

# Chapter 7 Control

## Part 4

### 7.4 Intelligent Control



# Outline

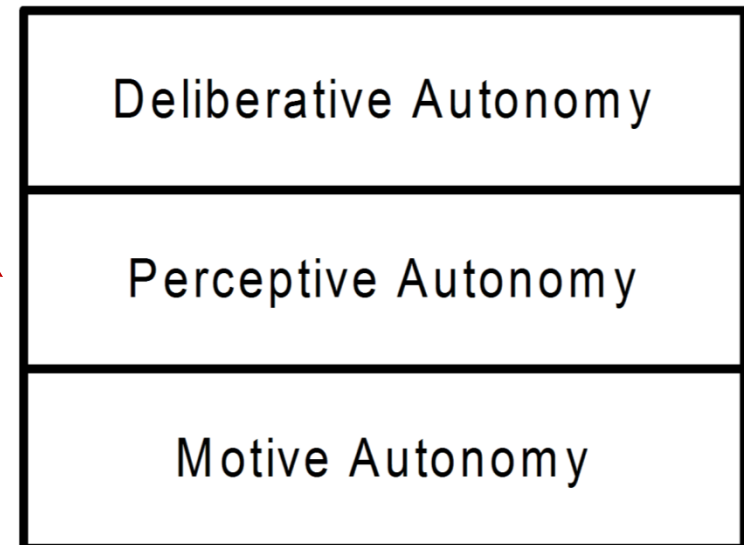
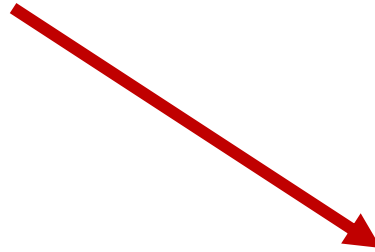
- 7.4 Intelligent Control
  - 7.4.1 Introduction
  - 7.4.2 Evaluation
  - 7.4.3 Representation
  - 7.4.4 Search
  - Summary

# Outline

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# Hierarchy

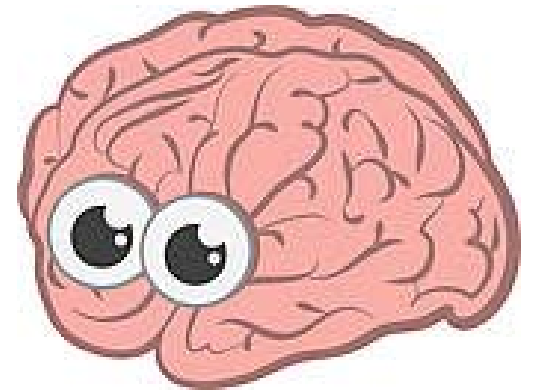
- We are here now ...
- Responsible for responding to the immediate environment.
- Requires feedback of the state of the environment (e.g. perception).
- May only need relative position estimates.



# 7.4.1.1 Intelligent Predictive Control

(Perceptive)

- By assumption: Environment is partially unknown and must be measured.
- Don't know beforehand where the obstacles are - or you would have planned around them already.
- “Intelligent” means understanding your surroundings. Hence:
  - IC must be perceptive.
- Perception is discussed later.
  - Here, we will use an environmental model that was produced by perception.



# 7.4.1.1 Intelligent Predictive Control

## (Predictive)

- Latencies and robot dynamics mean it takes time for actions to take effect.
- Robot may also be under-actuated.
- Elements in the scene may be dynamic.
- Hence IC must be predictive.



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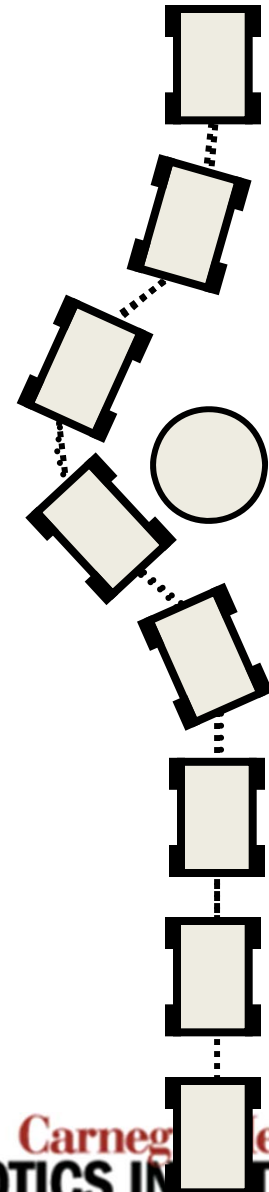
# 7.4.1.1 Intelligent Predictive Control (Reactive)

- However, perception must be done continuously because effective sensor range is limited by:
  - Missing parts (occlusion, limited sensor range)
  - Uncertainty
- Also, prediction of dynamic obstacles is only valid for short periods of time.
- Must:
  - perceive continuously
  - react to what you can see.
  - do it all over again high frequency.



# 7.4.1.1.1 Generic Intelligent Control Loop

- 1: Consider “all” options for proceeding through space.
- Check each for problems.
- Eliminate those options which are definitely (or probably) problems.
- If any options remain, pick the best from the perspective of mission execution. Goto 1:
- If none remain, do something which reduces your losses
- If you survive that, ask for help, or execute other recovery mechanisms.





# 7.4.1.1.1 Generic Intelligent Control Loop

(Elements of Effective IPC)

- A model of your capacity to move
  - Motion prediction
- A model of the state of the environment
  - Representation
- A capacity to evaluate alternatives for
  - Trajectory evaluation
- A capacity to search through the space of possible motions
  - Optimal control

## 7.4.1.2 Formulation as Optimal Control

(Objectives and Constraints)

- Motions can be ranked based on cost/utility and satisfaction of hard constraints:
- Simple case:
  - Score each motion (utility)
  - Do not hit obstacles (constraint)
- However, **obstacles** can also be encoded as **cost** of traversal and there are **motions** which do not satisfy feasibility **constraints**.

# 7.4.1.2.1 Optimal Control Formulation

(Objectives and Constraints)

- Objectives to Minimize
  - Risk level
  - Path following error
  - Path length to goal
  - Integral speed error.
- Constraints
  - Dynamics (“feasible”)  $\dot{\underline{x}} = f(\underline{x}, \underline{u}, t)$  ;  $\underline{u} \in U$
  - Don’t hit obstacles (“admissible”)  $\underline{x}(t) \notin O$
  - Don’t tip over (“stability”)

# 7.4.1.2.1 Optimal Control Formulation

(Equations)

- Over Time (Trajectory)

$$J[\underline{x}, t_f] = \phi(\underline{x}(t_f)) + \int_{t_0}^{t_f} L(\underline{x}, \underline{\dot{x}}, t) dt$$

$$\underline{\dot{x}} = f(\underline{x}, \underline{u}, t) \quad ; \quad \underline{u} \in U$$

$$\underline{x}(t_0) \in S \quad \underline{x}(t_f) \in G$$

- Over Space (Path)

$$J[\underline{x}, s_f] = \phi(\underline{x}(s_f)) + \int_{s_0}^{s_f} L(\underline{x}, \underline{u}, s) ds$$

$$\underline{x}(s_0) \in S \quad \underline{x}(s_f) \in G$$

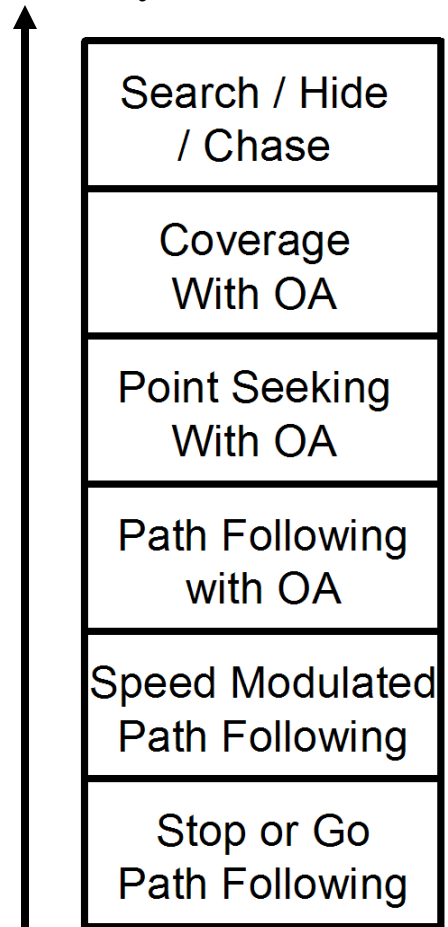
$$\underline{\dot{x}} = f(\underline{x}, \underline{u}, s) \quad ; \quad \underline{u} \in U$$

Line  
Integral

## 7.4.1.2.2 Encoding the Mission in the Objective

- The objective may impart differing levels of responsibility to intelligent control.
- 1) Fixed, detailed path - keep going or stop. AGVs do this in factories.
- 2) Fixed path with speed modulation. Following behavior is a special case of this.
- 3) Follow default path with deviation to avoid obstacles permitted.
- 4) Sparse waypoints to hit with complete authority to plan the paths between them.
- 5) Cover an entire area (e.g. mow the grass).
- 6) Search for something, run from something, or pursue something.

More  
Responsibility



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## 7.4.2 Evaluation

- Methods to **compute feasible trajectories** were covered earlier in motion prediction (dynamics).
- This section is about how to **evaluate trajectories**.

## 7.4.2.1 Cost of a Configuration

- In this view, a cost can be assigned to every configuration.
  - $L[x,u,t] \rightarrow L[x]$
- This is **different from cost of a point in the world**
  - Vehicle occupies volume
  - Volume depends on orientation
- Can be computed efficiently in some representations and some scenarios.



# 7.4.2.1 Cost of a Configuration

(Volume Cost)

- Integrate cost over the volume occupied.

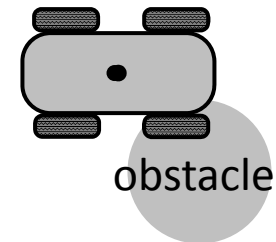
$$L[\underline{x}(s)] = \int_V c(x, y, z) dV$$

- Or intersect vehicle and obstacles.

$$L[\underline{x}(s)] = \bigcap \{o(x, y, z) \cap v(x, y, z)\}$$

- This can be pre-computed in static, known worlds.

- Can also be expressed in configuration space as  $L(x)$



An area or a volume property

$$J[\underline{x}(t)] = \Phi[\underline{x}(t_0), \underline{x}(t_f)] + \int_{t_0}^{t_f} L[\underline{x}(t)] dt$$

# 7.4.2.1 Cost of a Configuration

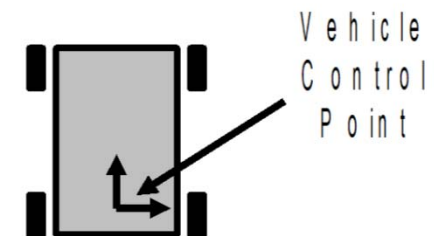
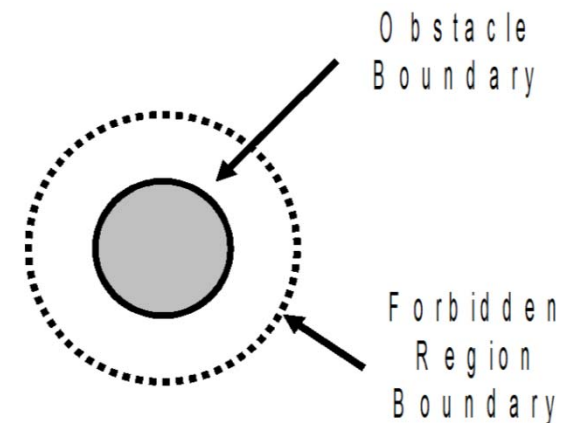
(Precision Intersection Calculations)

- Obstacles may place different constraints on **parts of** the vehicle.
  - Point hazards:
    - no part of the vehicle can drive over a 20 foot tall tree.
  - Wheel hazards:
    - wheels cannot drive over a hole - but the undercarriage can.
- Other conditions may depend on orientation.
  - Pose hazards:
    - a slope may be a problem in one orientation but not in another.

# 7.4.2.1 Cost of a Configuration

## (Volume of a Vehicle)

- A real vehicle is not a point.
  - A bad **point** in task space often corresponds to a bad **region** in state space (or configuration space).
  - Must account for the **width and length** of the vehicle.
- A real vehicle is **not a pancake**.
  - Overhanging obstacles occur in factories, warehouses, forests, buildings (tables).
  - Must account for the **height** of the space underneath them.



