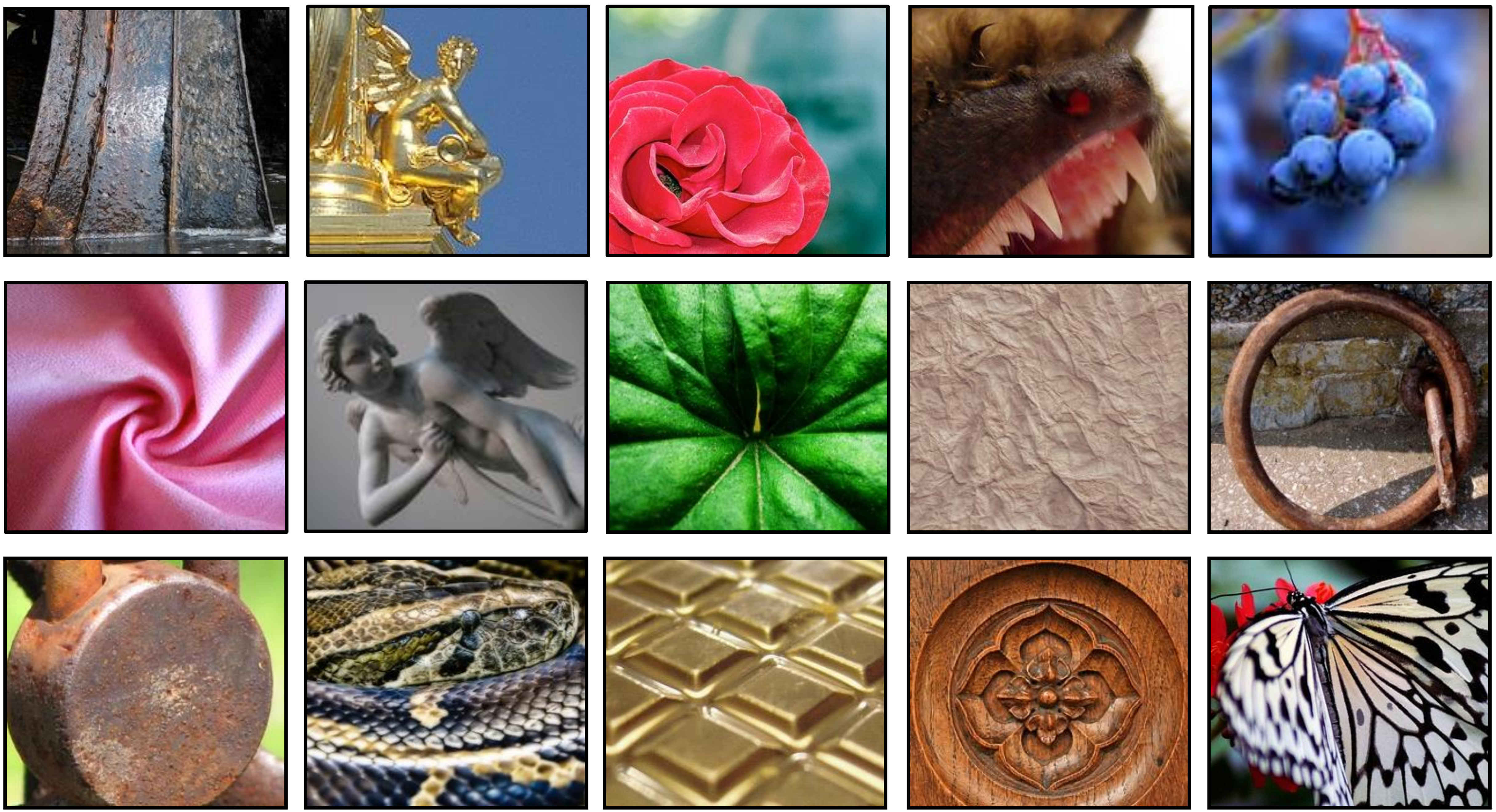
Slide credits

Most of these slides were adapted from:

- Srinivasa Narasimhan (16-385, Spring 2014).
- Todd Zickler (Harvard University).
- Steven Gortler (Harvard University).

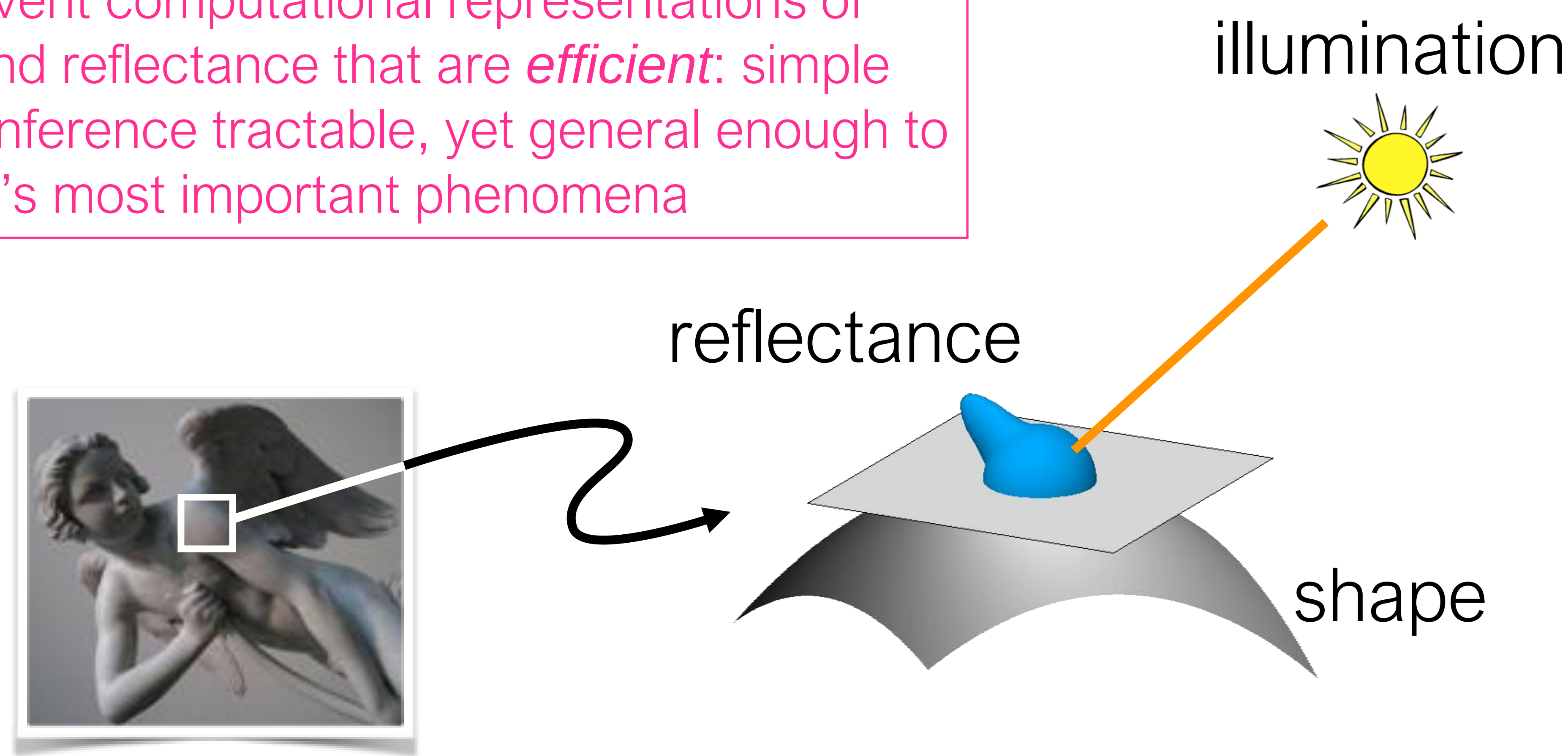
Appearance

Appearance



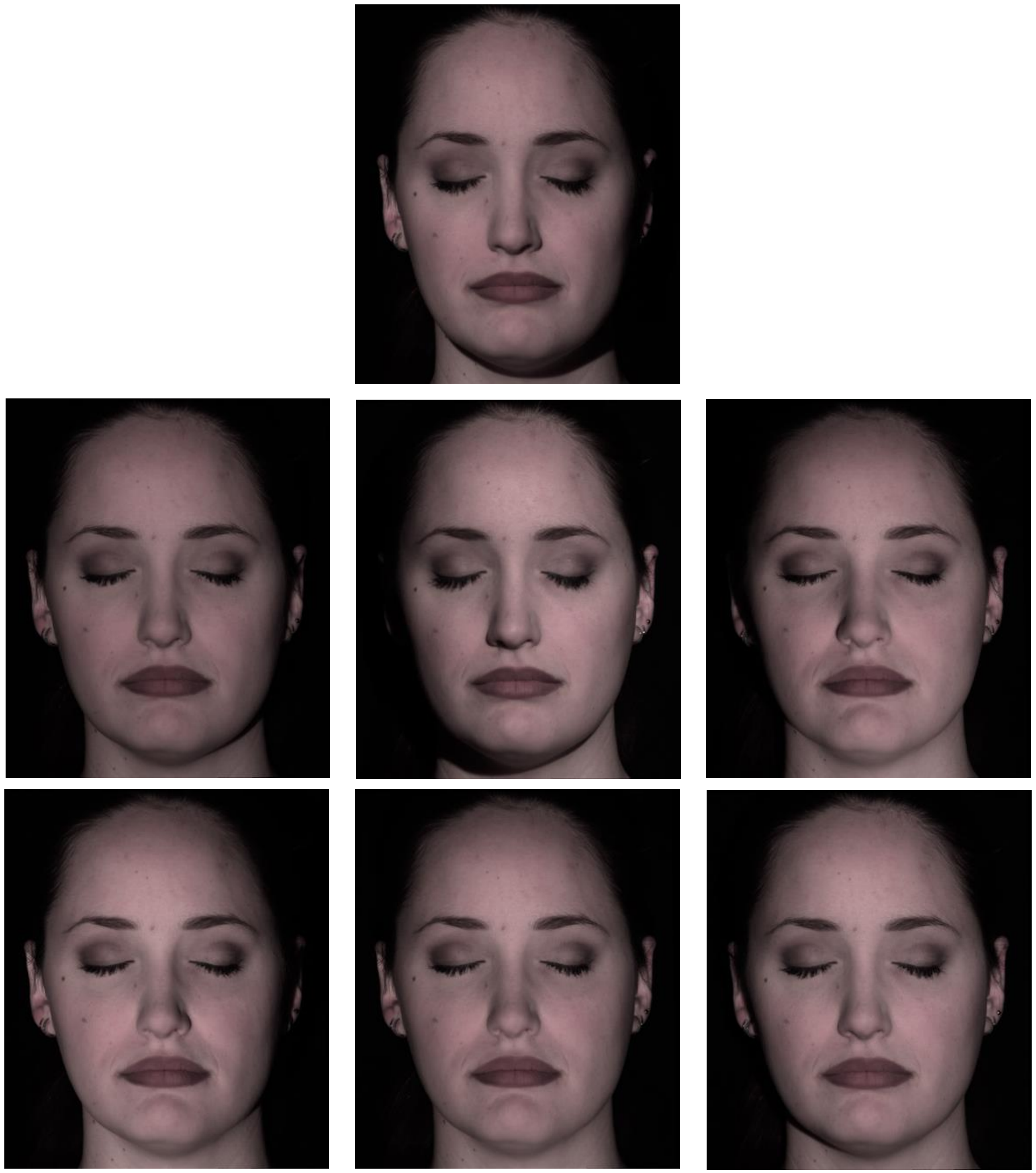
"Physics-based" computer vision
(a.k.a "inverse optics")

Our challenge: Invent computational representations of shape, lighting, and reflectance that are ***efficient***: simple enough to make inference tractable, yet general enough to capture the world's most important phenomena



$\mathbf{I}$ ⟹ shape, illumination, reflectance

Example application: Photometric Stereo



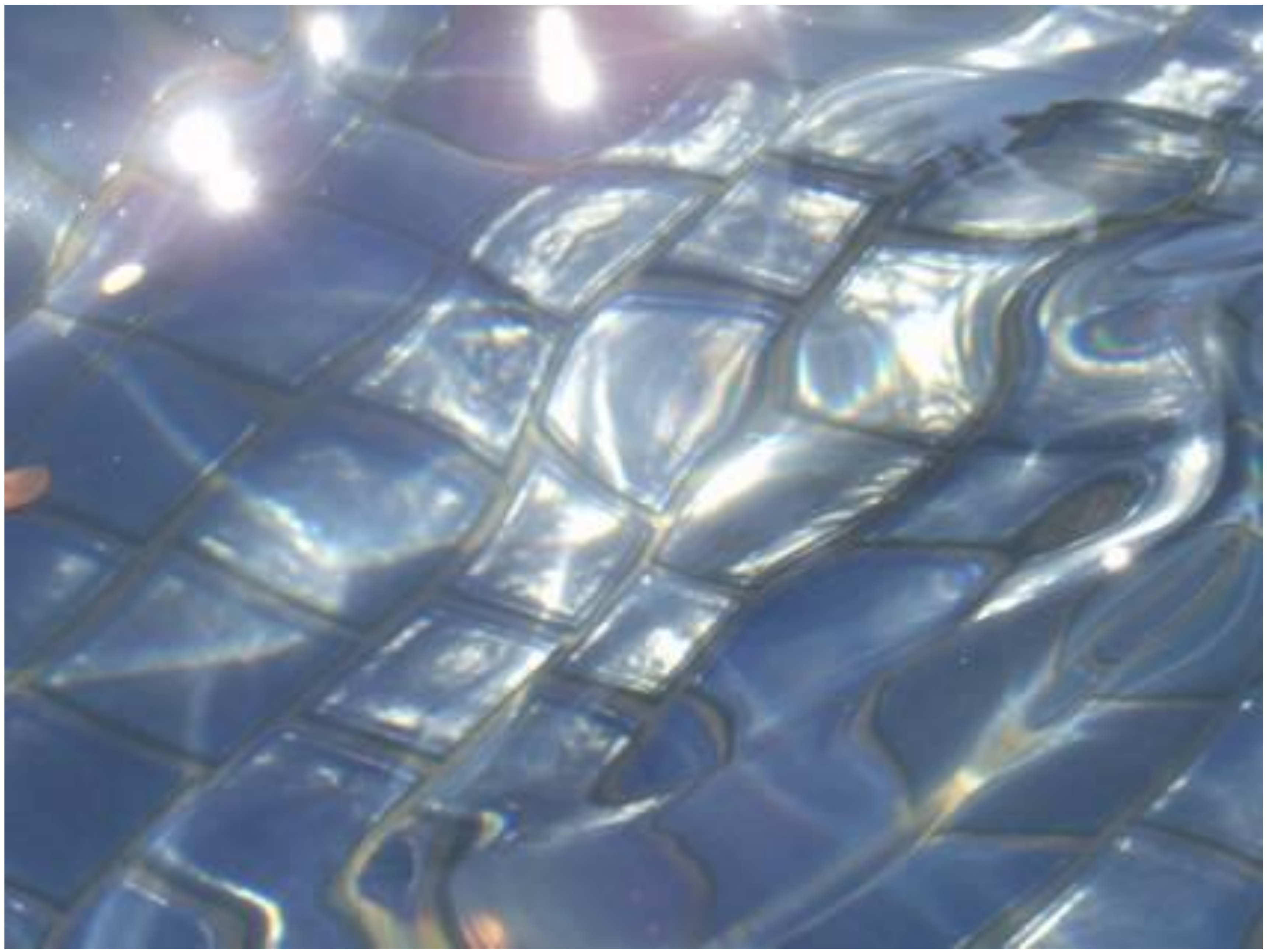
Why study the physics (optics) of the world?

Lets see some pictures!

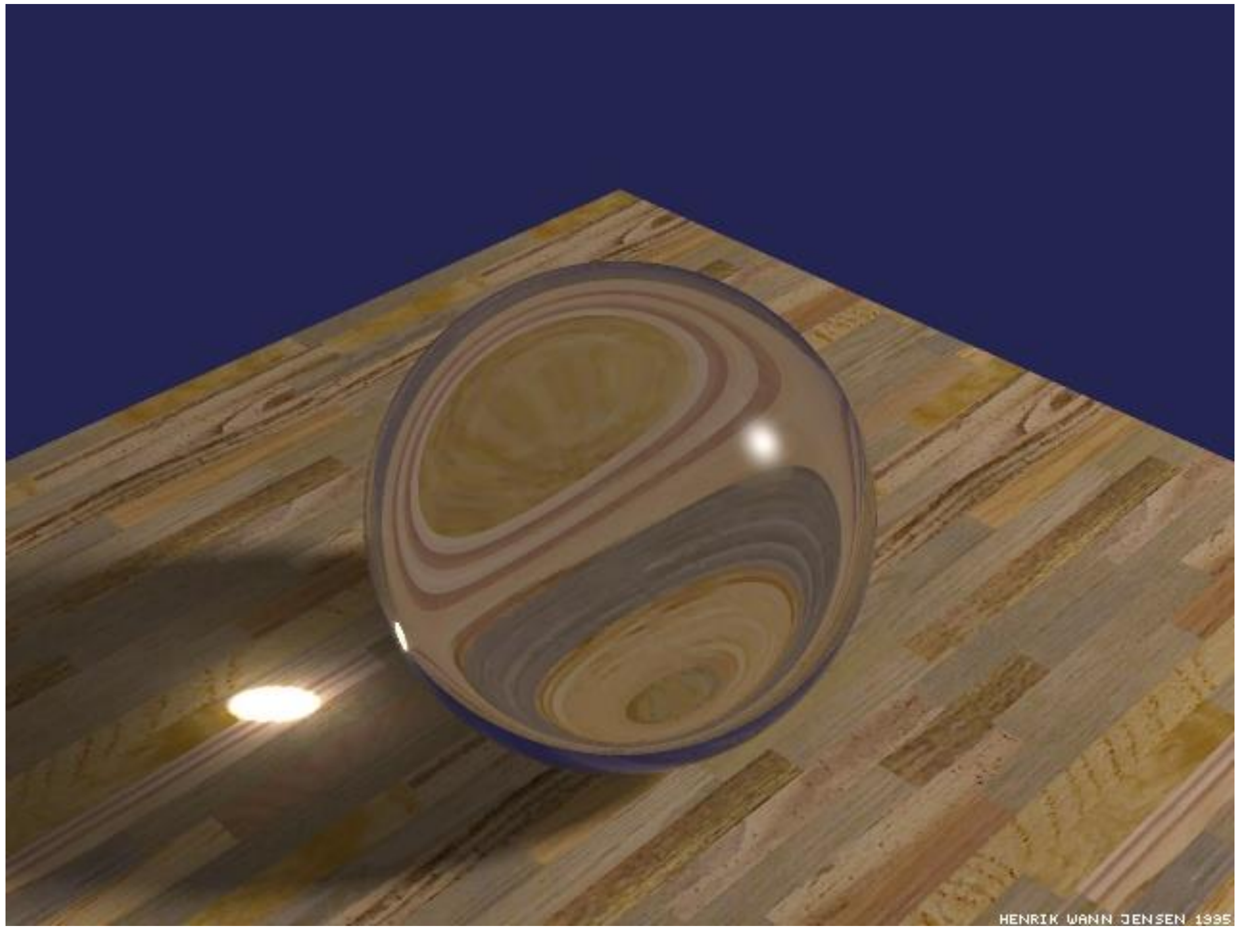
Light and Shadows


Reflections


Refractions



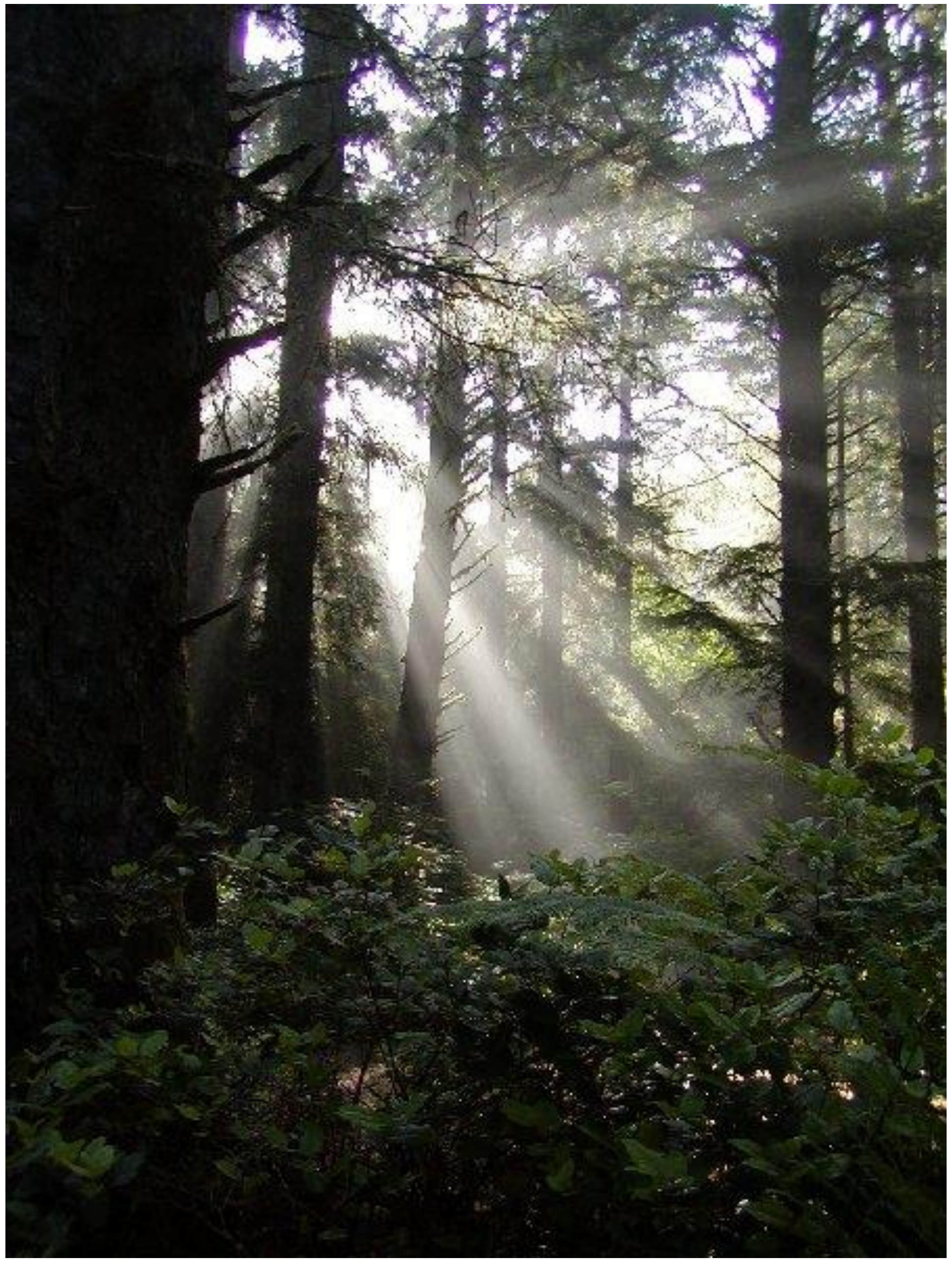

HENRIK WANN JENSEN 1995


HENRIK WANN JENSEN 1995

Interreflections


Mies Courtyard House with Curved Elements

Scattering

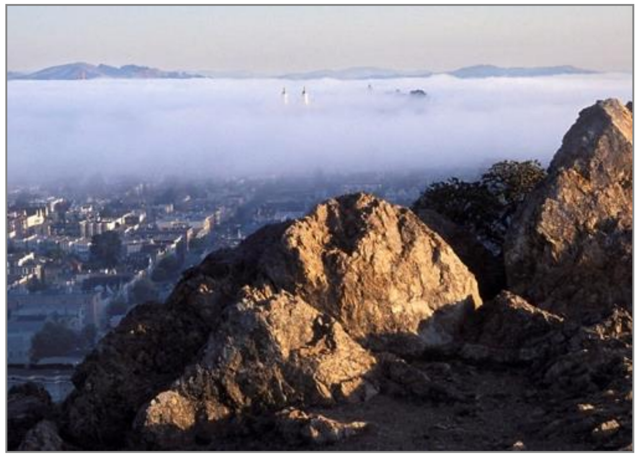


More Complex Appearances


RENDERED BY HENRIK WANN JENSEN - 2001


RENDERED BY HENRIK WANN JENSEN - 2001





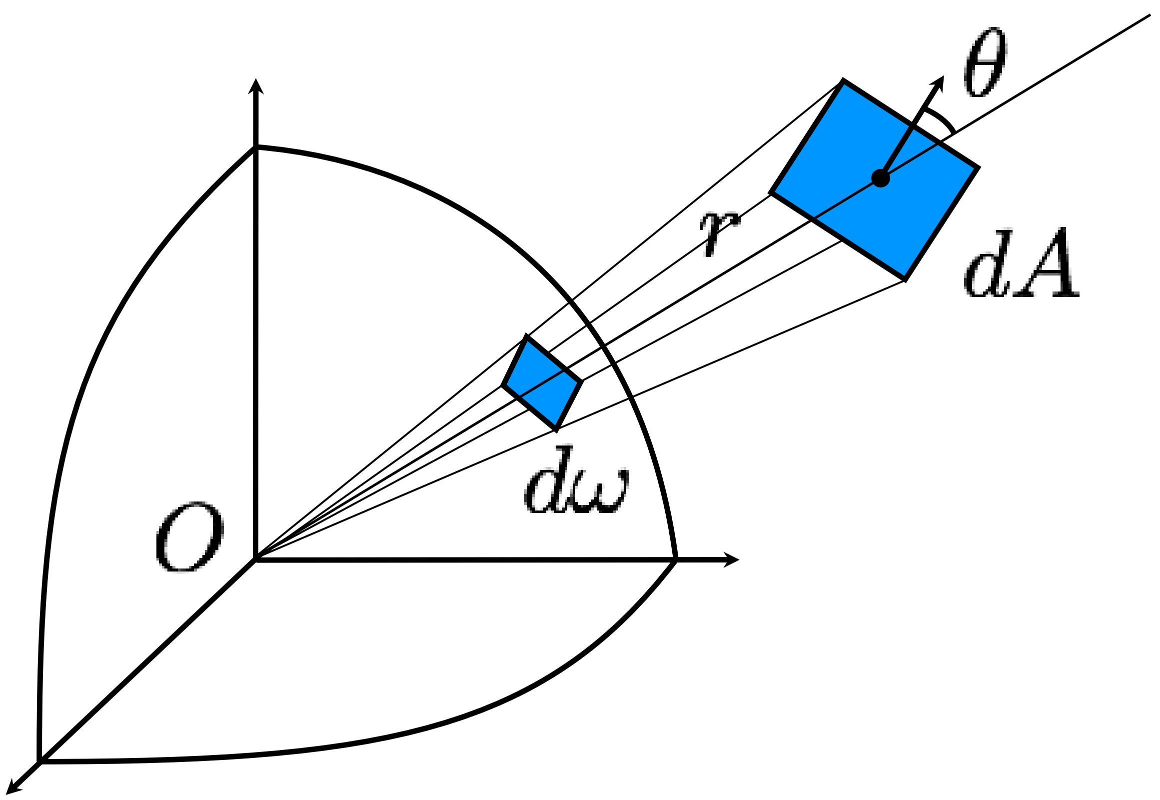




Measuring light and radiometry

Solid angle

- The *solid angle* subtended by a small surface patch with respect to point O is the area of its central projection onto the unit sphere about O



