Chapter 2
Math Fundamentals

Part 3

2.6.1 Kinematic Models of Video Cameras
2.6.2 Kinematic Models of Laser Rangefinders

Outline

- 2.6.1 Kinematic Models of Video Cameras
- 2.6.2 Kinematic Models of Laser Rangefinders

Outline

- 2.6.1 Kinematic Models of Video Cameras
  - Perspective Projection
- 2.6.2 Kinematic Models of Laser Rangefinders

Video Cameras

- Image formation in cameras follows the perspective projection.
- It is nonlinear.
- Unique in two ways:
  - reduces the dimension of the input vector by one
  - it requires a post normalization step to re-establish a unity scale factor.

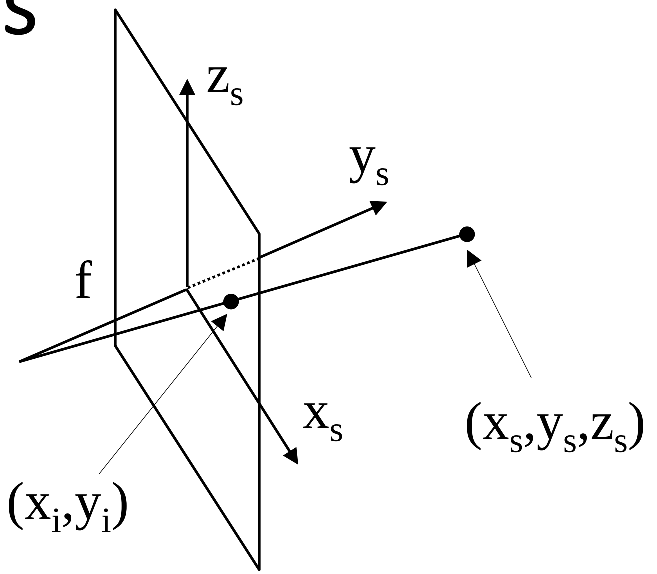
Video Cameras

- By similar triangles:

$$x_i = \frac{x_s f}{y_s + f} = \frac{x_s}{1 + y_s / f}$$

$$z_i = \frac{z_s f}{y_s + f} = \frac{z_s}{1 + y_s / f}$$

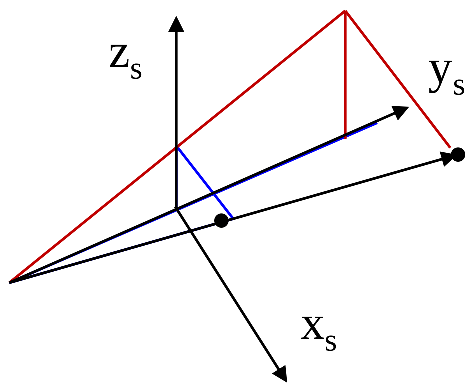
$$y_i = 0$$



- As a homogeneous transform:

$$\begin{bmatrix} x_i \\ y_i \\ z_i \\ w_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{f} & 0 & 1 \end{bmatrix} \begin{bmatrix} x_s \\ y_s \\ z_s \\ 1 \end{bmatrix}$$

The nonlinearity culprit



How can you tell this not invertible?

Outline

- 2.6.1 Kinematic Models of Video Cameras
- 2.6.2 Kinematic Models of Laser Rangefinders

Laser Rangefinders

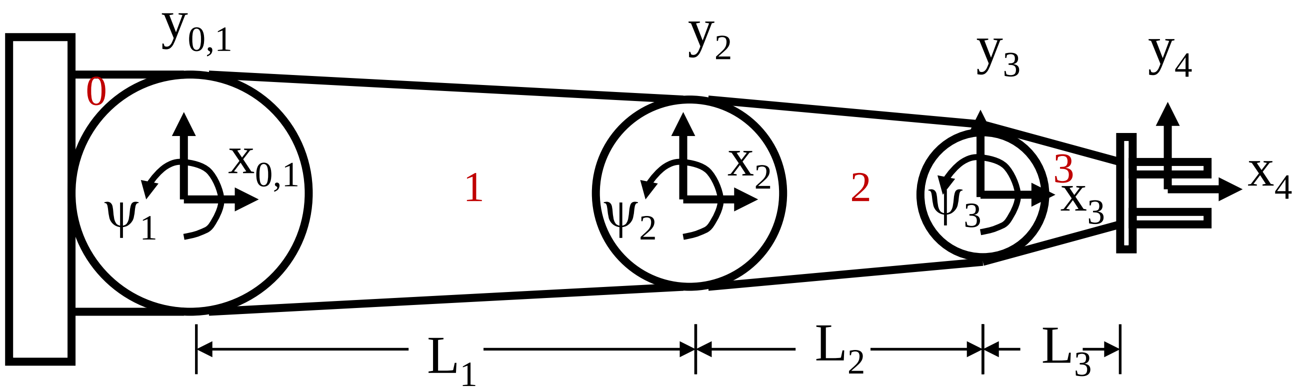
- Two kinds
  - Scanning devices use actuated mirrors to steer the beam.
  - Flash ladars which work like cameras.
- For the former, model what happens when a unit vector is reflected off of all of the mirrors involved.