## 7.4.2.2 Cost of a State

(State Dependence)

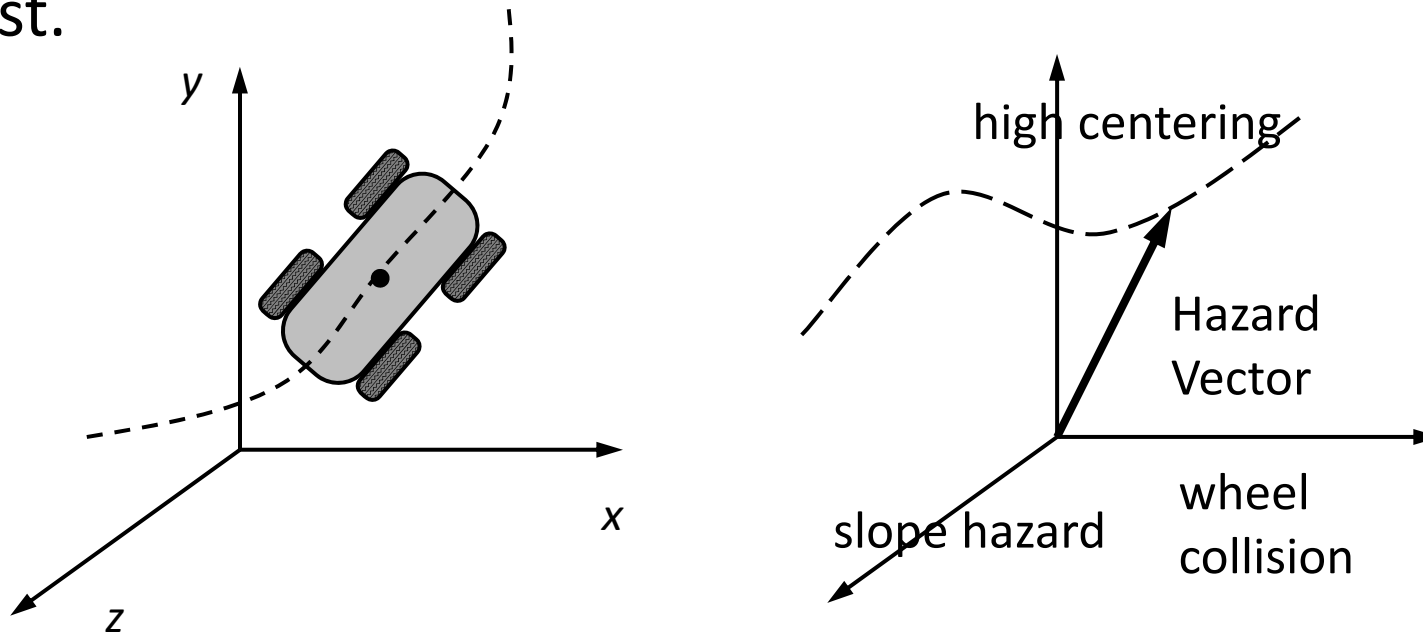
- There are many situations to be avoided
  - They depend on **more than** pose (e.g.  $V$ )
  - They are more properly expressed as a field over state space.
- Examples
  - Rollover relates to lateral acceleration, gravity.
  - Obstacle impact force depends on speed.

# 7.4.2.2.1 Types of State Dependent Hazards

- Hazards include:
  - loss of control:
    - Yaw instability (skidding); Steep slopes (braking)
  - loss of contact:
    - Rollover, high centering
  - loss of traction:
    - Ice, mud, entrapment hazards
  - collisions: application of damaging forces.
    - Will depend on speed (higher  $V$  often worse)
  - risks: uncertain situations to be avoided

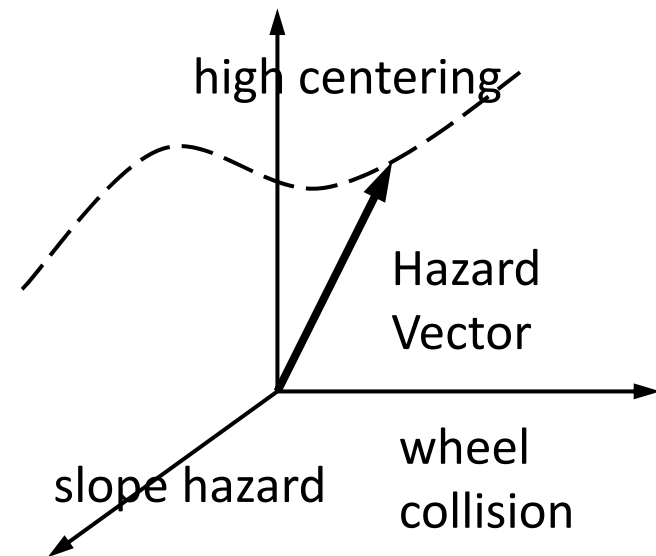
## 7.4.2.2 Hazard Space

- Hazardous states form clusters in state space corresponding to different types of interactions.
- As the vehicle moves in task space, the tip of some kind of hazard vector sweeps through hazard space.
- Comparing options requires the reduction of all hazard attributes for each time on a trajectory to a single scalar cost.



## 7.4.2.2.3 Consistent Hazard Units

- Must reduce all hazards to some **consistent units**. This may include any or all of:
  - Severity: 20 degrees is twice as bad as 10?
  - Distance: to go around versus effective distance over.
  - Energy consumed
  - Uncertainty
- Then the length of the vector is meaningful.



## 7.4.2.3 Cost of a Path

- In optimal control terms, this is our old friend:

$$J = \phi[\mathbf{x}(t_f)] + \int_{t_0}^{t_f} L(\underline{\mathbf{x}}, \underline{\mathbf{u}}, t) dt$$

- Maybe just add up the hazard score along the path?
- Probably makes sense to **weight less as distance increases?**



# 7.4.2.4 Models Used in Objectives and Constraints

- Models may be used in the computation of any or all of:
  - Motion Generation
  - Constraints
  - Objective functions

	<b>Attribute Used to generate a motion</b>	<b>Attribute Used in a Constraint</b>	<b>Attribute Used in Objective Function</b>
<b>Vehicle Model</b>	State (for motion prediction)	volume of vehicle (for collision constraints)	power consumption wheel slip maneuver aggressiveness
<b>Environment Model</b>	Terrain shape or mechanical properties.	volume of obstacles (for collision constraints)	Proximity to obstacles

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# 7.4.3 Representation

(Trajectory Evaluation)

- Methods to **compute feasible trajectories** were covered earlier in motion prediction (dynamics).
- This section is about how to **evaluate trajectories**.
- Before we can cost a path, we must **cost a point** on a path. That means we must model:
  - The path
  - The vehicle
  - The environment

## 7.4.3.1.1 Motion Constraints

- Dynamic constraints:

$$\dot{\underline{x}} = f(\underline{x}, \underline{u}, t)$$

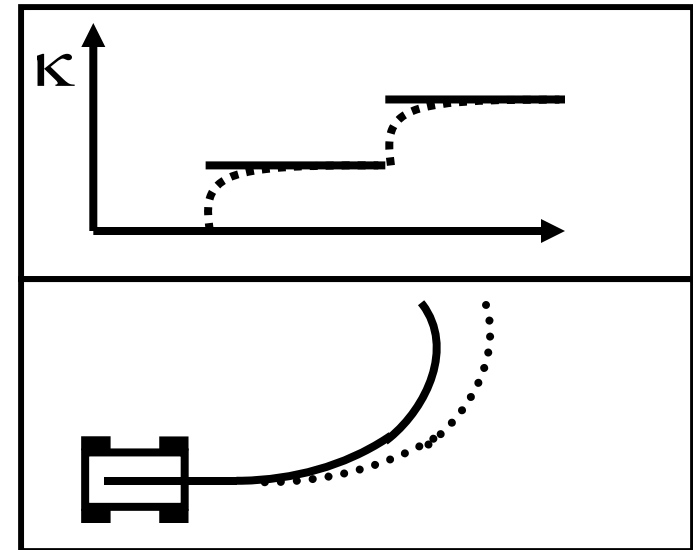
- Input/actuation constraints  $\underline{u} \in U$

$$|\kappa(s)| < \kappa_{\max}$$

$$|\dot{\kappa}(s)| < \dot{\kappa}_{\max}$$

## 7.4.3.1.2 Representation of Paths and Trajectories

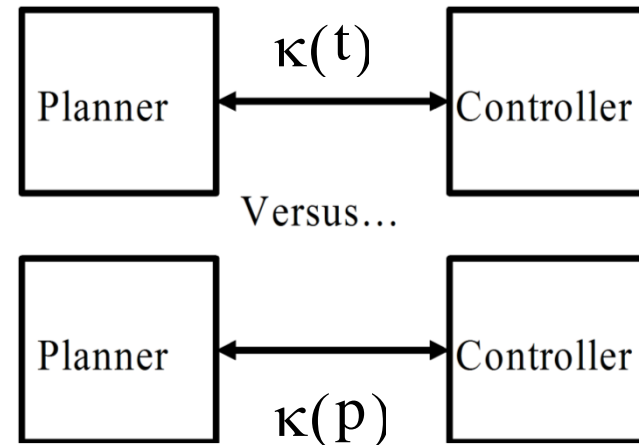
- Two options at least are:
  - **Input history**:  $\underline{u}(t)$
  - **State history**:  $\underline{x}(t)$
- $u \rightarrow x$  is easy
- $x \rightarrow u$  is **hard** (but we know how)
- Sampled versions of these are common:
  - Sequence of curvatures is a sampled  $\underline{u}(t)$
  - Ordered sequences of cells or points is a sampled  $\underline{x}(t)$



— Command  
..... Response

# 7.4.3.1.3 Compactness and Completeness of Motion Representations

- Compact representations can **streamline communications** to low level control.

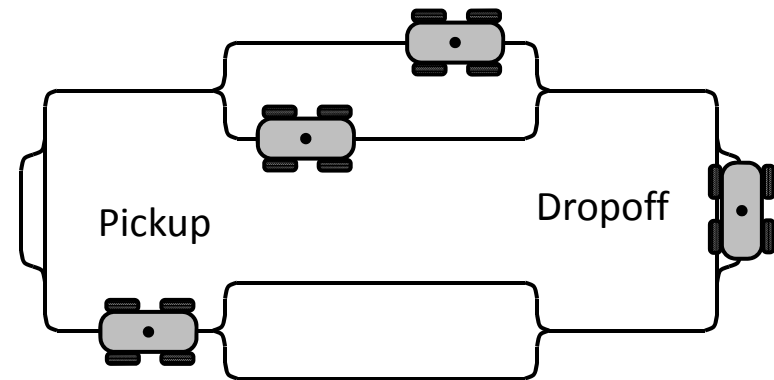


Commanding an entire trajectory (rather than instantaneous curvature) means you only have to do so occasionally.

It also **gives lower levels the opportunity to do predictive control.**

# Motivation: Offline Representations

- Trajectories sometimes must be stored in some off-line representation.
- For AGVs, **clothoids, lines, and arcs** are a common library of trajectory shapes.
- Complex shapes can always be approximated by a sequence of simpler ones.
- Its a good thing if the representation is both **complete and compact**



AGV guidepaths can be represented as a sequence of trajectories rather than a large set of points or poses..

## 7.4.3.2 Representing Configurations

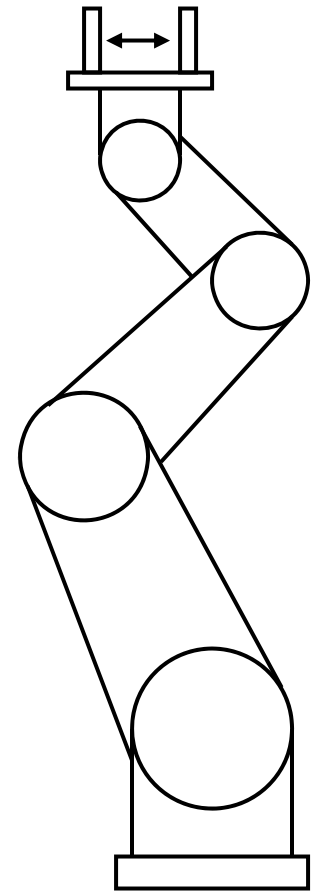
- State space is good for
  - predicting motion.
  - assessing certain hazards.
  
- Configuration space is good for encoding
  - Articulation
  - Occupied Volume



## 7.4.3.2 Representing Configurations

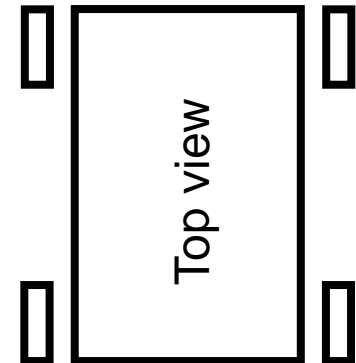
(C Space Definitions)

- A **configuration** of an object is a **specification** of the position of every point on the object (with respect to a fixed frame of reference).
- A Configuration Space is the space (i.e. set) of all configurations of the object.
- Informally, this is a set of generalized coordinates which completely **determine** the position of every point on the object.