Depends on:

- orientation of patch
- distance of patch

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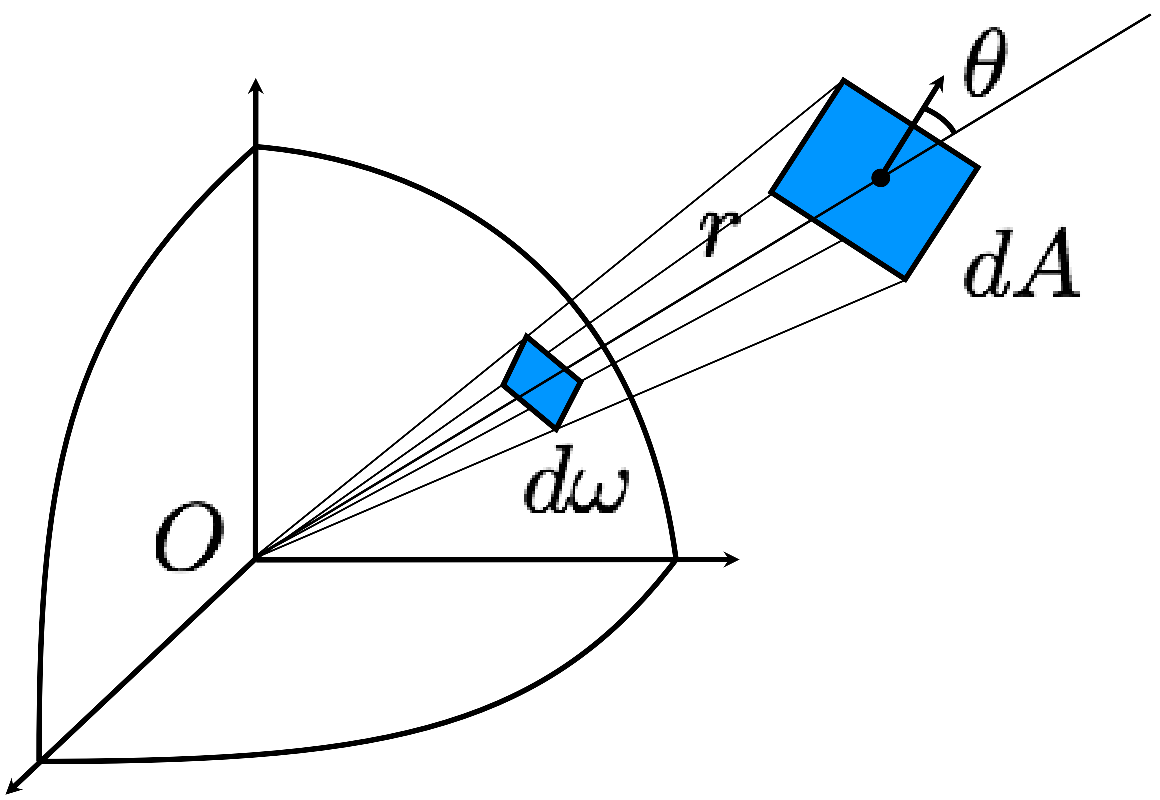

Depends on:

- orientation of patch
- distance of patch

One can show:

$$d\omega = \frac{dA \cos\theta}{r^2}$$

Units: steradians [sr]

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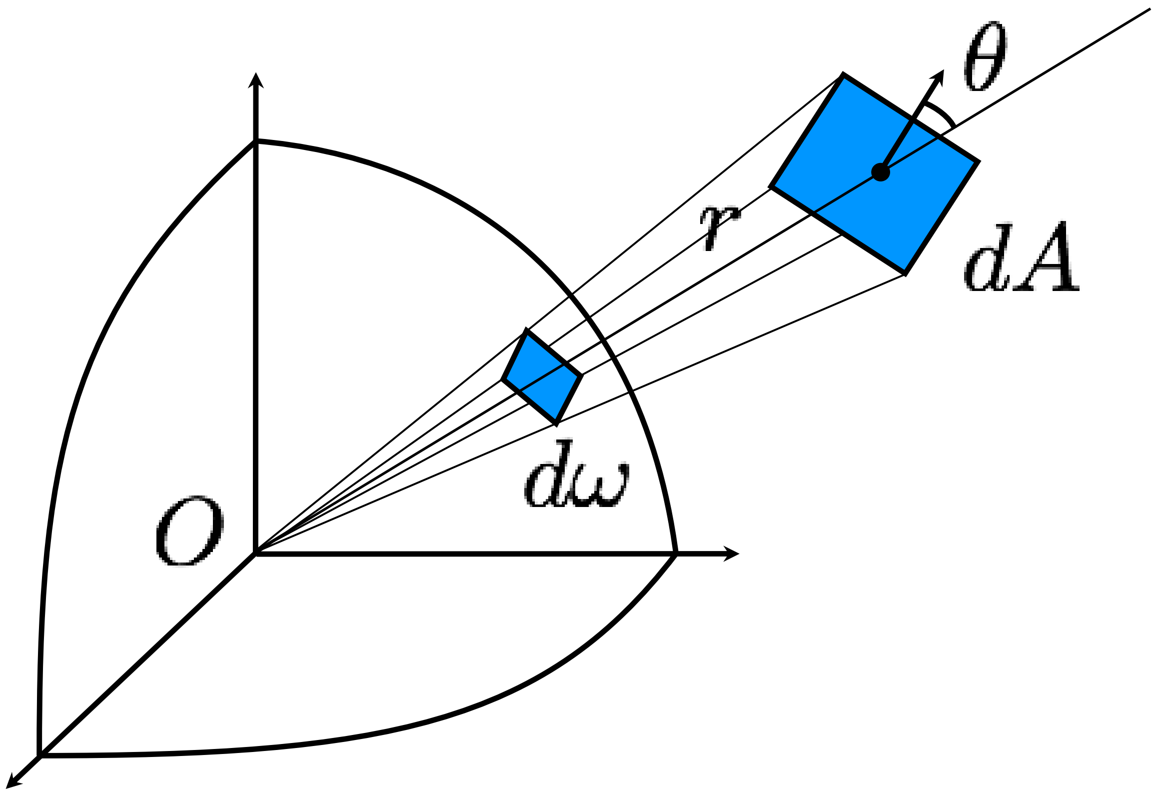


Depends on:

- orientation of patch
- distance of patch

One can show:

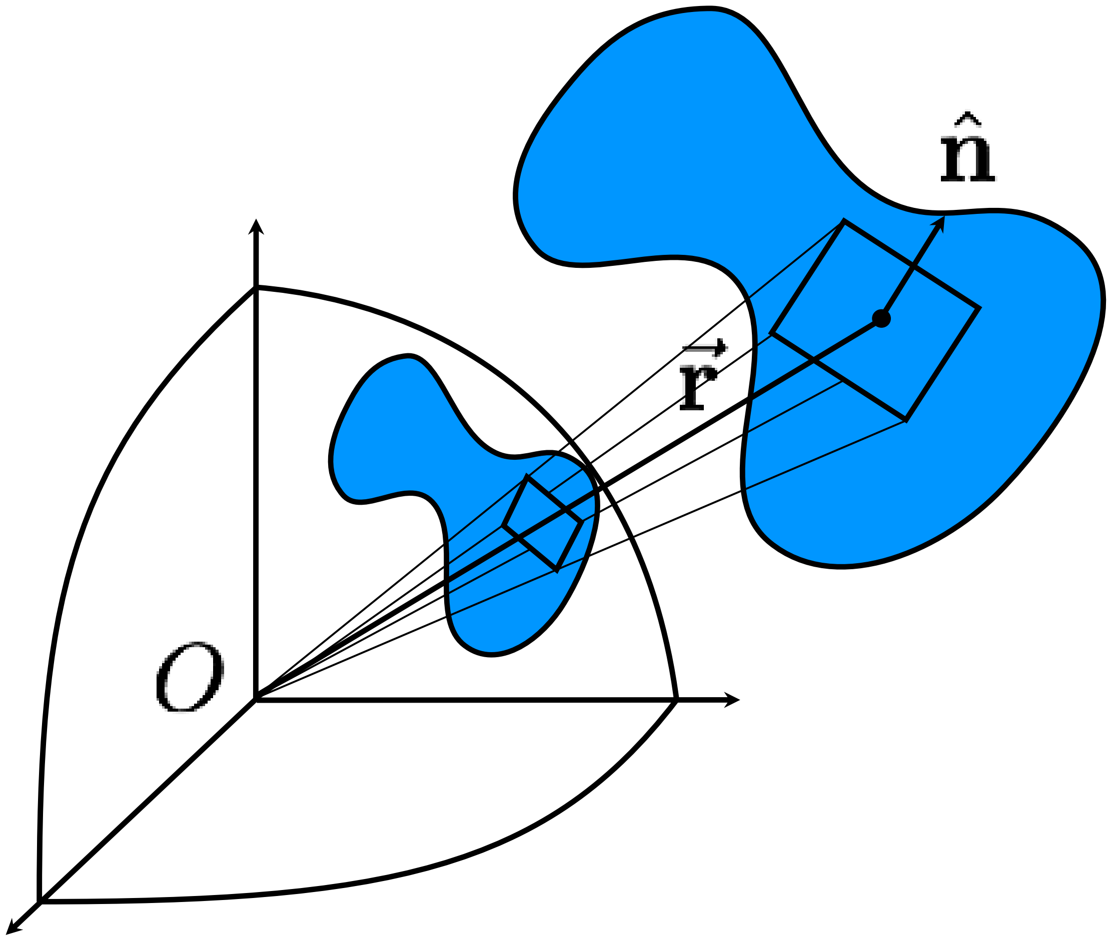
"surface foreshortening"

$$d\omega = \frac{dA \cos\theta}{r^2}$$

Units: steradians [sr]

Solid angle

- To calculate solid angle subtended by a surface S relative to O you must add up (integrate) contributions from all tiny patches (nasty integral)



$$\Omega = \iint_S \frac{\vec{\mathbf{r}} \cdot \hat{\mathbf{n}} \; dS}{|\vec{\mathbf{r}}|^3}$$

One can show:

"surface foreshortening"

$$d\omega = \frac{dA \cos\theta}{r^2}$$

Units: steradians [sr]

Question

- Suppose surface S is a hemisphere centered at O. What is the solid angle it subtends?

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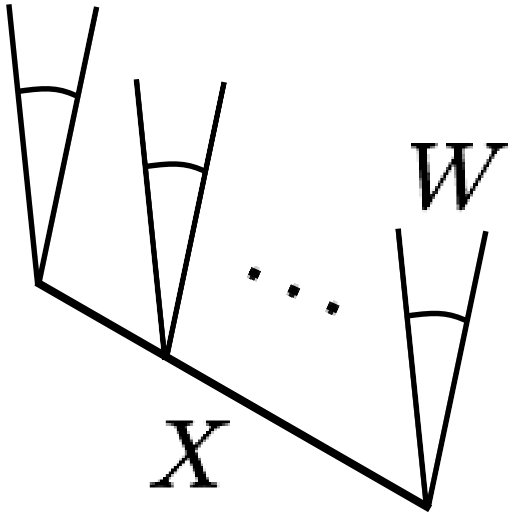
- Answer: 2\pi (area of sphere is 4\pi*r^2; area of unit sphere is 4\pi; half of that is 2\pi)

Quantifying light: flux, irradiance, and radiance

- Imagine a sensor that counts photons passing through planar patch X in directions within angular wedge W
- It measures *radiant flux* [watts = joules/sec]: rate of photons hitting sensor area
- Measurement depends on sensor area |X|

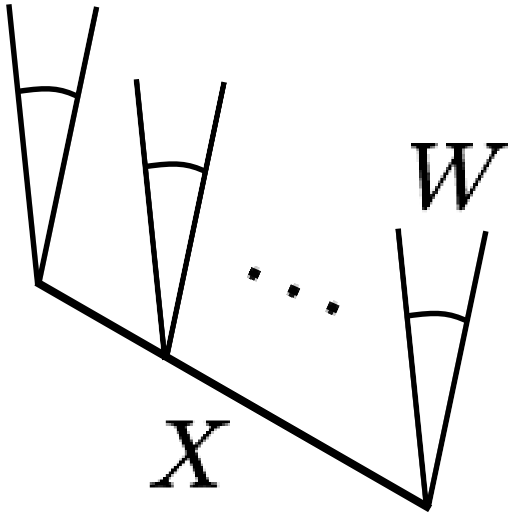


* shown in 2D for clarity; imagine three dimensions

radiant flux $\Phi(W, X)$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]



$$\frac{\Phi(W, X)}{|X|}$$

Quantifying light: flux, irradiance, and radiance

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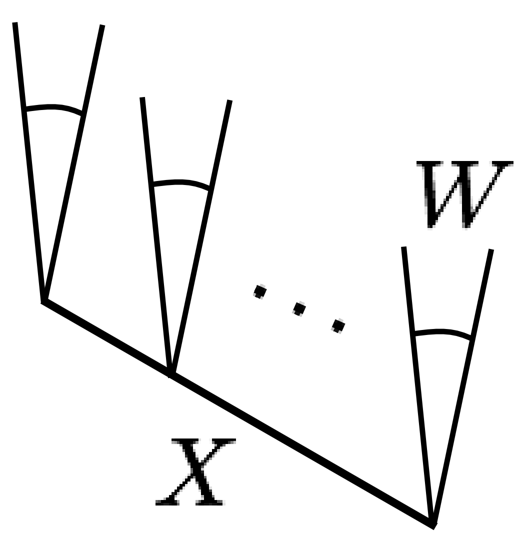
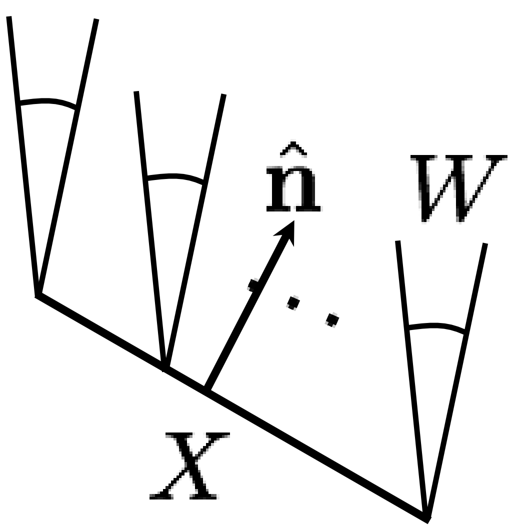


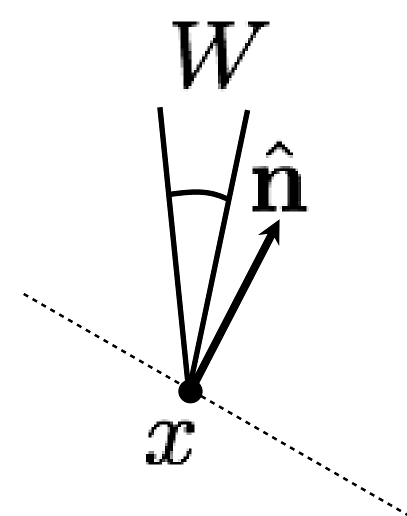
$$\lim_{X \longrightarrow x} \frac{\Phi(W, X)}{|X|}$$

Quantifying light: flux, irradiance, and radiance

- *Irradiance*:
  A measure of incoming light that is independent of sensor area |X|
- Units: watts per square meter [W/m^2]
- Depends on sensor direction normal.



$$\lim_{X \longrightarrow x}$$



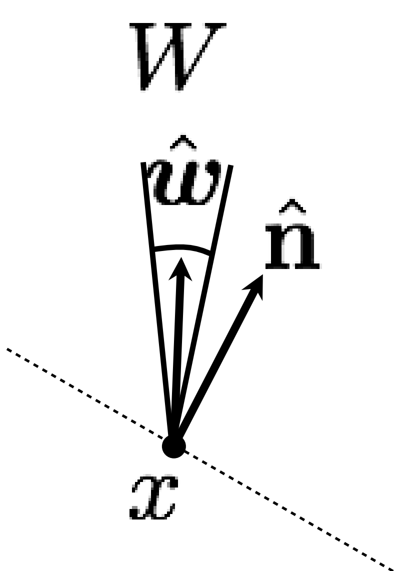
$$\frac{\Phi(W, X)}{|X|}$$

$$E_{\hat{\mathbf{n}}}(W, x)$$

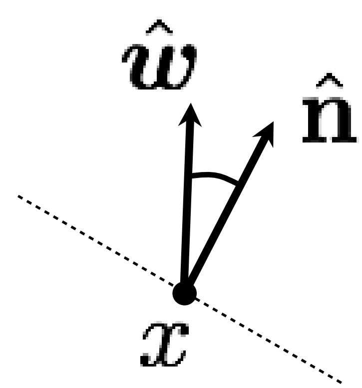
- We keep track of the normal because a planar sensor with distinct orientation would converge to a different limit
- In the literature, notations n and W are often omitted, and values are implied by context

Quantifying light: flux, irradiance, and radiance

- *Radiance*:
  A measure of incoming light that is independent of sensor area |X|, orientation n, and wedge size (solid angle) |W|
- Units: watts per steradian per square meter [W/(m^2·sr)]



$$\lim_{W \longrightarrow \hat{\omega}}$$



$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|}$$

$$L_{\hat{\mathbf{n}}}(\hat{\omega}, x)$$

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

Quantifying light: flux, irradiance, and radiance

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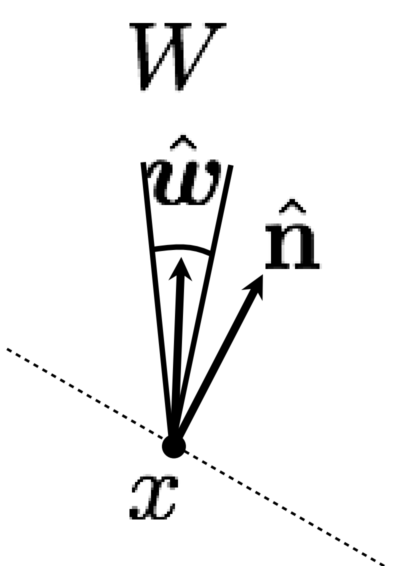
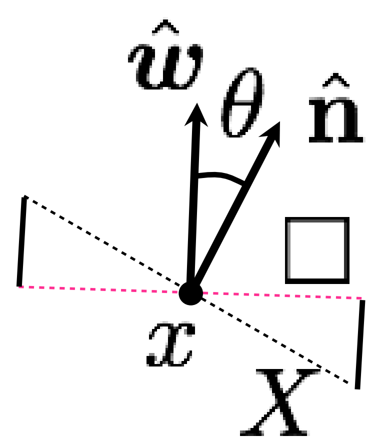


$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \quad \lim_{W \longrightarrow \hat{\omega}} \quad L_{\hat{\mathbf{n}}}(\hat{\omega}, x)$$



$$\cos\theta = \frac{\square/2}{|X|/2}$$

$$\longrightarrow \square = |X| \cos\theta$$

"foreshortened area"

- Has correct units, but still depends on sensor orientation
- To correct this, convert to measurement that would have been made if sensor was perpendicular to direction ω

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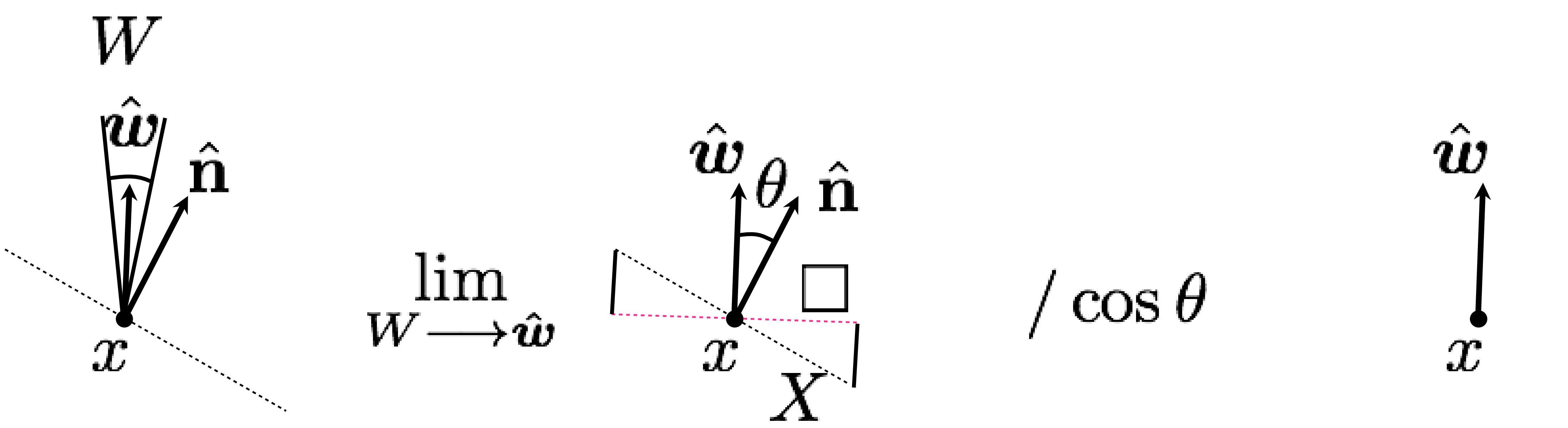

$$\frac{E_{\hat{\mathbf{n}}}(W, x)}{|W|} \qquad L_{\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}, x) \qquad L(\hat{\boldsymbol{\omega}}, x)$$