Configurations

Outline

- 2.6.1 Kinematic Models of Video Cameras
- 2.6.2 Kinematic Models of Laser Rangefinders
  - The Reflection Operator
  - Kinematics of the Azimuth Scanner
  - Summary

Contrast with Robot Kinematics



Link	ϕ	u	ψ	w
0	0	0	ψ_1	0
1	0	L_1	ψ_2	0
2	0	L_2	ψ_3	0
3	0	L_3	0	0

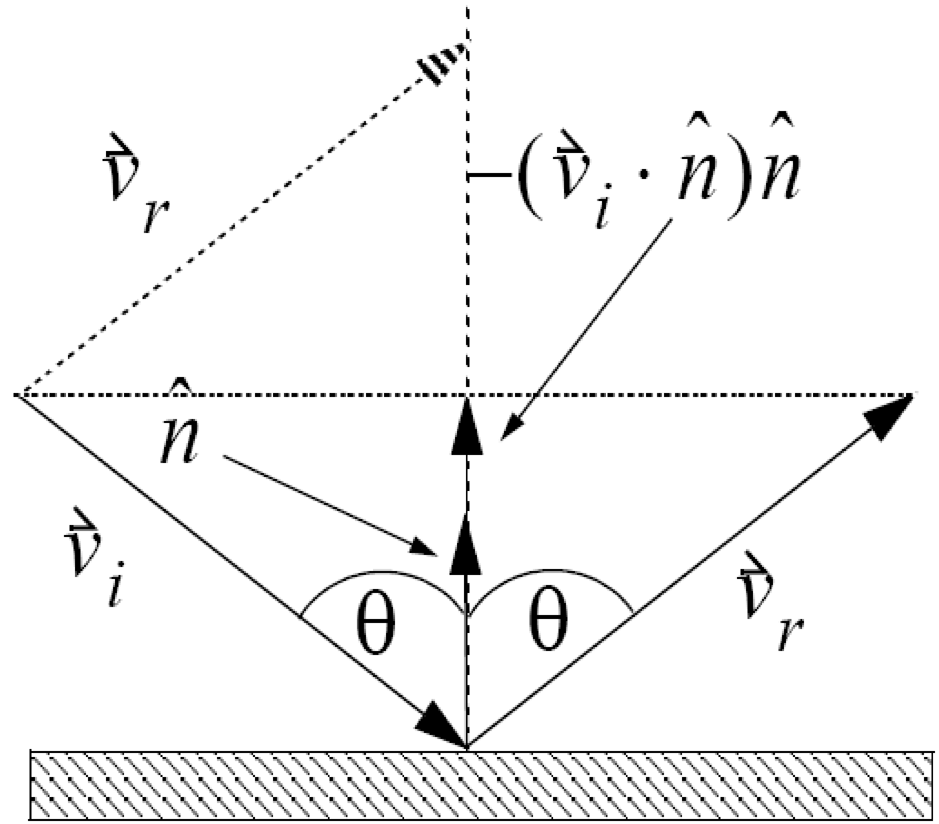
$$T_4^0 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Robots -> fundamental operator is a rotation
- Mirrors -> fundamental operator is a reflection

Reflection Operator

- Subtract twice the projection of incident ray onto mirror normal.

$$\vec{v}_r = \vec{v}_i - 2(\vec{v}_i \cdot \hat{n})\hat{n}$$
$$\vec{v}_r = \vec{v}_i - 2v_i \cos\theta \hat{n}$$
$$\vec{v}_r = \vec{v}_i - 2(\hat{n} \otimes \hat{n})\vec{v}_i$$
$$\vec{v}_r = \text{Ref}(\hat{n})\vec{v}_i$$



