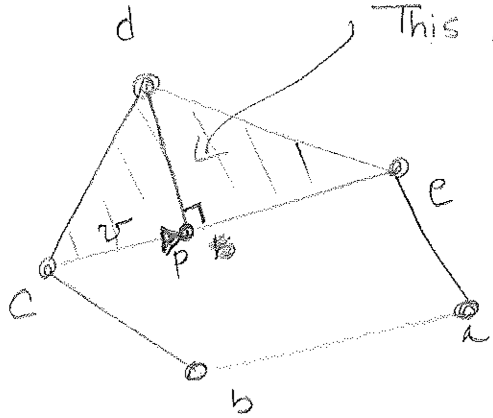
Homework #2 Solutions

1\. a.)

This area is the error we want



AREA $= \frac{1}{2} \|b\| \, \|h\|$

base　　height

Alternatively

$$\frac{\|(e-c) \times (d-c)\|}{2}$$

$$\|b\| = \|e-c\|$$

$$\|h\| = \|d-p\|$$

$$p = c + \left[(d-c) \cdot \hat{v}\right] \hat{v}$$

$$\hat{v} = (e-c) / \|e-c\|$$

$$\text{AREA} = \frac{1}{2} \|e-c\| \left\| \left(d - c - \left[\frac{(d-c)\cdot(e-c)}{\|e-c\|} \right] \frac{(e-c)}{\|e-c\|} \right) \right\|$$