## 7.4.3.2.2 C-Space Dimension

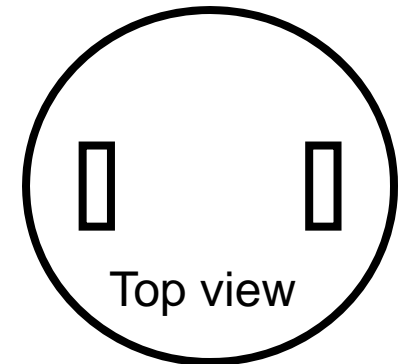
- The number of generalized coordinates required is the **dimension** of the C Space.
  - Articulations add to the C space dimension.
  - Constraints reduce it.



## 7.4.3.2.2 C-Space Dimension

(Computing C Space Dimension)

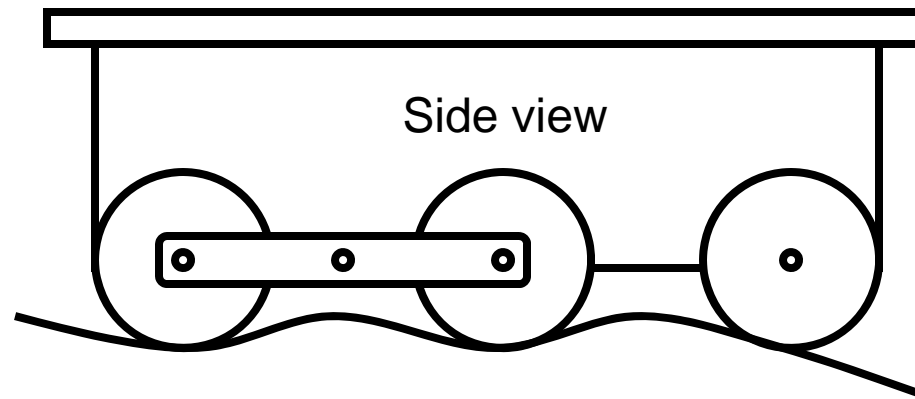
- Start by **adding the number of spatial dof of each rigid body** comprising the object.
- Then, **impose the constraints of articulation**
  - kneebone connected to the....
  - → including terrain following



## 7.4.3.2.2 C-Space Dimension

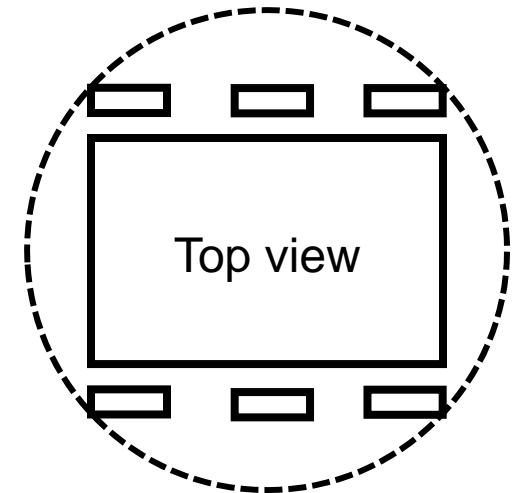
(Symmetry)

- Some dof (e.g. wheels) do not change the occupied volume when they articulate.
- Hence, irrelevant to collision detection.
- Usually, we **remove them** from the representation.
  - More efficient for Planning
  - Formally however, these dof are still dimensions of C space

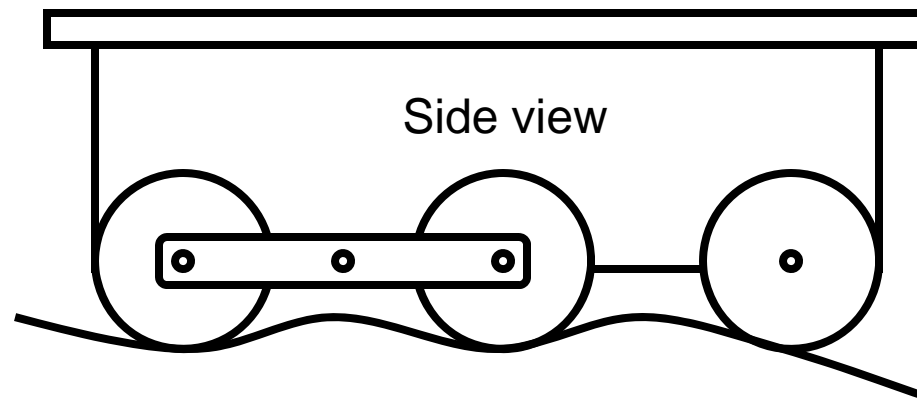
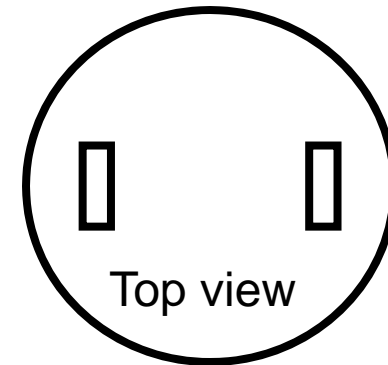
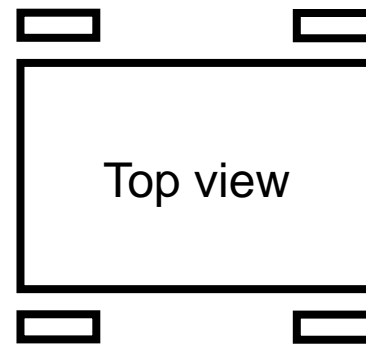
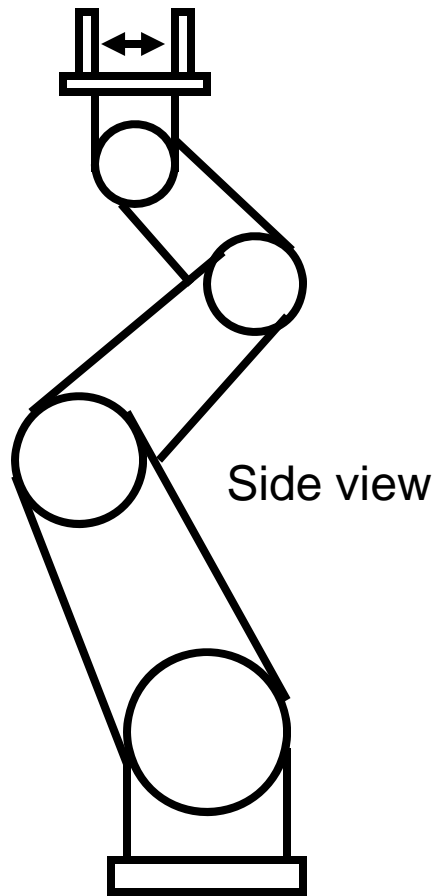


# Computation

- Computational complexity of search is directly related to:
  - the dimension of C Space
  - the complexity of the obstacles
- At times, it is valuable to **approximate a robot shape by a symmetric one** (say, by a circle) in order to reduce the dimension of C space.



# How Many DOF Should We Represent?



# 7.4.3.3 Representing the Environment

(Environmental Attributes for Costing)

- This section is about
  - Encoding spatial properties of the environment for trajectory evaluation purposes.
- This section is **not about representing elevation** for purposes predicting motion.
  - However, both can sometimes be stored in the same data structure.

# 7.4.3.3 Representing the Environment

(Discriminators for Representation)

- Dynamic range (discrete / continuous / mixture)
- Memory requirements
- Efficiency of intersection calculations
  - Affects collision detection efficiency
- Gradient information available ?
  - Affects search efficiency



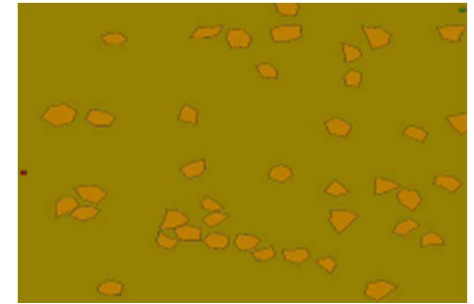
## 7.4.3.3 Representing the Environment

(Obstacles” and Costs)

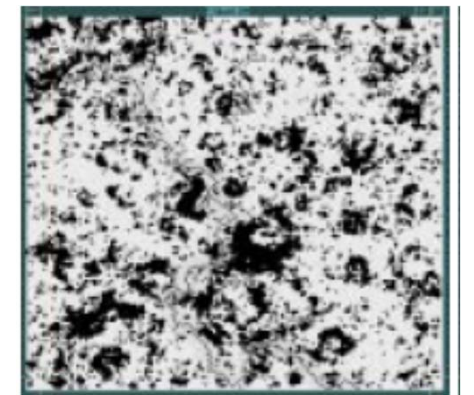
- Literally “obstacles” are impediments to motion.
  - You cannot drive through them
- Robotics thinks of them as places you should not drive
  - **Even if** you can drive through them
- Assigning a cost or “relative obstacle severity” is also common.

# 7.4.3.3.1 Set and Field Representations

- Sets → map an object index onto a region of space
  - Typically cost is uniform in the region and binary (meaning obstacle or not)
  - Represent obstacles as objects (which happen to occupy space)
  - Represent position, and perhaps shape as volume or boundary.
  - Can be memory efficient but computationally expensive.
- Fields → map a point in space onto a cost
  - Represent large spatial region as a raster or array.
  - Associate a utility or cost with every point in the workspace.
  - Can be memory inefficient but computationally cheaper than sets.



Set of Binary Obstacles

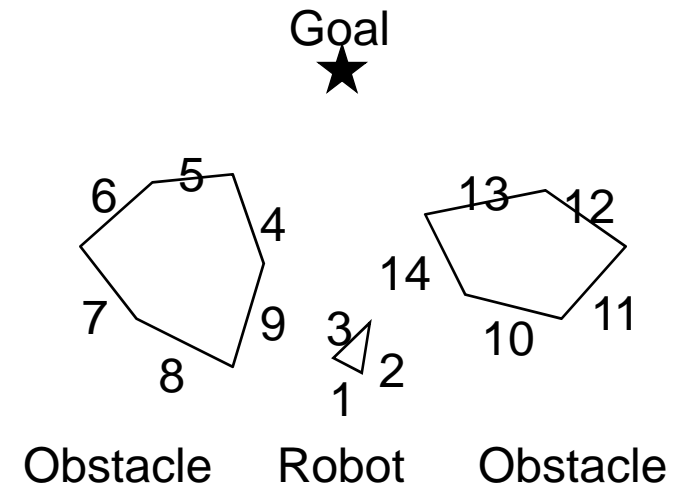


Cost Field

## 7.4.3.3.2 Shape Representations.

(Boundary Representations [for Sets])

- Collision checking involves checking for boundary intersections.
  - $\text{Num\_robot\_edges} * \text{num\_obstacle\_edges}$  computations without bounding boxes.
- Hence, the planning computational complexity **depends on the number of obstacle edges.**
- Bounding shapes can be used to quickly eliminate unnecessary intersection checks.



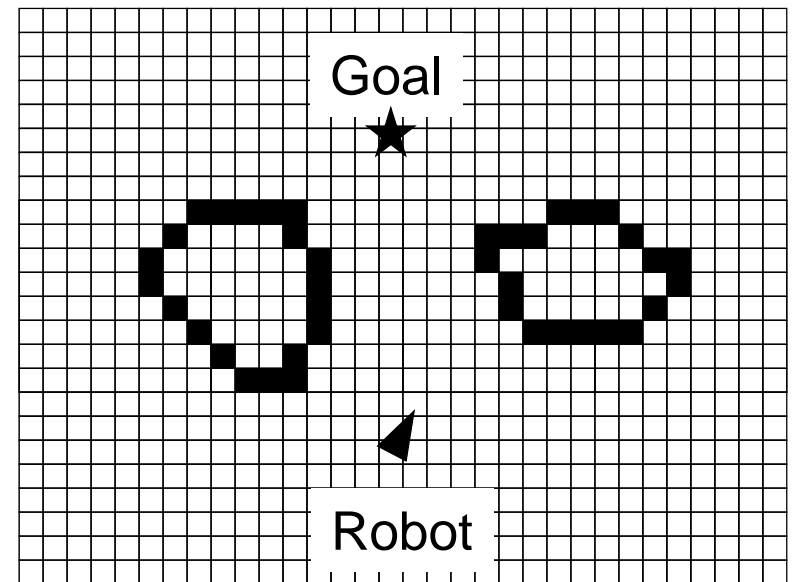
## 7.4.3.3.3 Obstacles Versus Free Space

- It may be better to represent free space rather than obstacles.