Reflection Operator

$$\vec{v}_r = \vec{v}_i - 2(\vec{v}_i \cdot \hat{n})\hat{n}$$

$$\vec{v}_r = \vec{v}_i - 2v_i \cos\theta \hat{n}$$

$$\vec{v}_r = \vec{v}_i - 2(\hat{n} \otimes \hat{n})\vec{v}_i$$

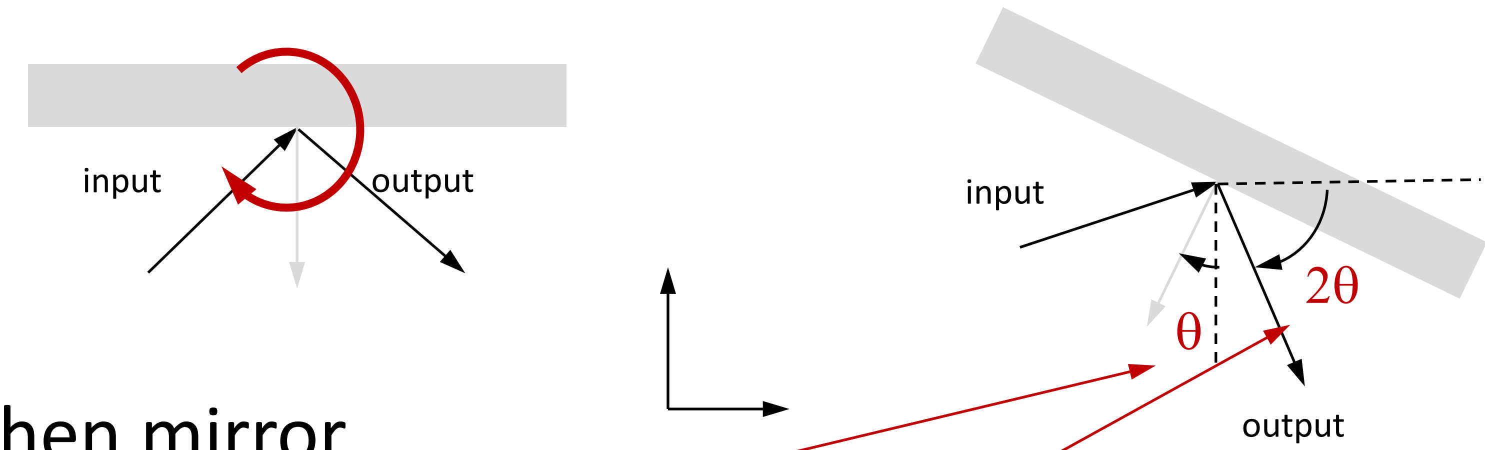
$$\vec{v}_r = \mathrm{Ref}(\hat{n})\vec{v}_i$$

$$\mathrm{Ref}(\hat{n}) = I - 2(\hat{n} \otimes \hat{n}) = \begin{bmatrix} 1 - 2n_x n_x & -2n_x n_y & -2n_x n_z \\ -2n_y n_x & 1 - 2n_y n_y & -2n_y n_z \\ -2n_z n_x & -2n_z n_y & 1 - 2n_z n_z \end{bmatrix}$$

Outer Product

Matrix Reflection Operator
(Householder Transform)