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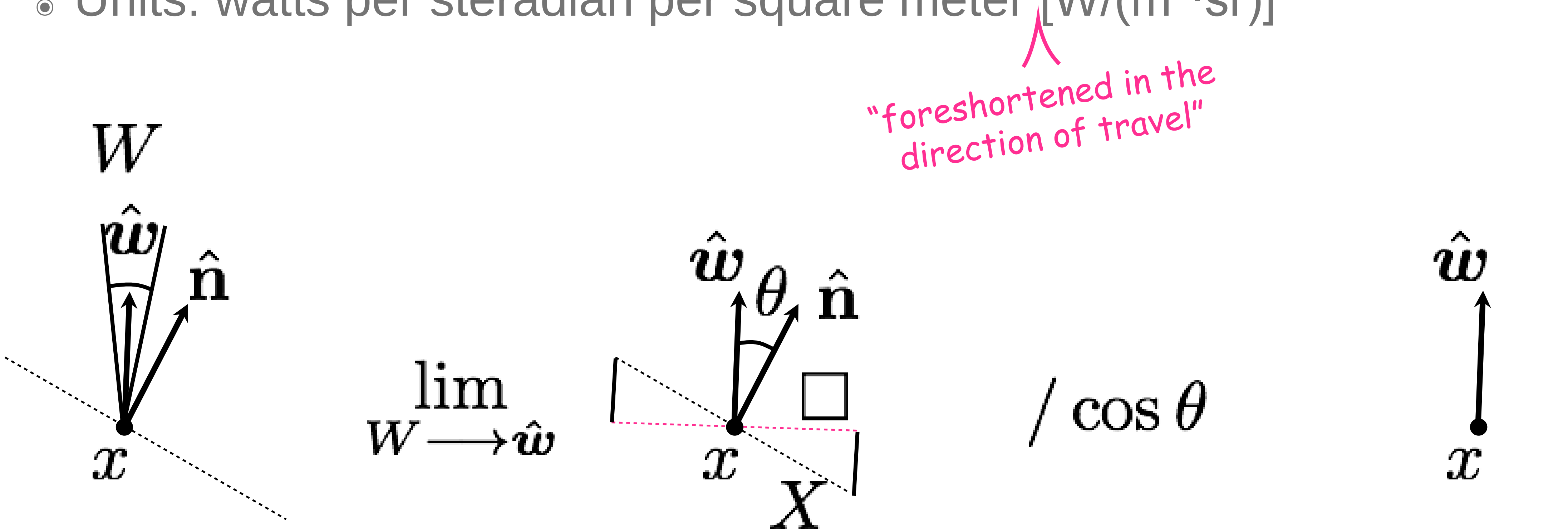

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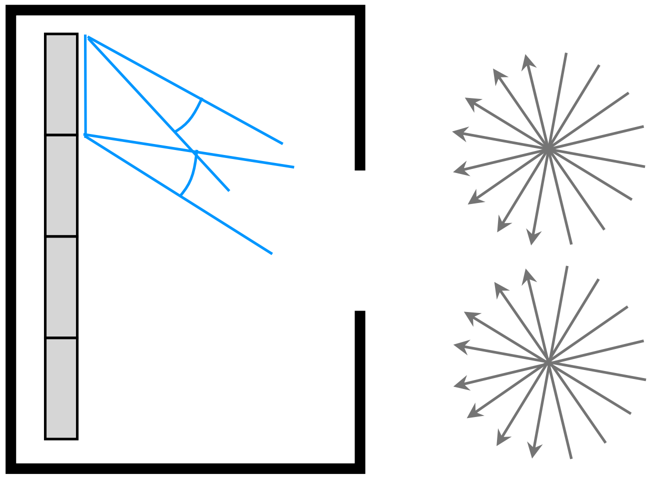
Quantifying light: flux, irradiance, and radiance

- Attractive properties of radiance:
  - Allows computing the radiant flux measured by *any* finite sensor



Quantifying light: flux, irradiance, and radiance

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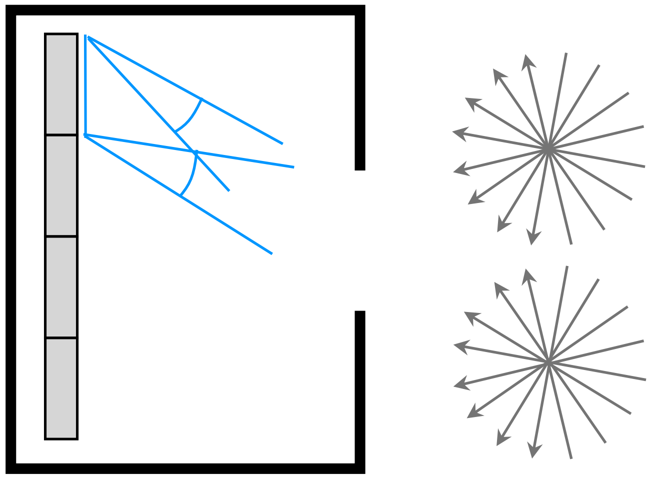
$$\Phi(W, X) = \int_X \int_W L(\hat{\omega}, x) \cos\theta d\omega dA$$



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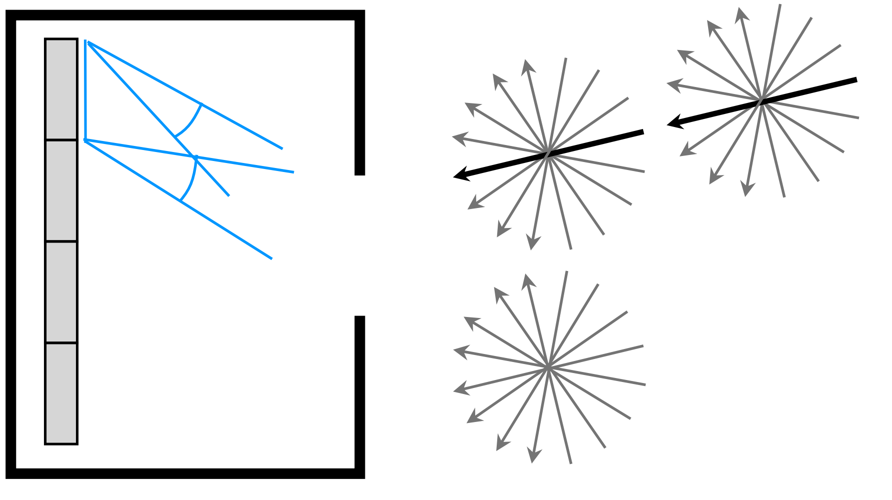
  - Constant along a ray in free space

$$L(\hat{\omega}, x) = L(\hat{\omega}, x + \hat{\omega})$$



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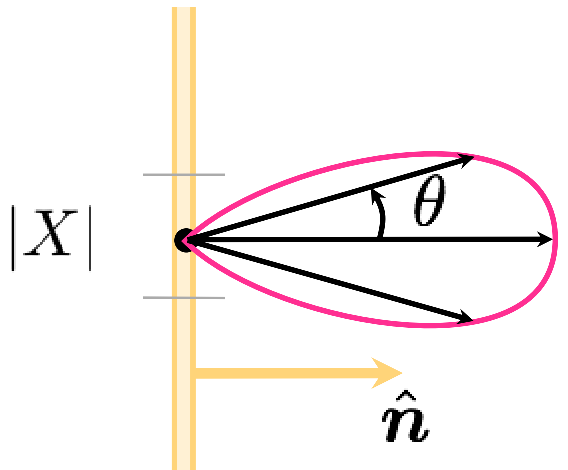
  - Constant along a ray in free space

$$L(\hat{\boldsymbol{\omega}}, x) = L(\hat{\boldsymbol{\omega}}, x + \hat{\boldsymbol{\omega}})$$

  - A camera measures radiance (after a one-time radiometric calibration). So RAW pixel values are proportional to radiance.
    - "Processed" images (like PNG and JPEG) are not linear radiance measurements!!

Question

- Most light sources, like a heated metal sheet, follow Lambert's Law



"Lambertian area source"

$$J(\hat{\boldsymbol{\omega}}) = J_o \langle \hat{\boldsymbol{\omega}}, \hat{\boldsymbol{n}} \rangle = J_o \cos\theta$$

radiant intensity [W/sr]

- What is the radiance $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x})$ of an infinitesimal patch [W/sr·m^2]?

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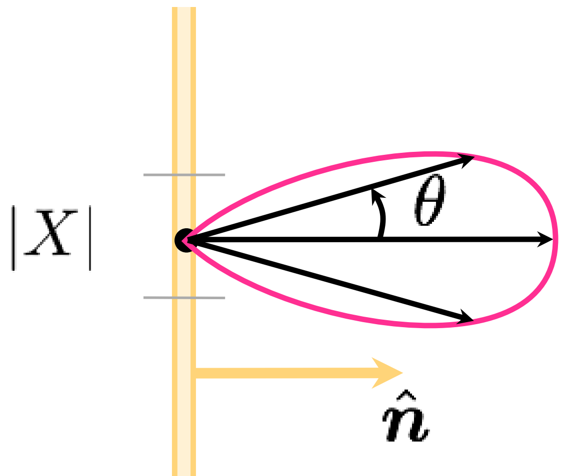

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Answer: $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x}) = J_o / |X|$ (independent of direction)

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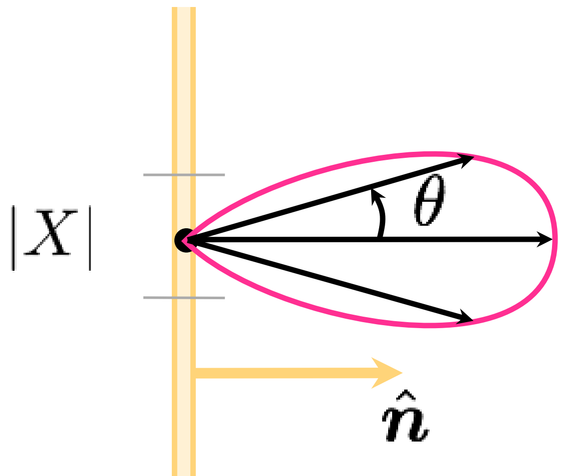

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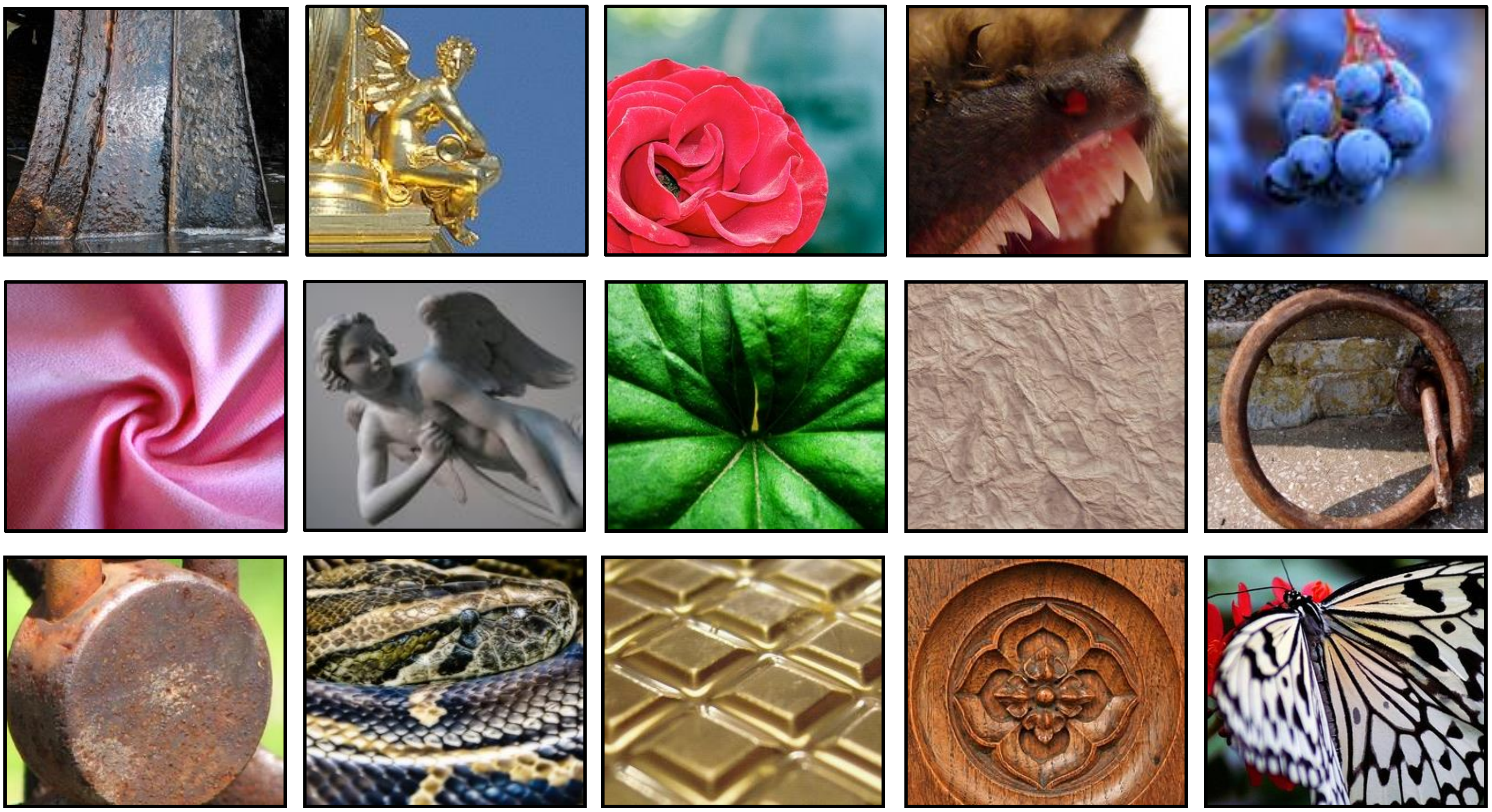
Answer: $L(\hat{\boldsymbol{\omega}}, \boldsymbol{x}) = J_o / |X|$ (independent of direction)

"Looks equally bright when viewed from any direction"

Appearance



"Physics-based" computer vision
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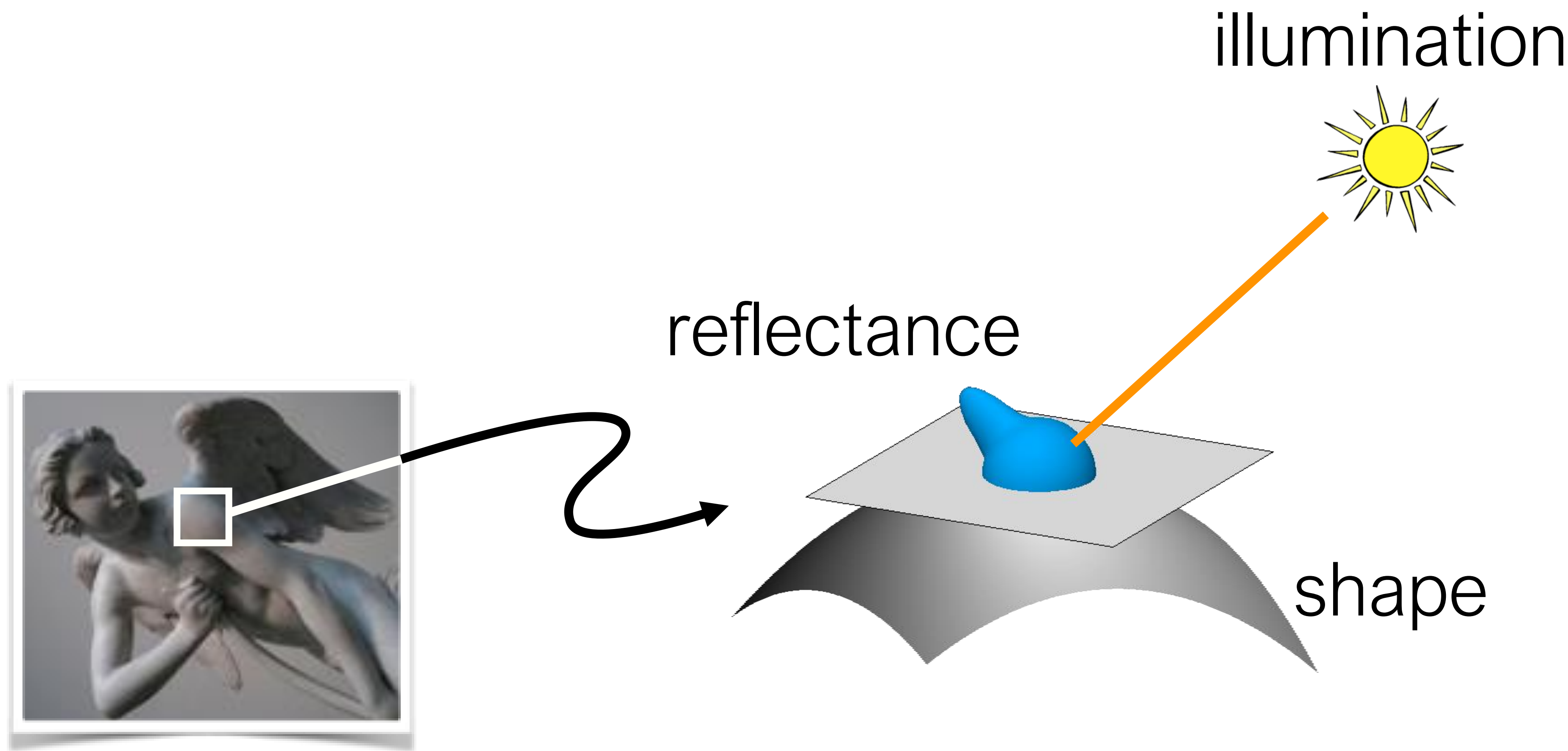

$\mathbf{I} \Longrightarrow$ shape, illumination, reflectance

Reflectance and BRDF

Reflectance

- Ratio of outgoing energy to incoming energy at a single point
- Want to define a ratio such that it:
  - converges as we use smaller and smaller incoming and outgoing wedges
  - does not depend on the size of the wedges (i.e. is intrinsic to the material)

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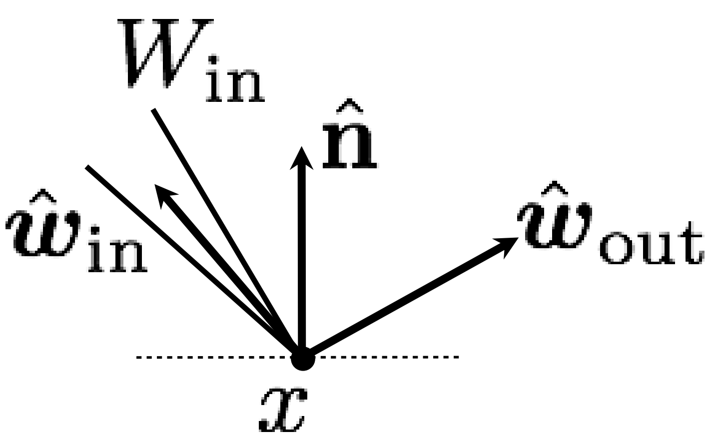


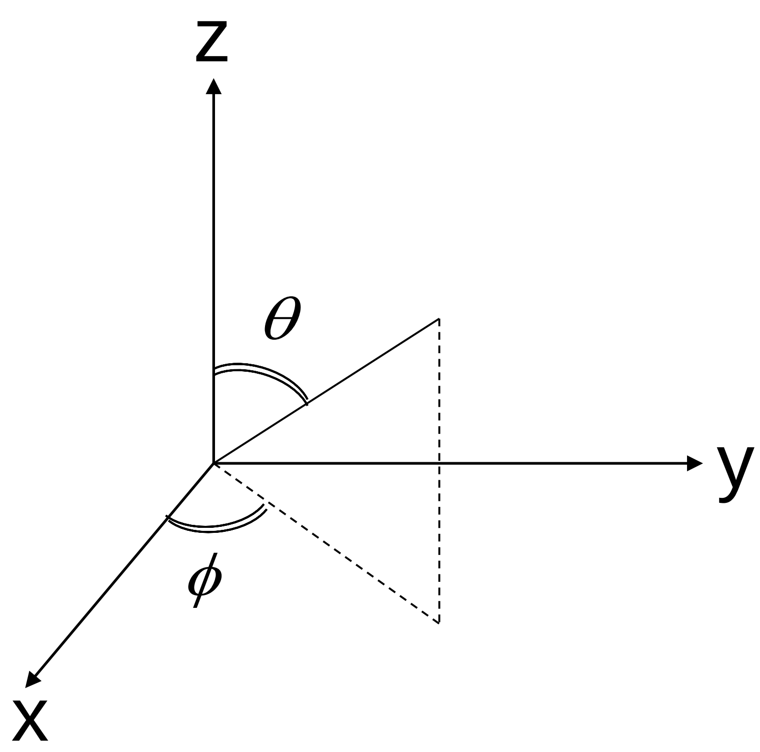
$$\lim_{W_{\text{in}} \longrightarrow \hat{\boldsymbol{w}}_{\text{in}}}$$

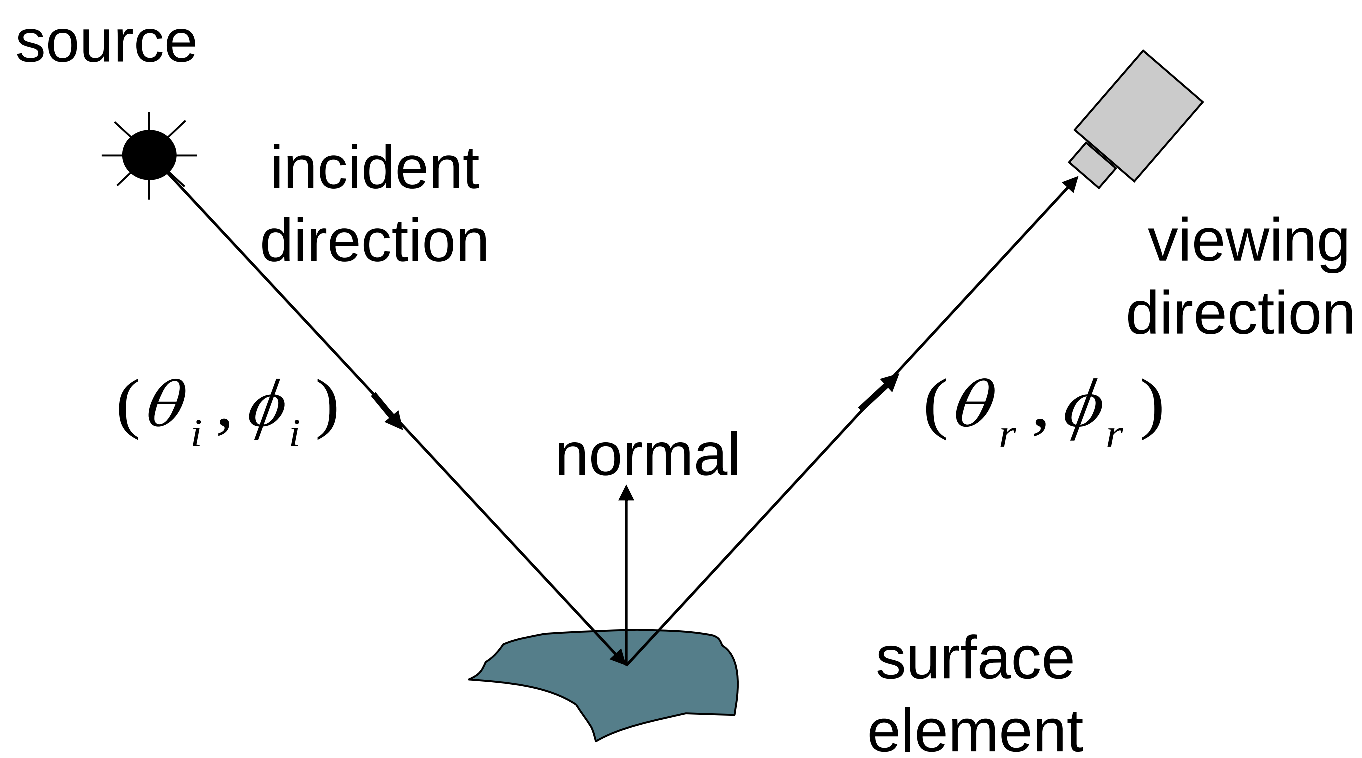
$$f_{x,\hat{\mathbf{n}}}(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}})$$

$$f_{x,\hat{\mathbf{n}}}(W_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) = \frac{L^{\text{out}}(x, \hat{\boldsymbol{\omega}}_{\text{out}})}{E^{\text{in}}_{\hat{\mathbf{n}}}(W_{\text{in}}, x)}$$