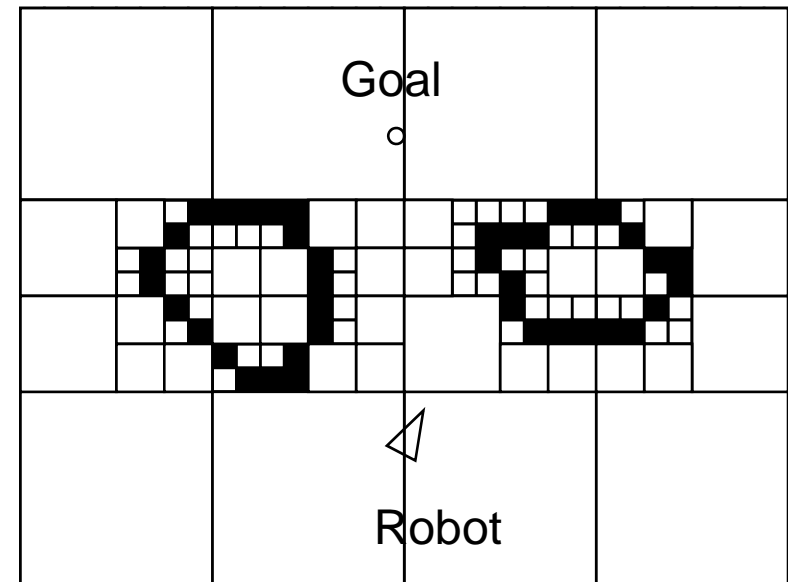
## 7.4.3.3.4 Sampled Versus Continuum

- Sampled may be best alternative in complex environments
  - i.e. whose continuous representation would be large.
- Intersection calculation is an **AND of two rasters**.
- Planning computational complexity tends to **depend on the resolution** of the representation.



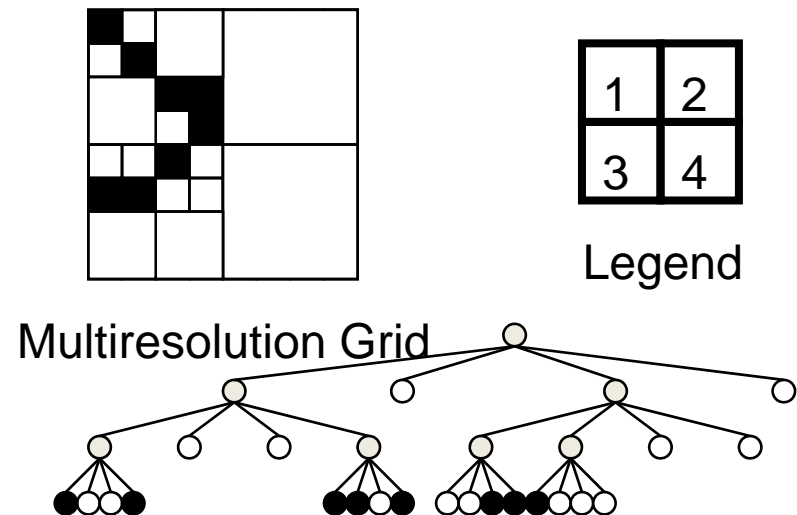
## 7.4.3.3.5 Hierarchy and Quadtrees

- A popular approach to reducing memory is a kind of hierarchical grid called a quadtree (octree in 3D).



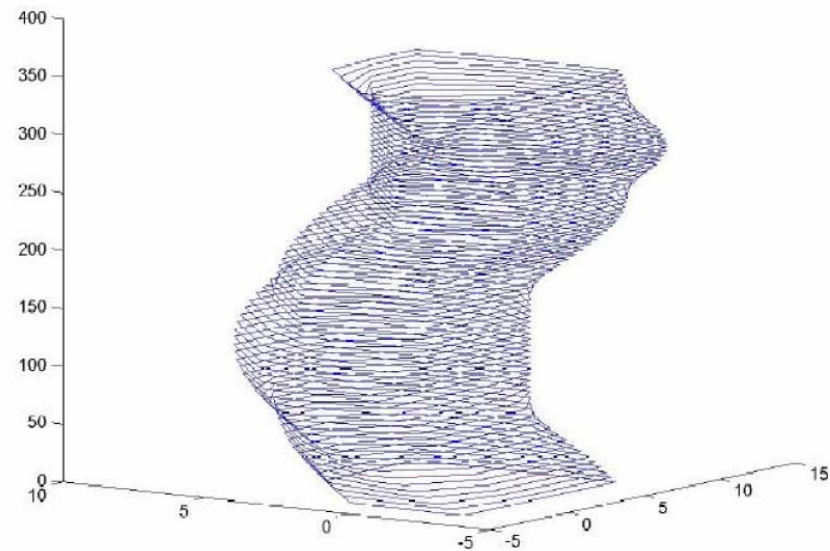
## 7.4.3.3.5 Hierarchy and Quadtrees

- A tree of nodes that are:
  - filled,
  - unfilled
  - partially filled
- Only the partially filled ones at each level are elaborated.



## 7.4.3.4 Derived Spatial Representations

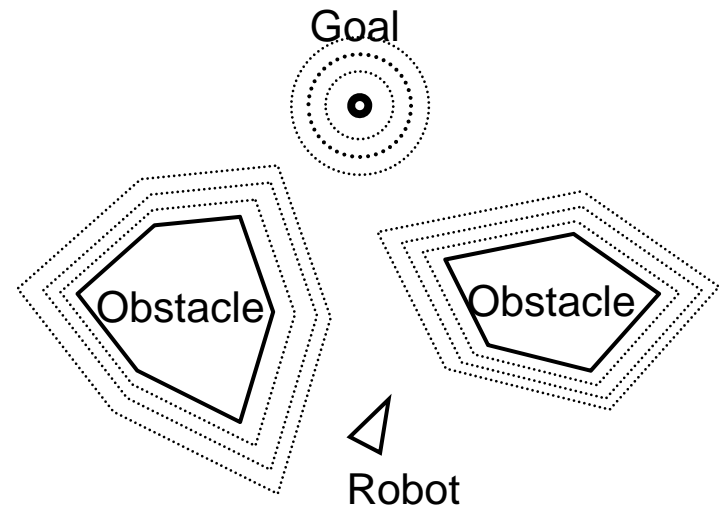
- Representations **derived from** the basic geometric or cost information can be useful.
- May make collision checking more efficient.





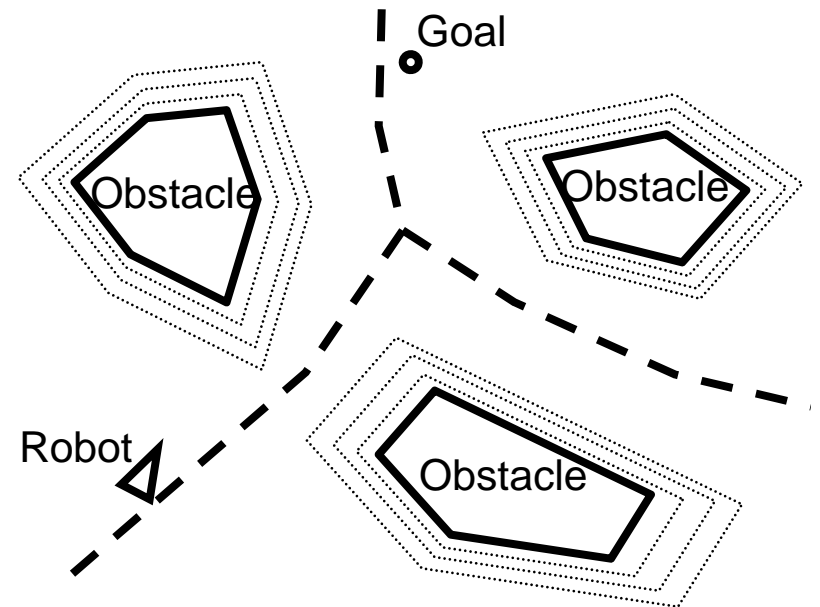
# 7.4.3.4.1 Potential (& Proximity) Fields

- Proximity (minimum distance to a collision) is a special derived field.
- Individual potentials can simply be added or perhaps combined in more principled ways.
  - For example, a proximity field can be formed as the min distance to any collision.
- Controls, policies, inputs etc. are derived from the field at the present position (e.g follow the gradient).
  - Such representations have a well defined gradient and they can be used in relaxation based search as well as sequential search.



## 7.4.3.4.2 Voronoi Diagrams

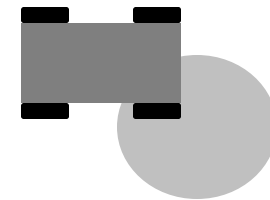
- Subspaces of the original space.
- Can be generated from a field representation.
- Set of **all points which are equidistant from at least two** obstacle boundaries.
- Local maxima in the proximity field.



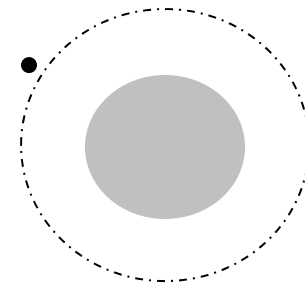
# 7.4.3.4.3 C-Space Obstacles

(Mapping Volumes to C Space)

- C Space obstacle = set of configurations where a collision in the workspace takes place
  - Compute it with boundaries
  - Compute it with volume intersection
- Precomputes the intersection calculation
  - So its only done once.
- **Can also be done** as a volume integral **for continuous costs**
- Not worth it
  - When the environment is dynamic or sensed with noise
  - When only a small region of the workspace will need to be tested



