HW3

1.
(a) A cluttered scene, where it is difficult to see between objects for any given camera point.

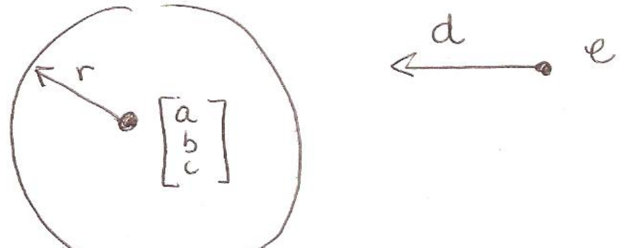
(b.) A scene with fog or any other scattering medium, where light dissapates with distance.

(c) A scene with mirror reflections, where small changes in camera position may change the image dramatically

(d.) A scene that changes over time — leaves blowing in the wind, ocean waves, a crowded street, etc...

... and so on ...

2.



(a) $(x-a)^2 + (y-b)^2 + (z-c)^2 - r^2 = 0$

intersect with $e + dt$

$$(e_x + d_x t - a)^2 + (e_y + d_y t - b)^2 + (e_z + d_z t - c)^2 - r^2 = 0$$

Let $A = d_x^2 + d_y^2 + d_z^2$

$B = \left[(e_x - a)d_x + (e_y - b)d_y + (e_z - c)d_z\right]2$

$C = (e_x - a)^2 + (e_y - b)^2 + (e_z - c)^2 - r^2$

Then $At^2 + Bt + C = 0$

Solve for t

$$t = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

If $4AC > B^2$ there is no intersection

If $4AC = B^2$ and $-B/2A \geq 0$

then $e + d\left(-B/2A\right)$ is the first intersection

If $4AC < B^2$ then solve for

$$t_1 = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \qquad t_2 = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

If $t_1 < t_2$

If $t_1 > 0$, then t_1 is the first intersection

else if $t_2 > 0$, then t_2 is the first intersection

else no intersection

else

If $t_2 > 0$, t_2 is the first intersection

else if $t_1 > 0$, t_1 is the first intersection

else no intersection

Let t_i be the parameter value for the first intersection.

The intersection point is

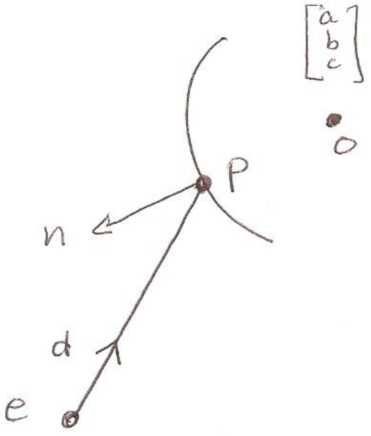
$e + d\,t_i$

Key points $\Rightarrow$

1. check interior of square root for valid values of t

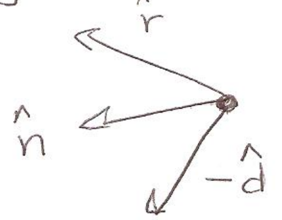
2. Make sure to pick the smallest positive value of t as the first intersection

(b.)



Let $p = e + d\,t_i$
where t_i is the point of first intersection

Let $o = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ (sphere center)



Normal $\hat{n} = \dfrac{p-o}{\|p-o\|} = \dfrac{p-o}{r}$

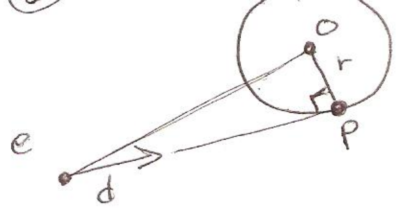
$\hat{d} = d/\|d\|$

$-\hat{d} + \hat{r} = 2(\hat{n} \cdot -\hat{d})\,\hat{n}$

$\boxed{\hat{r} = \hat{d} - 2(\hat{n}\cdot\hat{d})\,\hat{n}}$

$\boxed{\text{Ray} = e + d t_i + \hat{r}\,t}$

(c.)



ii. $t = -B/2A$

From 2a.

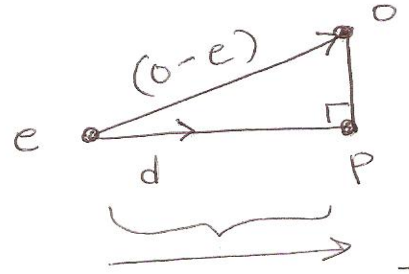
$B = [(e-o)\cdot d]\,2$

$A = \|d\|^2$

$\boxed{t = \dfrac{-(e-o)\cdot d}{\|d\|^2}}$

ok

iii.