- Notations x and n often implied by context and omitted; directions \omega are expressed in local coordinate system defined by normal n (and some chosen tangent vector)
- Units: sr^{-1}
- Called Bidirectional Reflectance Distribution Function (BRDF)

BRDF: Bidirectional Reflectance Distribution Function





$E^{surface}(\theta_i, \phi_i)$ Irradiance at Surface in direction (θ_i, ϕ_i)

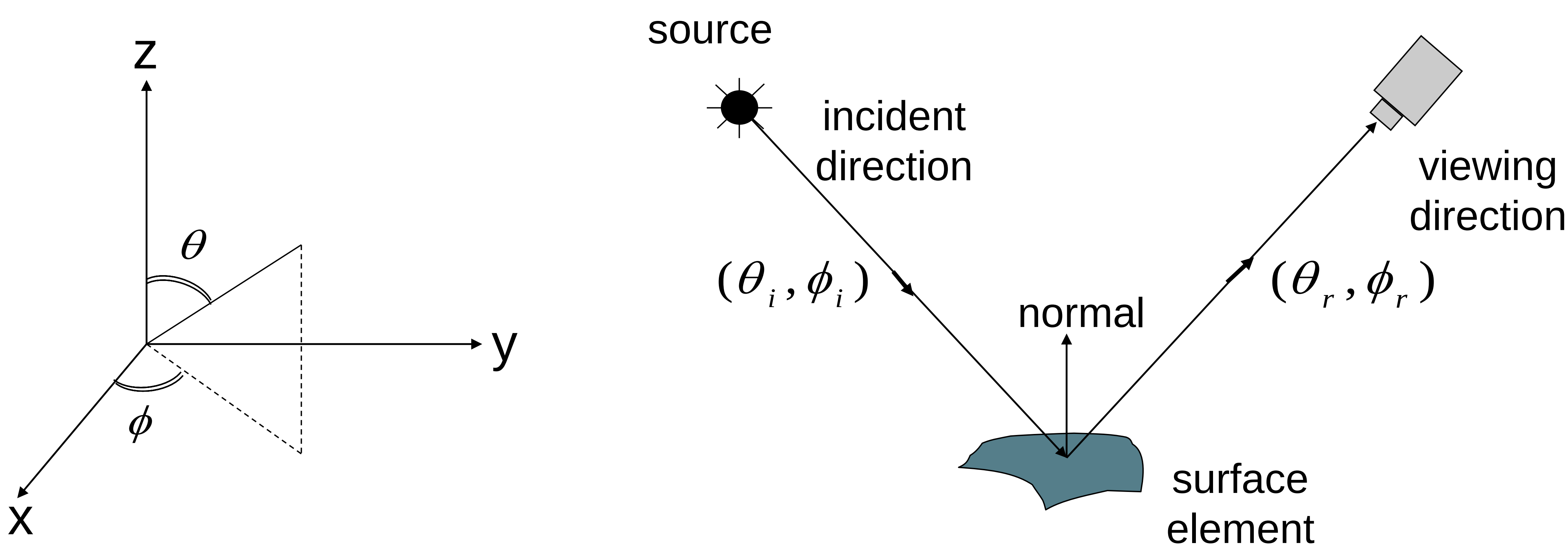
$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

$$\text{BRDF}: f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

Reflectance: BRDF

- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties

Important Properties of BRDFs

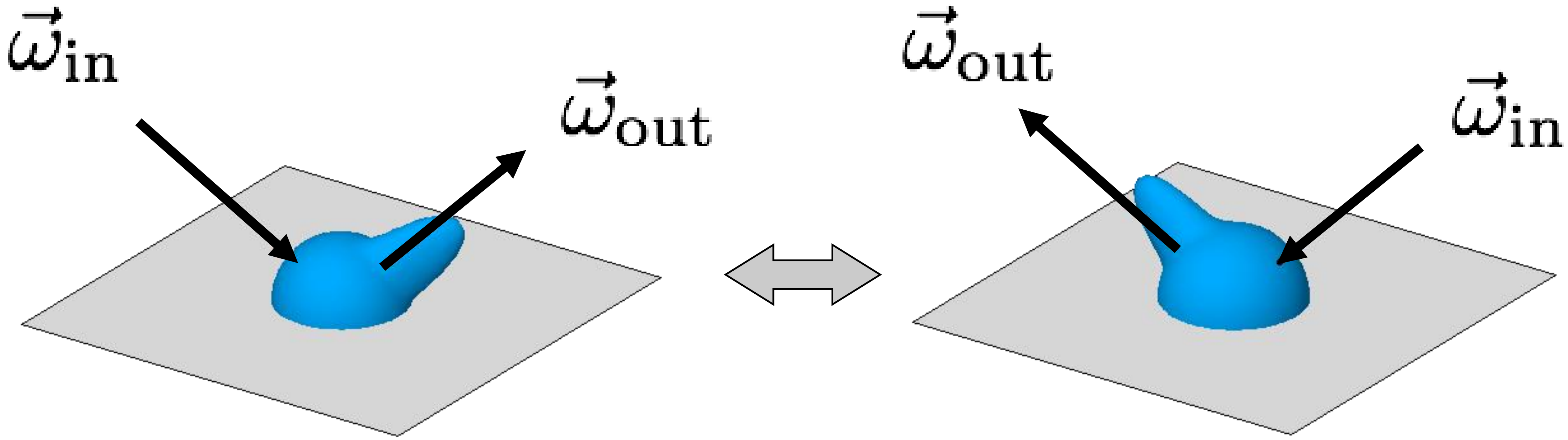


- Conservation of Energy:

$$\forall \hat{\boldsymbol{\omega}}_{\mathrm{in}}, \quad \int_{\Omega_{\mathrm{out}}} f(\hat{\boldsymbol{\omega}}_{\mathrm{in}}, \hat{\boldsymbol{\omega}}_{\mathrm{out}}) \cos\theta_{\mathrm{out}} d\hat{\boldsymbol{\omega}}_{\mathrm{out}} \leq 1$$

Why smaller than or equal?

Property: “Helmholtz reciprocity”

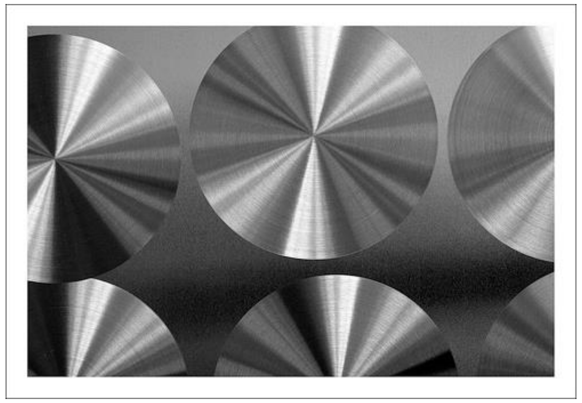


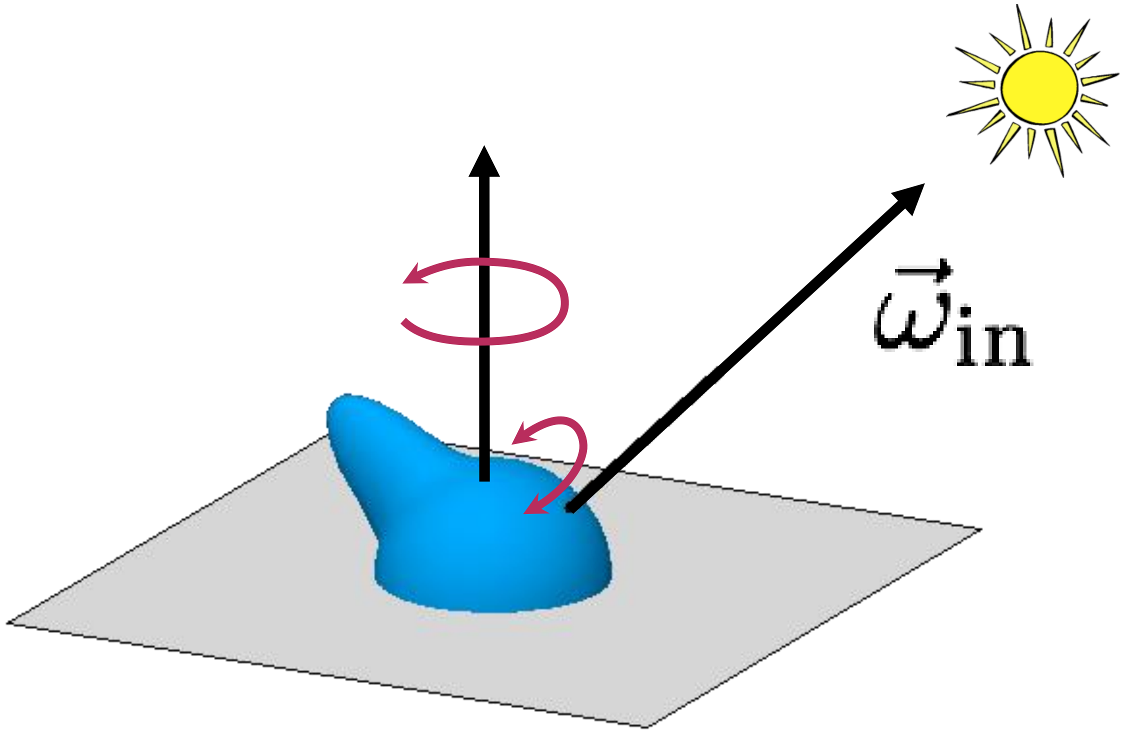
- Helmholtz Reciprocity: (follows from 2nd Law of Thermodynamics)

BRDF does not change when source and viewing directions are swapped.

$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}) = f_r(\vec{\omega}_{\text{out}}, \vec{\omega}_{\text{in}})$$

Common assumption: Isotropy





$f_r(\vec{\omega}_{\text{in}}, \cdot)$

[Matusik et al., 2003]

BRDF does not change when surface is rotated about the normal.

4D → 3D

$$f_r(\sout{\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Can be written as a function of 3 variables : $f\,(\theta_i, \theta_r, \phi_i - \phi_r)$

Reflectance: BRDF

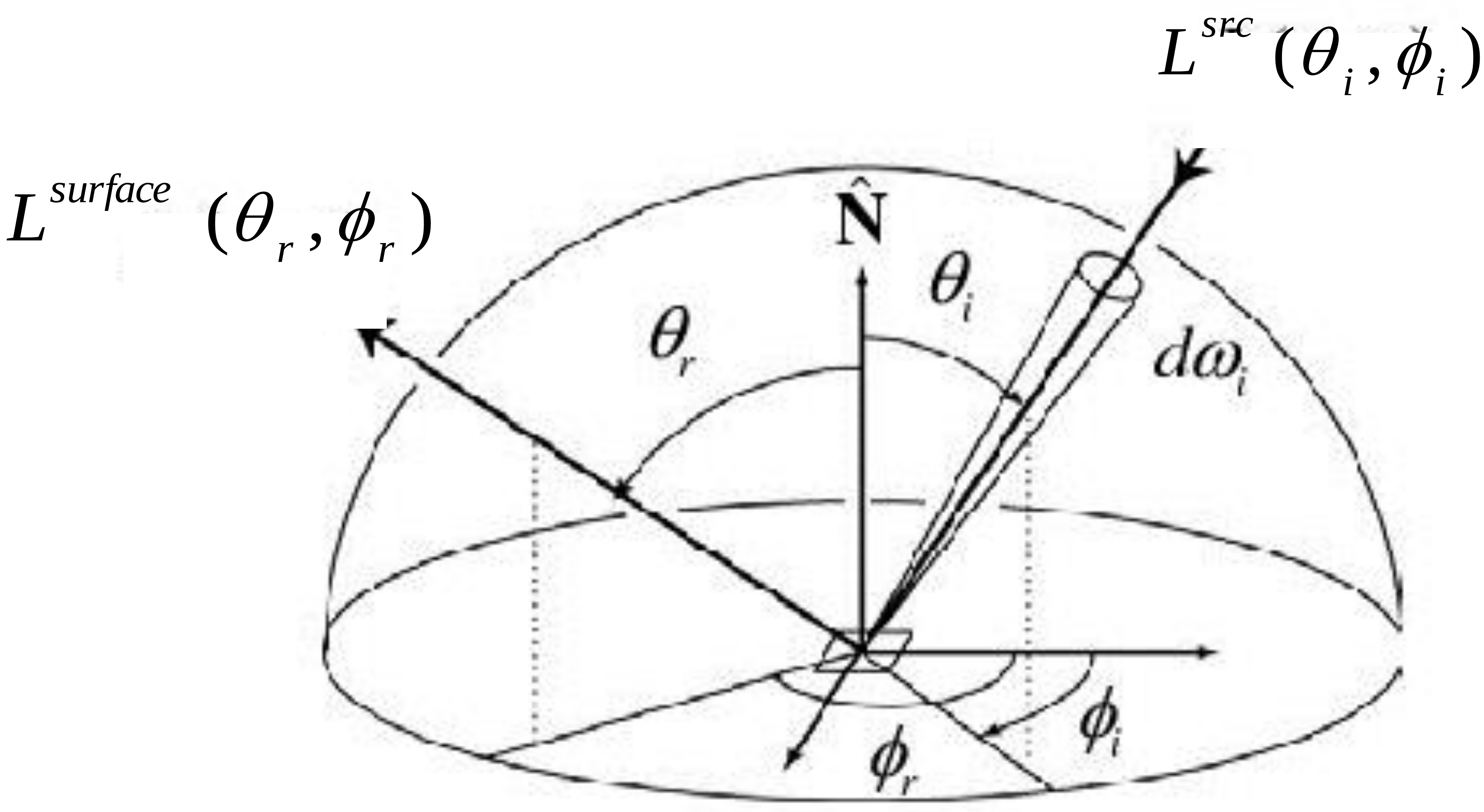
- Units: sr^{-1}
- Real-valued function defined on the double-hemisphere
- Has many useful properties
- Allows computing output radiance (and thus pixel value) for *any* configuration of lights and viewpoint

$$L^{\text{out}}(\hat{\omega}) = \int_{\Omega_{\text{in}}} f(\hat{\omega}_{\text{in}}, \hat{\omega}_{\text{out}}) L^{\text{in}}(\hat{\omega}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\omega}_{\text{in}}$$

reflectance equation

Why is there a cosine in the reflectance equation?

Derivation of the Reflectance Equation



From the definition of BRDF:

$$L^{surface}(\theta_r, \phi_r) \quad = \quad E^{surface}(\theta_i, \phi_i)\, f(\theta_i, \phi_i; \theta_r, \phi_r)$$

Derivation of the Scene Radiance Equation

From the definition of BRDF:

$$L^{surface}(\theta_r,\phi_r) = \underline{E^{surface}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)$$

Write Surface Irradiance in terms of Source Radiance:

$$L^{surface}(\theta_r,\phi_r) = \underline{L^{src}(\theta_i,\phi_i)}\, f(\theta_i,\phi_i;\theta_r,\phi_r)\, \underline{\cos\theta_i d\omega_i}$$

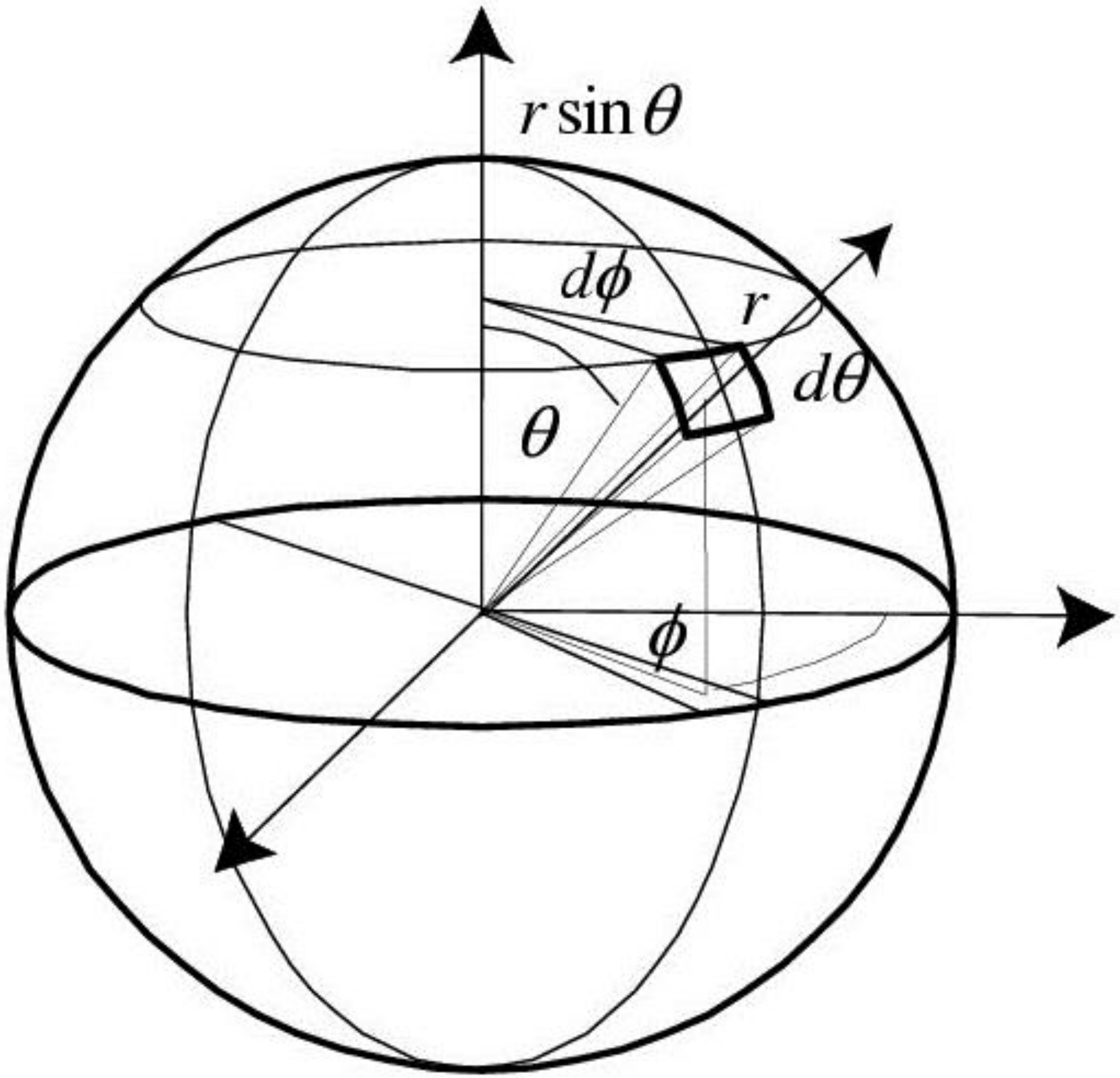
Integrate over entire hemisphere of possible source directions:

$$L^{surface}(\theta_r,\phi_r) = \int_{2\pi} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\, \underline{d\omega_i}$$

Convert from solid angle to theta-phi representation:

$$L^{surface}(\theta_r,\phi_r) = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i\, \underline{\sin\theta_i d\theta_i d\phi_i}$$

Differential Solid Angles

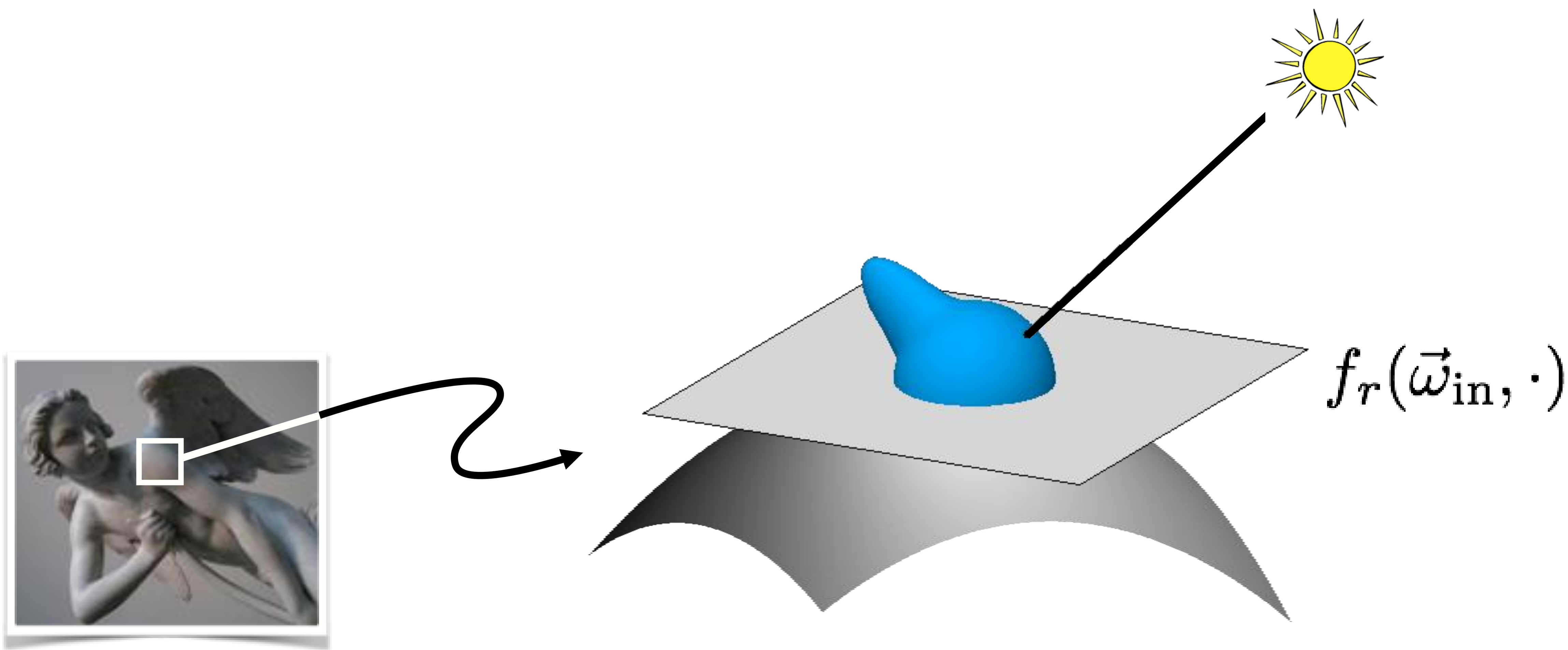


$$dA = (r\,d\theta)(r\sin\theta\,d\phi)$$
$$= r^2 \sin\theta\,d\theta\,d\phi$$

$$d\omega = \frac{dA}{r^2} = \sin\theta\,d\theta\,d\phi$$

$$S = \int_0^{\pi}\int_0^{2\pi} \sin\theta\,d\theta\,d\phi = 4\pi$$

BRDF



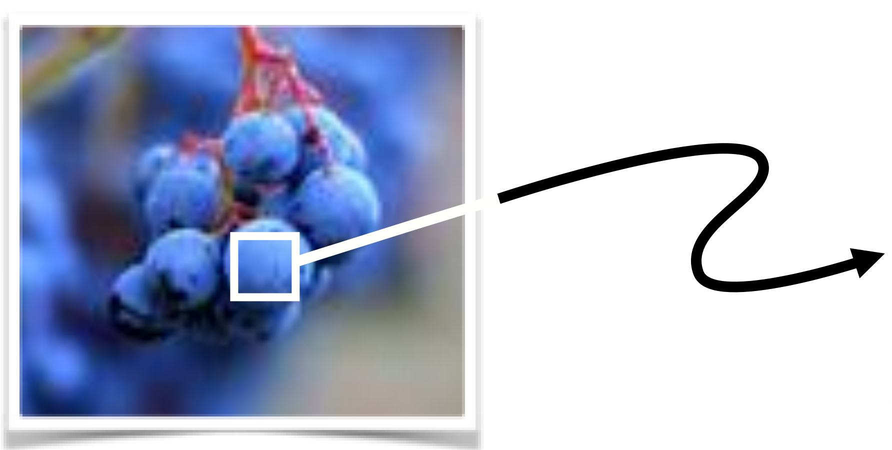
$$f_r(\vec{\omega}_{\mathrm{in}}, \vec{\omega}_{\mathrm{out}})$$