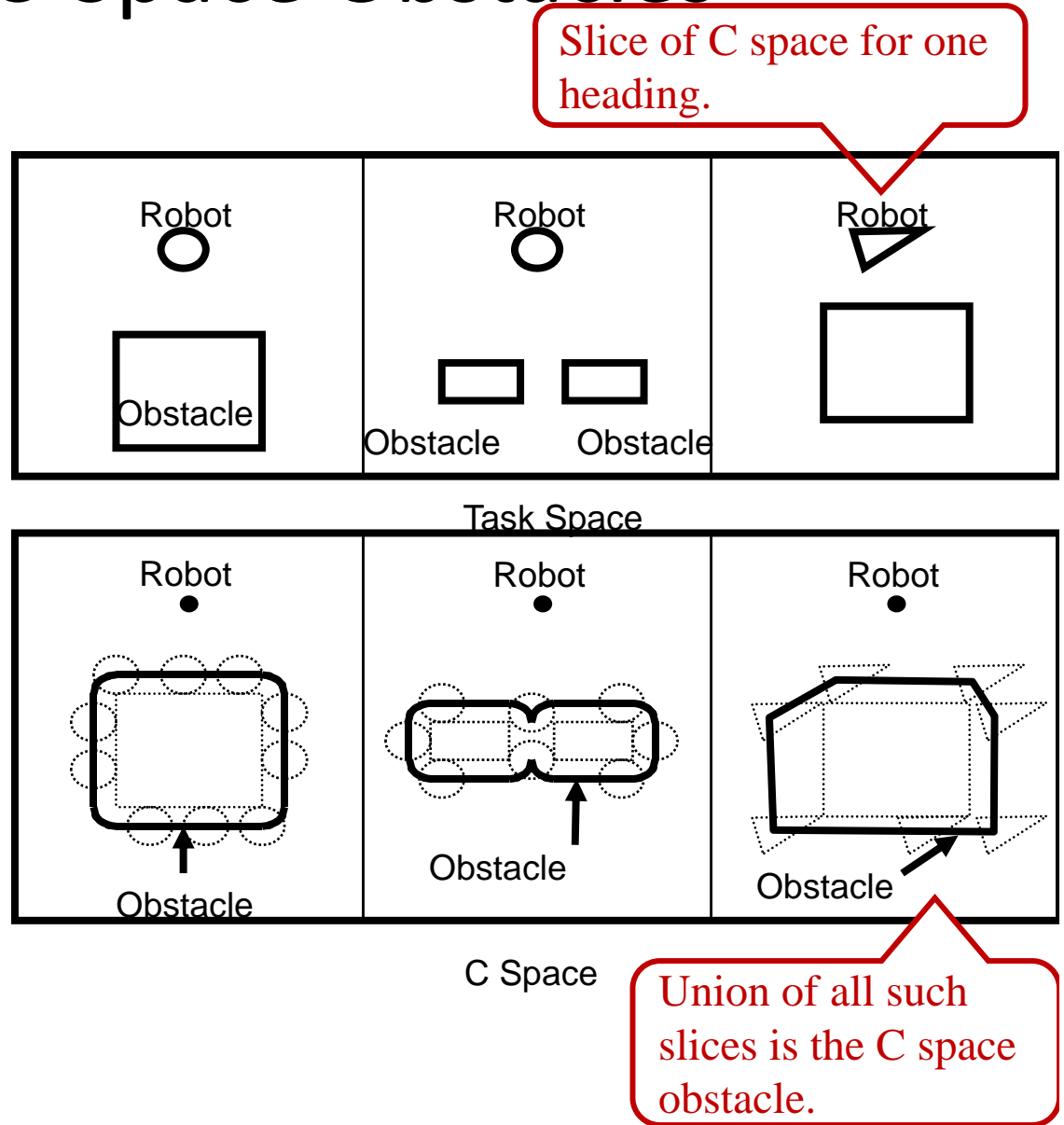
Workspace



Cspace

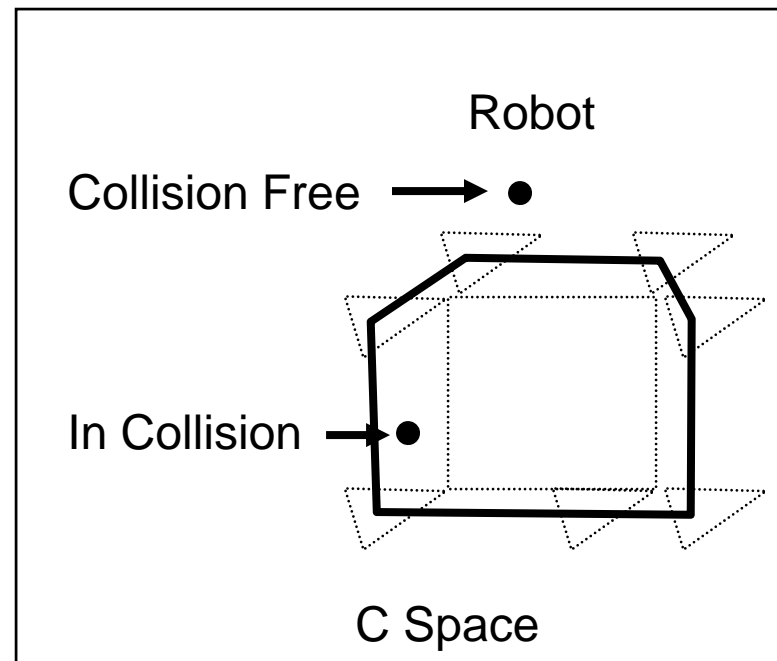
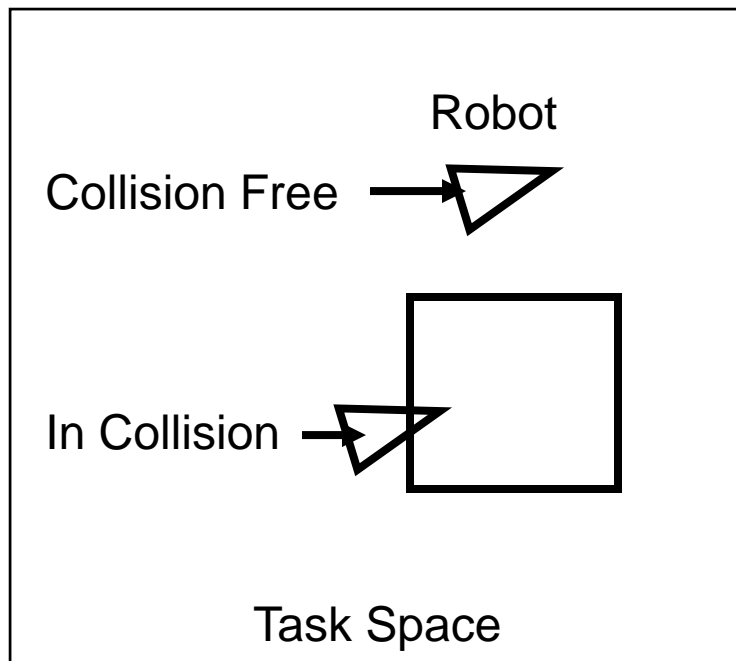
# 7.4.3.4.3 C-Space Obstacles

- Boundaries of obstacles in the environment can be converted to equivalent obstacles in C Space.
- For every point on the boundary of an obstacle, **compute every configuration of the robot which can be in contact with that point.**
- Its a property of both the robot shape and the obstacle shape.



## 7.4.3.4.3 C-Space Obstacles

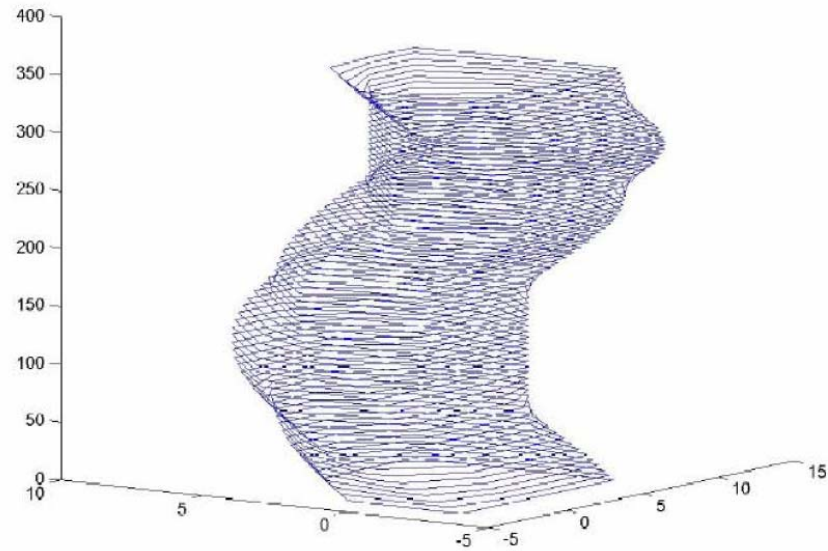
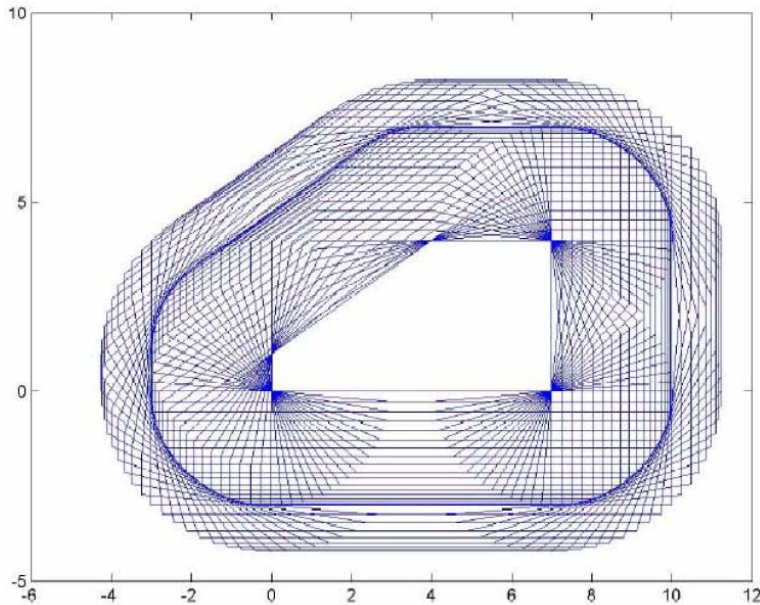
- Now, collisions can be detected cheaply:
  - ask if the **point robot** is inside a C space obstacle.



# 7.4.3.4.3 C-Space Obstacles

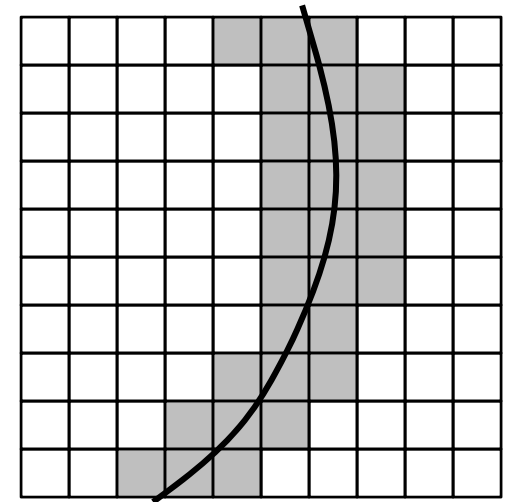
(Dimension = 3)

- C Space obstacles can sometimes be hard to compute explicitly.



# C Space and Precomputation

- Cost precomputation → when the environment is static and known
- Cell precomputation → can be done even if costs are changing.



# State Space $\rightarrow$ Workspace Mapping

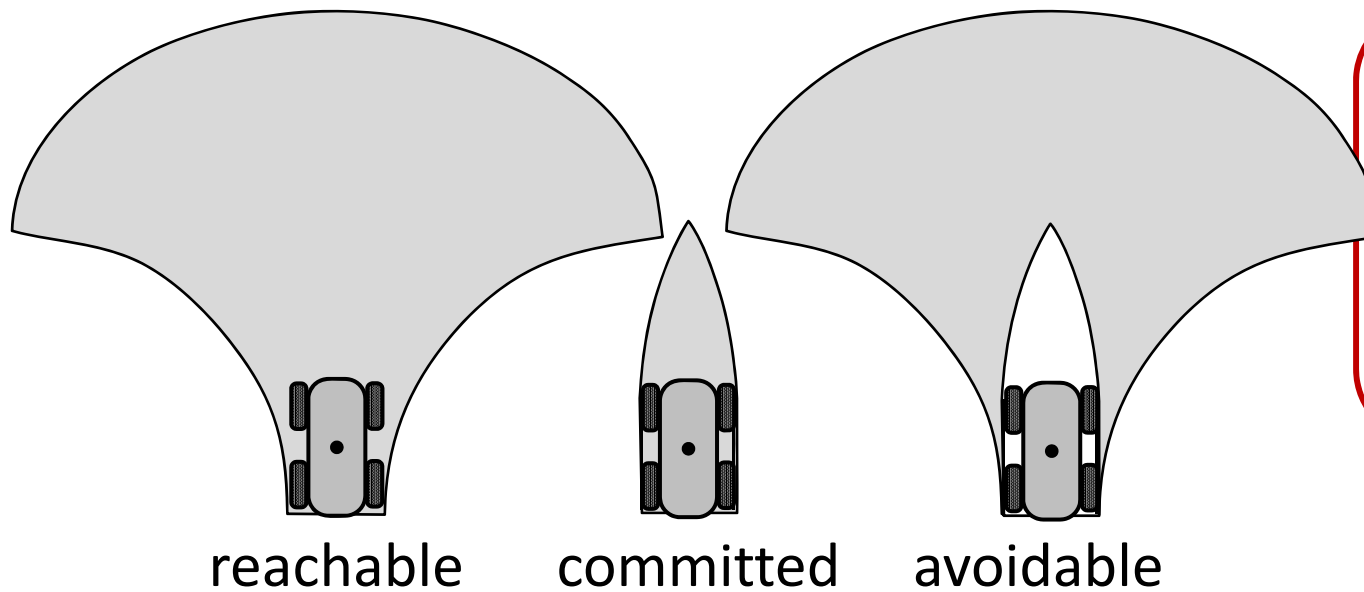
- Just like
  - obstacles in the workspace can be mapped to regions in C space
  - Regions and hazards in workspace can be mapped to state space



## 7.4.3.4.4 Partitions of State Space and Work Space (Committed Motion)

- Can segment space in a limited horizon reachable region.
  - Reachable = some point on the vehicle can reach it for **some** motion.
  - Committed = some point on the vehicle will reach it for **every** motion.
  - Avoidable = Reachable – committed.