The distance from e to p is

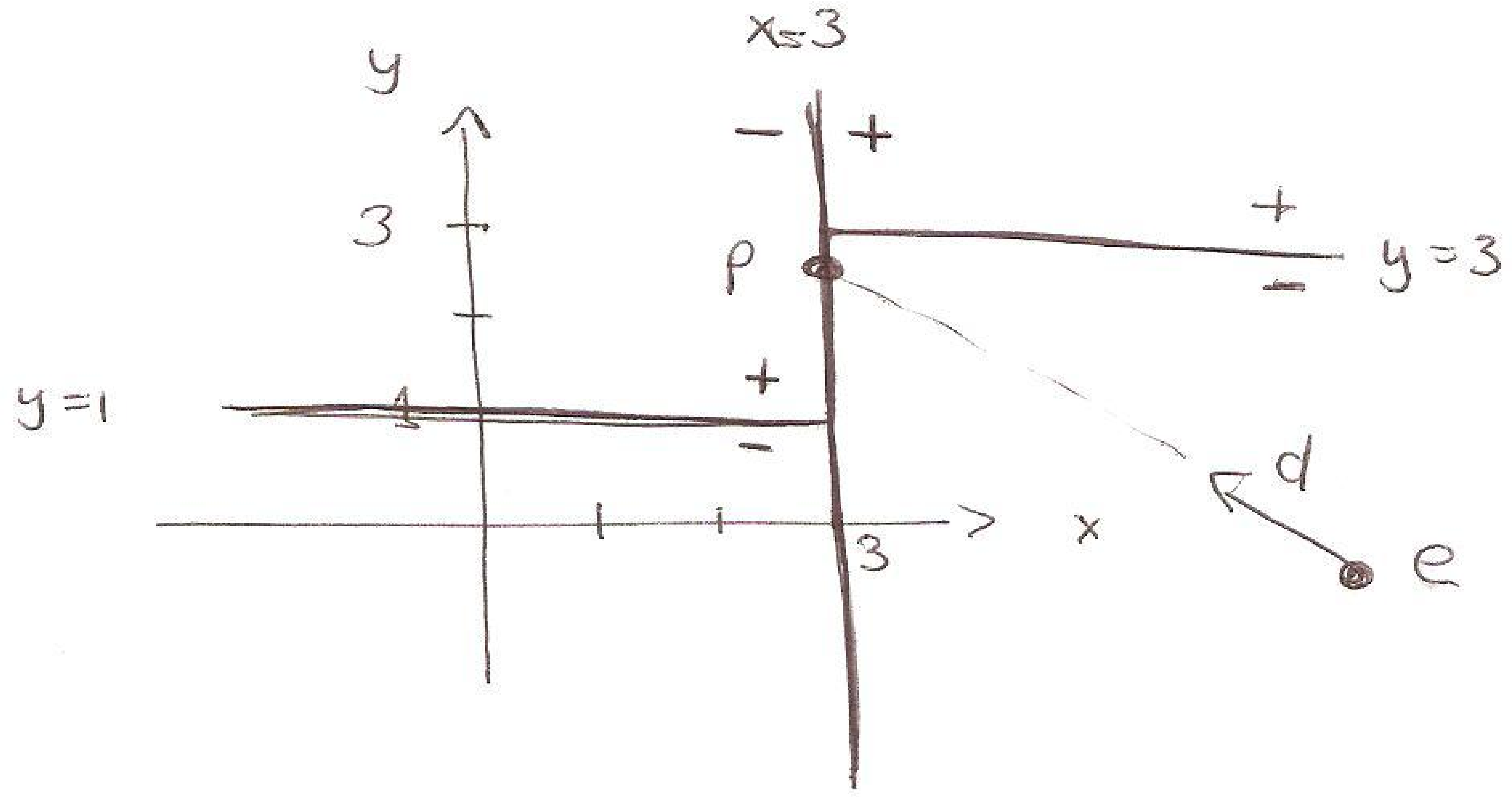
$\dfrac{(o-e)\cdot d}{\|d\|} = \dfrac{-(e-o)\cdot d}{\|d\|}$