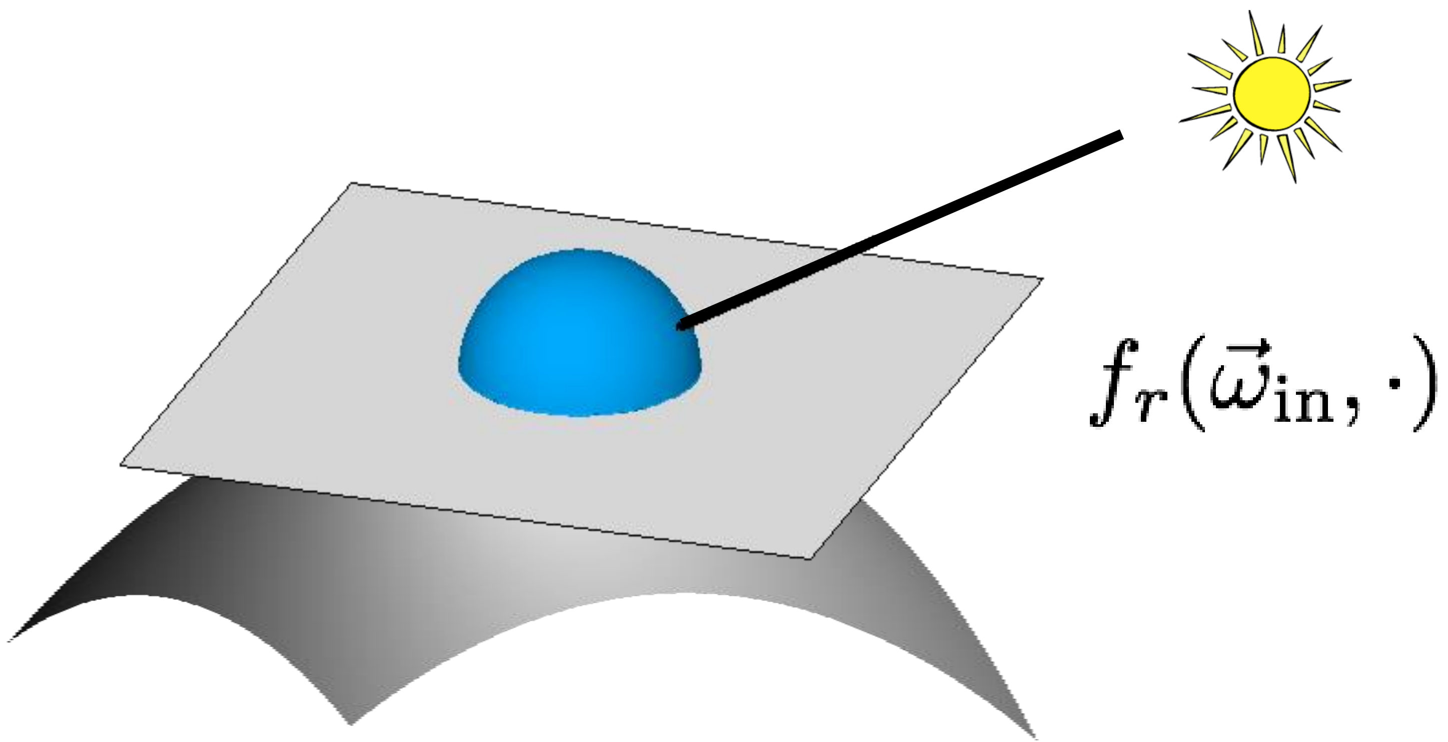
Bi-directional Reflectance Distribution Function (BRDF)

BRDF

Lambertian (diffuse) BRDF: energy equally distributed in all directions

What does the BRDF equal in this case?

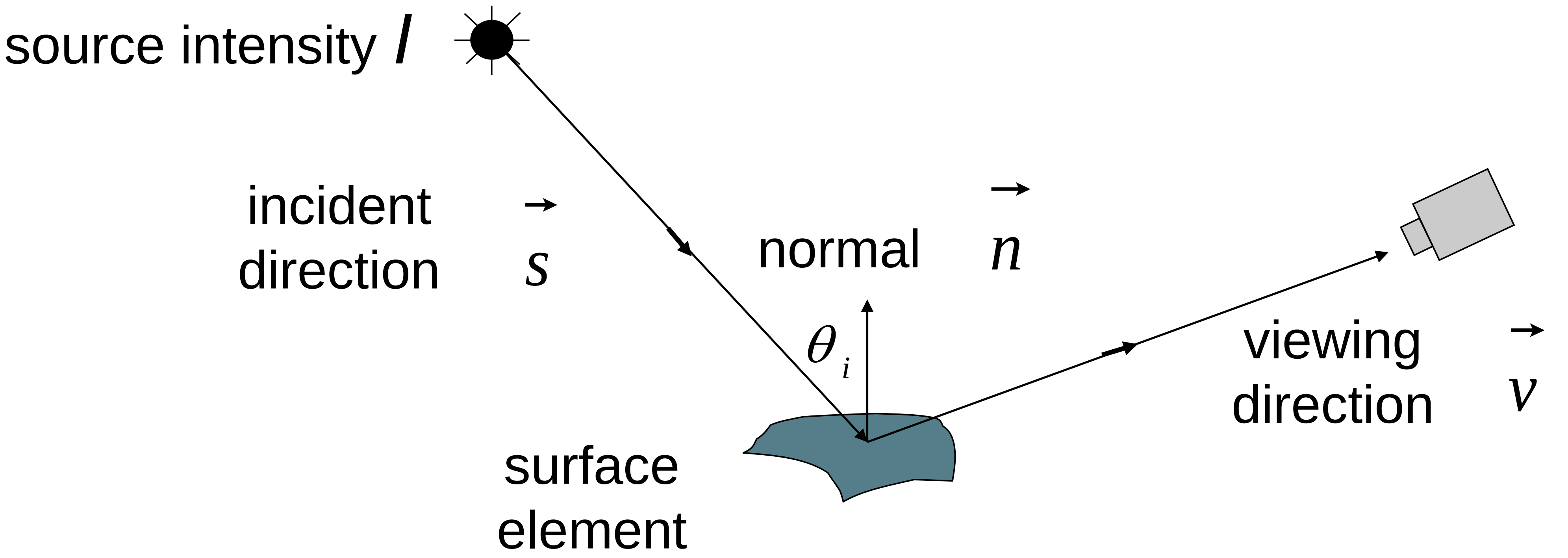




$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Diffuse Reflection and Lambertian BRDF

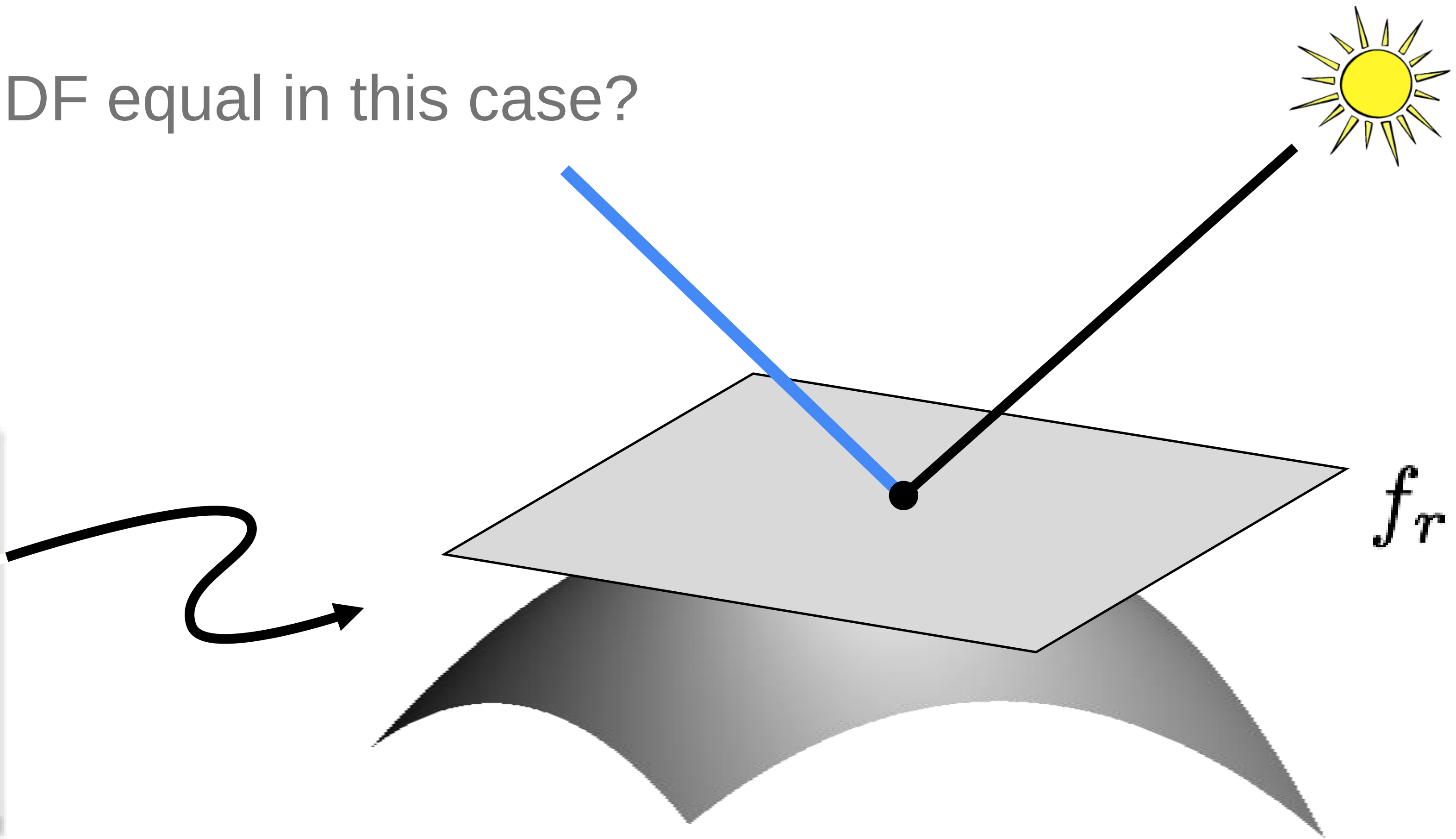


- Surface appears equally bright from ALL directions! (independent of $\vec{v}$)
- Lambertian BRDF is simply a constant : $f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{\rho_d}{\pi}$ ← albedo
- Most commonly used BRDF in Vision and Graphics!

BRDF

Specular BRDF: all energy concentrated in mirror direction

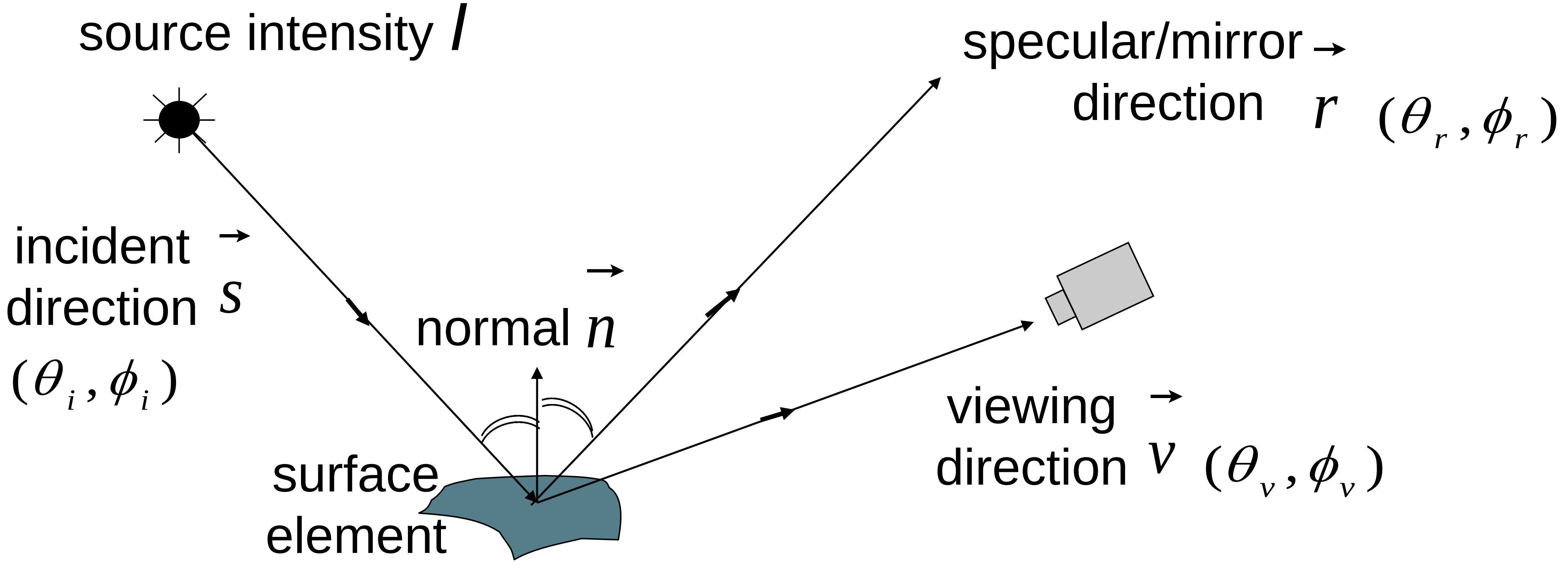
What does the BRDF equal in this case?



$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Specular Reflection and Mirror BRDF



- Valid for very smooth surfaces.

- All incident light energy reflected in a SINGLE direction (only when $\vec{v} = \vec{r}$).

- Mirror BRDF is simply a double-delta function :

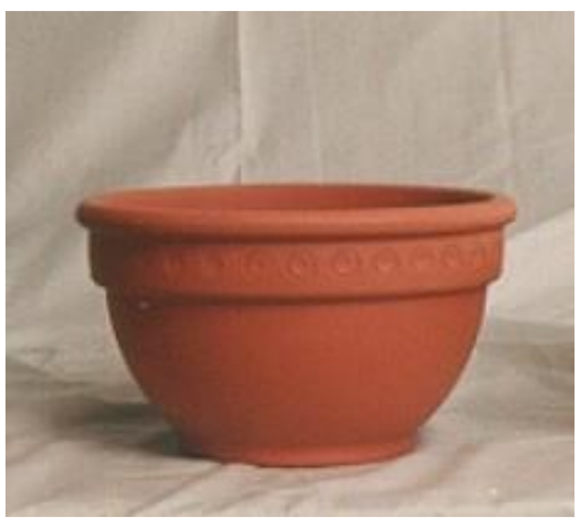
specular albedo

$$f(\theta_i, \phi_i; \theta_v, \phi_v) = \rho_s \; \delta(\theta_i - \theta_v) \; \delta(\phi_i + \pi - \phi_v)$$

Example Surfaces

Body Reflection:

Diffuse Reflection
Matte Appearance
Non-Homogeneous Medium
Clay, paper, etc



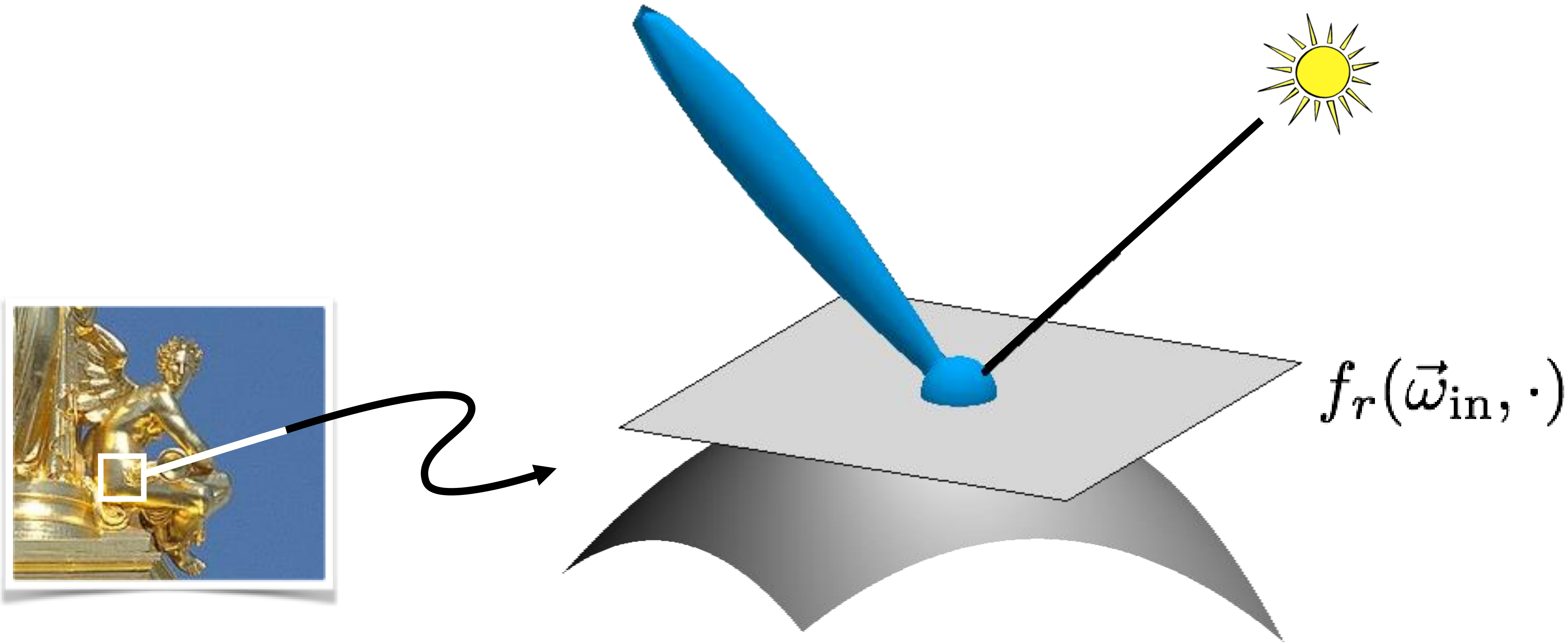
Surface Reflection:

Specular Reflection
Glossy Appearance
Highlights
Dominant for Metals

Many materials exhibit both Reflections:

BRDF

Glossy BRDF: more energy concentrated in mirror direction than elsewhere



$$f_r(\vec{\omega}_{\text{in}}, \vec{\omega}_{\text{out}})$$

Bi-directional Reflectance Distribution Function (BRDF)

Trick for dielectrics (non-metals)

- BRDF is a sum of a Lambertian diffuse component and non-Lambertian specular components
- The two components differ in terms of color and polarization, and under certain conditions, this can be exploited to separate them.

$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

Trick for dielectrics (non-metals)

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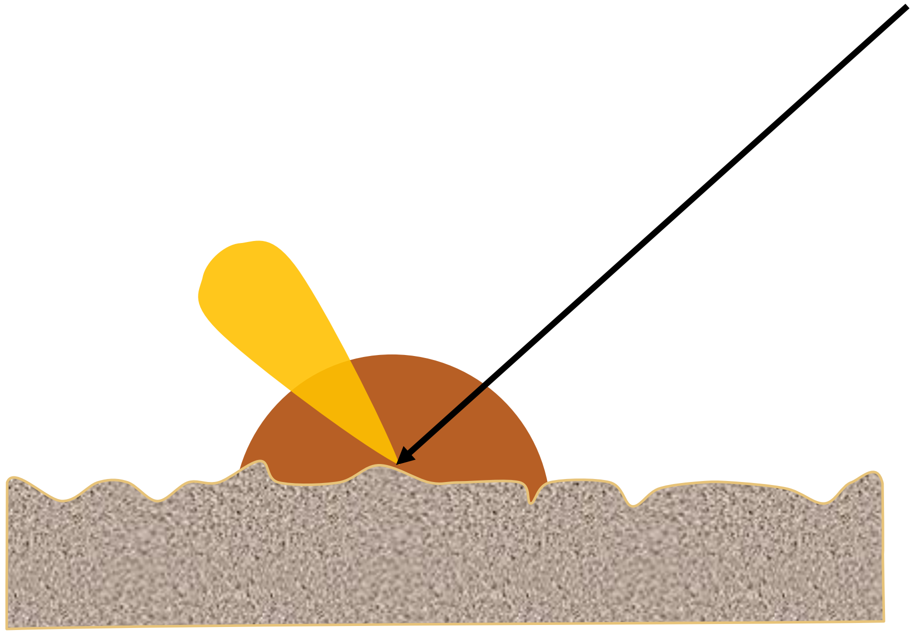
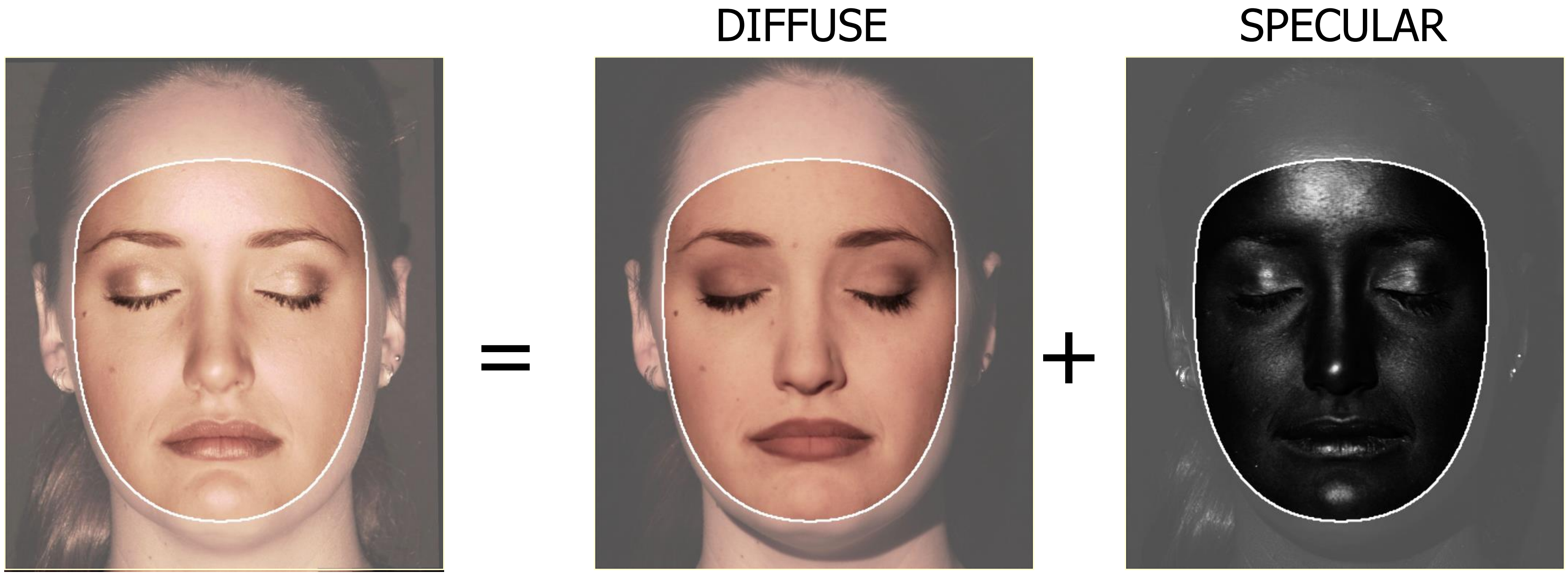
$$f(\vec{\omega}_i, \vec{\omega}_o) = f_d + f_s(\vec{\omega}_i, \vec{\omega}_o)$$

Often called the *dichromatic BRDF*:

- Diffuse term varies with wavelength, constant with polarization
- Specular term constant with wavelength, varies with polarization