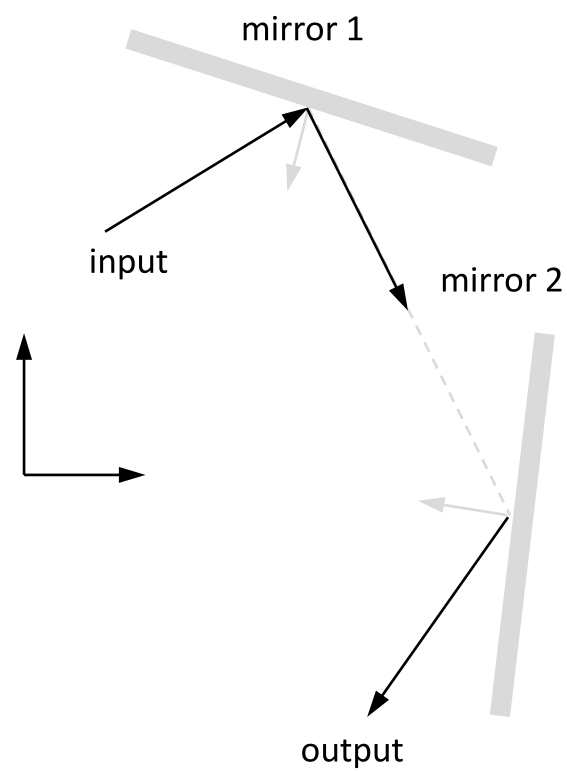
Mirror Gain



- When mirror rotates through angle θ
- Beam rotate through angle of 2θ

Box 2.6 Kinematic Modelling of Rangefinders

- 1: Choose coordinates fixed to sensor housing.
- 2: Express beam leaving laser diode as a unit vector.
- 3: Express normal of mirror 1 in terms of its rotation angle.
- 4: Reflect the beam off mirror 1
- 5: Express normal of mirror 2 in terms of its rotation angle.
- 6: Reflect result of step 4 off mirror 2
- 7: Result is the orientation of the beam expressed in terms of the mirror articulation angles.



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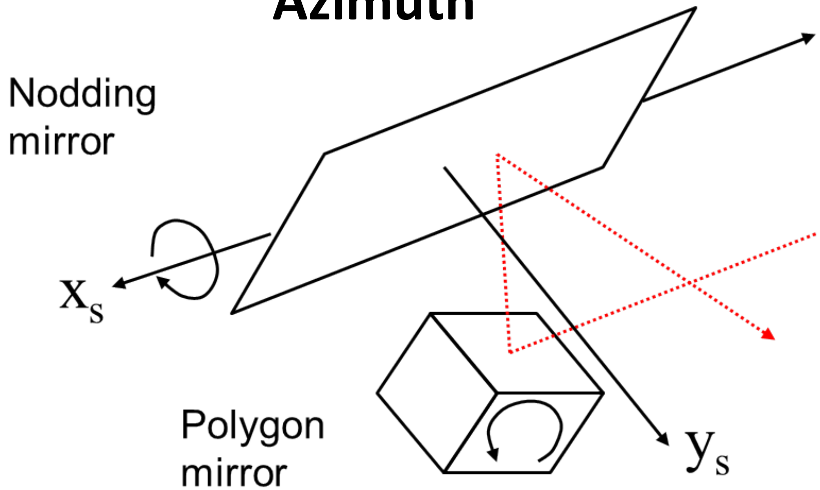
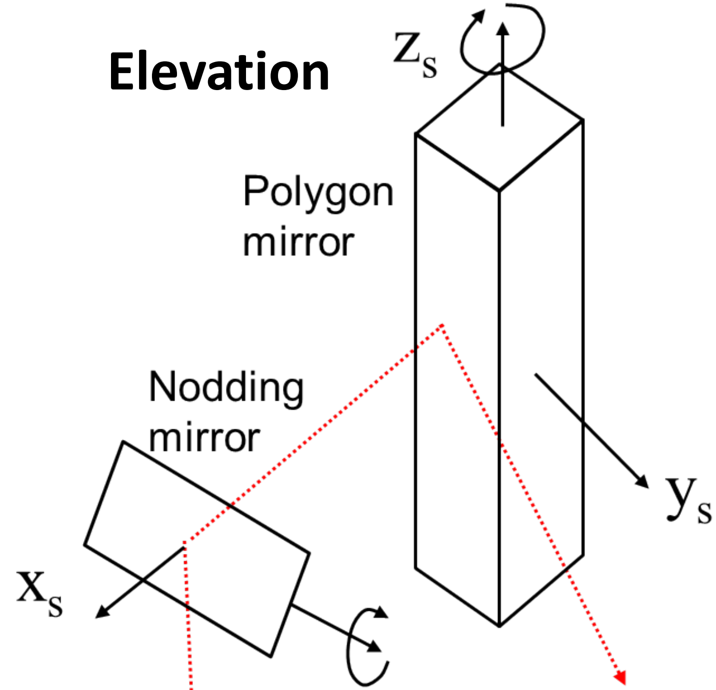
Scanning Mechanisms