$(p-e) = \dfrac{(o-e)\cdot d}{\|d\|}\left(\dfrac{d}{\|d\|}\right)$

This distance is also $\|d\|\,t$

$\|d\|\,t = \dfrac{-(e-o)\cdot d}{\|d\|} \Rightarrow \boxed{t = \dfrac{-(e-o)\cdot d}{\|d\|^2}}$

(3) (a)



(6.) Compute the intersection parameter t

$e_x + d_x t = 3$ $\qquad$ $\boxed{t = \dfrac{3 - e_x}{d_x}}$

If $e_x \geq 3$ ~~trace $X \geq 3$ branch first~~

If $t > 0$

trace $X \geq 3$ branch with $t_+ = [0, t]$

trace $X < 3$ branch with $t_- = [t, \infty]$

else

trace $X \geq 3$ branch with $t_+ = [0, \infty]$

else

If $t > 0$

trace $X < 3$ branch with $t_- = [0, t]$

trace $X \geq 3$ branch witt $t_+ = [t, \infty]$

else

trace $X < 3$ branch with $t_- = [0, \infty]$

Left branch

(c) Compute intersection ~~time~~ parameter

$e_y + d_y t = 1$ $\quad \boxed{t = \dfrac{1 - e_y}{d_y}}$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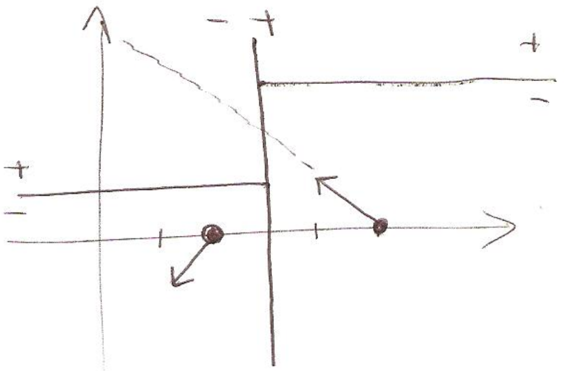
If (dy > 0)    // y<1 then y≥1
    If (t < tmin)  trace y≥1   t+ = [tmin, tmax]
    else if (t > tmax)  trace y<1   t- = [tmin, tmax]
    else
        trace y<1   t- = [tmin, t]
        trace y≥1   t+ = [t, tmax]
else    // dy<0 => y≥1 then y<1
    If (t< tmin)  trace y<1   t- = [tmin, tmax]
    else if (t > tmax)  trace y≥1   t+ = [tmin, tmax]
    else
        trace y≥1   t+ = [tmin, t]
        trace y<1   t- = [t, tmax]
```

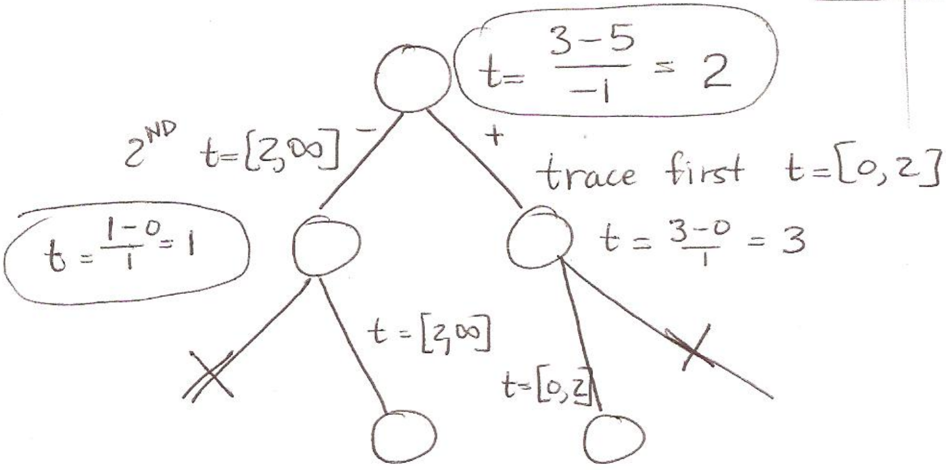
The right branch is handled similarly

Key point

To get this question right, you **must** keep track of param t values as you traverse the tree

(d) $e = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ $d = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$





Compute $t = 2$ as intersection w/ 1^{st} splitting plane

Descend right branch $\boxed{x \geq 3}$ w/ $\underline{t = [0, 2]}$