## 7.4.3.4.4 Partitions of State Space and Work Space (Committed Motion)



Committed region :

- a) grows quickly with speed
- b) Bends with initial curvature

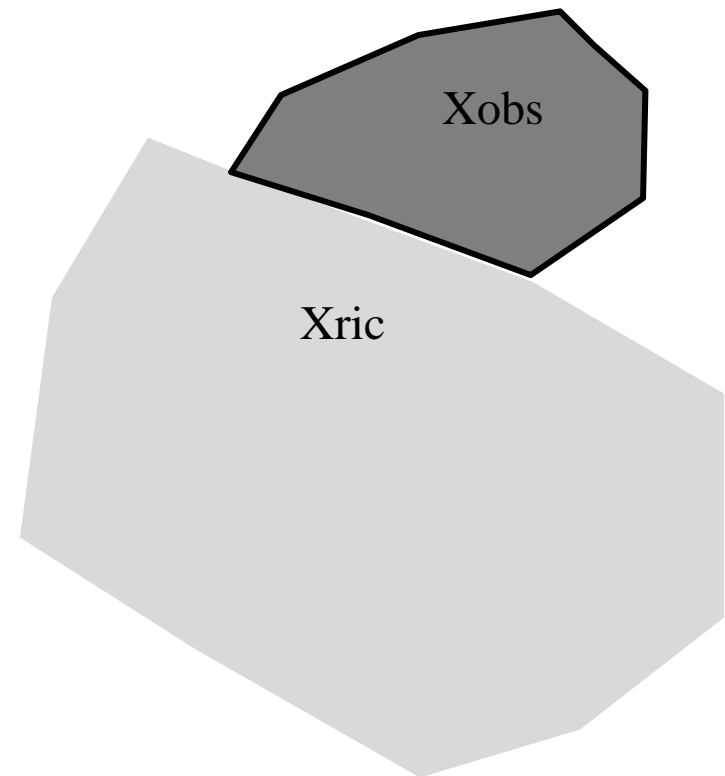
Farther  
forward  
Close to  
centerline

Farther  
backward  
Far from  
centerline

Such a figure can also  
be drawn for different  
classes of trajectory  
(stopping, turning)

## 7.4.3.4.4 Partitions of State Space and Work Space (Regions of Inevitable Collision)

- State space obstacles.
- RIC = set of **all initial states** from which entry into an obstacle **must eventually occur**.
  - Pick a state in  $X_{obs}$
  - Solve DE backwards for all possible controls to get there.
  - All states occupied for some  $X_{obs}$  and all controls are in  $X_{ric}$



## 7.4.3.4.5 Incorporating Risk and Uncertainty (Sources of Uncertainty)

- Interpretation: **Assessments** of hazards are not accurate.
- Sensing: Localization error implies obstacles may be **incorrectly located** relative to the vehicle.
- Motion: There is no guarantee that motion prediction/control will **do what was planned** or predicted.

## 7.4.3.4.5 Incorporating Risk and Uncertainty (Techniques for Coping)

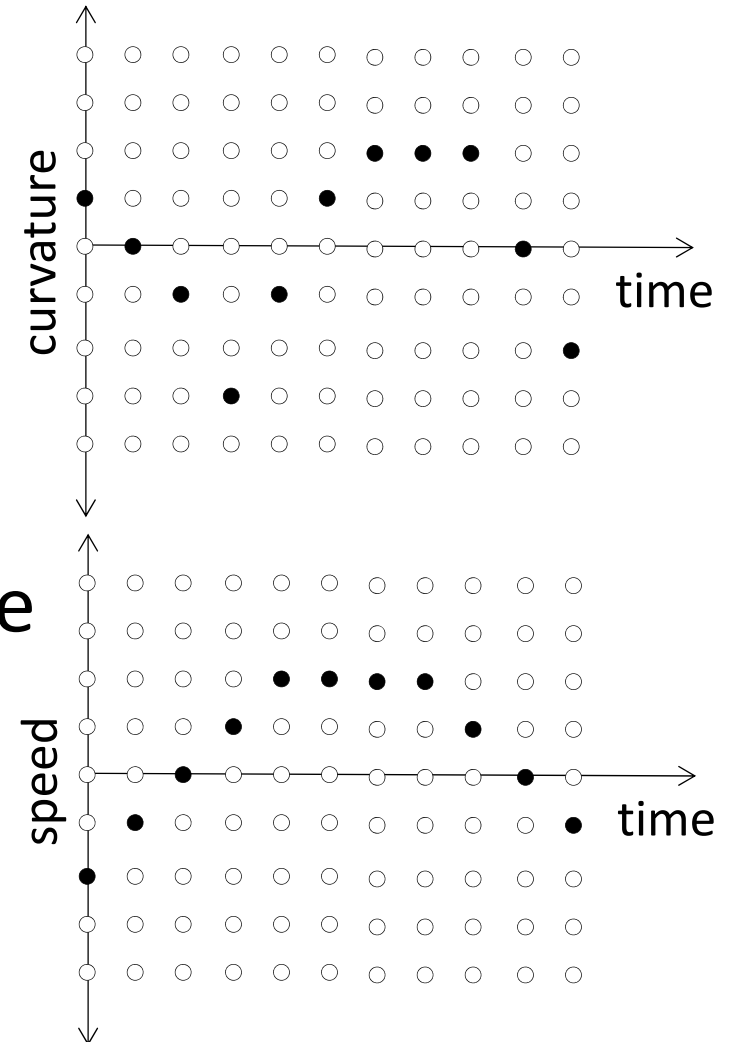
- Margin: Use a **deliberately oversized vehicle** when assessing collisions.
  - This means you won't be able to squeeze through tight spaces.
- Oversize the Obstacles. **Filter** the environmental representation to cause high cost to bleed into adjacent areas.
- Explicitly compute motion uncertainty and map uncertainty and **do both** of the above.

# Outline

- 7.4 Intelligent Control
  - 7.4.1 Introduction
  - 7.4.2 Evaluation
  - 7.4.3 Representation
  - 7.4.4 Search
  - Summary

## 7.4.4.1 Sampling, Discretization, and Relaxation (Input Discretization and Parameterization)

- In full generality, there is a function space  $u(t)$  to search.
- Discretization and parameterization are two options.
- For 10 signal levels and 40 time samples, there are  $10^{40}$  alternatives.
  - Not feasible to search at 10 Hz.



## 7.4.4.1 Sampling, Discretization, and Relaxation (Continuum vs Sampling)

- Sampling
  - Avoids local minima
  - Inefficient/impossible in dense obstacles
- Relaxation (Continuum)
  - Finds only local minima
  - Very efficient in dense obstacles
  - Requires gradient information.



## 7.4.4.2 Constraint Ordering

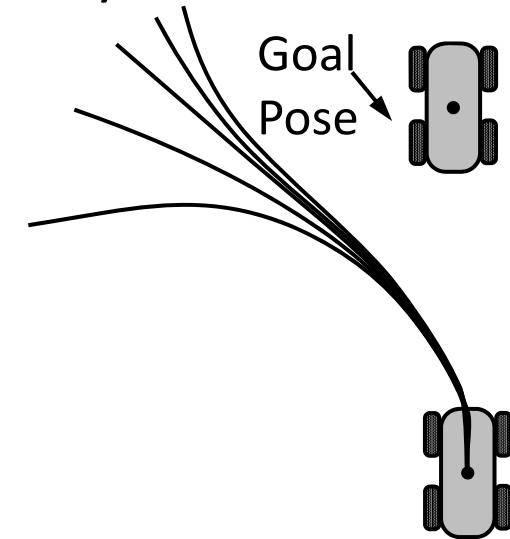
(Ordering Feasibility and Admissability Constraints)

- Option 1:
  - Find good places
  - See if you can go there
- Option 2:
  - Find places you can go
  - See if they are good.
- Imposing the **most limiting constraint first** is often most efficient.

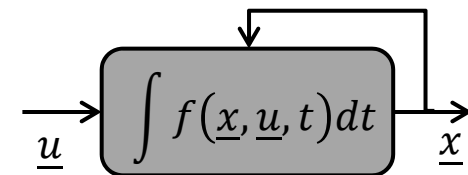
# 7.4.4.2 Constraint Ordering

(Constraint Ordering Dilemma)

- In which space should we conduct search (= express alternatives)?
- Easy in Input/Control/Action Space:
  - Dynamic feasibility.
  - Actuation limits (e.g turn radius).
- Easy in Work/State/C Space:
  - Obstacle Intersection
  - Following Global Guidance
  - Enforcing workspace constraints.
  - Ensuring good separation.
- What's **easy in one is hard in the other.**



Forward:  $\dot{\underline{x}} = f(\underline{x}, \underline{u}, t)$

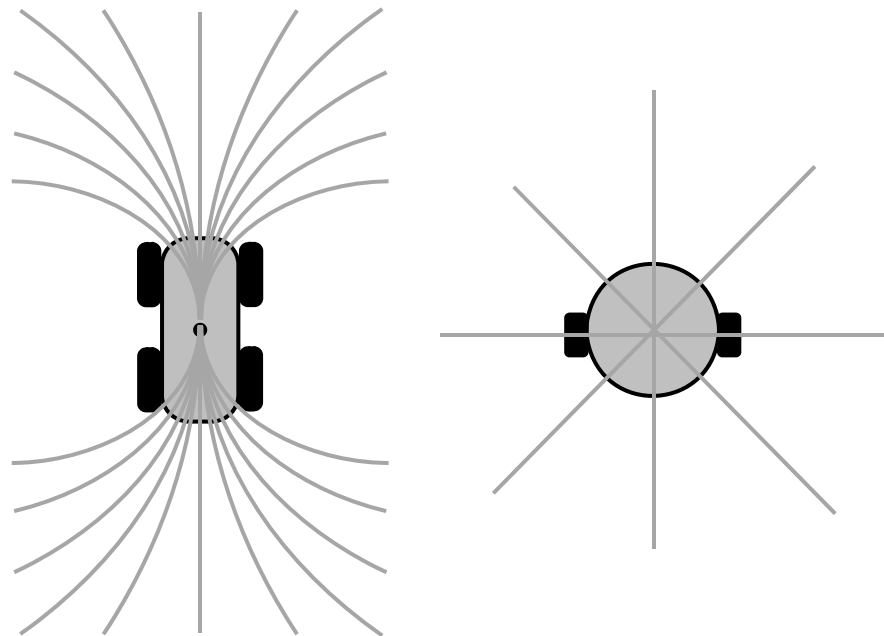


Inverse:  $\underline{u} = ?(\underline{x})$



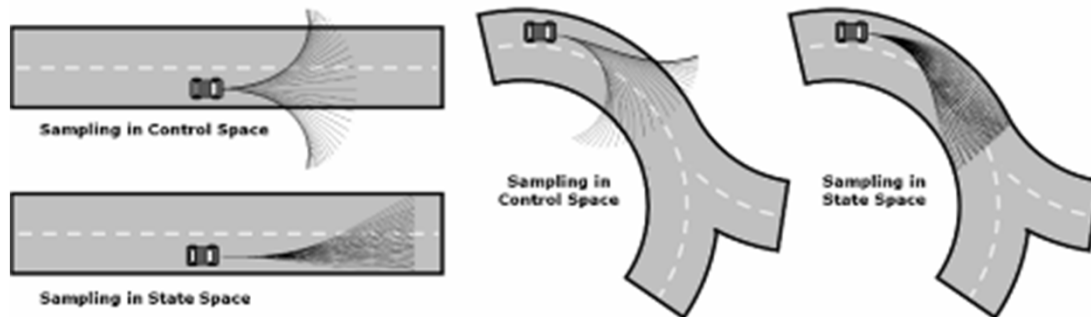
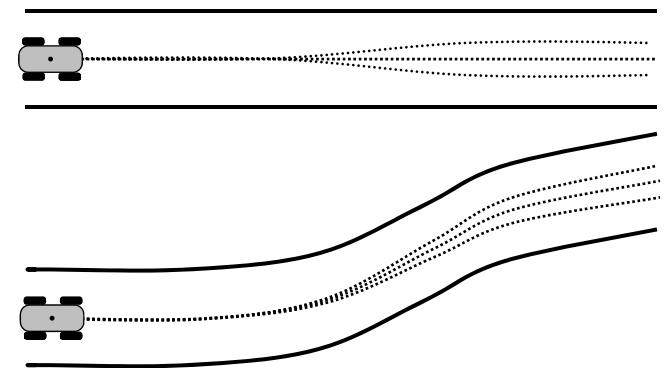
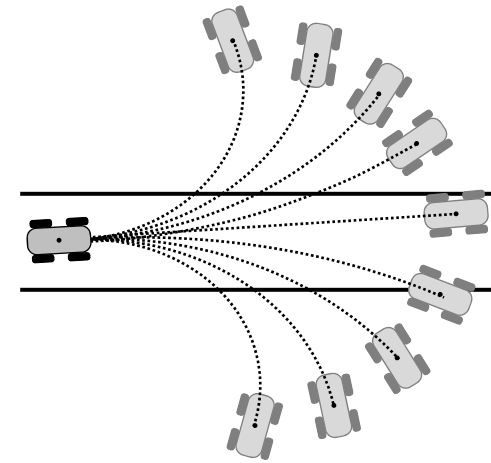
## 7.4.4.2.1 Search Coordinates

- There is value in artificially limiting mobility.
- Almost any vehicle can be driven by considering only arcs.
- A differentially steered vehicle can be driven by considering compositions of point turns and line segments



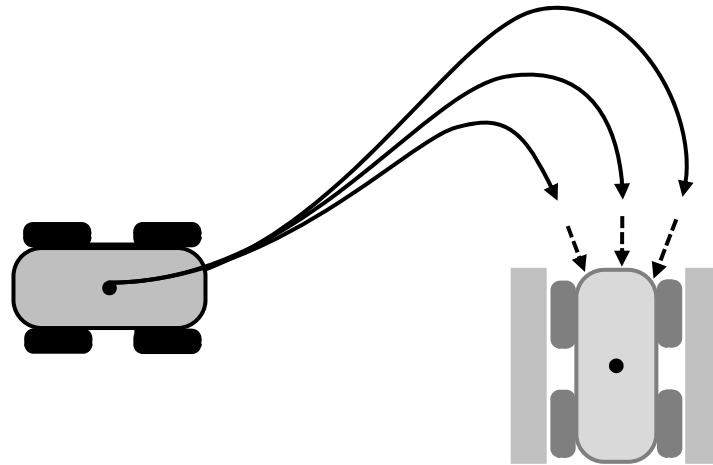
# 7.4.4.2.2 Environmental Constraints and Guidance

- Sometimes there is value in limiting maneuverability artificially to **respect and exploit** environmental structure.
  - Admissibility first.
- This focuses the search and eliminates wasted computation.