Azimuth



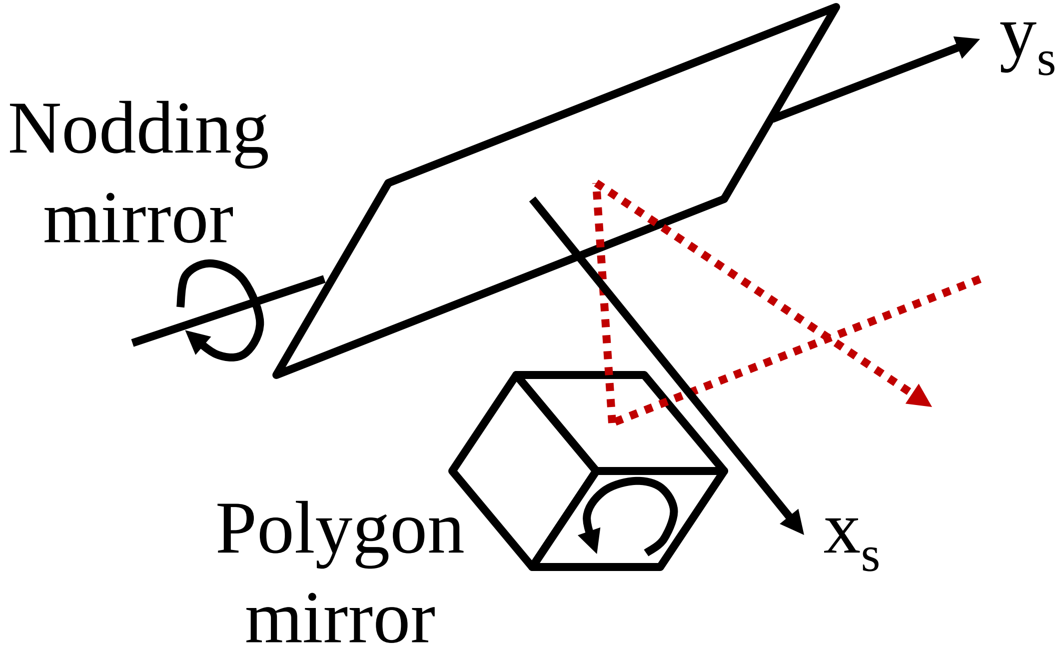
Elevation



- Enter along x
- Reflect around "z"
- Reflect around "y"
- Leave along "y"

- Enter along z
- Reflect around "-x"
- Reflect around "xy"
- Leave along "y"