Compute $t = 3$ as intersection w/ $y = 3$ splitting plane

Descend $\boxed{y < 3}$ branch w/ $\underline{t = [0, 2]}$

Descend $\boxed{x < 3}$ branch w/ $\underline{t = [2, \infty]}$

Compute $t = 1$ as intersection w/ $y = 1$ splitting plane

Descend $\boxed{y \geq 1}$ branch w/ $\underline{t = [2, \infty]}$

(e) $e = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $d = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

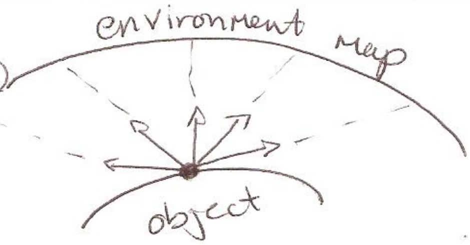
Compute $t = \frac{3-2}{-1} = -1$ as intersection w/ $x = 3$ plane

Descend $\boxed{x < 3}$ branch w/ $\underline{t = [0, \infty]}$

Compute $t = \frac{1-0}{-1} = -1$ as intersection w/ $y = 1$ plane

Descend $\boxed{y < 1}$ branch w/ $\underline{t = [0, \infty]}$

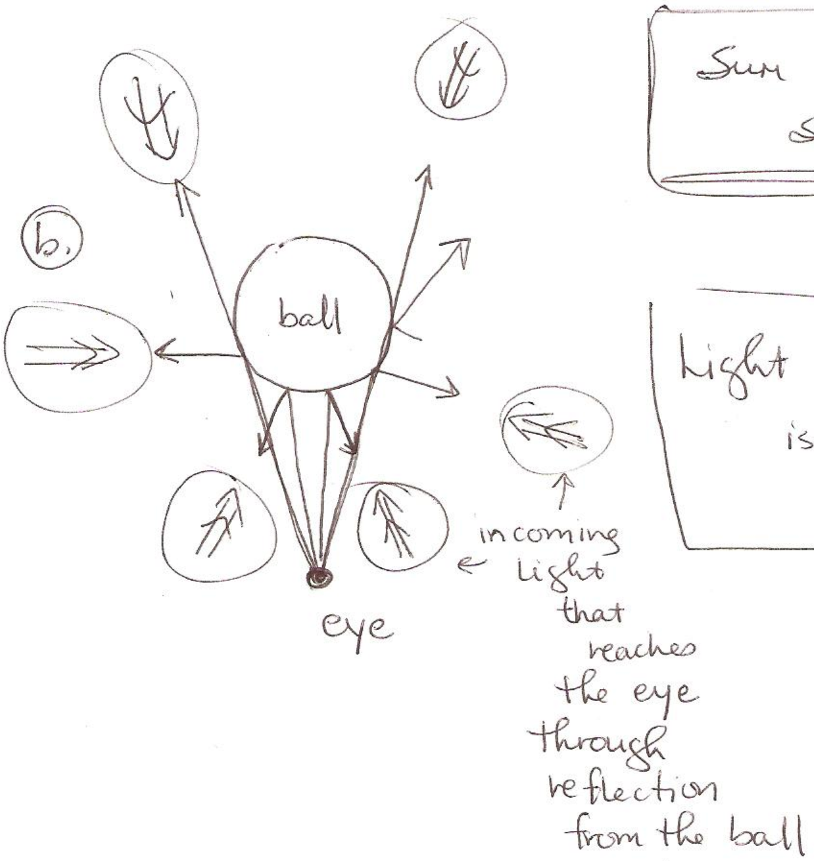
4 (a)



Sample the environment map at many points as shown in the diagram to collect a good representation of incoming light from all directions.

Sum up $K_d L_d (n \cdot l)$ from each sample

(b.)



Light from almost all directions is captured, as shown in the figure

Only light that would pass through the ball to reach the eye & light that would be occluded by the camera is missing

5. (a.) No – only diffuse effects

(b.) Yes – effect of light transfer from one diffuse surface to another

(c.) yes – diffuse reflection varies w/ surface normal

(d.) no – this is a specular effect

(e.) yes – most ambient glow is due to diffuse surface interreflections

6. (a.) yes – through direct lighting calculation

(b.) yes – through diffuse reflections in photon tracing

(c.) yes – as in (b)

(d.) no – rays are only cast from eye, not traced further & photon tracing does not capture this specular effect well

(e) yes – as in (b)