b.) until SIMPLE-ENOUGH (POLY) DO

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        MIN_AREA = GET_ERROR (POLY.VERTEX[0])
        BEST_VERTEX = POLY.VERTEX[0]
        FOR  VERT = POLY.VERTEX[1] to POLY.LAST_VERTEX
             AREA = GET_ERROR (VERT)
             IF AREA < MIN_AREA
                 MIN_AREA = AREA
                 BEST_VERTEX = VERT
        REMOVE_VERTEX(POLY, BEST_VERTEX)
```

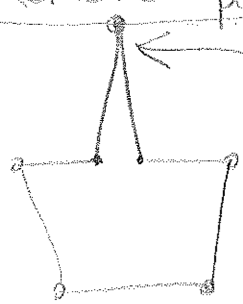
c.)

Indexed vertex list				Neighbor list		
0	x_0	y_0	z_0	0	1	5
1	x_1	y_1	z_1	1	0	2
	⋮			⋮		

Maintain neighbor list for easy computation of error due to vertex removal

d.) Displace neighboring vertices upon vertex removal to preserve total area of the shape.

e.) Remove point at min. distance from the new shape.



Small area loss but vertex is important for preserving perceived shape of object

Min distance captures this.

2 a.)

$$\begin{bmatrix} 1 & t & t^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix}$$

$$(1-2t+t^2)\,P_0 + (2t-2t^2)\,P_1 + t^2\,P_2$$

b.)

$$\begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

c.)

$$\begin{pmatrix} (1-2t+t^2) \\ (2t-2t^2) \\ t^2 \end{pmatrix}$$

d.) Interpolates p_0 at $t=0$

$$P(t) = (1-2t+t^2)p_0 + (2t-2t^2)p_1 + t^2 p_2$$

$\rightarrow P(0) = (1)p_0$

Interpolates p_2 at $t=1$

$\rightarrow P(1) = (1)p_2$

e.) i. Yes, $\boxed{\text{Let } p_2 = p_3}$

$$P_{012}(t) = (1-2t+t^2)p_0 + (2t-2t^2)p_1 + t^2 p_2$$
$$P_{345}(t) = (1-2t+t^2)p_3 + (2t-2t^2)p_4 + t^2 p_5$$

$$\boxed{\begin{aligned} P_{012}(1) &= (1)p_2 = p_2 \\ P_{345}(0) &= (1)p_3 = p_2 \end{aligned}} \Big\} \text{ C0 continuity} \checkmark$$

ii.

$$P'_{012}(t) = (-2+2t)p_0 + (2-4t)p_1 + 2tp_2$$
$$P'_{345}(t) = (-2+2t)p_2 + (2-4t)p_4 + 2tp_5$$

$$P'_{012}(1) = (-2)p_1 + 2p_2$$
$$P'_{345}(0) = (-2)p_2 + 2p_4$$

Set $-2p_1 + 2p_2 = -2p_2 + 2p_4$

$$4p_2 = 2p_1 + 2p_4 \qquad (p_1 + p_4) = 2p_2$$

YES

Set p_1 & p_4 so that $\boxed{(p_2 - p_1) = (p_4 - p_2)}$

iii.

$$P''_{012}(t) = 2p_0 - 4p_1 + 2p_2$$
$$P''_{345}(t) = 2p_2 - 4p_4 + 2p_5$$

We would need

$$2p_0 - 4p_1 + \cancel{2p_2} = \cancel{2p_2} - 4p_4 + 2p_5$$

$$\boxed{p_0 - 2p_1 = -2p_4 + p_5}$$

YES, but

if p_0 & p_5 are fixed, this would determine p_1 & p_4

if p_0 & p_5 are fixed, we must have flexibility to change both p_7 & p_8

f.) Setting p_1 fully determines p_4, p_7 etc.

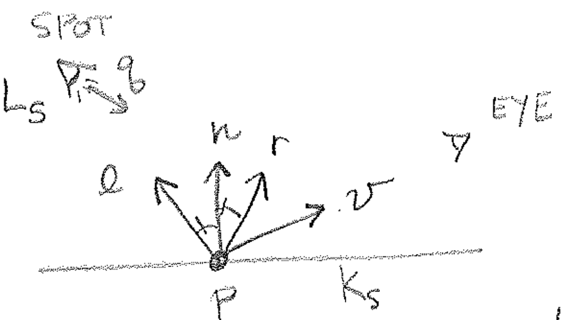
" the tangent at the first join point " the tangents at all later join points

g.) There is <u>not</u> flexibility enough to interpolate all end control points & maintain C2 continuity (see argument in e. iii)

h.) The artist can control all tangents at join points,

3.

α



a.)

K_s

γ

depends on $(r \cdot v)^\gamma$

K_s

$L_s(-q \cdot l)^\alpha$

$$\boxed{L_s K_s (-q \cdot l)^\alpha (r \cdot v)^\gamma} = \text{Specular Intensity}$$

Note: Full credit for writing complete expression for Phong Illum at p

Note: adding attenuation coefficients is fine

b.)
- L_s (specular) intensity of light source
- q direction of spotlight
- K_s specular reflection coefficient
- l direction from p toward light
- α controls spotlight falloff
- r direction of perfect reflection from l about surface normal
- v direction from p toward viewer
- γ controls shininess of material

c.) $\boxed{K_s, \gamma}$ (opt. n)

d.) $\boxed{L_s, \alpha}$ (opt. q, attenuation coef)

4. a.)

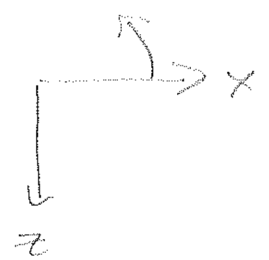
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b.)

$$\begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c.)

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$x \Rightarrow -z$

$z \Rightarrow x$

d.)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix} = \hat{u}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \hat{v}$$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \hat{\omega}$$

Let $\omega = \hat{u} \times \hat{y}$ (then normalize)

And $\hat{v} = \hat{\omega} \times \hat{u}$

$$\hat{u} \times \hat{y} = \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{matrix} 1 & -1 & 1 \\ 0 & 1 & 0 \end{matrix}$$

$$\hat{\omega} \times \hat{u} = \frac{1}{\sqrt{6}} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{matrix} -1 & 0 & 1 \\ 1 & -1 & 1 \end{matrix}$$

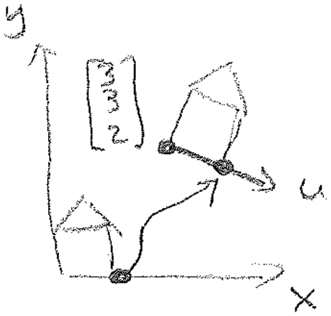
Checking: $\hat{u}\cdot\hat{v} = 0$ $\quad\hat{u}\cdot\hat{w} = 0$ $\quad\hat{u}\cdot\hat{w} = 0$

$$\hat{u}\times\hat{v} = \frac{1}{\sqrt{18}}\begin{bmatrix}-3\\0\\3\end{bmatrix} = \hat{w}$$ ok

1 -1 1
1 2 1

Result matrix
$$\begin{bmatrix}\hat{u}_x & \hat{v}_x & \hat{w}_x & 3\\ \hat{u}_y & \hat{v}_y & \hat{w}_y & 3\\ \hat{u}_z & \hat{v}_z & \hat{w}_z & 2\\ 0 & 0 & 0 & 1\end{bmatrix}$$

e.)



$$\begin{bmatrix}3\\3\\2\\1\end{bmatrix} + \hat{u}$$ by reference to the picture

f.) $$\begin{bmatrix}3\\3\\2\\1\end{bmatrix} + \hat{v}$$ by reference to the picture

ok

g.) $$\begin{bmatrix}| & | & | & 3\\ u & v & w & 3\\ | & | & | & 2\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \begin{bmatrix}u_x+3\\u_y+3\\u_z+2\\1\end{bmatrix}$$ ok

$$\begin{bmatrix}| & | & | & 3\\ u & v & w & 3\\ | & | & | & 2\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\1\\0\\1\end{bmatrix} = \begin{bmatrix}v_x+3\\v_y+3\\v_z+2\\1\end{bmatrix}$$