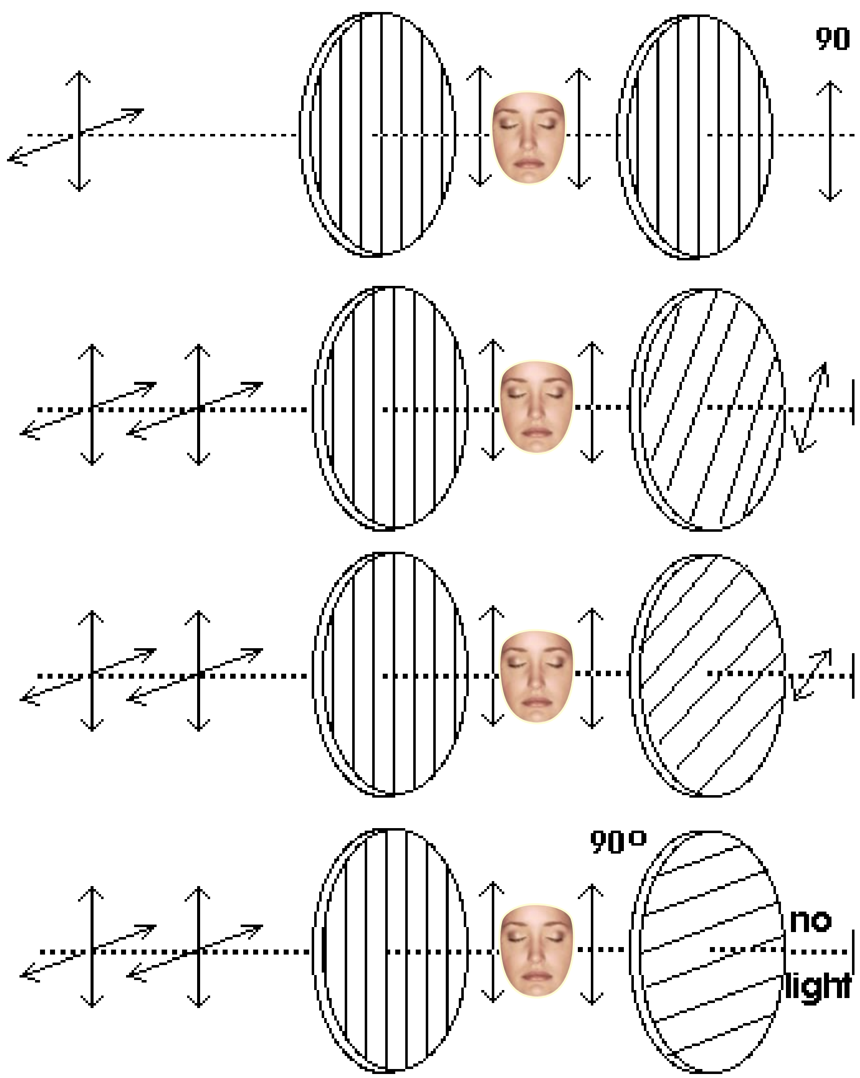


Trick for dielectrics (non-metals)



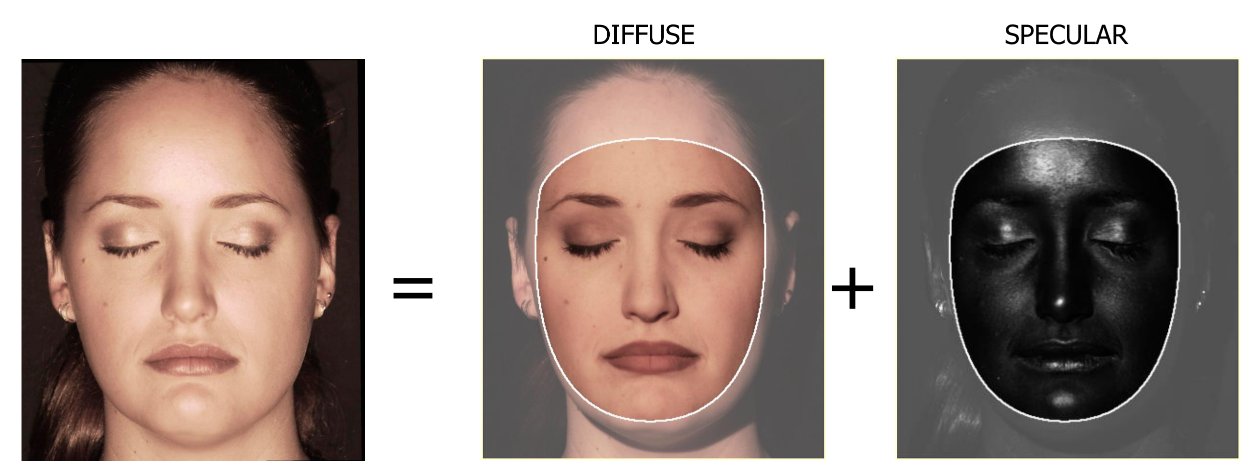
- In this example, the two components were separated using linear polarizing filters on the camera and light source.

Trick for dielectrics (non-metals)

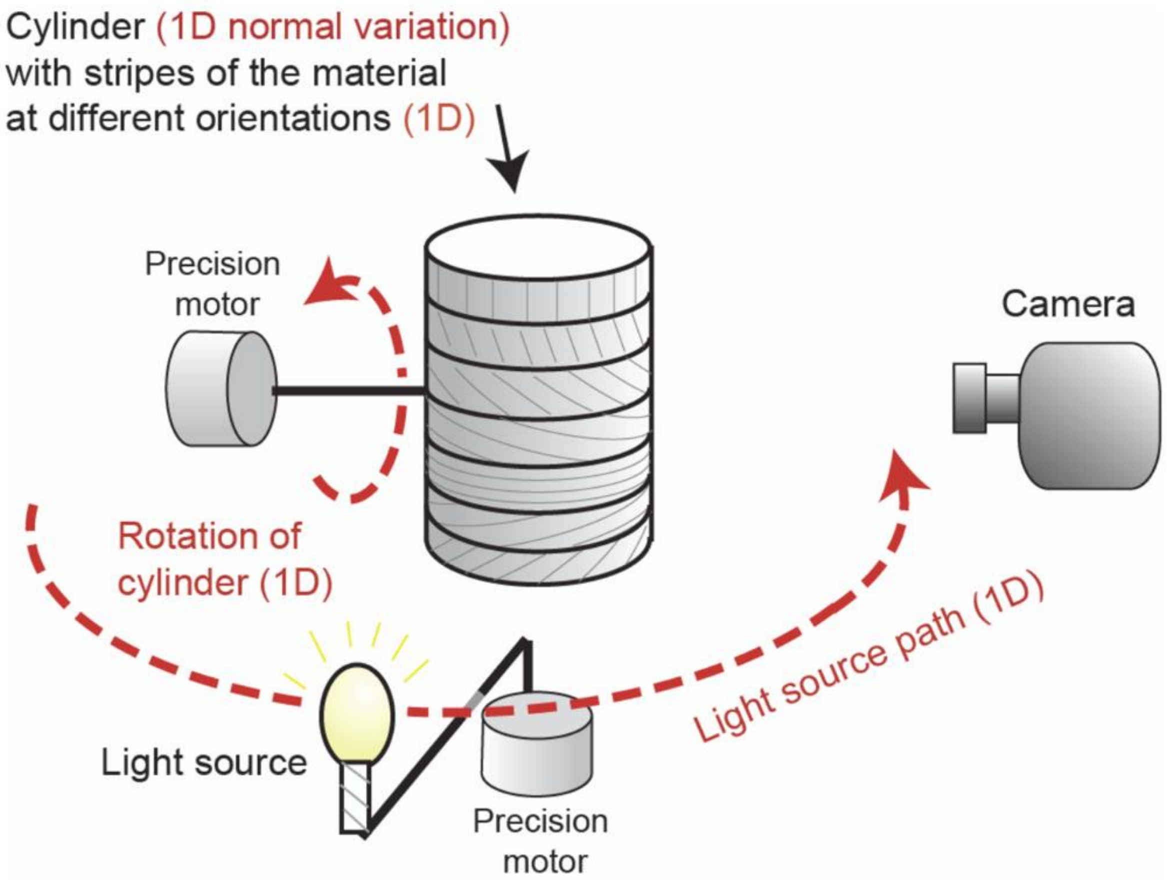


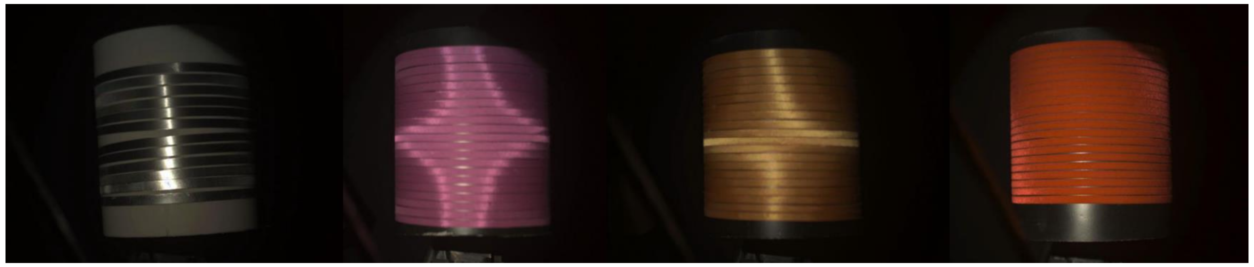
$$I_{\|} = \frac{1}{2} I_{\text{diffuse}} + I_{\text{specular}}$$

$$I_{\perp} = \frac{1}{2} I_{\text{diffuse}}$$

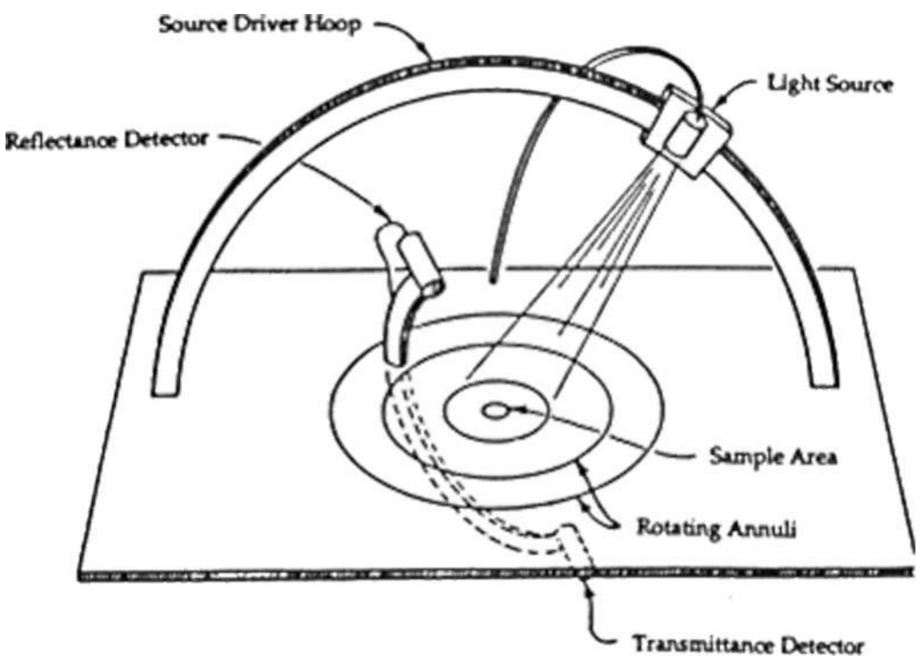


Tabulated 4D BRDFs (hard to measure)





[Ngan et al., 2005]



Gonioreflectometer

Low-parameter (non-linear) BRDF models

- A small number of parameters define the (2D,3D, or 4D) function
- Except for Lambertian, the BRDF is non-linear in these parameters
- Examples:

Where do these constants come from?

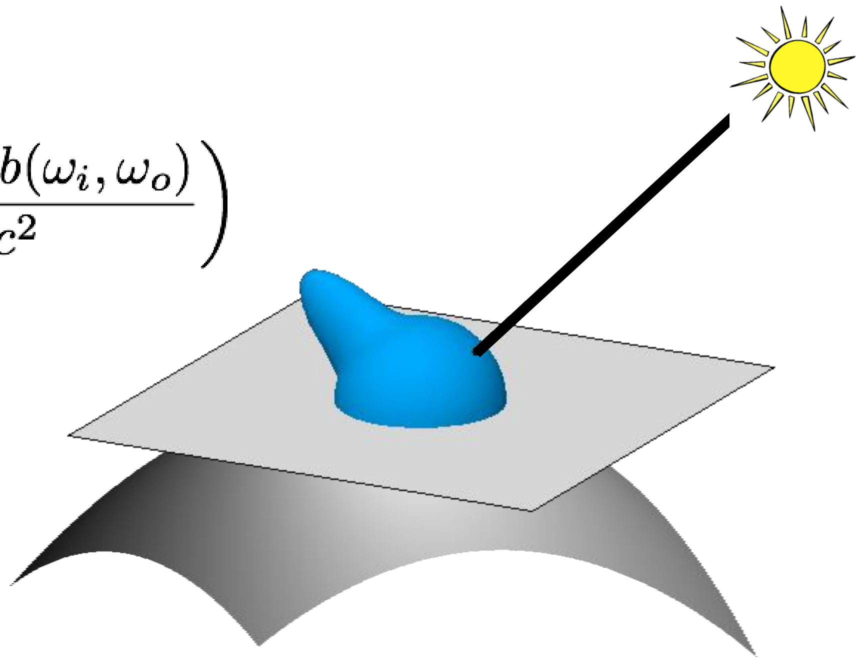
Lambertian: $f(\omega_i, \omega_o) = \frac{a}{\pi}$

Phong: $f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c \left(2\langle \omega_i, n \rangle \langle \omega_o, n \rangle - \langle \omega_i, \omega_o \rangle\right)$

Blinn: $f(\omega_i, \omega_o) = \frac{a}{\pi} + b \cos^c b(\omega_i, \omega_o)$

Lafortune: $f(\omega_i, \omega_o) = \frac{a}{\pi} + b(-\omega_i^\top A \omega_o)^k$

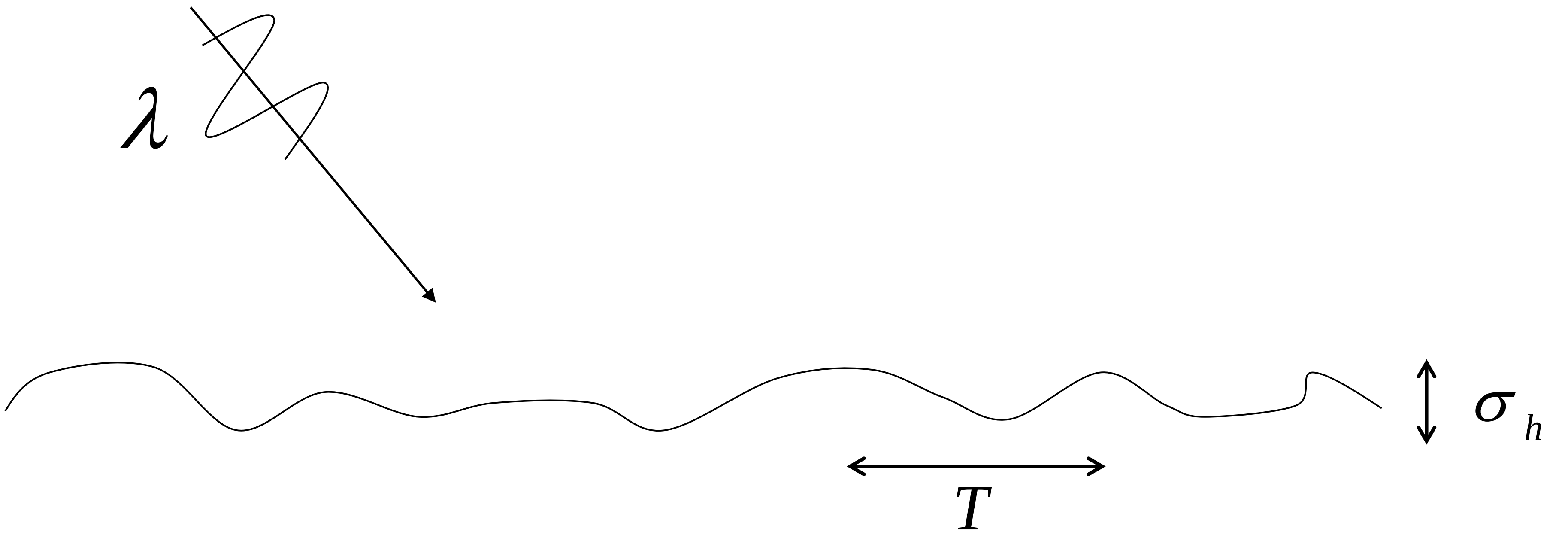
Ward: $f(\omega_i, \omega_o) = \frac{a}{\pi} + \frac{b}{4\pi c^2 \sqrt{\langle n, \omega_i \rangle \langle n, \omega_o \rangle}} \exp\left(\frac{-\tan^2 b(\omega_i, \omega_o)}{c^2}\right)$



α is called the *albedo*

Reflectance Models

Reflection: An Electromagnetic Phenomenon



Two approaches to derive Reflectance Models:

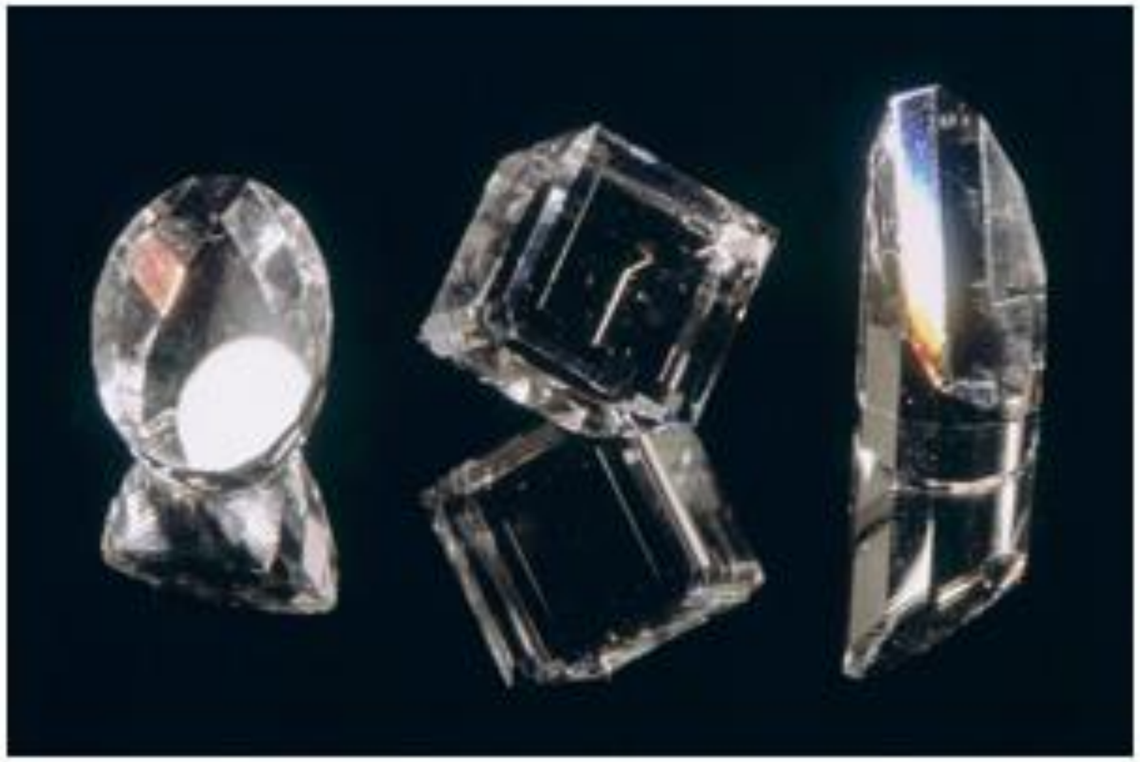
– Physical Optics (Wave Optics)

– Geometrical Optics (Ray Optics)

Geometrical models are approximations to physical models

But they are easier to use!

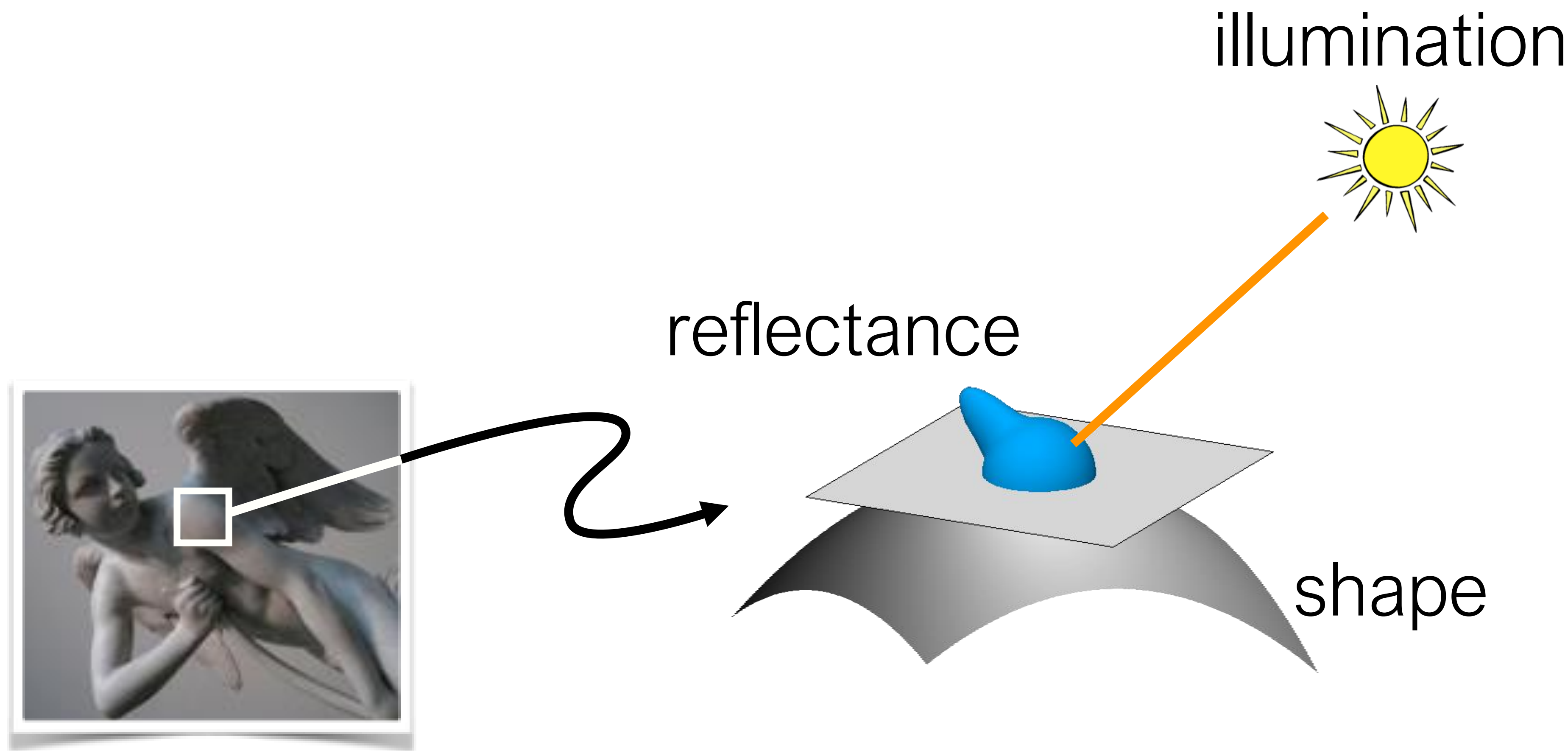
Reflectance that Require Wave Optics






Light sources

"Physics-based" computer vision
(a.k.a "inverse optics")



$\mathbf{I} \Longrightarrow$ shape, illumination, reflectance

Lighting models: Plenoptic function

- Radiance as a function of position and direction
- Radiance as a function of position, direction, and time
- Spectral radiance as a function of position, direction, time and wavelength

$$L(x, \omega, t, \lambda)$$

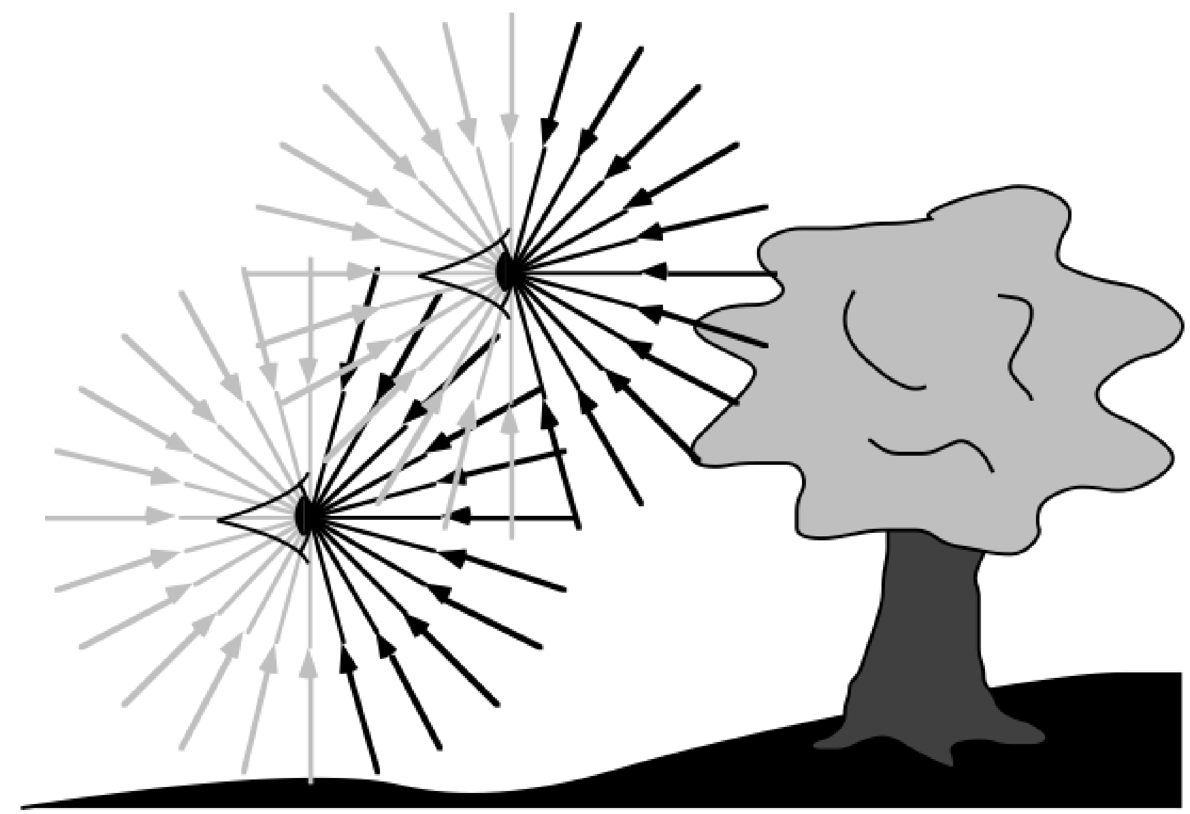


Fig.1.3
The plenoptic function describes the information available to an observer at any point in space and time. Shown here are two schematic eyes-which one should consider to have punctate pupils-gathering pencils of light rays. A real observer cannot see the light rays coming from behind, but the plenoptic function does include these rays.