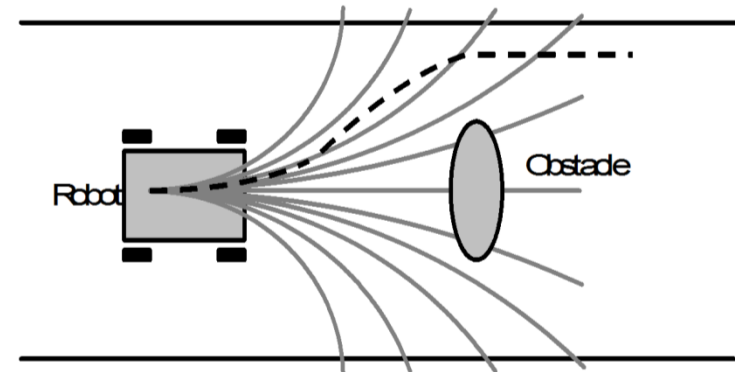
## 7.4.4.2.2 Environmental Constraints and Guidance (Global Guidance)

- Need to hit the terminal state fairly precisely to make the tight maneuver.



## 7.4.4.2.2 Environmental Constraints and Guidance (Workspace Constraints with Obstacles)

- Here workspace constraints and **admissibility** leave only a **small region** in trajectory space.
  - feasible
  - admissable



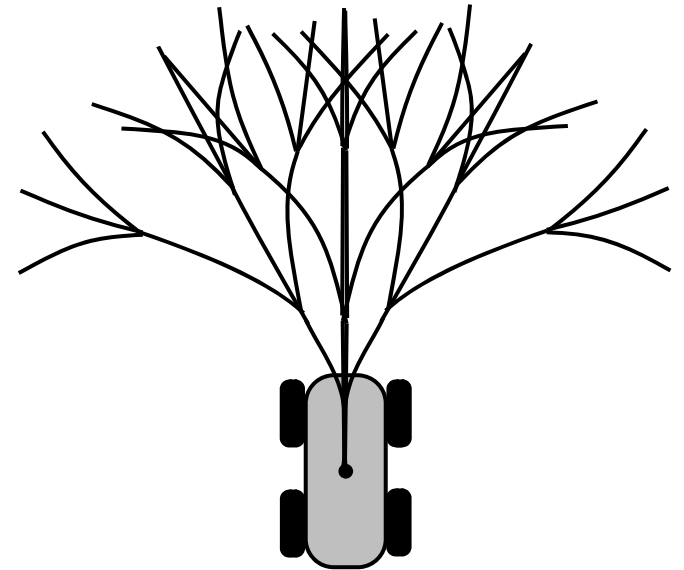
The problem of avoiding the obstacle and staying on the road is solvable - but solution is not in the space of arcs. Only a compound turn (left, then right) will work.

## 7.4.4.3 Efficient Search

- Above section tries to conduct the search in a space that satisfies the constraints intrinsically.
- This section looks at how to accelerate the search itself.

## 7.4.4.3.1 Mitigating Effects

- Distinct inputs do not necessarily generate very distinct outputs.
- The environmental representation is not of infinite resolution.
  - So a continuum search is not necessary.
- Often there are many solutions and any one is good enough.
- Sometimes can search in priority order (e.g. straight first) in sparse obstacles.





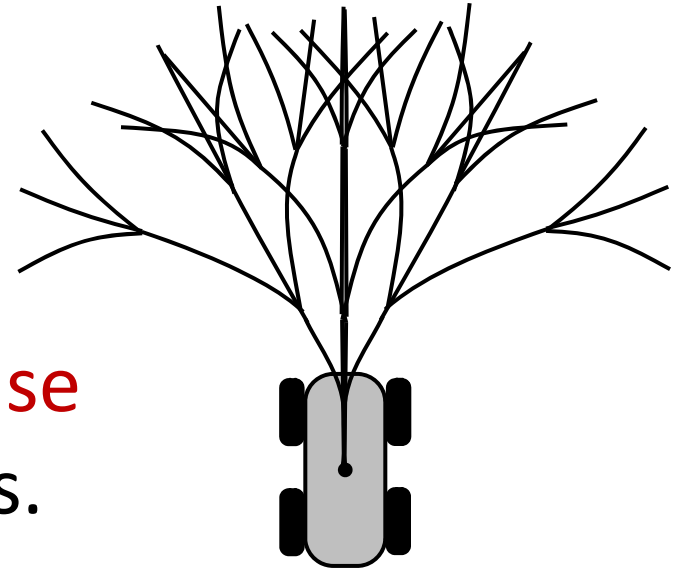
## 7.4.4.3.2 Reusing Computations

- When the environment is cluttered, search efficiency matters more.
- Can exploit tree structure to **reuse** the component path evaluations.
- Total length opposite is:

$$(3 + 3^2 + 3^3) \frac{s}{3} = (1 + 3 + 3^2)s = \boxed{13s}$$

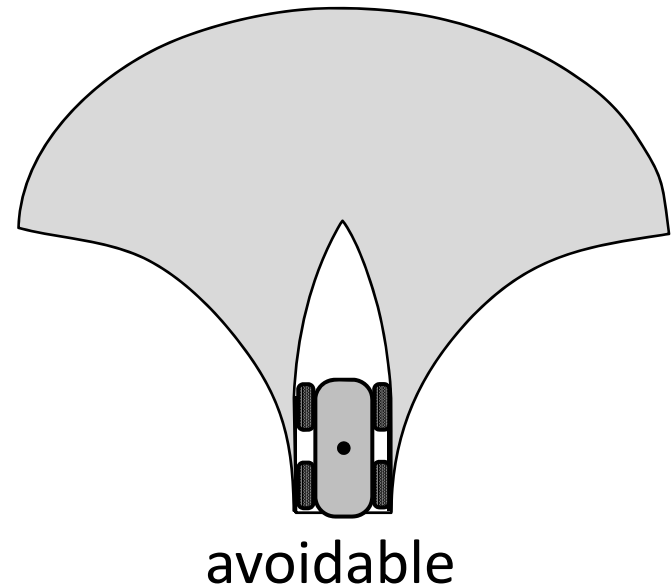
- Whereas 27 paths require:

$$3^3 s = \boxed{27s}$$



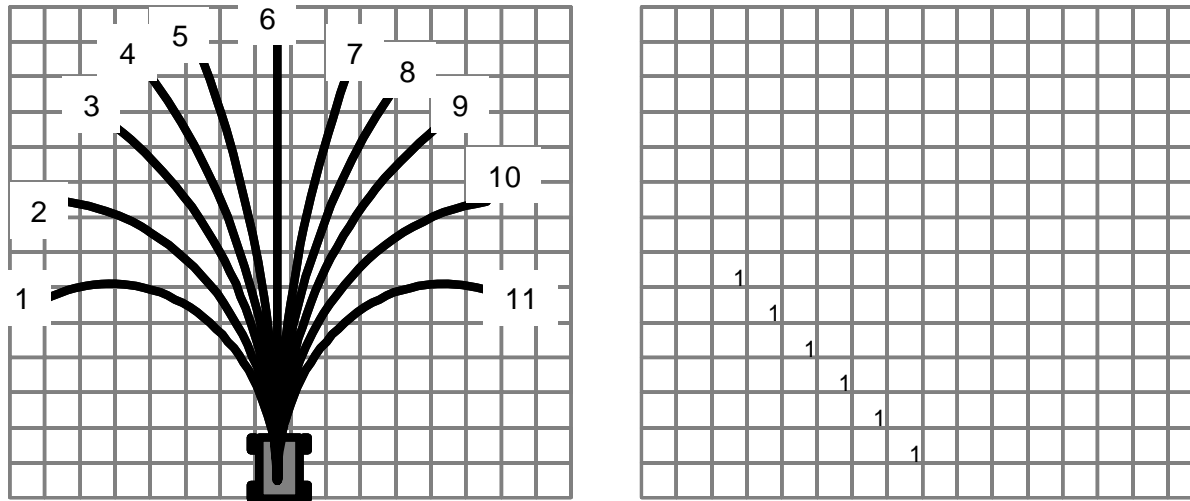
## 7.4.4.3.3 Exploiting Committed Motion

- Makes no sense to search for obstacles in committed region.
- Makes a big difference at high speeds.



# Trajectory Caching

- Invert the input-to-state mapping in a lookup table:
  - Assumes fixed terrain shape
  - May be more efficient to visit each map cell rather than each possible input.



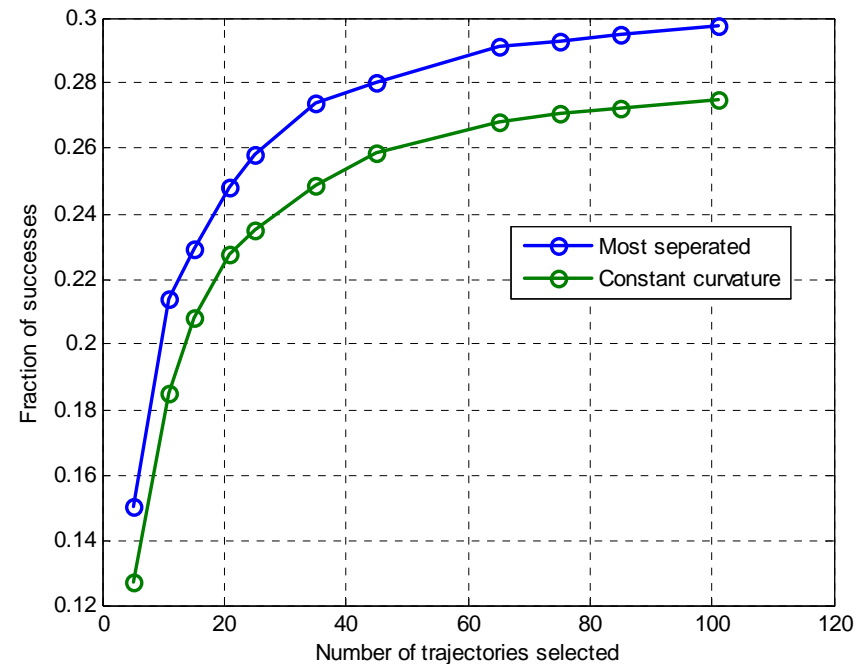
- This gives “Input space obstacles” (c.f. C space).

## 7.4.4.4 Search Space Design

- Tradeoffs and desirable characteristics of the search space itself.

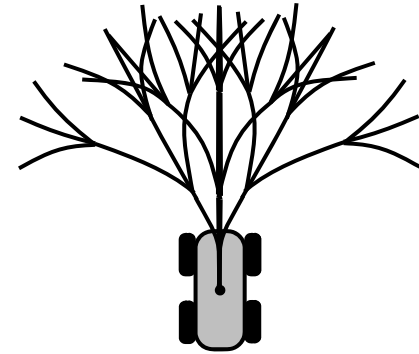
## 7.4.4.4.1 Mutual Separation

- Not all search spaces are created equal.
  - More “separated” is statistically better.
- What’s more...
  - Relaxation of a finite set of alternatives can improve matters dramatically.

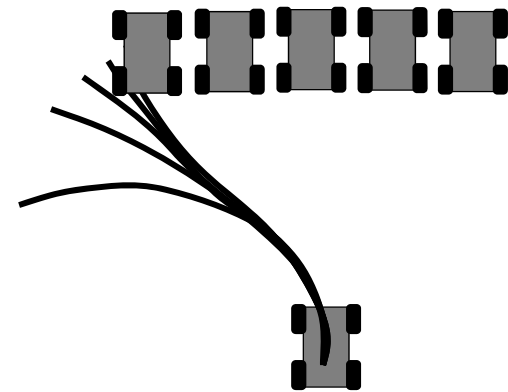


# Feasibility Versus Separation

- Would like to:
  - Span the space of feasible motions (arcs do not!)
  - While sampling as uniformly as possible.
- Problem:
  - First is easy to do in input space.
  - Second is easy to do in state space.



