Homework #1 Solutions

1. (10 points) The standard graphics pipeline is highly optimized to display a large number of triangles (or other simple geometry) as quickly as possible, incorporating simple transformations, lighting, shading, and texturing.

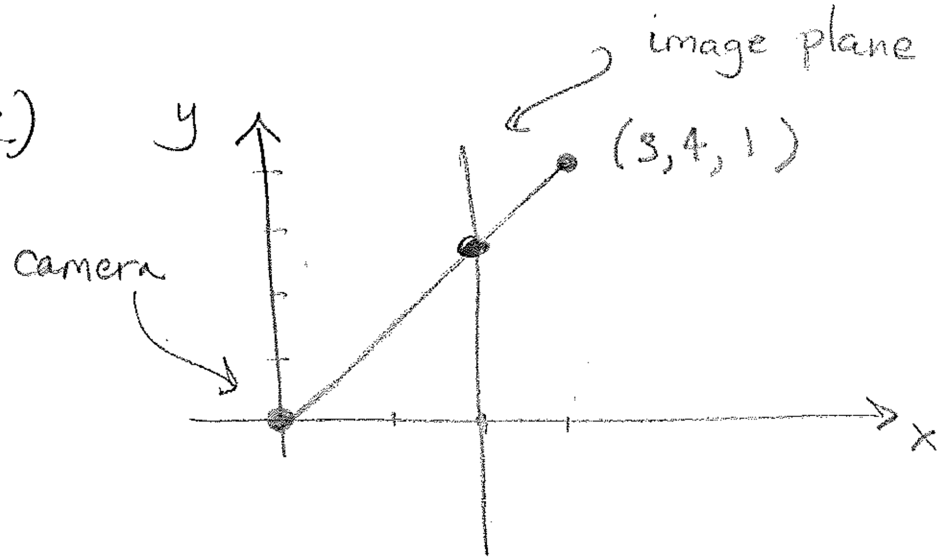
   Important concepts:
   - Triangles come into the graphics card independently, or as strips or fans. There is no notion of global geometry, such as what is connected to what or behind what other thing.
   - The graphics hardware supports a collection of basic operations

2. (10 points) See attached

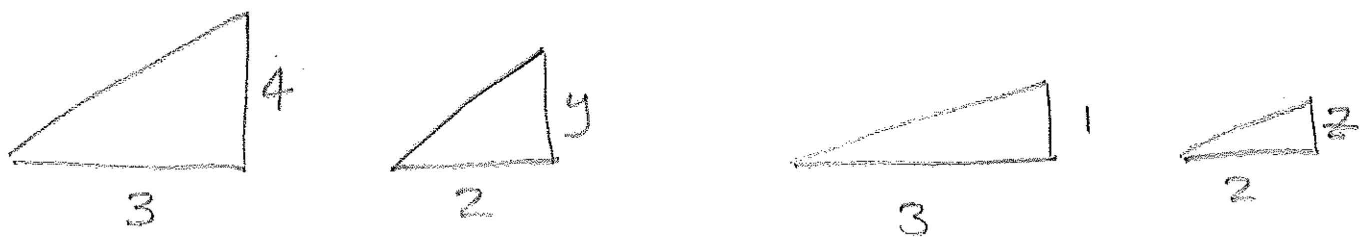
3. (10 points) We should take human perception into account because:
   a. People perceive brightness on a log scale, i.e., we must effectively continue to double brightness to achieve equal increments in apparent brightness
   b. People can perceive a much greater ratio of brightnesses than can be shown on a typical display
   c. People perceive edges (color differences) much more clearly than specific colors
   d. …

4. (16 points) See attached

2)



By similar triangles:



$4/3 = y/2$

$y = 8/3$

$\frac{1}{3} = \frac{z}{2}$

$z = 2/3$

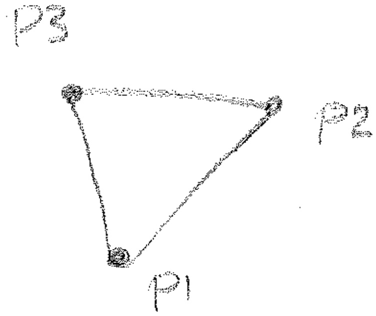
$\boxed{\text{Projected point} = [2, 8/3, 2/3]}$