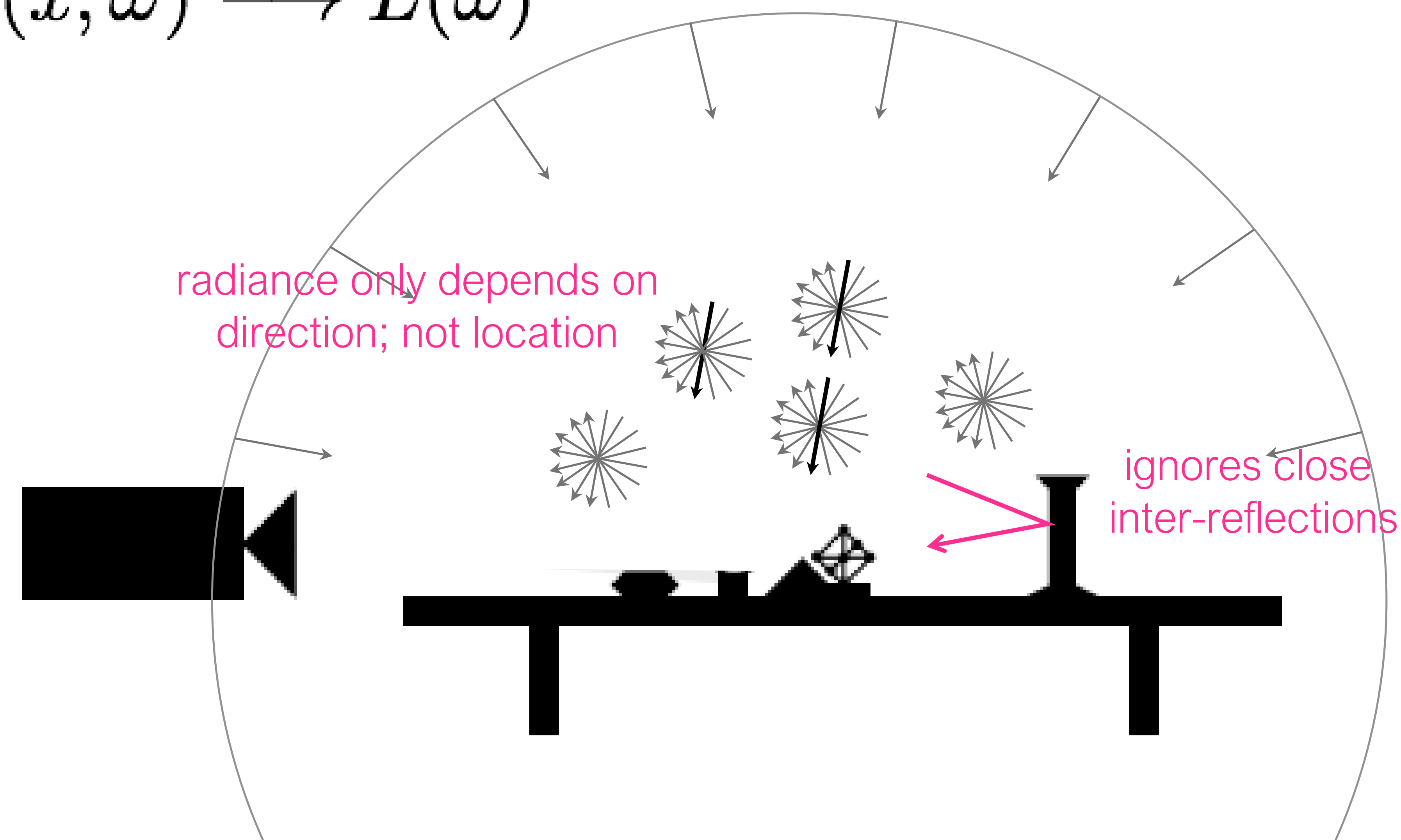
[Adelson and Bergen, 1991]

Lighting models: far-field (or directional) approximation

- Assume that, over the observed region of interest, all source of incoming flux are relatively far away

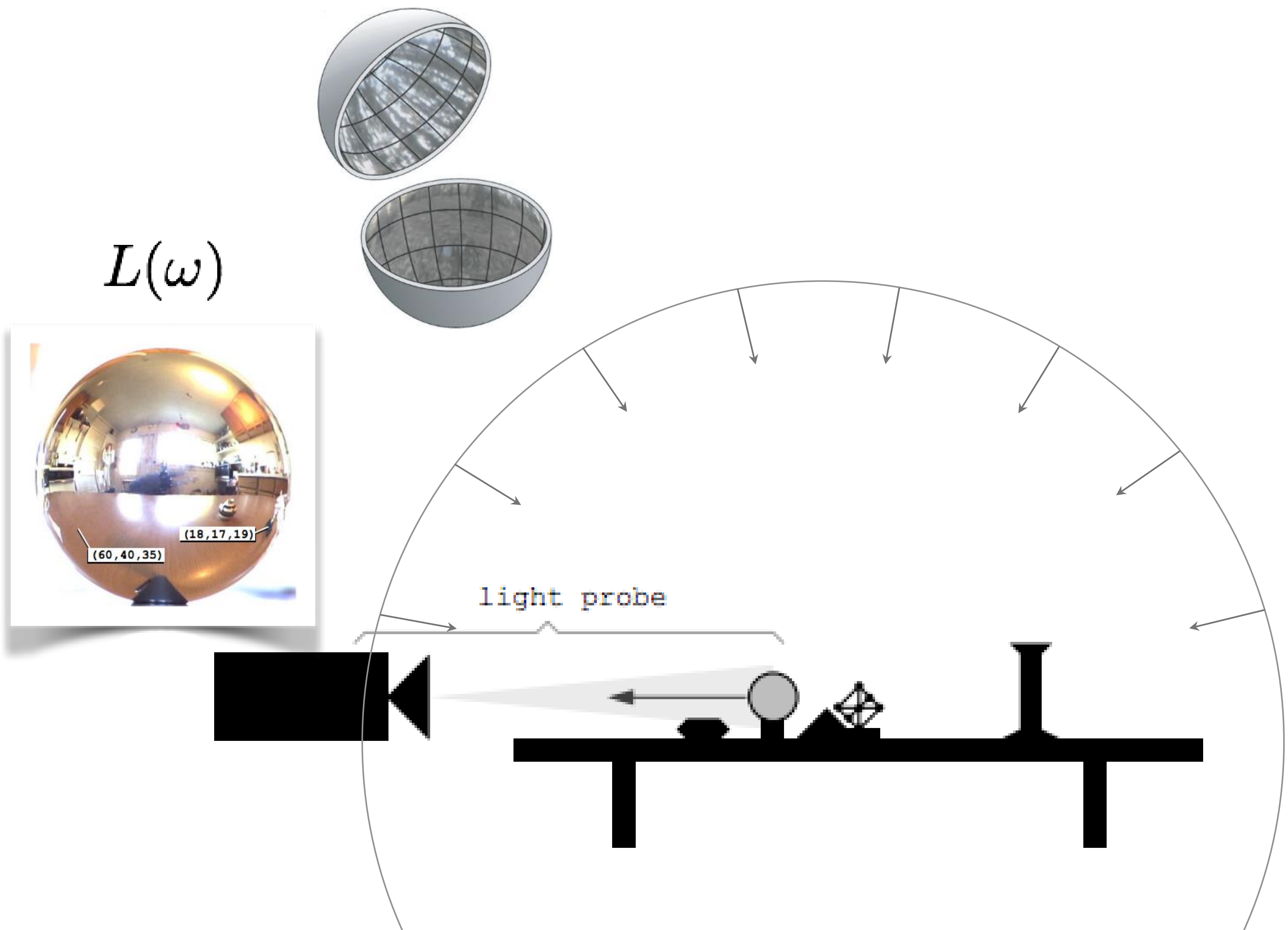
$$L(x, \omega, t, \lambda) \longrightarrow L(\omega, t, \lambda)$$

$$L(x, \omega) \longrightarrow L(\omega)$$



[Debevec, 1998]

Application: augmented reality



[Debevec, 1998]

Application: augmented reality

(a) Background photograph

(b) Camera calibration grid and light probe

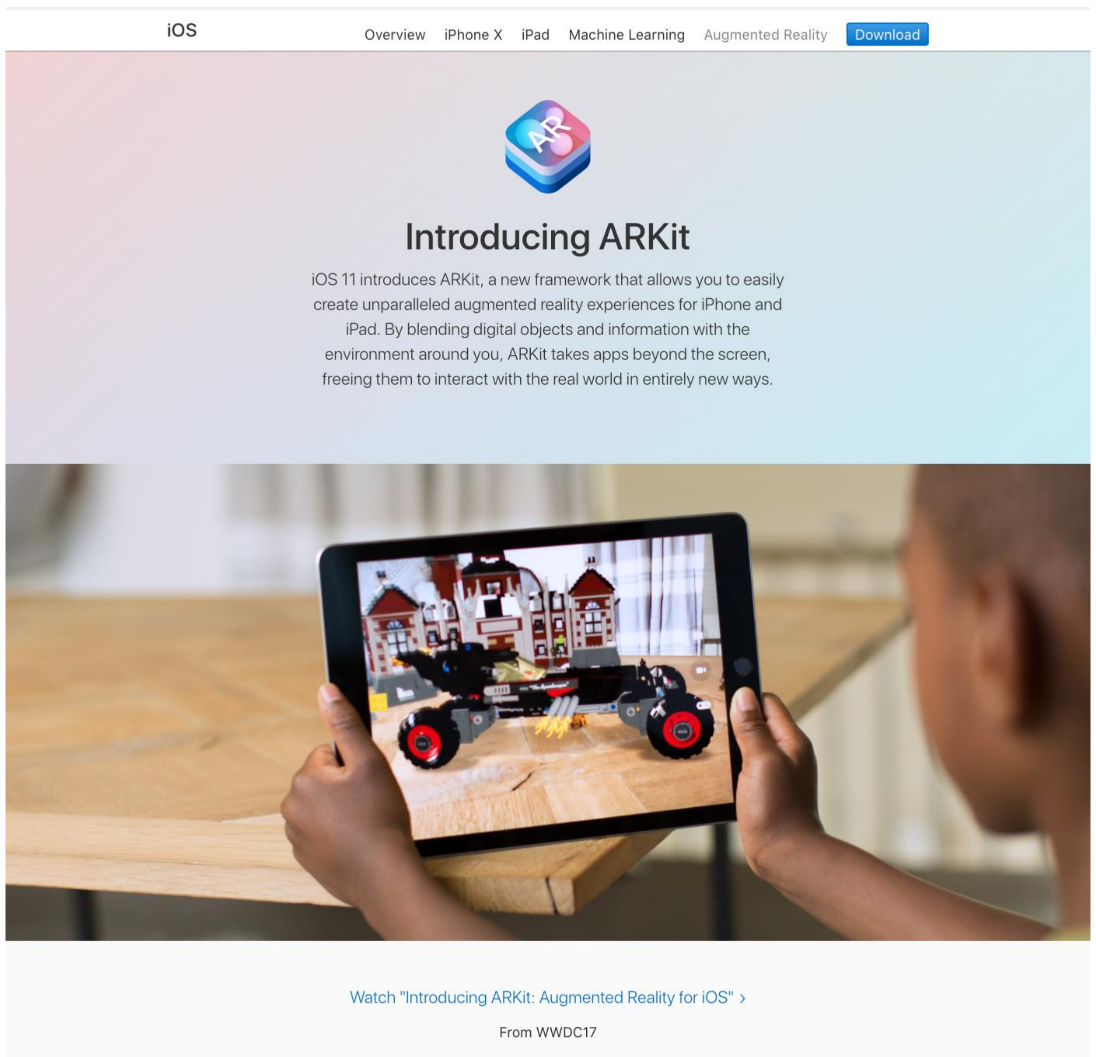
(g) Final result with differential rendering

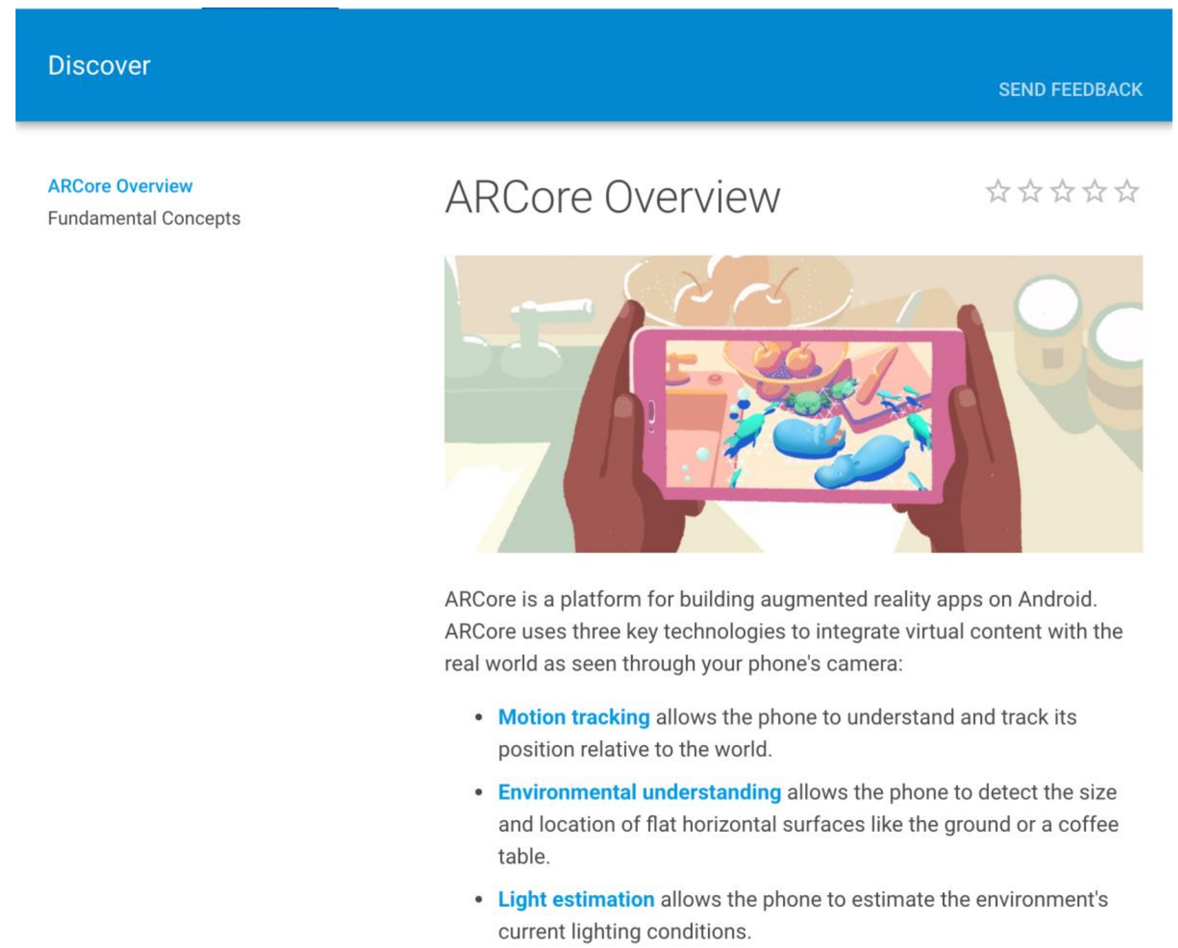
[Debevec, 1998]

Application: augmented reality

http://gl.ict.usc.edu/LightStages/

Application: augmented reality





[https://developer.apple.com/arkit/] [https://developers.google.com/ar/]

Lighting models: far-field approximation

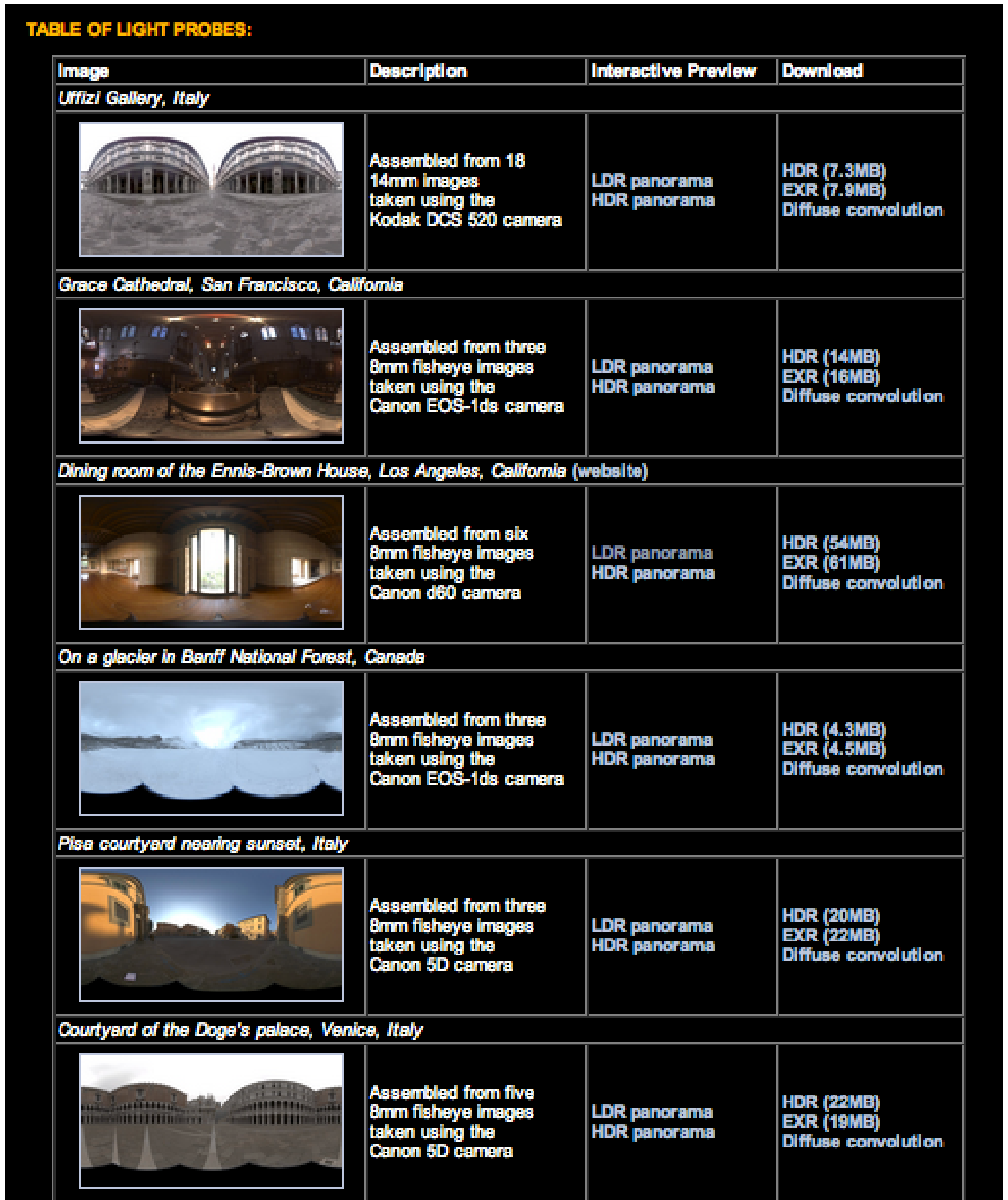


TABLE OF LIGHT PROBES:

Image	Description	Interactive Preview	Download
Uffizi Gallery, Italy			
	Assembled from 18 14mm images taken using the Kodak DCS 520 camera	LDR panorama HDR panorama	HDR (7.3MB) EXR (7.9MB) Diffuse convolution
Grace Cathedral, San Francisco, California			
	Assembled from three 8mm fisheye images taken using the Canon EOS-1ds camera	LDR panorama HDR panorama	HDR (14MB) EXR (16MB) Diffuse convolution
Dining room of the Ennis-Brown House, Los Angeles, California (website)			
	Assembled from six 8mm fisheye images taken using the Canon d60 camera	LDR panorama HDR panorama	HDR (54MB) EXR (61MB) Diffuse convolution
On a glacier in Banff National Forest, Canada			
	Assembled from three 8mm fisheye images taken using the Canon EOS-1ds camera	LDR panorama HDR panorama	HDR (4.3MB) EXR (4.5MB) Diffuse convolution
Pisa courtyard nearing sunset, Italy			
	Assembled from three 8mm fisheye images taken using the Canon 5D camera	LDR panorama HDR panorama	HDR (20MB) EXR (22MB) Diffuse convolution
Courtyard of the Doge's palace, Venice, Italy			
	Assembled from five 8mm fisheye images taken using the Canon 5D camera	LDR panorama HDR panorama	HDR (22MB) EXR (19MB) Diffuse convolution

- One can download far-field lighting environments that have been captured by others

  [http://gl.ict.usc.edu/Data/HighResProbes/]

- A number of apps and software exist to help you capture capture your own environments using a light probe

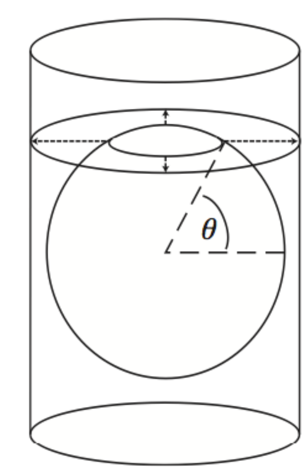


Figure 6. To produce the equal-area cylindrical projection of a spherical map, one projects each point on the surface of the sphere horizontally outward onto the cylinder, and then unwraps the cylinder to obtain a rectangular "panoramic" map.