Azimuth Scanner

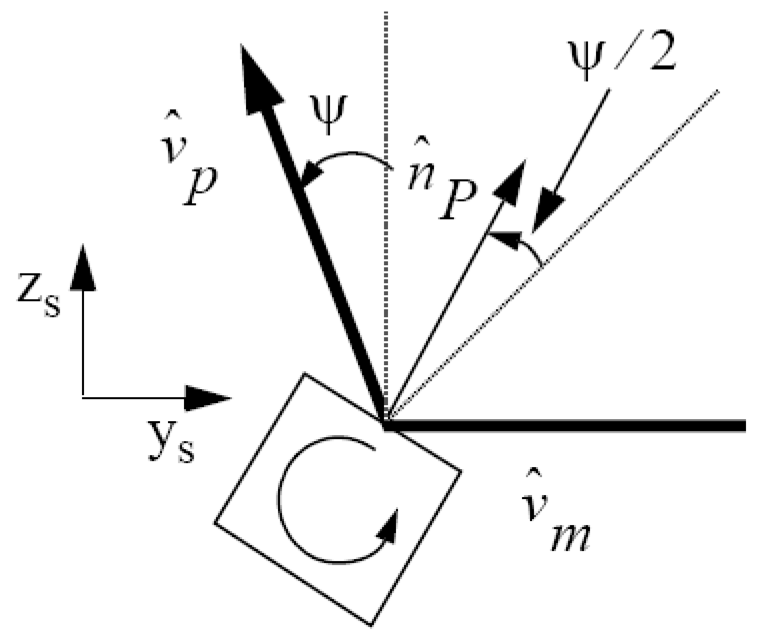


Azimuth Scanner

$$\hat{v}_m = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}^T$$

$$\hat{v}_p = \mathrm{Ref}(\hat{n}_P)\hat{v}_m$$

$$\hat{v}_p = \begin{bmatrix} 0 & -s\psi & c\psi \end{bmatrix}^T$$

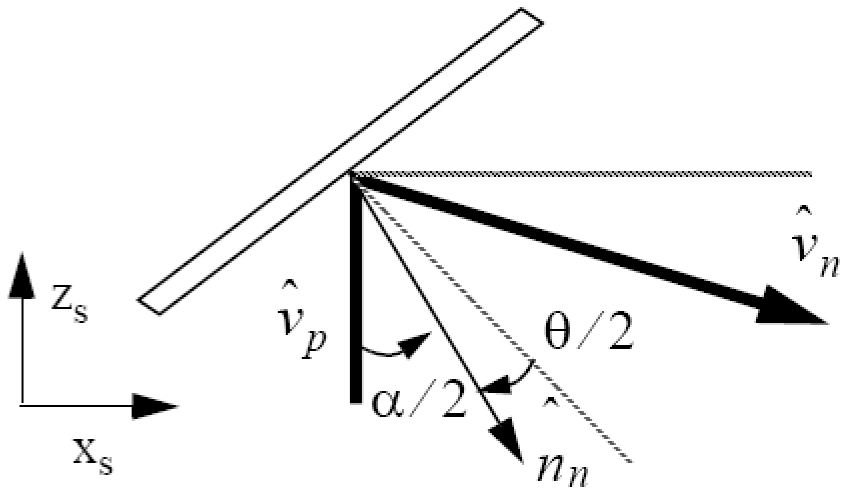


Azimuth Scanner

$$\hat{v}_p = \begin{bmatrix} 0 & -s\psi & c\psi \end{bmatrix}^T$$

put: $$\frac{\alpha}{2} = \frac{\pi}{4} - \frac{\theta}{2}$$

$$\hat{n}_n = \begin{bmatrix} s\frac{\alpha}{2} & 0 & -c\frac{\alpha}{2} \end{bmatrix}^T$$

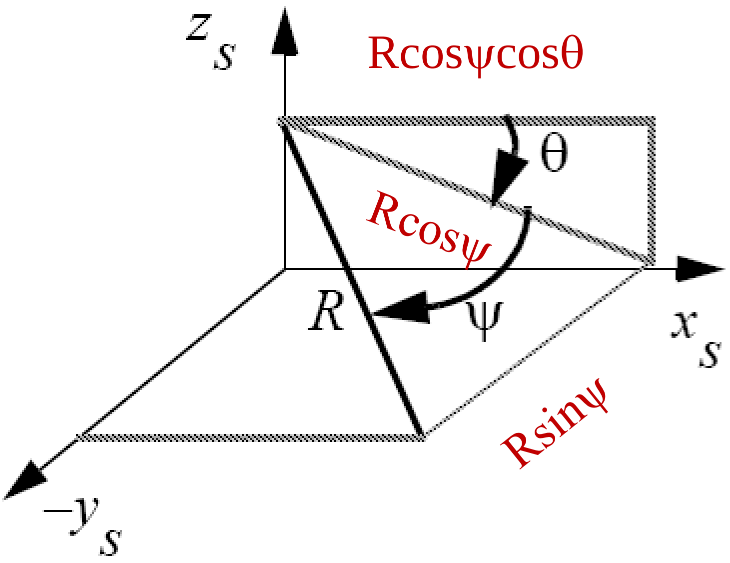


$$\hat{v}_n = \begin{bmatrix} 0 \\ -s\psi \\ c\psi \end{bmatrix} + 2c\psi c\frac{\alpha}{2}\begin{bmatrix} s\frac{\alpha}{2} \\ 0 \\ -c\frac{\alpha}{2} \end{bmatrix} = \begin{bmatrix} 2c\psi c\frac{\alpha}{2}\left(s\frac{\alpha}{2}\right) \\ -s\psi \\ c\psi - 2c\psi c\frac{\alpha}{2}\left(c\frac{\alpha}{2}\right) \end{bmatrix} = \begin{bmatrix} c\psi s\alpha \\ -s\psi \\ -c\psi c\alpha \end{bmatrix} = \begin{bmatrix} c\psi s\left(\frac{\pi}{2} - \theta\right) \\ -s\psi \\ -c\psi c\left(\frac{\pi}{2} - \theta\right) \end{bmatrix}$$

$$\boxed{\hat{v}_n = \begin{bmatrix} [c\psi c\theta] & -[s\psi] & -[c\psi s\theta] \end{bmatrix}^T}$$

Azimuth Scanner

$$\underline{v}_s = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} Rc\psi c\theta \\ -Rs\psi \\ -Rc\psi s\theta \end{bmatrix}$$



- Equivalent to a rotation about y by θ and then a rotation about the new z axis by -ψ.