4. a. (4 points)

$p1 = [1, 2, 5]$

$p2 = [2, 2, 4]$

$p3 = [3, 4, 5]$



We need a normal

$$n = (p2 - p1) \times (p3 - p1)$$

$$n = [1, 0, -1] \times [2, 2, 0] \qquad \begin{matrix} 1 & 0 & -1 \\ 2 & 2 & 0 \end{matrix}$$

$$n = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$$

$$(p - p1) \cdot n = 0$$

$$\boxed{\left(p - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}\right) \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 0}$$

b. (2 points)

$$\left(p1 - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}\right) \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = \left(\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}\right) \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 0$$

$$\left(p2 - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}\right) \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = \left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right) \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 0$$

$$\left(p3 - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}\right) \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 0$$

c.) (4 points)

Intersect $a + tb$ with $\left(p - \begin{bmatrix}1\\2\\5\end{bmatrix}\right) \cdot n = 0$

$$\left(a + tb - \begin{bmatrix}1\\2\\5\end{bmatrix}\right) \cdot n = 0$$

$$t\,(b \cdot n) + \left(a - \begin{bmatrix}1\\2\\5\end{bmatrix}\right) \cdot n = 0$$

$$\boxed{t = \frac{\left(\begin{bmatrix}1\\2\\5\end{bmatrix} - a\right) \cdot n}{(b \cdot n)}} \qquad \boxed{n = \begin{bmatrix}2\\-2\\2\end{bmatrix}}$$

d.) (2 points)

We must check $\boxed{b \cdot n = 0}$

geometric: which implies the line is parallel to the plane

mathematical: & results in a divide by zero in our computation of t

e.) (2 points)

$$t = \frac{\left(\begin{bmatrix}1\\2\\5\end{bmatrix} - \begin{bmatrix}0\\0\\0\end{bmatrix}\right) \cdot \begin{bmatrix}2\\-2\\2\end{bmatrix}}{\begin{bmatrix}1\\1\\1\end{bmatrix} \cdot \begin{bmatrix}2\\-2\\2\end{bmatrix}} = \frac{2-4+10}{2-2+2} = \underline{4}$$

$$\text{intersection} = a + tb = \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}1\\1\\1\end{bmatrix} 4 = \boxed{\begin{bmatrix}4\\4\\4\end{bmatrix}}$$

f.) (2 points)

$$\left(p - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}\right) \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 0$$

$$\left(\begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}\right) \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 0$$

$$\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = 6 - 4 - 2 = 0$$ ✓

g.) (4 points)

We need a local coordinate frame

$$\text{origin} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} \quad u = p2 - p1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad v = n \times u = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$$

$$\boxed{p1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad p2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}} \qquad p3 \Rightarrow \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

$$au + bv = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

$$\boxed{p3 = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}}$$

$$p4 \Rightarrow \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$$au + bv = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$$

$$\boxed{p4 = \begin{bmatrix} 2 \\ \frac{1}{2} \end{bmatrix}}$$

Implicits

$$P_{12} = p \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \quad P_{12}(p3) = \tfrac{1}{2} \quad P_{12}(p4) = \tfrac{1}{2} \quad \boxed{\gamma = 1}$$

$$P_{13} = p \cdot \begin{bmatrix} -\frac{1}{2} \\ 1 \end{bmatrix} = 0 \quad P_{13}(p2) = -\tfrac{1}{2} \quad P_{13}(p4) = -\tfrac{1}{2} \quad \boxed{\beta = 1}$$

$$P_{23} = (p - p2) \cdot \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} = 0 \quad P_{23}(p1) = \tfrac{1}{2} \quad P_{23}(p4) = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} = -\tfrac{1}{2}$$

$$\boxed{\alpha = -1}$$

Checking

$$-1 \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \boxed{\begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}}$$ OK