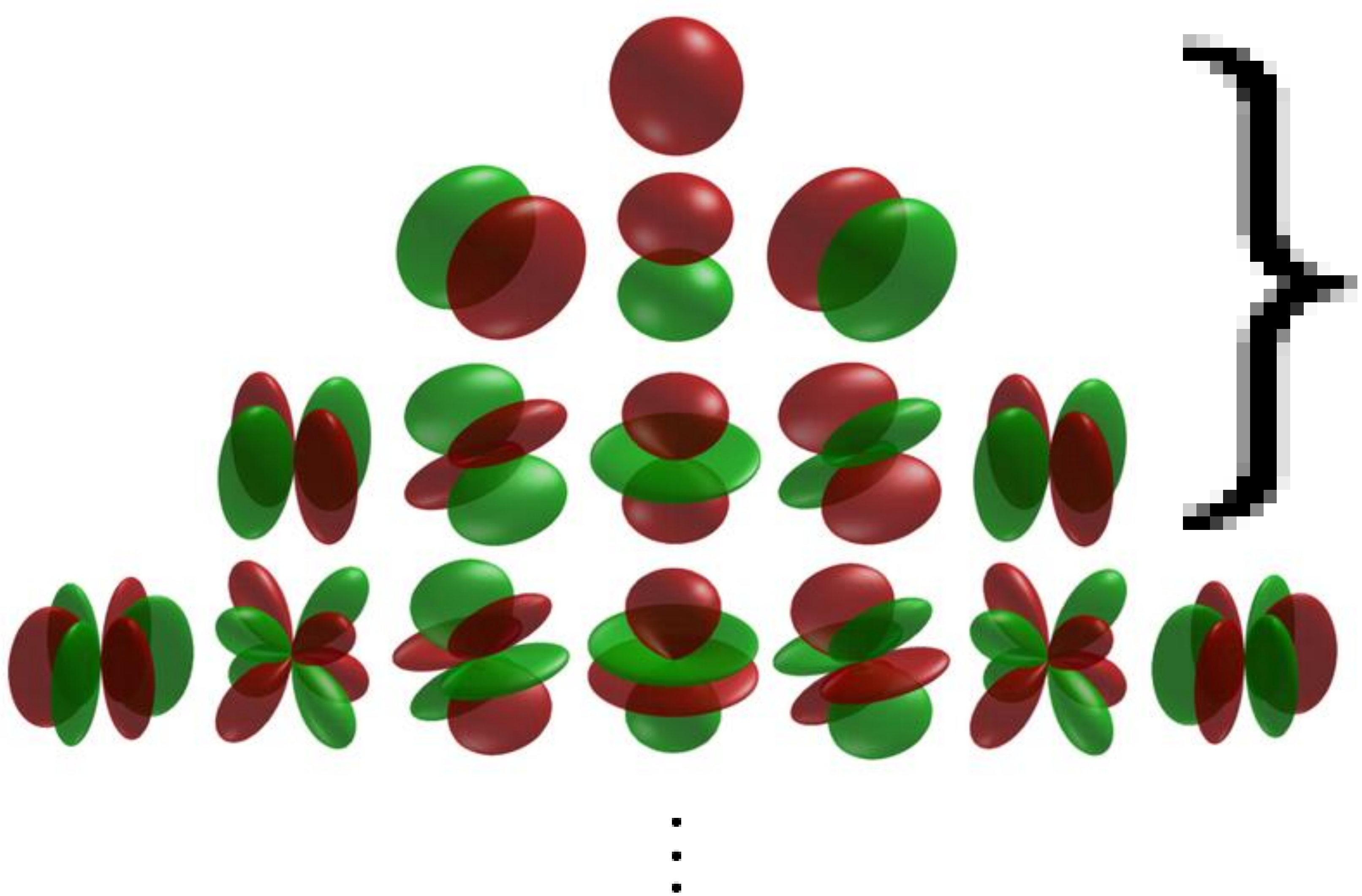
Application: inferring outdoor illumination

From a single image (left), we estimate the most likely sky appearance (middle) and insert a 3-D object (right). Illumination estimation was done entirely automatically.

[Lalonde et al., 2009]

A further simplification: Low-frequency illumination

$$L(\omega) = \sum_i a_i Y_i(\omega)$$



First nine basis functions are sufficient for re-creating Lambertian appearance

[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]

Low-frequency illumination

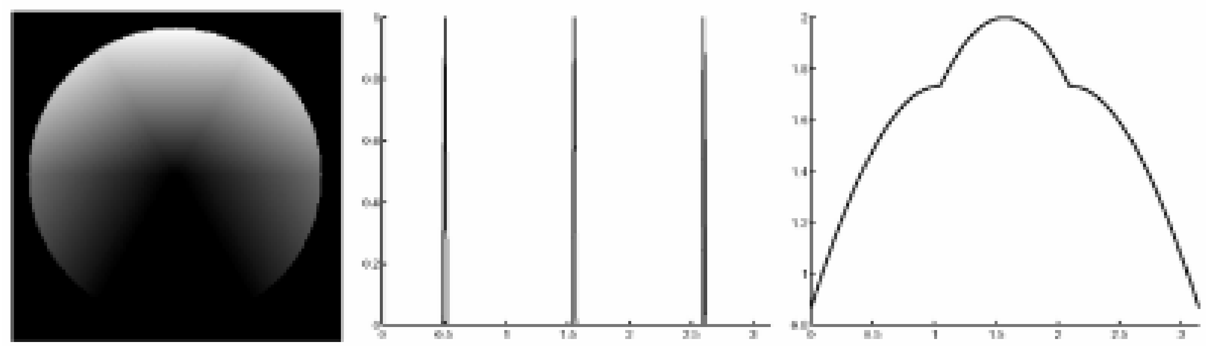


Fig. 2. On the left, a white sphere illuminated by three directional (distant point) sources of light. All the lights are parallel to the image plane, one source illuminates the sphere from above and the two others illuminate the sphere from diagonal directions. In the middle, a cross-section of the lighting function with three peaks corresponding to the three light sources. On the right, a cross-section indicating how the sphere reflects light. We will make precise the intuition that the material acts as a low-pass filtering, smoothing the light as it reflects it.

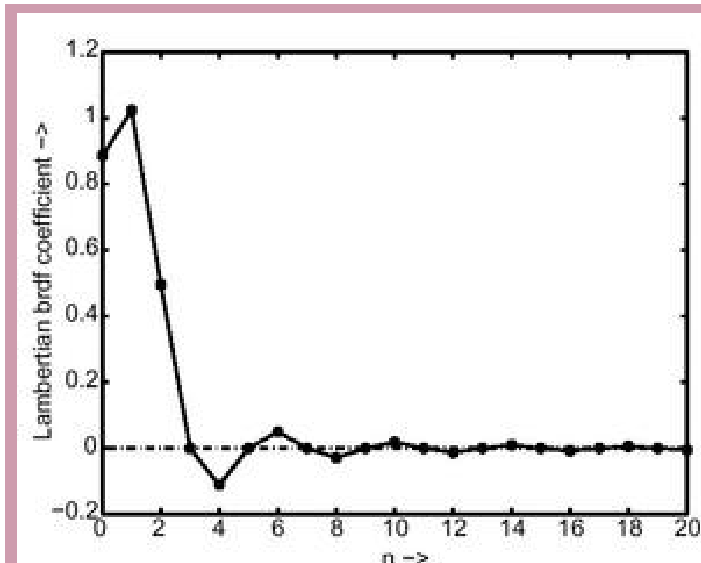


Figure 3. Plot of spherical harmonic terms in Lambertian BRDF filter.

[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]

Low-frequency illumination

$$L(\omega) = \sum_i a_i Y_i(\omega)$$

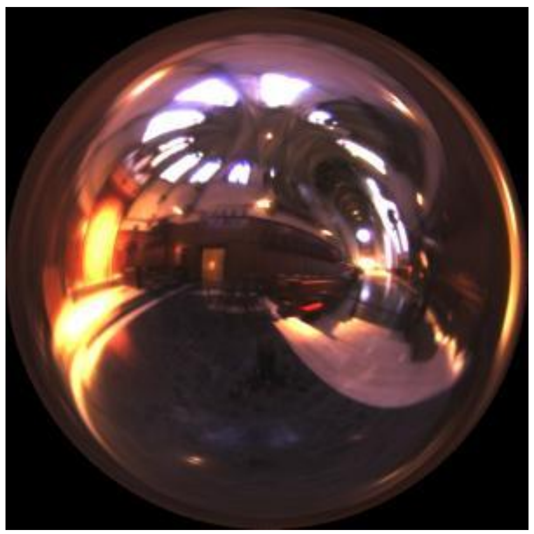
⇩ Truncate to first 9 terms

$$\vec{\ell} = (\ell_1, \ldots, \ell_9)$$

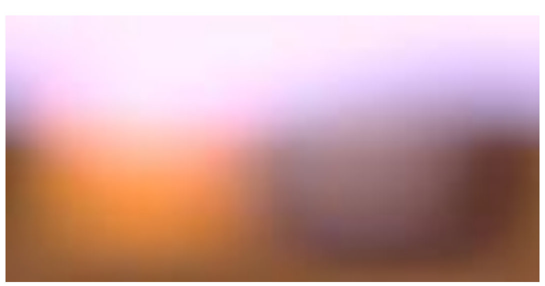
[Ramamoorthi and Hanrahan, 2001; Basri and Jacobs, 2003]

Application: Trivial rendering

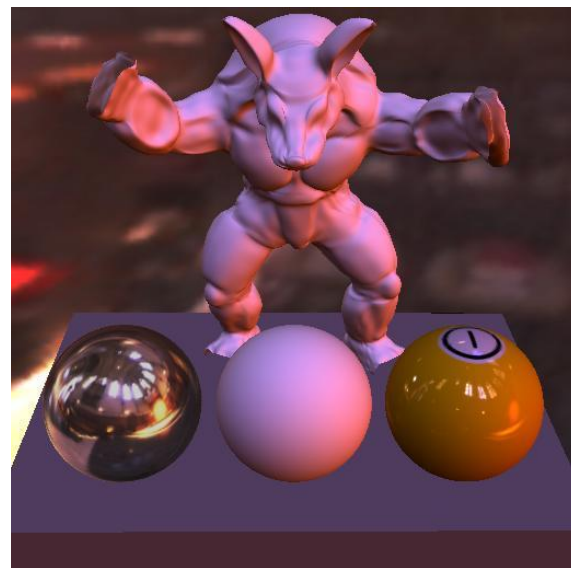
Capture light probe



Low-pass filter (truncate to first nine SHs)



Rendering a (convex) diffuse object in this environment simply requires a lookup based on the surface normal at each pixel



White-out: Snow and Overcast Skies

CAN’T perceive the shape of the snow covered terrain!

CAN perceive shape in regions lit by the street lamp!!

WHY?

Diffuse Reflection from Uniform Sky

$$L^{surface}(\theta_r,\phi_r) = \int_{-\pi}^{\pi}\int_{0}^{\pi/2} L^{src}(\theta_i,\phi_i)\, f(\theta_i,\phi_i;\theta_r,\phi_r)\cos\theta_i \sin\theta_i\, d\theta_i\, d\phi_i$$

- Assume Lambertian Surface with Albedo = 1 (no absorption)

$$f(\theta_i,\phi_i;\theta_r,\phi_r) = \frac{1}{\pi}$$

- Assume Sky radiance is constant

$$L^{src}(\theta_i,\phi_i) = L^{sky}$$

- Substituting in above Equation:

$$\boxed{L^{surface}(\theta_r,\phi_r) = L^{sky}}$$

Radiance of any patch is the same as Sky radiance !! (white-out condition)

Even simpler:
Directional lighting

- Assume that, over the observed region of interest, all source of incoming flux is from one direction

$$L(x, \omega, t, \lambda) \longrightarrow L(x, t, \lambda) \longrightarrow s(t, \lambda)\delta(\omega = \omega_o(t))$$

$$L(x, \omega) \longrightarrow L(\omega) \longrightarrow s\delta(\omega = \omega_o)$$

- Convenient representation

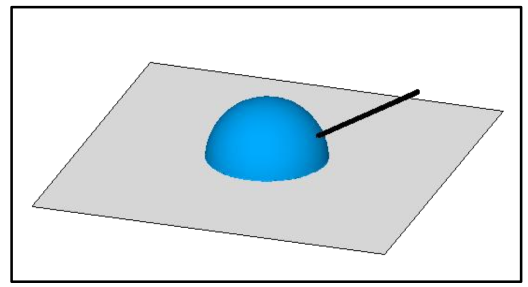
$$\vec{\boldsymbol{\ell}} = (\ell_x, \ell_y, \ell_z)$$

"light direction" $\hat{\boldsymbol{\ell}} = \dfrac{\vec{\boldsymbol{\ell}}}{||\vec{\boldsymbol{\ell}}||}$

"light strength" $||\vec{\boldsymbol{\ell}}||$

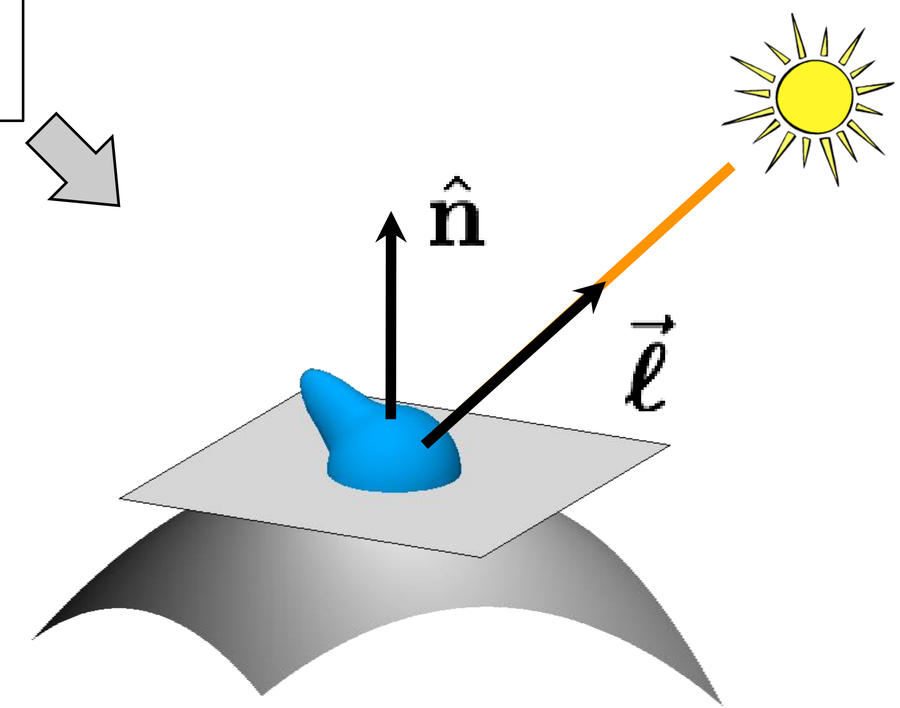
Simple shading

ASSUMPTION 1:
LAMBERTIAN



ASSUMPTION 2:
DIRECTIONAL LIGHTING

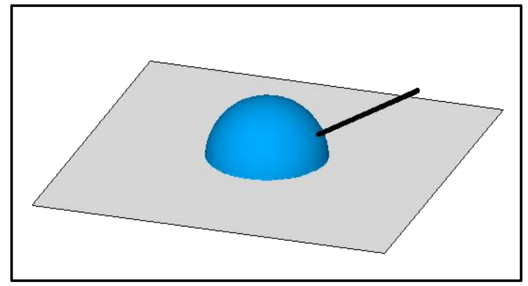
$$L^{\text{out}}(\hat{\boldsymbol{\omega}}) = \int_{\Omega_{\text{in}}} f(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) L^{\text{in}}(\hat{\boldsymbol{\omega}}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\boldsymbol{\omega}}_{\text{in}}$$



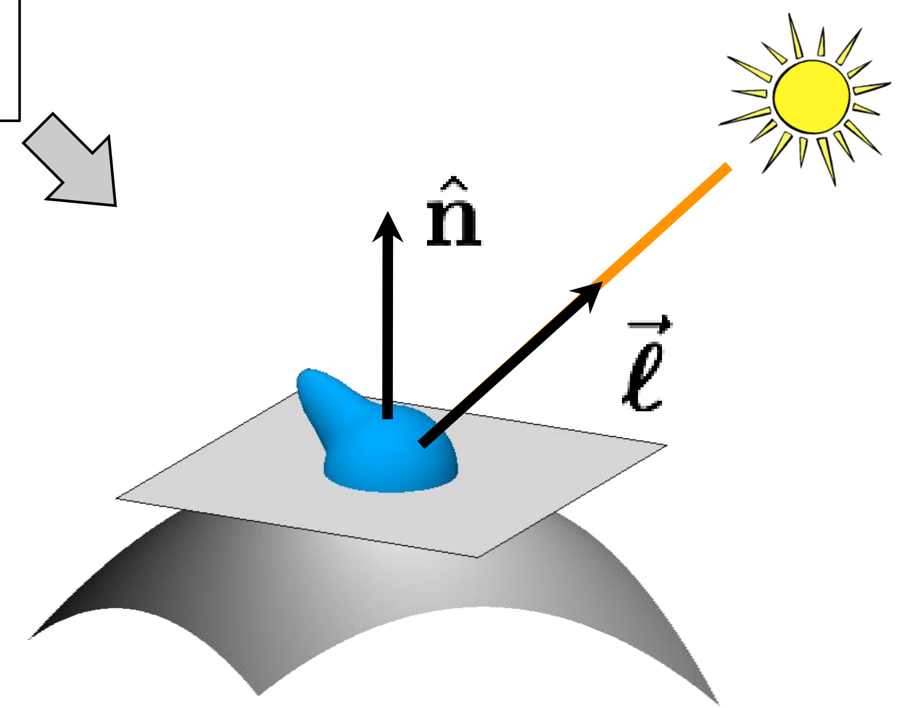
$$I = a\hat{\mathbf{n}}^\top \vec{\boldsymbol{\ell}}$$

"N-dot-l" shading

ASSUMPTION 1:
LAMBERTIAN



ASSUMPTION 2:
DIRECTIONAL LIGHTING

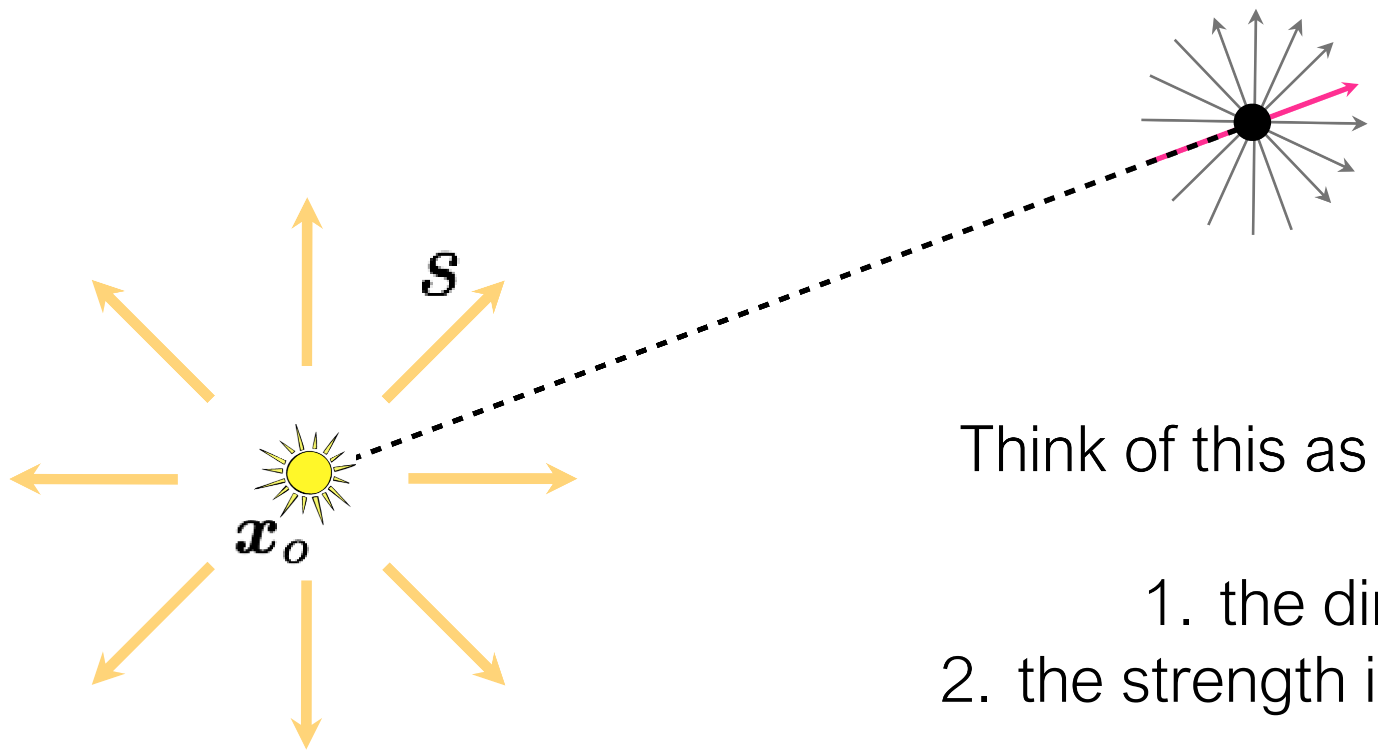


$$L^{\text{out}}(\hat{\boldsymbol{\omega}}) = \int_{\Omega_{\text{in}}} f(\hat{\boldsymbol{\omega}}_{\text{in}}, \hat{\boldsymbol{\omega}}_{\text{out}}) L^{\text{in}}(\hat{\boldsymbol{\omega}}_{\text{in}}) \cos\theta_{\text{in}} d\hat{\boldsymbol{\omega}}_{\text{in}}$$

$$I = a\hat{\mathbf{n}}^{\top}\vec{\boldsymbol{\ell}}$$

An ideal point light source

$$L(\boldsymbol{x}, \boldsymbol{\omega}) = \frac{s}{||\boldsymbol{x} - \boldsymbol{x}_o||^2} \;\; \delta\left(\boldsymbol{\omega} = \frac{\boldsymbol{x} - \boldsymbol{x}_o}{||\boldsymbol{x} - \boldsymbol{x}_o||}\right)$$



Think of this as a spatially-varying directional source where

1. the direction is away from x_o
2. the strength is proportional to 1/(distance)^2

Summary of some useful lighting models

- plenoptic function (function on 5D domain)
- far-field illumination (function on 2D domain)
- low-frequency far-field illumination (nine numbers)
- directional lighting (three numbers = direction and strength)
- point source (four numbers = location and strength)

References

Basic reading:

- Szeliski, Section 2.2.
- Gortler, Chapter 21.
  This book by Steven Gortler has a great *introduction* to radiometry, reflectance, and their use for image formation.

Additional reading:

- Arvo, "Analytic Methods for Simulated Light Transport," Yale 1995.
- Veach, "Robust Monte Carlo Methods for Light Transport Simulation," Stanford 1997.
  These two thesis are foundational for modern computer graphics. Among other things, they include a thorough derivation (starting from wave optics and measure theory) of all radiometric quantities and associated integro-differential equations. You can also look at them if you are interested in physics-based rendering.
- Dutre et al., "Advanced Global Illumination," 2006.
  A book discussing modeling and simulation of other appearance effects beyond single-bounce reflectance.
- Weyrich et al., "Principles of Appearance Acquisition and Representation," FTCGV 2009.
  A very thorough review of everything that has to do with modeling and measuring BRDFs.
- Walter et al., "Microfacet models for refraction through rough surfaces," EGSR 2007.
  This paper has a great review of physics-based models for reflectance and refraction.
- Matusik, "A data-driven reflectance model," MIT 2003.
  This thesis introduced the largest measured dataset of 4D reflectances. It also provides detailed discussion of many topics relating to modelling reflectance.
- Rusinkiewicz, "A New Change of Variables for Efficient BRDF Representation," 1998.
- Romeiro and Zickler, "Inferring reflectance under real-world illumination," Harvard TR 2010.
  These two papers discuss the isotropy and other properties of common BRDFs, and how one can take advantage of them using alternative parameterizations.
- Shafer, "Using color to separate reflection components," 1984.
  The paper introducing the dichromatic reflectance model.
- Stam, "Diffraction Shaders," SIGGRAPH 1999.
- Levin et al., "Fabricating BRDFs at high spatial resolution using wave optics," SIGGRAPH 2013.
- Cuypers et al., "Reflectance model for diffraction," TOG 2013.
  These three papers describe reflectance effects that can only be modeled using wave optics (and in particular diffraction).