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Inverse Kinematics

$$\underline{v}_s = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} R c\psi c\theta \\ -R s\psi \\ -R c\psi s\theta \end{bmatrix}$$

$$\begin{bmatrix} R \\ \psi \\ \theta \end{bmatrix} = \begin{bmatrix} \sqrt{x_s^2 + y_s^2 + z_s^2} \\ \operatorname{atan}(-y_s / \sqrt{x_s^2 + z_s^2}) \\ \operatorname{atan}(-z_s / x_s) \end{bmatrix}$$

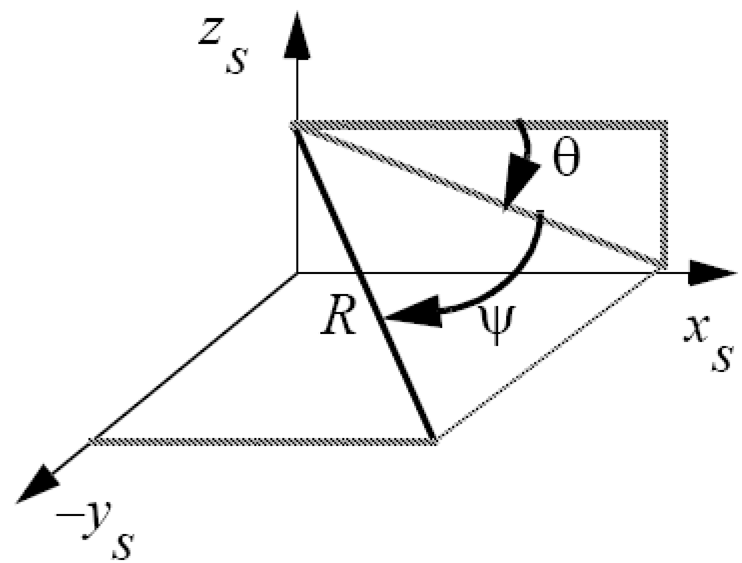


Image of Flat Terrain

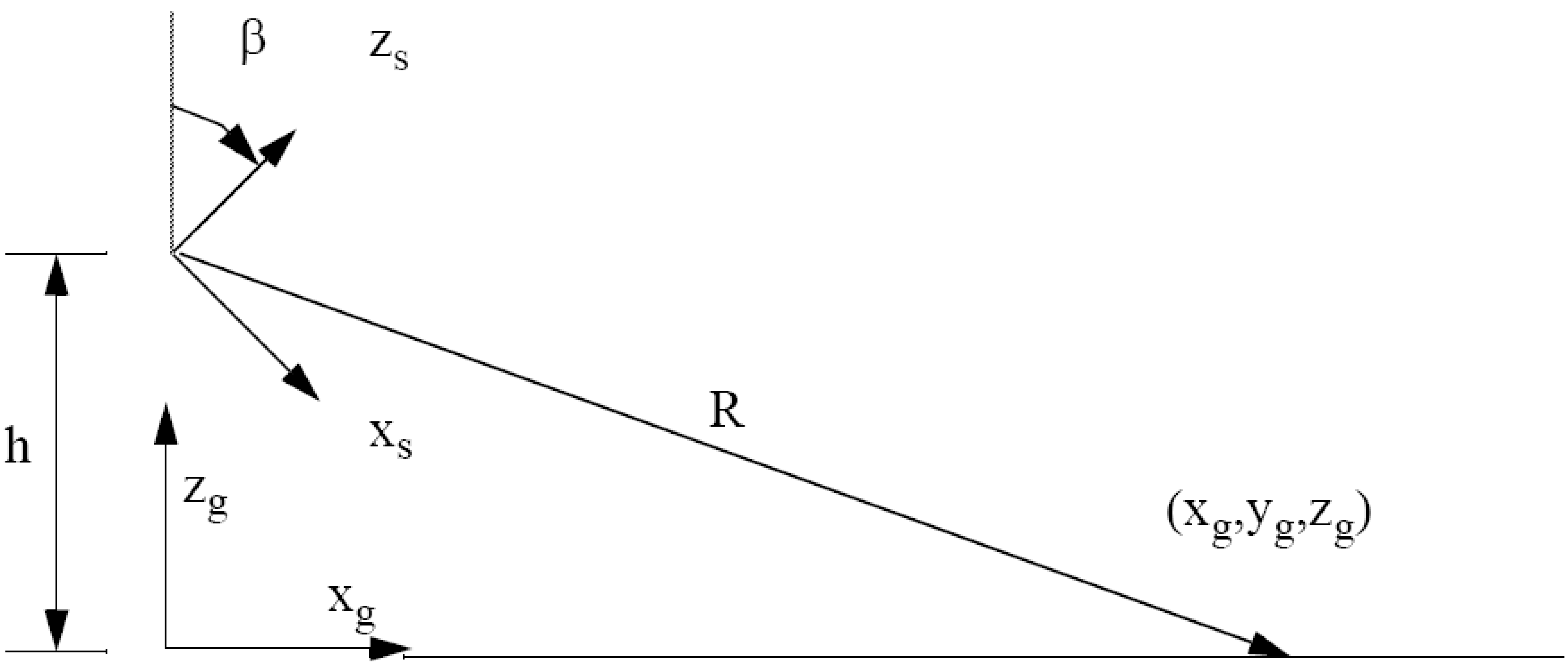
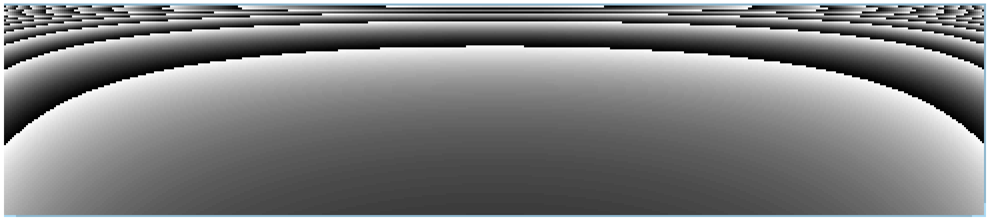