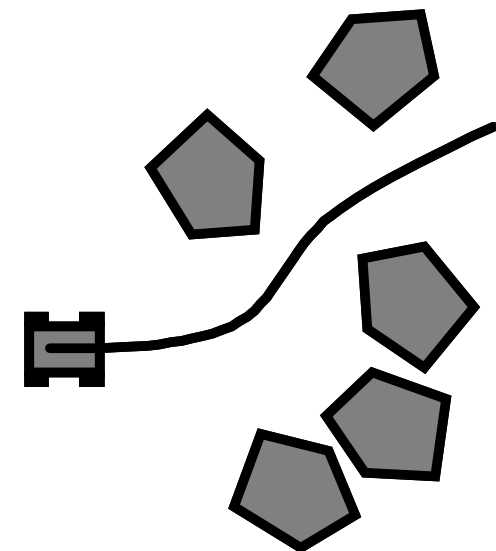
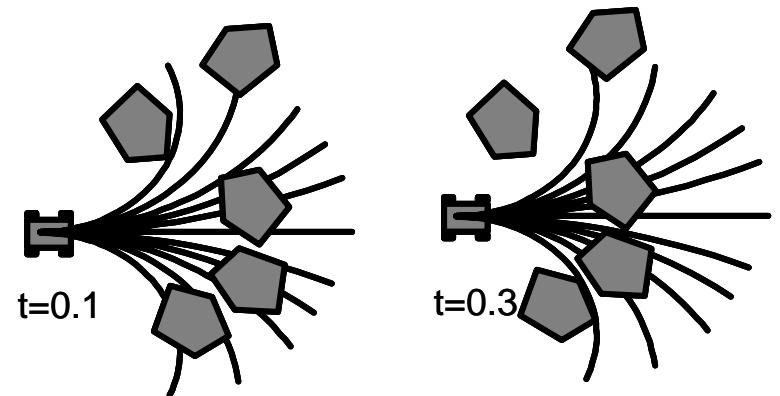
Feasible, Not Well Separated



Well Separated,. Not Feasible

## 7.4.4.4.2 Completeness

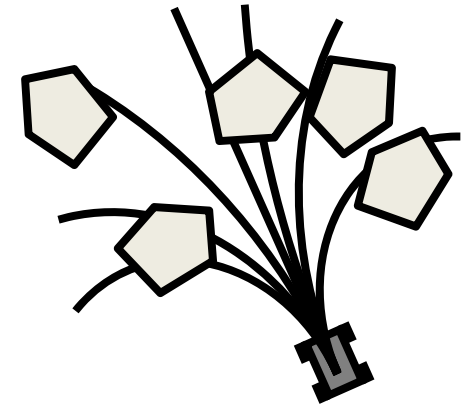
- The natural assumption that:
  - any set of reasonable trajectories ...
  - ... searched often enough ...
  - ... can generate any path is ...
- **WRONG**
- Solutions must be safe as far as the stopping distance even if only a small amount will be executed.
  - Because you may not have the option to change your mind.



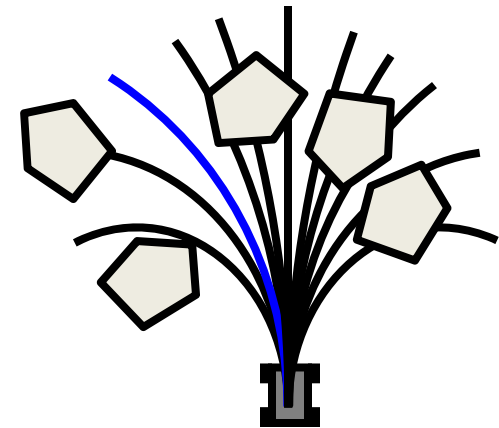
## 7.4.4.4.3 Robustness to Control Uncertainty

via Persistence

- Plan instability can cause a rare solution to be lost. Two solutions:
- Persistence:
  - Make sure the next search includes the last solution in case it's the only one.
- **Relaxation**
  - Deform the search space to regenerate the old solution.



$t=0.3$



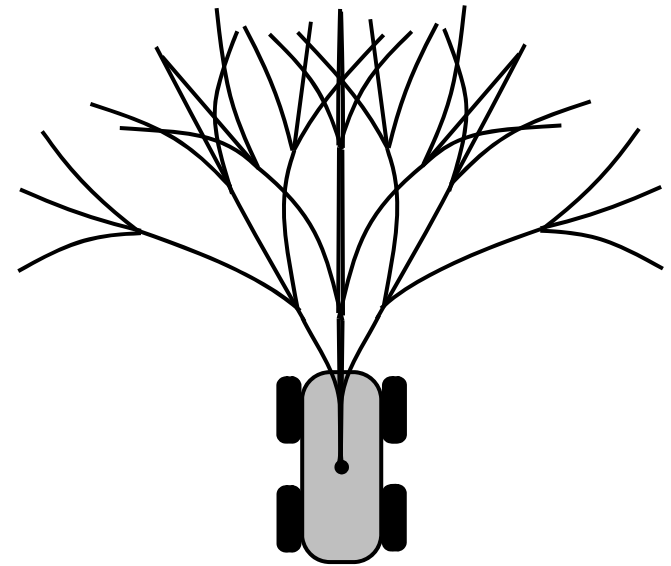
$t=0.1$



## 7.4.4.4.3 Robustness to Control Uncertainty

(Search Space Persistence)

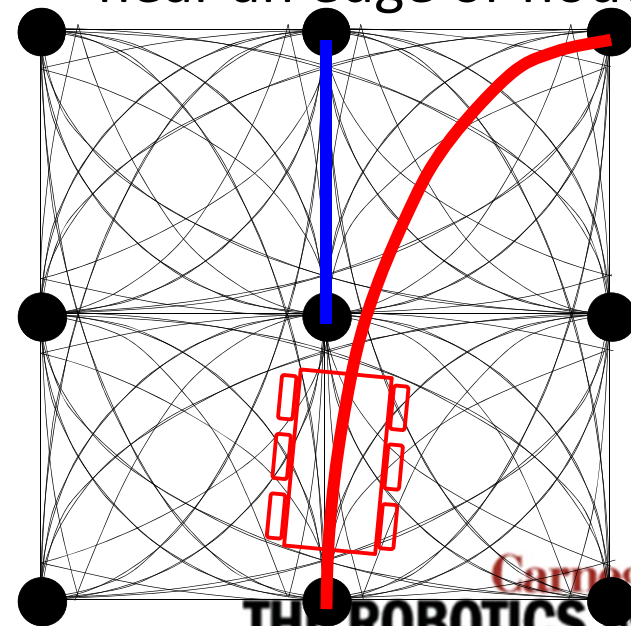
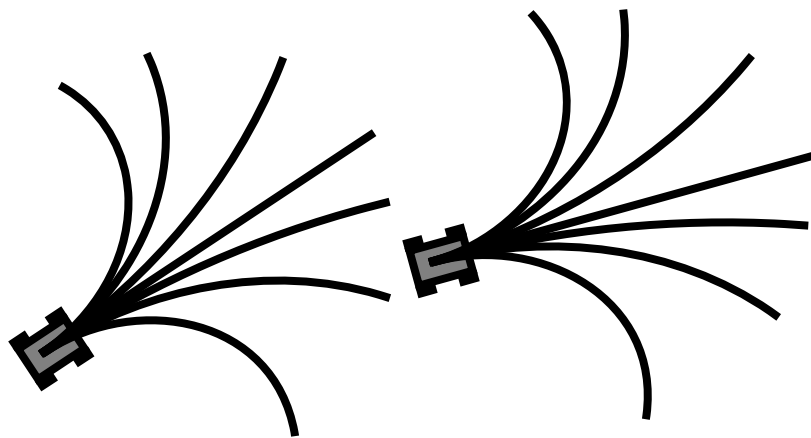
- Simple technique is to start next iteration from intended pose rather than actual pose.
- Special case:
  - Execute one segment at right and then replan from the fork point.



# 7.4.4.4.3 Robustness to Control Uncertainty

(Search Space Persistence)

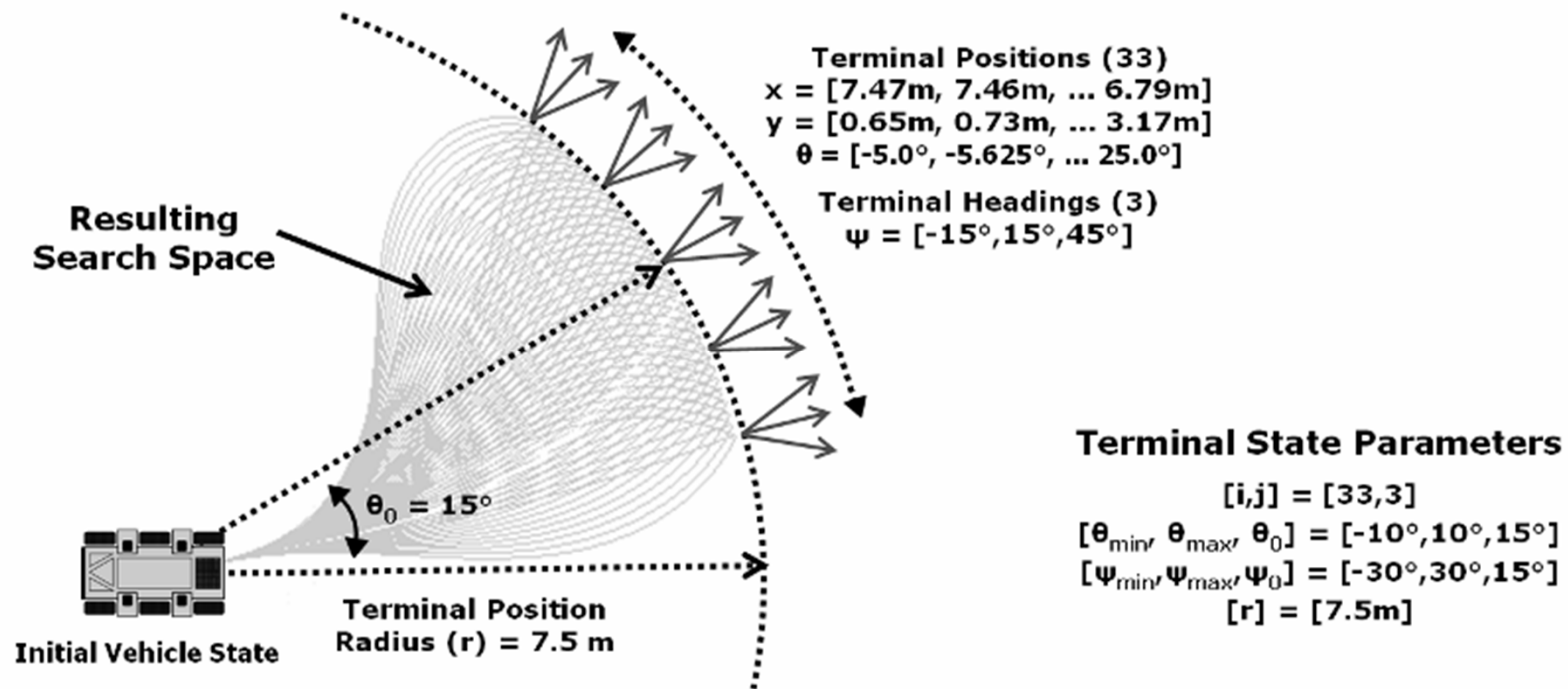
- Search spaces fixed to robot (RHMPC)...
  - are not stable
  - hard to reuse computation
  - BUT robot is always on an edge and node.
- Search spaces fixed to ground....
  - are stable
  - easy to reuse computation
  - BUT robot may not be near an edge or node.



# Ultimate OA System

- **Feasible:** Generates feasible motions only.
- **Admissible:** Exploits global guidance and satisfies global constraints.
- **Efficient:** Samples feasible set uniformly in the workspace.

# Ultimate System ?



- Shape of feasible set in workspace is computed off-line and stored in lookup tables.
- Impose workspace constraints on that.
- Sample regularly in state space with trajectory generation.

# Outline

- 7.4 Intelligent Control
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# Summary

- Avoiding obstacles is a kind of planning problem.
  - Motion prediction.
  - Trajectory Evaluation
  - Search
- Its a real-time problem.
  - If the choice is between smart and fast, semi-dumb robots rule here.
- Dynamics matter in many ways.
- Cleverness of several kinds are possible.

# Summary

- Alternative courses of action are evaluated based on models of environmental interaction.
- A constrained optimization formulation applies.
  - Obstacles and dynamics are constraints
  - Feasible paths evaluated for utility.
- A large number of options exist for the representations used in planning models.
  - Each has its own issues and advantages from the perspective of computational complexity.