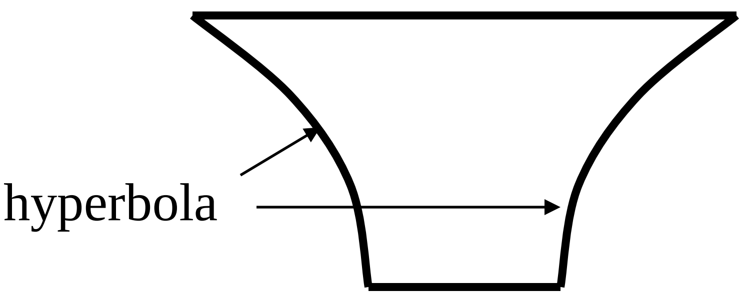
Image of Flat Terrain

Image Plane



$$R = h/(c\psi s\theta\beta)$$

Ground Plane



$$x_g = h/t\theta\beta$$

$$y_g = -ht\psi/s\theta\beta$$

$$z_g = 0$$

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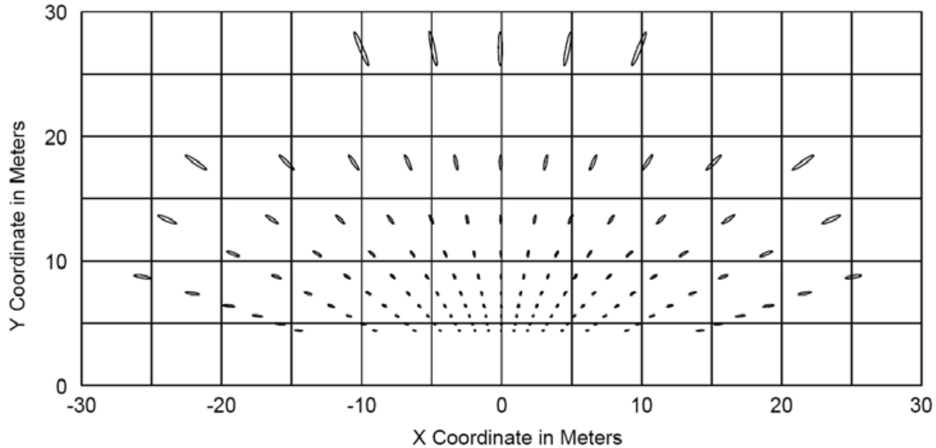
Resolution

- Linearize fwd kinematics:

$$\begin{bmatrix} dx_g \\ dy_g \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{-h}{(s\theta\beta)^2} \\ \dfrac{-h(\sec\psi)^2}{s\theta\beta} & \dfrac{ht\psi c\theta\beta}{(s\theta\beta)^2} \end{bmatrix} \times \begin{bmatrix} d\psi \\ d\theta \end{bmatrix}$$



- Jacobian Determinant:

$$dx_g dy_g = \left[\frac{(h\sec\psi)^2}{(s\theta\beta)^3}\right] d\psi d\theta$$

- Approximation:

$$|J| \approx \frac{\boxed{R^3}}{h}$$

Laser spot size / spacing
Grows with
Cube of Range

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Summary

- Video cameras are modeled by a perspective projection.
- Laser rangefinder models are nonlinear and cannot be represented by a constant homogeneous transform - like a camera.
- However, our mechanism modeling rules apply perfectly (see text) and one can also use a reflection operator to model them.